Gaussian process regression for profile fitting

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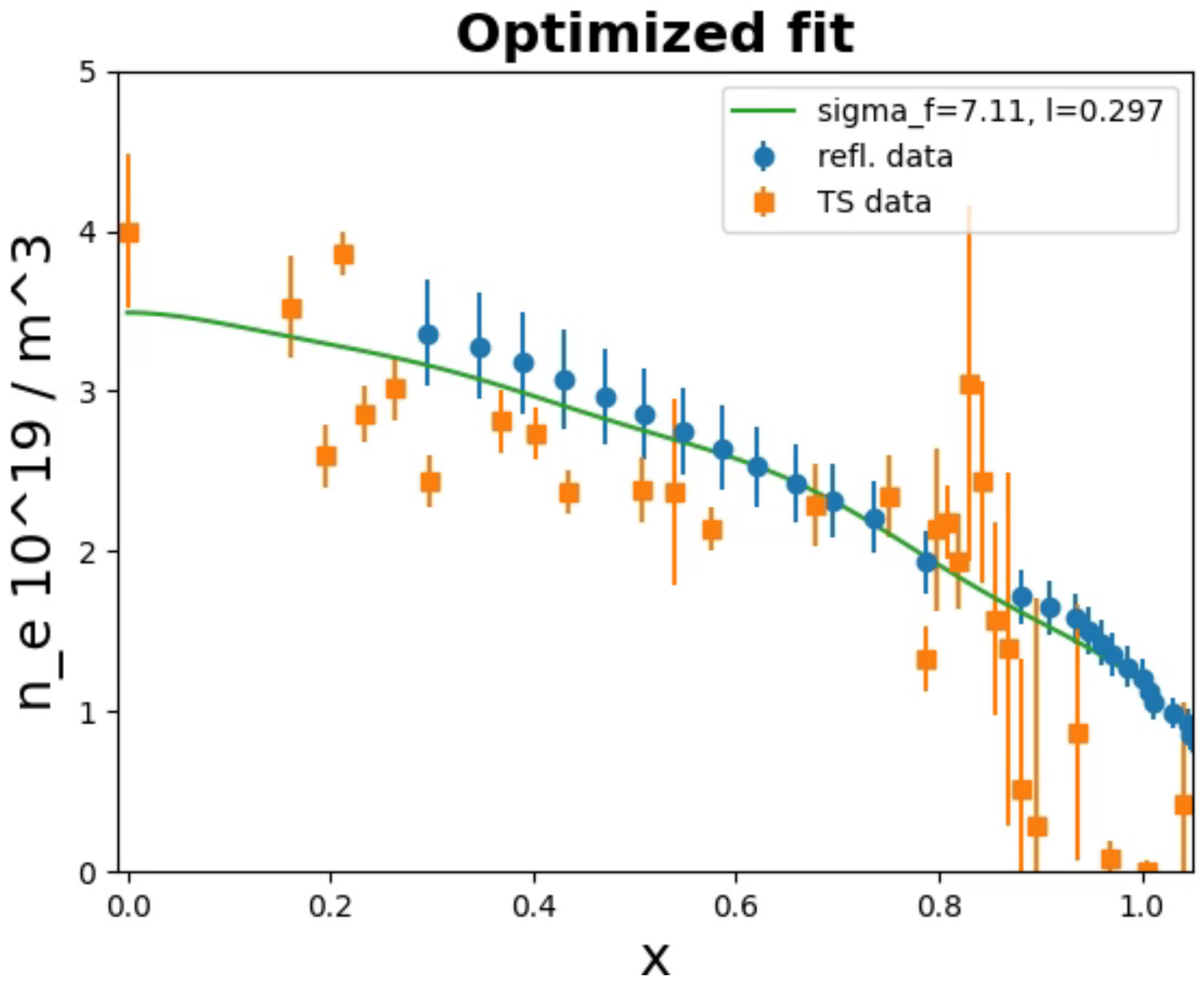
Presented at NSTX-U Physics Meeting

