Equilibrium with FLOW and Applications to MAST and Its Stability

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Motivation

- Flows, either driven or intrinsic, are commonly observed in tokamak experiments.
- Toroidal flows produce quantitative modifications in plasma equilibrium profiles.
- Poloidal flows produce qualitative modification in the equilibrium profiles when they exceed the poloidal sound speed (not covered in this presentation).
- A detailed understanding of equilibrium in the presence of flow an essential step in the study of modern tokamak experiments.

Outline

1. Equilibrium Model
2. Application to MAST

Outline

MHD Equilibrium Equations with Flow.

The equilibrium is described by the usual set of ideal MHD/GCP equations:

- Continuity: $\nabla \cdot (\rho \underline{V}) = 0$
- Momentum: $\rho \underline{V} \cdot \nabla \underline{V} = \underline{J} \times \underline{B} - \nabla \cdot \underline{\underline{P}}$

  $\underline{\underline{P}} \equiv p_\perp \underline{\underline{I}} + \Delta \underline{B}\underline{B}$ $\qquad \Delta \equiv (p_\parallel - p_\perp)/B^2$
- Maxwell equations and ideal Ohm's law:

  $\nabla \times (\underline{V} \times \underline{B}) = 0$ $\qquad \nabla \cdot \underline{B} = 0$

Magnetic Field and Plasma Velocity Are Determined from the Equilibrium Solution.

The magnetic field is written in terms of magnetic poloidal flux and free functions:

$$\underline{B}_\theta = \frac{\nabla\psi \times \hat{e}_\varphi}{R} \qquad B_\varphi = \frac{1}{R}\frac{F(\psi) + R^2\Phi(\psi)\Omega(\psi)}{1 - \Phi(\psi)^2/\rho - \Delta}$$

Plasma velocity is expressed as:

$$\underline{V} = \frac{\Phi(\psi)}{\rho}\underline{B} + R\Omega(\psi)\hat{e}_\varphi \equiv M_{A\theta}\underline{V}_A + R\Omega(\psi)\hat{e}_\varphi$$

The Equilibrium Problem Is Reduced to a Set of Two Equations.

In the presence of plasma rotation, the equilibrium problem is solution of the (algebraic) Bernoulli equation and of a modified (PDE) Grad-Shafranov equation[1,2,3]:

$$\frac{1}{2}\left[\frac{\Phi(\psi)B}{\rho}\right]^2 - \frac{1}{2}[R\Omega(\psi)]^2 + W(\rho,|B|,\psi) = H(\psi), \quad \text{(Bernoulli)}$$

$$\nabla\cdot\left[\left(1-M_{Ap}^2-\Delta\right)\left(\frac{\nabla\psi}{R^2}\right)\right] = -\frac{\partial p_\parallel}{\partial\psi} - \frac{B_\varphi}{R}\frac{dF(\psi)}{d\psi} -$$
$$(\underline{V}\cdot\underline{B})\frac{d\Phi(\psi)}{d\psi} \quad - R\rho V_\varphi\frac{d\Omega(\psi)}{d\psi} - \rho\frac{dH(\psi)}{d\psi} + \rho\frac{\partial W}{\partial\psi}.$$

(Grad-Shafranov)

[1] E. Hameiri Phys. Fluids, **26**, 230 (1983)
[2] S. Semenzato et al., Comput. Phys. Rep., **1**, 389 (1984)
[3] R. Iacono et al., Phys. Fluids B, **2**, 1794 (1990)

The Equilibrium Is Defined by Free Functions.

- In order to calculate the equilibrium solution, it is necessary to assign SIX free functions:

$$
\begin{array}{lcl}
\Phi(\psi) & \rightarrow & \text{Field-Aligned Flow} \\
\Omega(\psi) & \rightarrow & \text{Toroidal Flow} \\
F(\psi) & \rightarrow & \text{Toroidal Field / Poloidal Current} \\
\partial p_{\|}/\partial \psi & \rightarrow & \text{Parallel Pressure} \\
H(\psi) & \rightarrow & \text{Bernoulli Function} \\
\partial W/\partial \psi & \rightarrow & \text{Enthalpy}
\end{array}
$$

- This is opposed to the TWO free functions that need to be assigned in the static, isotropic case (standard GS).

Only Five Free Functions Are Needed In the MHD Case.

- The (isotropic, single-fluid) MHD model is a special case of the previous system:

$$\frac{1}{2}\left[\frac{\Phi(\psi)B}{\rho}\right]^2-\frac{1}{2}[R\Omega(\psi)]^2+\frac{\gamma}{\gamma-1}S(\psi)\rho^{\gamma-1}=H(\psi), \qquad \text{(Bernoulli)}$$

$$\nabla\cdot\left[\left(1-\frac{\Phi^2(\psi)}{\rho}\right)\left(\frac{\nabla\psi}{R^2}\right)\right]=-\frac{B_\varphi}{R}\frac{dF(\psi)}{d\psi}-(\underline{V}\cdot\underline{B})\frac{d\Phi(\psi)}{d\psi}$$
$$-R\rho V_\varphi\frac{d\Omega(\psi)}{d\psi}-\rho\frac{dH(\psi)}{d\psi}+\frac{\rho^\gamma}{\gamma-1}\frac{dS(\psi)}{d\psi}. \qquad \text{(GS)}$$

- Free functions (determined by transport physics outside the MHD model) need to be assigned as input.
- We will only focus on the MHD model in the rest of this talk.

