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Simulations and Theory of Compressional and Global Alfvén Eigenmode Stability in NSTX and Beyond

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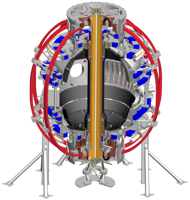

Motivation and Main Results

- **Motivation**: sub-cyclotron Alfvén Eigenmodes (AEs) have been experimentally linked to anomalous electron temperature flattening in NSTX
  - No theory quantitatively reproduces the observations
- **Goal**: predict instability conditions for realistic neutral beam (NBI) distributions using analytic theory and numerical simulations
- **Main result**: simple theory describes high frequency AE excitation and demonstrates how to stabilize modes with additional NBI source
  - Explains NSTX-U suppression of AEs with new beam source
  - Provides insight to control and study the associated electron energy transport

Outline

- Introduction: Alfvén Eigenmodes Linked to Anomalous Electron Transport
- Hybrid Simulations Reveal Complicated Stability Boundaries
- Simple Analytic Theory Explains Simulations
- Theory Yields Experimental Insights
- Injecting Multiple Beams Can Control Alfvén Eigenmodes

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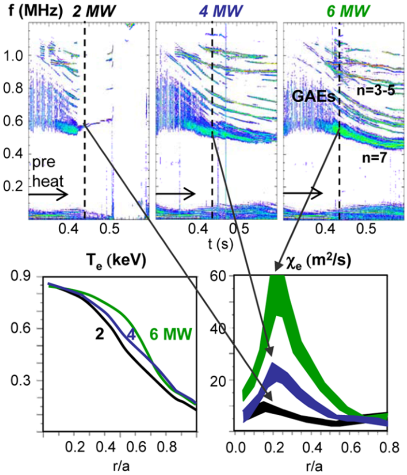
Anomalous T_e Flattening in NSTX Correlates with CAE/GAEs

- Beam-driven compressional (CAE) and global (GAE) Alfvén eigenmodes have been excited in NSTX(-U), MAST, DIII-D, AUG, and may be present in ITER
- Temperature profiles can not be explained by turbulence in gyrokinetic simulations
- Methods to control CAEs/GAEs are essential to studying and predicting the electron energy transport that they induce[1]

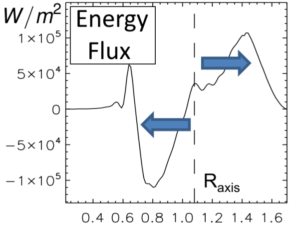


[1] D. Stutman *et al.* Phys. Rev. Lett. **102**, 115002 (2009)

How Can CAEs/GAEs Affect Temperature Profiles?

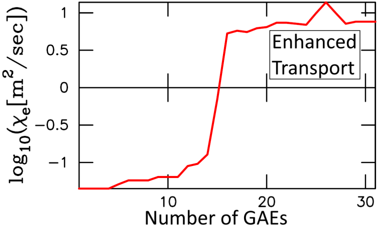
Energy Channeling

- AE in core can mode convert to KAW near edge, damping on electrons
- Modifies effective beam energy deposition profile[2,3]



Orbit Stochastization

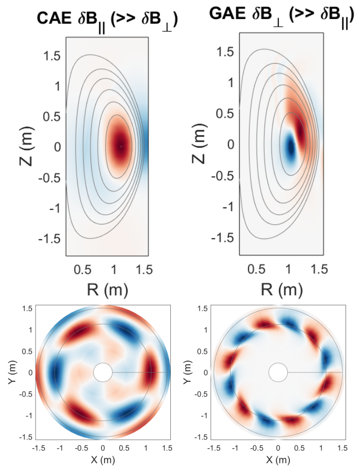
- Sufficiently many unstable AEs can stochasticize electron orbits
- Enhances diffusion, transporting energy away from the core[4]



[2] Y.I. Kolesnichenko *et al.* Phys. Rev. Lett. **104**, 075001 (2010)
[3] E.V. Belova *et al.* Phys. Rev. Lett. **115**, 015001 (2015)
[4] N.N. Gorelenkov *et al.* Nucl. Fusion **50**, 084012 (2010)