Jan. 25th, 2021

Problem: combining TS and refl. data for EAST

- EAST's Thomson scattering system has low time resolution and can often be noisy
- Reflectometer data is fast and smooth, but does not reach the core
- For the most accurate profile (required for LH propagation), measured data from both sources are combined



Gaussian process regression improves profile fitting

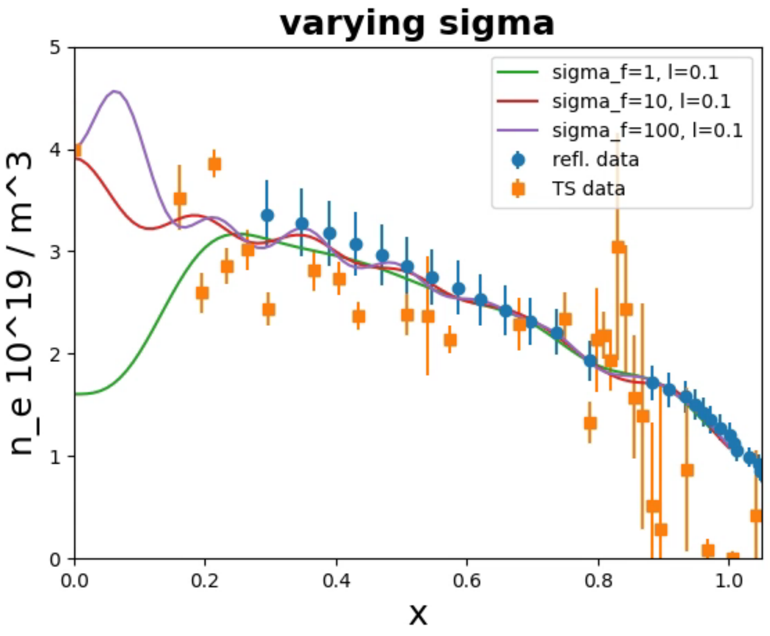
- Gaussian process regression (GPR) technique uses statistics to allow profile fitting to be more easily automated, with better results
  - [M.A. Chilenski *et al* 2015 *Nucl. Fusion* **55** 023012]
- Given a set of (possibly multi-dimensional) input and output, it can be used to predict new outputs with better spatial and/or temporal resolution
- Open-source Python package: **gptools**
- Symmetry constraints (zero slope at $\rho = 0$) can be easily enforced
- Provides an alternative option than fitting with splines, tanh, polynomial, etc.

Kernels translate input data to output predictions

- The covariance kernel describes how closely related input/output data should be
- 1D squared exponential kernel:

$$k_{SE}(x, x') = \sigma_f^2 \exp\left(-\frac{|x - x'|^2}{2l^2}\right)$$

- σ_f^2 is the "signal variance", sets how much the fitted curve can vary in *y*



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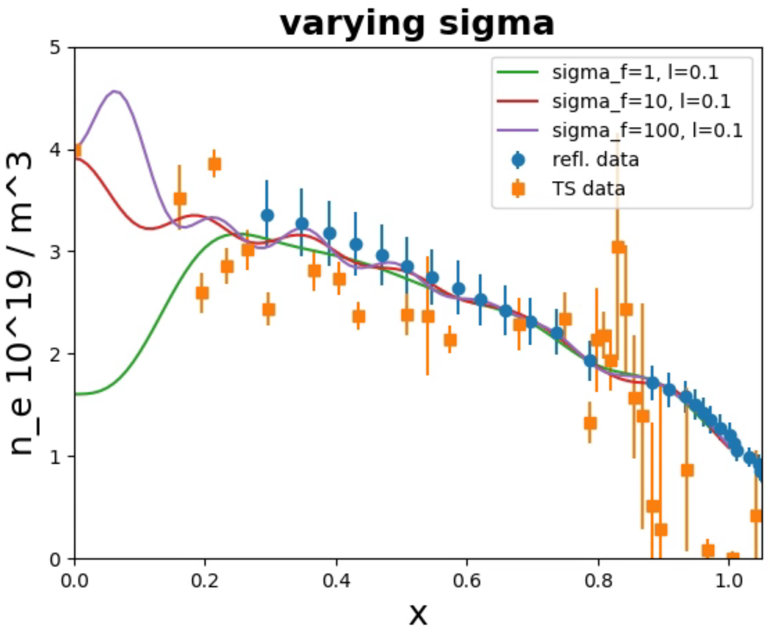
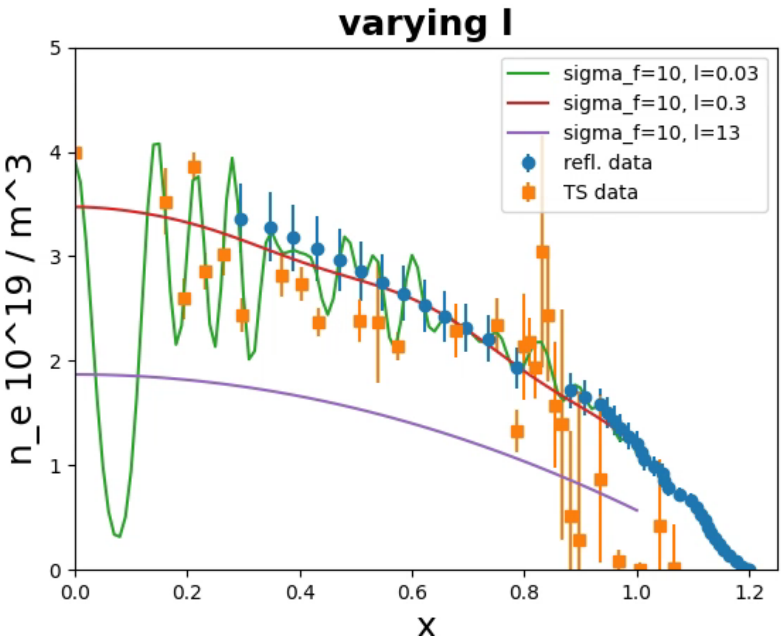
- σ_f^2 is the "signal variance", sets how much the fitted curve can vary in *y*
- *l* is the "covariance length scale", sets how much correlation in *x* is expected
  - Not the same thing as gradient length scale
- For *y(x, t)*, there would be l_x and l_t





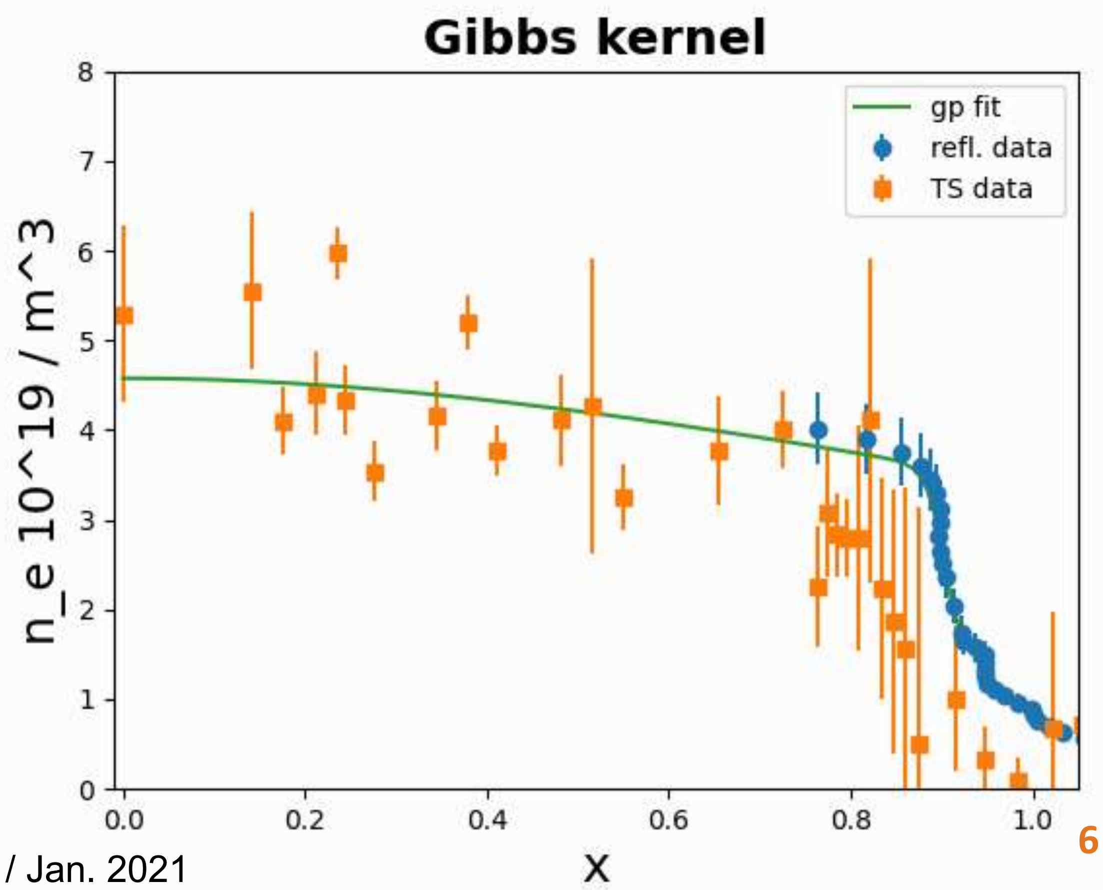
Gibbs kernel allows for varying covariance scale length