Outline

Equilibria Based On Experimental Measurements Can Be Computed.

- Toroidal velocity measurements are available for many experiments, in particular MAST.
- Poloidal velocity measurements are harder to obtain. Arguably, poloidal rotation is not important in MAST.
- FLOW[4] is **not** a reconstruction code. A multi-step process is needed to calculate equilibria from experimental measurements:
  1. An equilibrium reconstruction is run with a different code.
  2. Free functions $F(\psi)$ and $p(\psi)$ are read from a gfile.
  3. Velocity and density profile measurements (vs. R) are also needed.
  4. The input is converted into an appropriate format (a semi-automated task).

[4] L. Guazzotto et al., Phys. Plasmas **11**, 604 (2004)

Focus On Toroidal Flow for MAST

- The equilibrium problem simplifies considerably for zero poloidal velocity.
- In particular, the Bernoulli equation has an analytic solution. For isotropic plasmas:

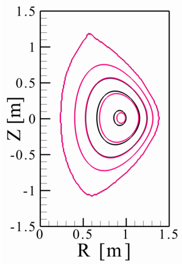
$$\rho = D(\psi) e^{\frac{(R^2 - R_0^2)\Omega^2(\psi)}{2T(\psi)}} \qquad \text{for isothermal plasmas,}$$

$$\rho = D(\psi) \left[1 + \frac{1}{2}(R^2 - R_0^2)\Omega^2(\psi) \frac{\gamma - 1}{\gamma} \frac{D(\psi)}{P(\psi)} \right]^{\frac{1}{\gamma - 1}} \qquad \text{if } \gamma \neq 1.$$

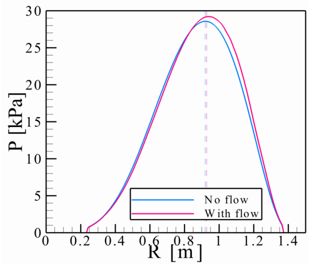
- $D(\psi)$ and $P(\psi)$ are auxiliary functions used to define the necessary free functions.
- Observe that the effect of toroidal rotation increases for small aspect ratio ($R/R_0 \sim 1 + \varepsilon \cos(\theta)$).

MAST Equilibria Are Calculated.

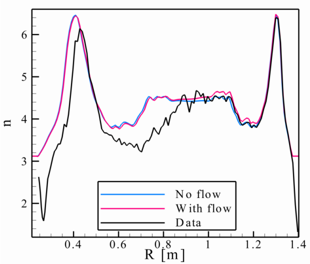
- Shot 24306 was used for our first test.
- Moderate rotation causes a shift of pressure, poloidal flux and density profiles.



Pressure and ψ comparison



Flow effect on pressure

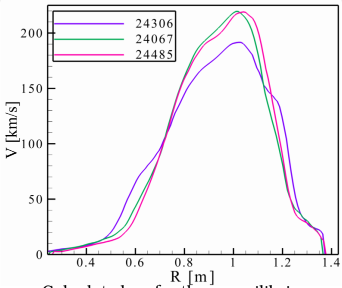


Flow effect on density (10^{19}m^{-3})

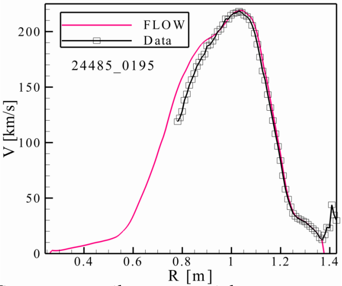
- We must be careful about distinguishing between the effect of rotation and total pressure!

MAST Equilibria Are Routinely Run.

- Different shots are analyzed.
- Arbitrary rotation profiles and levels can be included in the calculation.
- The calculated velocity matches the experimental one reasonably well (only one case shown).



Calculated v_φ for three equilibria



Comparison with experimental measurement

Stability (Future) Application

- The MARS stability code has been modified to include equilibrium rotation.
- The equilibrium with flow is calculated by FLOW.
- MAST and MAST-U are perfect candidates for exploring the effect of rotation on macroscopic stability.
- Benchmarks and comparisons with the results obtained by Dr. Berkery and Dr. Sabbagh will increase our confidence in and understanding of the results.

Possible Future Extensions

- Even though poloidal rotation may be small in MAST[5] and MAST-U, it would be interesting to include it in the equilibrium calculation.
- Even small poloidal rotation may affect stability in a meaningful way.
- Anisotropy can be included in the calculation (it's already in MARS!).
- The two-fluid version of FLOW[6] can also be applied to experimental equilibria.

[5] A R Field et al., 2009 Plasma Phys. Control. Fusion **51** 105002
[6] L. Guazzotto and R. Betti, Phys. Plasmas **22**, 092503 (2015)

Conclusions

- Equilibrium with flow is a more challenging calculation than static equilibrium.
- Since high rotation is found in MAST, equilibrium with flow is a necessary evil.
- A collaboration has been started to include rotation in the equilibrium calculations for MAST and MAST-U.
- A stability analysis will follow.