Sub-cyclotron Alfvén Eigenmodes in NSTX(-U)

- **Compressional Alfvén eigenmode** (CAE): ideal magnetosonic mode: $\omega \approx k v_A$
- **Global Alfvén eigenmode** (GAE): discrete shear Alfvén eigenmode existing below minimum of Alfvén continuum: $\omega \leq \left[k_{\|}(r) v_A(r)\right]_{\text{min}}$
- CAEs/GAEs interact with fast ions through Doppler-shifted cyclotron resonance $\omega - \langle k_{\|} v_{\|} \rangle - \langle k_{\perp} v_{\text{Dr}} \rangle = \ell \langle \omega_{ci} \rangle$
- Observed to propagate both **co-** ($k_{\|} > 0, \ell = 0$) and **cntr-** ($k_{\|} < 0, \ell = 1$) to the beam/plasma current with $|n| = 3 - 15$, $\omega/\omega_{ci} \approx 0.1 - 1.2$
  - Do not typically drive fast ion transport, unlike lower frequency modes (TAEs, RSAEs, BAEs, ...)



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Hybrid Simulation Method

- Hybrid MHD and Particle code (HYM)
- Initial value code in tokamak geometry
- Thermal plasma: single fluid resistive MHD model
- Beam ions: full orbit kinetic particles with δF scheme
  - Captures Doppler-shifted cyclotron resonance which drives the modes
- Equilibrium includes fast ion effects self-consistently[5]
- Thermal plasma and beam ions coupled through current in momentum equation

$$\rho\frac{d\boldsymbol{V}}{dt} = -\nabla P + \underbrace{(\boldsymbol{J}-\boldsymbol{J_b}) \times \boldsymbol{B} - en_b(\boldsymbol{E} - \eta\delta\boldsymbol{J})}_{\text{current coupling}} + \mu\Delta\boldsymbol{V}$$

[5] E.V. Belova *et al.* Phys. Plasmas **10**, 3240 (2003)

Fast Ion Distribution Model

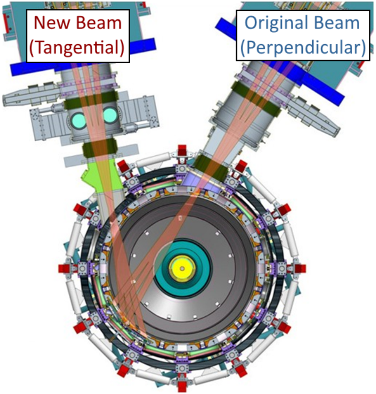
- Equilibrium distribution $F_0 = F_1(v)F_2(\lambda)F_3(p_\phi)$
  - Trapping parameter $\lambda = \mu B_0/\mathcal{E} \approx v_\perp^2/v^2$

$$F_1(v) = \frac{1}{v^3 + v_c^3}$$

$$F_2(\lambda) = \exp\left(-(\lambda - \lambda_0)^2/\Delta\lambda^2\right)$$

$$F_3(p_\phi) = \left(\frac{p_\phi - p_{\min}}{m_i R_0 v - q_i\psi_0 - p_{\min}}\right)^\sigma$$

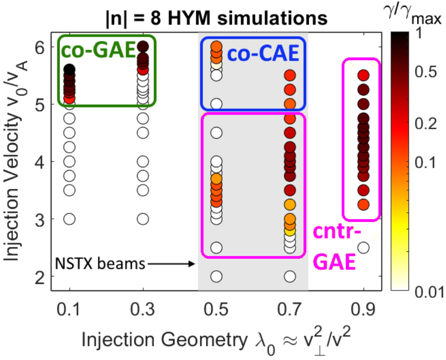
$$\text{for } v < v_0 \quad \text{and} \quad p_\phi > p_{\min}$$



- NSTX: $v_0/v_A \lesssim 5$, $\lambda_0 = 0.5 - 0.7$ for original beam
- NSTX-U: $v_0/v_A \lesssim 2$, $\lambda_0 = 0$ for new beam
- Parameters matched to TRANSP ($\Delta\lambda \approx 0.3$)