- To allow for two different scale lengths in the pedestal vs the core, use Gibbs 1D kernel:

$$k_G(x,x') = \sigma_f^2 \left(\frac{2l(x)\,l(x')}{l^2(x)+l^2(x')} \right)^{\frac{1}{2}} \exp\left(- \frac{|x-x'|^2}{l^2(x)+l^2(x')} \right)$$

$$l(x) = \frac{l_1+l_2}{2} - \frac{l_1-l_2}{2} \tanh \frac{x-x_0}{l_w}$$

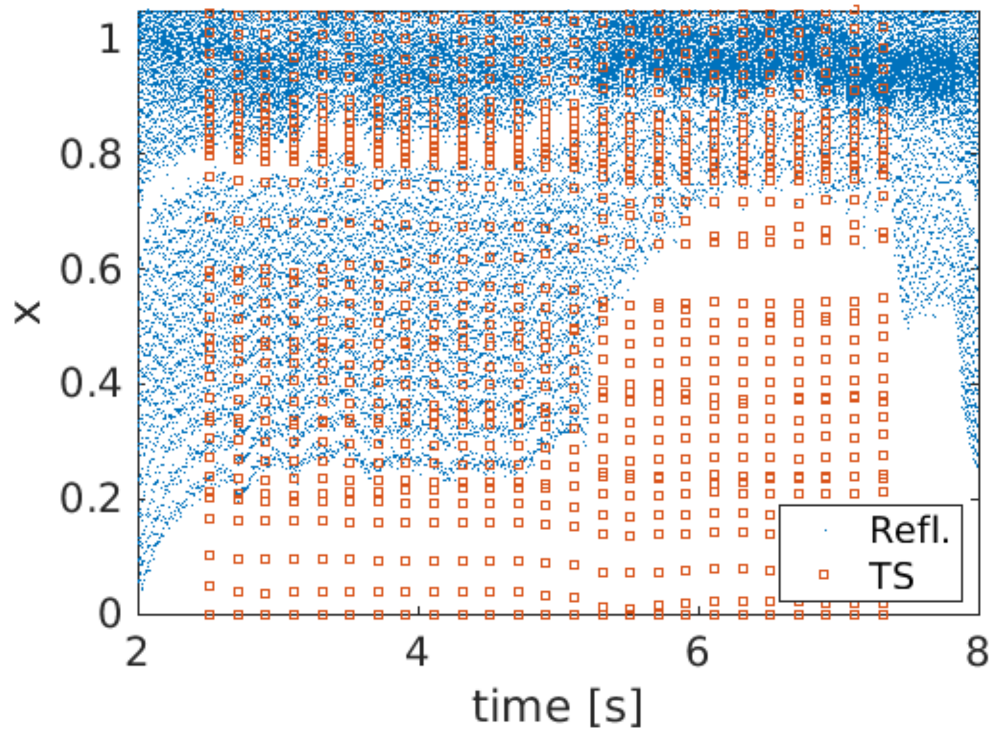
where l_1 is the core length scale, l_2 is the edge length scale, and x_0 and l_w are the center and width of the transition