Hybrid Simulations Predict Rich Mixture of CAEs/GAEs

- Parameter scan of injection geometry (λ_0) and velocity (v_0/v_A) reveals complicated stability boundaries for different mode types[6]
  - Simulated $|n| = 1 - 12$ separately
- GAEs excited at lower beam energy ($v_0/v_A \gtrsim 2.5$) than CAEs ($v_0/v_A \gtrsim 4$), typically with larger growth rates
- co-GAEs excited with very tangential beams
  - Anomalous cyclotron resonance ($\ell = -1$)
  - May exist in future NSTX-U experiments



- Colored dot: growth rate of most unstable mode in each simulation
- White dot: no unstable modes

[6] J.B. Lestz *et al.* arXiv:2101.05976 (2021)

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Simple Conditions Derived for Net Drive

- Fast ion drive depends on gradients of the distribution

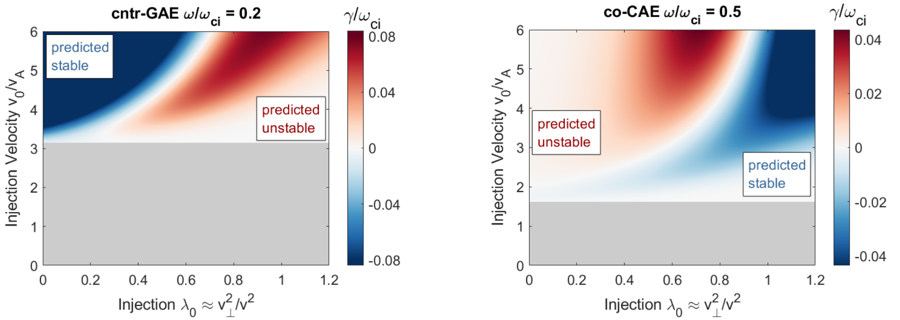
$$\gamma \propto \int h(\lambda, v) \left[\left(\frac{\ell \omega_{ci}}{\omega} - \lambda \right) \frac{\partial}{\partial \lambda} + \frac{v}{2} \frac{\partial}{\partial v} \right] f_b(\lambda, v)\, d^2\boldsymbol{v} > 0 \text{ for instability}$$

- cntr-propagating CAE/GAEs $(\ell = +1)$ driven by $\partial f_b / \partial \lambda > 0$
  - $v_0 < v_{\|,\mathrm{res}} / (1 - \lambda_0 \langle \bar{\omega}_{ci} \rangle)^{3/4}$ necessary for instability[7]
- co-propagating CAEs $(\ell = 0)$ driven by $\partial f_b / \partial \lambda < 0$
  - $v_0 > v_{\|,\mathrm{res}} / \left(1 - \frac{\langle \bar{\omega}_{ci} \rangle}{2} \left[\lambda_0 + \sqrt{\lambda_0^2 + 8 \Delta \lambda^2 / 3} \right] \right)^{5/8}$ necessary for instability[8]
- Due to resonance condition, $v_{\|,\mathrm{res}}(\omega/\omega_{ci}, k_\| / k_\perp) = (\omega - \ell \langle \omega_{ci} \rangle) / k_\|$

[7] J.B. Lestz *et al.* Phys. Plasmas **27**, 022513 (2020)
[8] J.B. Lestz *et al.* Phys. Plasmas **27**, 022512 (2020)

Analytic Bounds Explain Simulation Results

- Numerically integrate full analytic expression for growth rate to predict instability
  - red: net fast ion drive, blue: net fast ion damping
  - gray: insufficient beam velocity for resonant interaction



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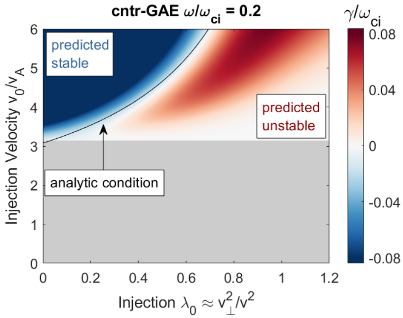

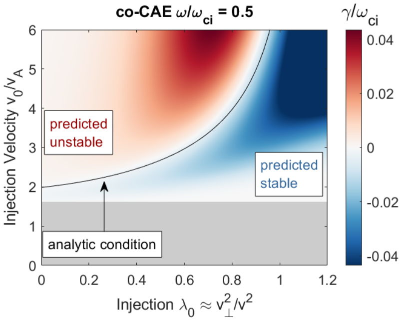

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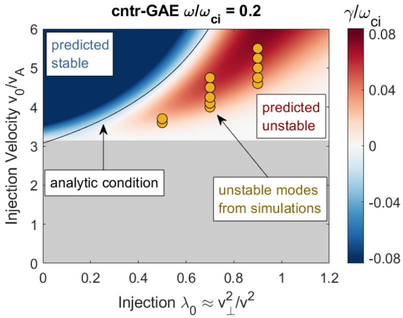
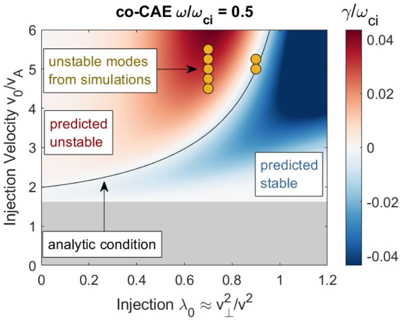
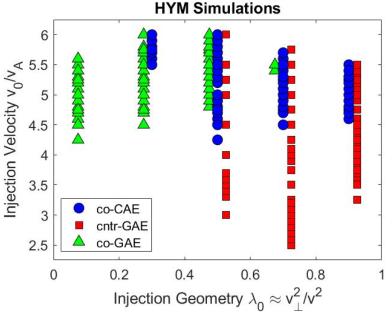
- black curve: approximate analytic conditions reproduce numerical calculation
- gold: unstable modes from HYM simulations agree with theory





Beam Parameters Determine Most Unstable Modes

- cntr-GAEs prefer $\lambda_0 \to 1$, whereas co-GAEs require small λ_0 for instability
  - Driven by opposite sign of $\partial f_0 / \partial \lambda$
- co-CAEs are less unstable due to a smaller coefficient multiplying growth rate
  - $\gamma_{\ell=0} \sim (\omega/\omega_{ci})\gamma_{\ell=\pm 1}$
- cntr-GAEs can be destabilized at small v_0/v_A
  - co-GAEs require large Doppler shift
  - co-CAEs suffer relatively large $\partial f_0 / \partial v$



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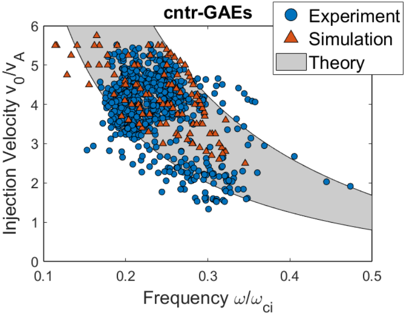
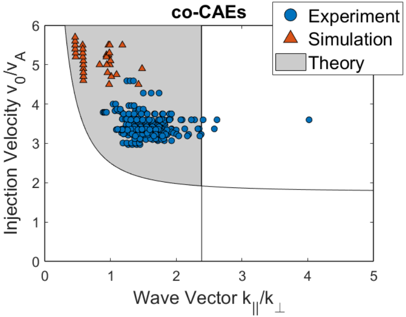
Agreement Between NSTX Experiments, Theory, and Simulations

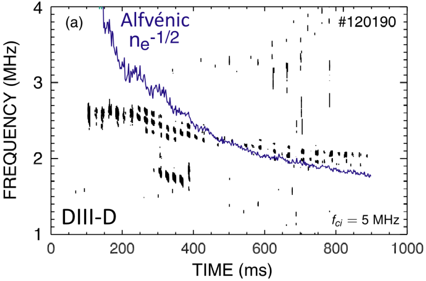
- For fixed NBI parameters, instability conditions constrain the spectrum of modes
- Cross-comparison with NSTX database of cntr-GAEs and co-CAEs demonstrates greater than 80% agreement with theory
- blue: NSTX observations, red: HYM simulations
  gray: unstable region predicted by theory