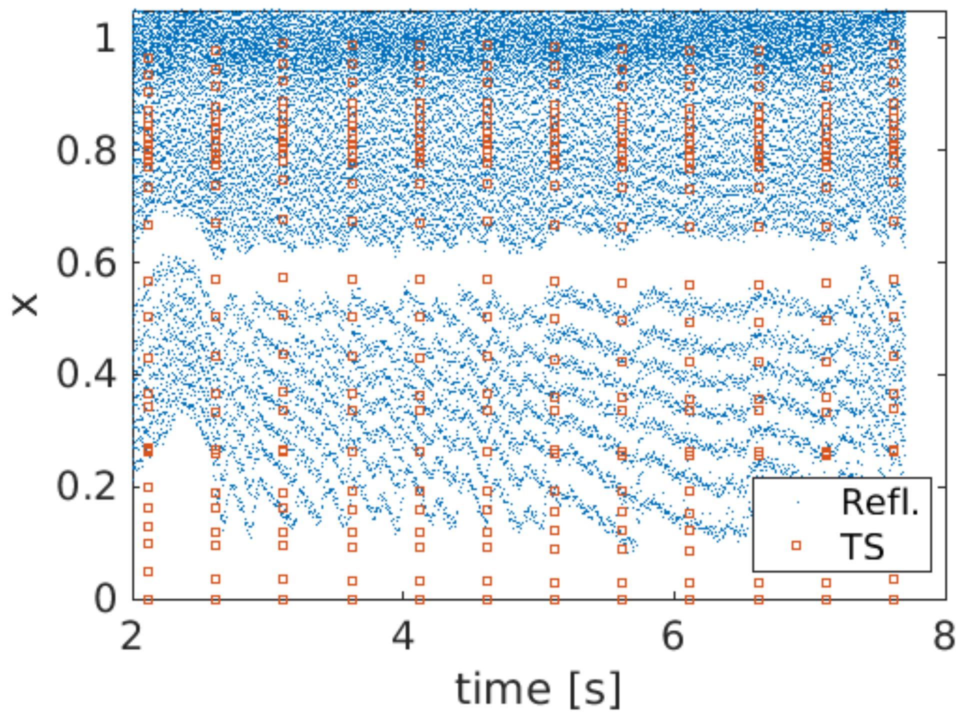
GPR can be used for 2D interpolation

- 2D interpolation can:
  - use TS data when reflectometer data is not available during H-mode
  - use faster refl. to fill in gaps in TS data

85127 n_e data locations



85611 n_e data locations



Conclusion & Future work

- Gaussian process regression (GPR) is a powerful tool that can automate profile fitting with good accuracy
  - Covariance length scales informed by physics
  - are robust in similar scenarios