Theory Explains Previous Observations on DIII-D

- DIII-D low field experiments ($v_0/v_A \approx 1.5$) observed cntr-modes with $\omega/\omega_{ci} \approx 0.6$
  - Tentatively identified as CAEs, with unexplained $k_\perp \rho_{\perp b} < 0.8$
  - Density scaling also not Alfvénic, conflicting with dispersion relation[9]



[9] W.W. Heidbrink *et al.* Nucl. Fusion **46**, 324 (2006)

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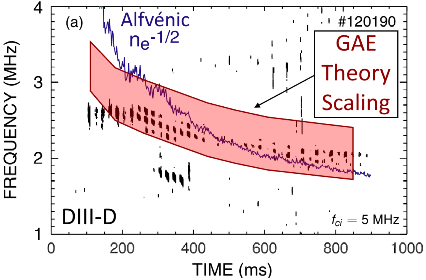
  - Tentatively identified as CAEs, with unexplained $k_\perp \rho_{\perp b} < 0.8$
  - Density scaling also not Alfvénic, conflicting with dispersion relation[9]

- Theory predicts narrow range of unstable GAEs: $\frac{1}{1+v_0/v_A} < \frac{\omega}{\omega_{ci}} < \frac{1}{1+v_0/v_A(1-\lambda_0)^{3/4}}$
  - Compatible with any value of $k_\perp \rho_{\perp b}$
  - Weaker density scaling agrees with data
- Theory predicts higher frequencies for larger λ_0
- Unstable CAEs would require much higher than observed frequencies



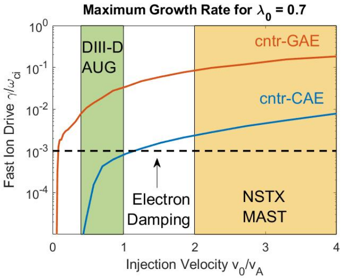
- High frequency observations in DIII-D may be more consistent with GAEs

[9] W.W. Heidbrink *et al.* Nucl. Fusion **46**, 324 (2006)

Why Are cntr-GAEs Preferred Over cntr-CAEs at Small v_0/v_A?

- Despite similar instability conditions, cntr-GAEs more common than cntr-CAEs
  - In NSTX ($v_0/v_A = 2 - 5$), both cntr-GAEs and cntr-CAEs were observed
  - In early NSTX-U ($v_0/v_A = 1.5 - 2.5$), almost exclusively cntr-GAEs
    - MAST had similar range, but different beam geometry – identified as CAEs
  - In DIII-D ($v_0/v_A = 0.5 - 1$), cntr-GAEs are most consistent with theory
  - In AUG ($v_0/v_A = 0.4 - 0.6$), cntr-GAEs seem more likely as well

- **Explanation**: theory predicts cntr-GAEs have growth rates 20 - 50 times larger than CAEs
  - CAEs also have an order of magnitude larger electron Landau damping rate than GAEs
    - At low v_0/v_A, this could stabilize CAEs



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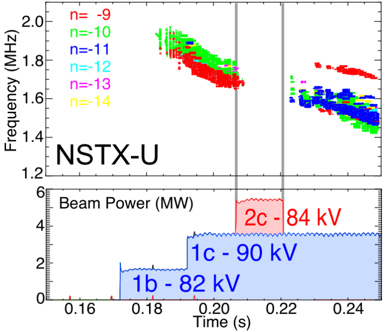
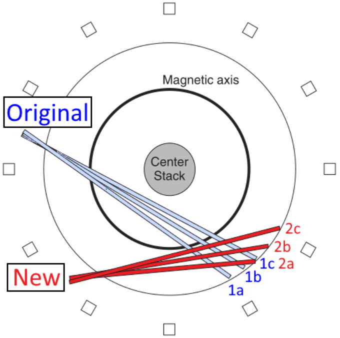
GAE Suppression Discovered on NSTX-U



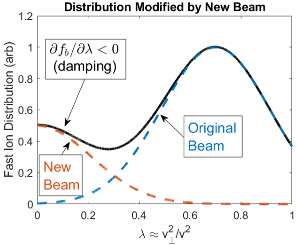


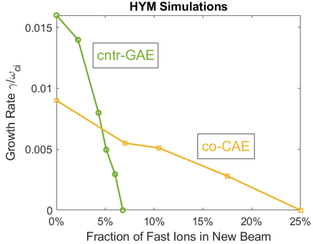
- NSTX-U found robust suppression of cntr-GAEs with addition of new off-axis/tangential beams[10]

[10] E.D. Fredrickson *et al.* Phys. Rev. Lett. **118**, 265001 (2017)

Analytic Theory Explains cntr-GAE Stabilization on NSTX-U

- Tangential injection flips sign of $\partial f_b/\partial\lambda \rightarrow$ damping
- **Stabilization**: damping from new beam balances drive from original beam
  - 7% of fast ions in new beam predicted for complete stabilization of cntr-GAEs
    - Very close to experiment and HYM simulations[11]
- Surprisingly, simulations show that tangential injection also stabilizes co-CAEs
  - Requires $\sim$ 25% fast ions in new beam
  - Theory predicts that very perpendicular injection should also stabilize co-CAEs, challenging to verify





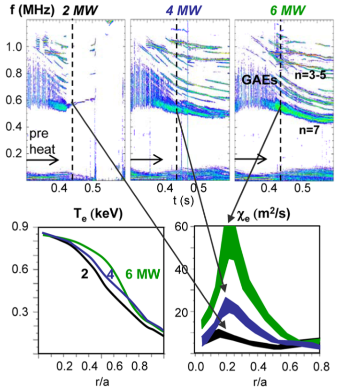
[11] E.V. Belova *et al.* Phys. Plasmas **26**, 092507 (2019)

Stabilization Techniques for Studying Electron Transport

1. Add a new beam in a different geometry (increase damping from $\partial f_b/\partial\lambda$)
   - To suppress **cntr**-GAEs/CAEs, add a more tangential beam
     - explains NSTX-U GAE suppression observations
   - To suppress **co**-CAEs, add a very tangential <u>or</u> perpendicular beam
     - driven by $\partial f_b/\partial\lambda < 0$, opposite condition for cntr-CAEs/GAEs
     - near marginal stability, large radiative damping is sensitive to beam distribution
   - To suppress **either**, counter-inject a new beam
     - Accesses new resonance for same mode, with opposite contribution to drive
2. Add a new beam at a different voltage without changing geometry
   - Adding a beam at a lower voltage should suppress **co**-CAEs
3. Add resonant particles which are stabilizing ($\lambda > \lambda_0$ for cntr-GAEs)
   - Can be achieved by lengthening the tail of the distribution – RF heating?

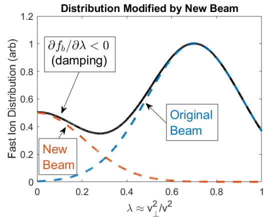
Testable Predictions for CAEs/GAEs in NSTX-U

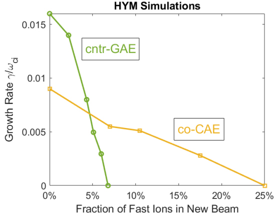
1. Test if T_e flattening responds to GAE stabilization with off-axis/tangential beams



Testable Predictions for CAEs/GAEs in NSTX-U

1. Test if T_e flattening responds to GAE stabilization with off-axis/tangential beams
2. Test co-CAE stabilization with tangential NBI
   - Requires low field (NSTX-like v_0/v_A) to excite co-CAEs at all
     - Predicted by simulations but beyond the scope of the simple theory.