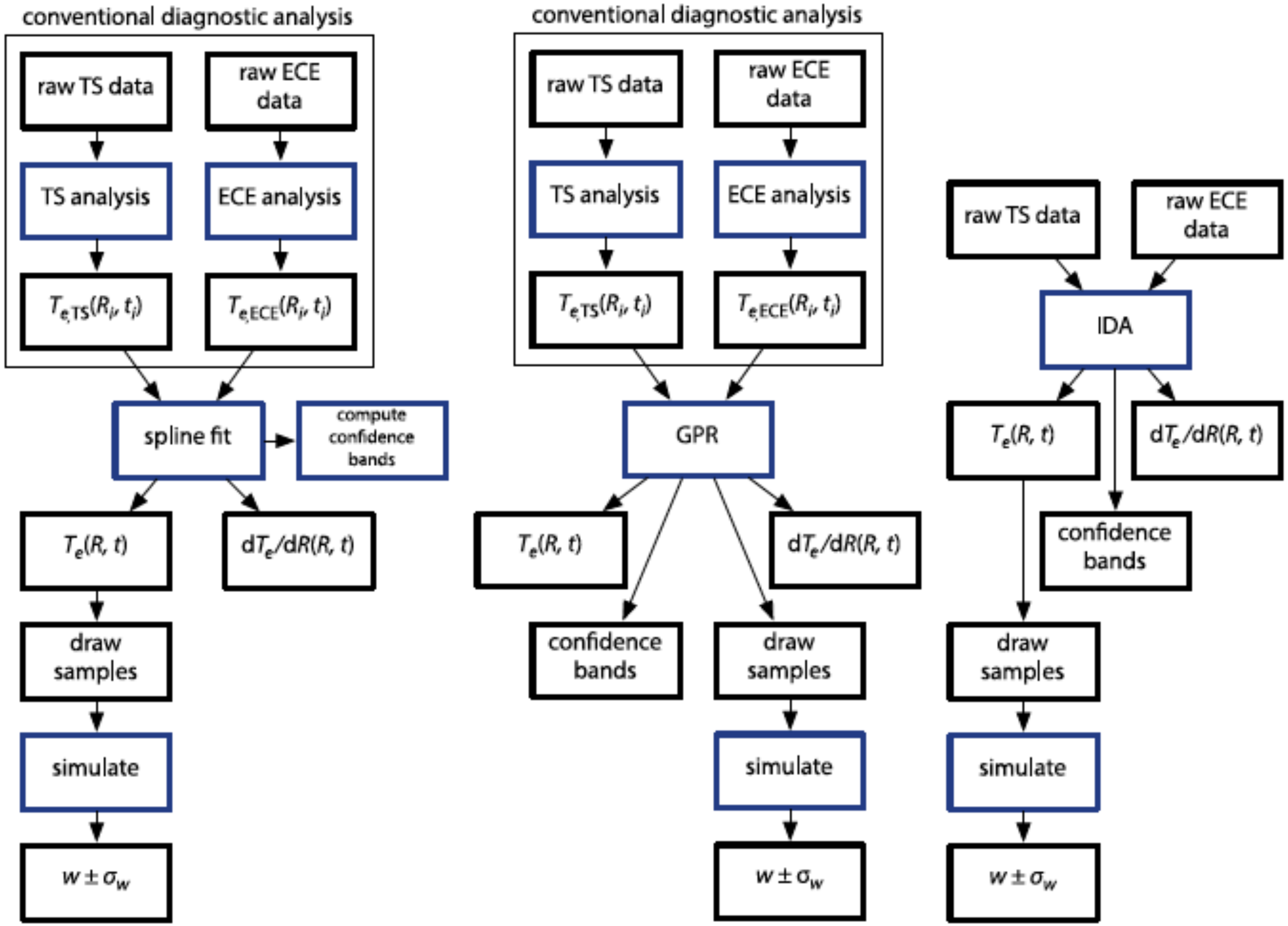
- Possible applications with gptools:
  - Incorporate into automated analysis of diagnostics for better resolution or better signal-to-noise
  - Apply *en masse* to a database of pedestal measurements to calculate pedestal height, width, etc. for a semi-empirical model

Additional resources

- Paper: M.A. Chilenski *et al* 2015 *Nucl. Fusion* **55** 023012
  - doi: https://doi.org/10.1088/0029-5515/55/2/023012
- Python code:
  - Python 2: https://pypi.org/project/gptools/
  - Python 3: https://github.com/wilkiechoi/gptools3
- User manual:
  - Webpage: https://gptools.readthedocs.io/en/latest/
  - PDF: https://buildmedia.readthedocs.org/media/pdf/gptools/latest/gptools.pdf

Backup Slides

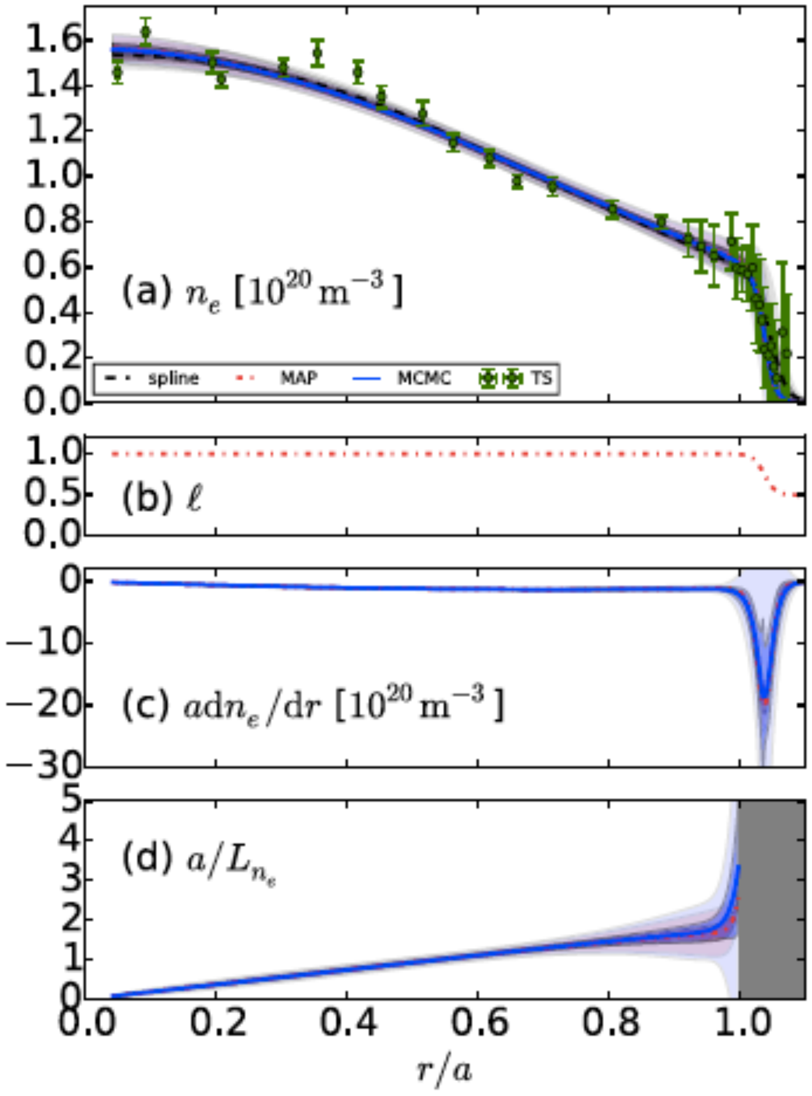
Comparison of workflow, fig. 4 from [Chilenski NF 2015]



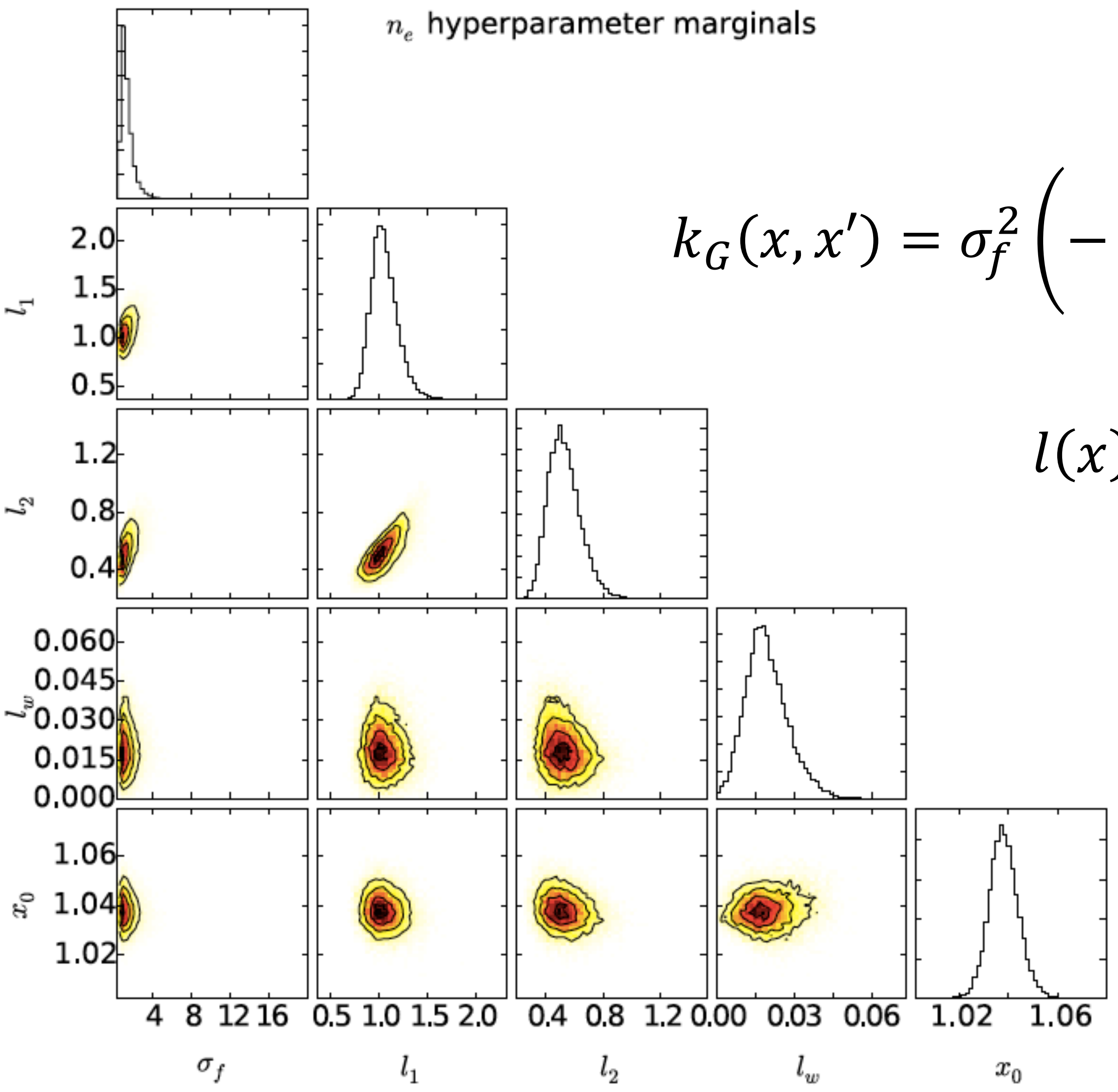
(a) spline (b) GPR (c) IDA (profiles only)

H-mode density profile, fig. 5 from [Chilenski NF 2015]

- Using gptools to fit C-Mod density profile
- Showing long covariance length scale in the core and shorter *l* at the edge
- Also calculating density gradient of the pedestal, important for transport analysis



Hyperparameters of fit, fig. 7 from [Chilenski NF 2015]



$$k_G(x, x') = \sigma_f^2 \left(- \frac{2l(x)\, l(x')^2}{l^2(x) + l^2(x')} \right)^{\frac{1}{2}} \exp\left(- \frac{|x - x'|^2}{l^2(x) + l^2(x')} \right)$$

$$l(x) = \frac{l_1 + l_2}{2} - \frac{l_1 - l_2}{2} \tanh \frac{x - x_0}{l_w}$$

- Resultant parameters of Gibbs kernel for density profile on previous slide