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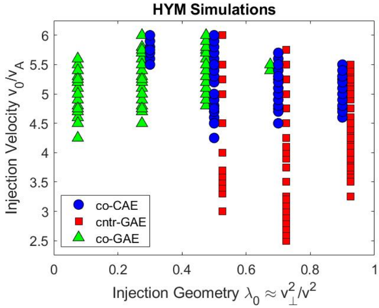
2. Test co-CAE stabilization with tangential NBI
   - Requires low field (NSTX-like v_0/v_A) to excite co-CAEs at all
     - Predicted by simulations but beyond the scope of the simple theory.
3. Try to excite high frequency co-GAEs, which were not previously observed
   - Requires low field and tangential NBI
     - Once excited, can also test if they can be suppressed by perp. NBI, as predicted



Summary

- CAEs/GAEs were investigated in NSTX(-U) with hybrid simulations and theory
- A simple analytic theory of sub-cyclotron Alfvén eigenmode instability has been developed for realistic NBI distributions
  - **Perpendicular injection**: drives cntr-propagating CAEs/GAEs and damps co-modes
  - **Tangential injection**: damps cntr-CAEs/GAEs and can drive or damp co-modes
  - Explains experimental observations and simulations of CAE/GAE excitation and stabilization in multiple devices (NSTX, NSTX-U, DIII-D, AUG)
- **Impact**: theory for control of CAEs/GAEs will enable investigation of their role in electron energy transport and help identify transport mechanisms
- **Future Applications**: (1) project to ITER (α distribution, multiple ion species), (2) try similar approach to interpret ion cyclotron emission (ICE), and (3) model sub-cyclotron instabilities driven by runaway electrons[12]

[12] C. Liu *et al.* Nucl. Fusion **61**, 036011 (2021)

Backup Slides

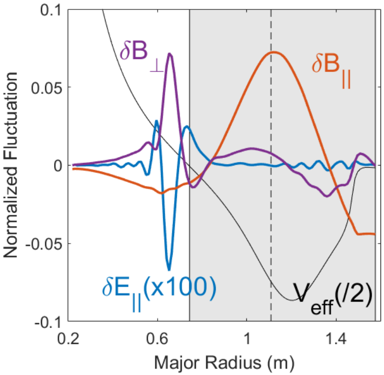
Compressional Alfvén Eigenmodes (CAE)

- Ideal magnetosonic mode in toroidal geometry
  - Compressional polarization
  - In uniform, low β limit, $\omega = kv_A$
- Localized by 2D wave equation

$$\left[\nabla_\perp^2 - V_{\text{eff}}(r,\theta)\right]\delta B_{||} = 0$$

$$V_{\text{eff}}(r,\theta) = k_{||}^2 - \frac{\omega^2}{v_A^2} \approx \left(\frac{n}{R}\right)^2 - \left(\frac{\omega}{v_A}\right)^2$$

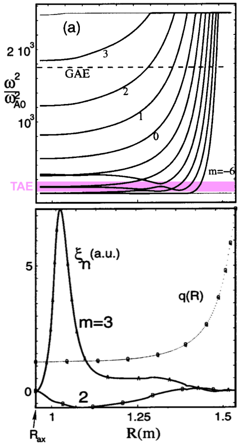
- $V_{\text{eff}} = 0$ coincides with Alfvén resonance, where CAE couples to kinetic Alfvén wave



$n = 4$ CAE calculated by HYM. $\delta B_{||}$ corresponds to the CAE. Coherent $\delta B_\perp$, $\delta E_{||}$ structures show the KAW.

Global Alfvén Eigenmodes (GAE)

- Discrete shear Alfvén eigenmode solutions may exist below minimum of Alfvén continuum
  - Approximate dispersion $\omega \leq \left[k_{\parallel}(r) v_A(r)\right]_{min}$
  - Weakly damped due to separation from continuum
- Dominant shear polarization: $\delta B_{\perp} \gg \delta B_{\parallel}$
  - In NSTX conditions, also have large compressional component $\delta B_{\parallel} \approx \delta B_{\perp}$ near edge
- CAEs/GAEs routinely observed in NSTX with $|n| = 3 - 12$ and $\omega/\omega_{ci} \approx 0.1 - 1.2$
  - ICE with $\omega > \omega_{ci}$ also present



GAE calculated by NOVA

HYM Physics Model

Fluid Thermal Plasma

$$\rho \frac{d\boldsymbol{V}}{dt} = -\nabla P + (\boldsymbol{J} - \boldsymbol{J_b}) \times \boldsymbol{B} - e n_b(\boldsymbol{E} - \eta\delta\boldsymbol{J}) + \mu\Delta\boldsymbol{V}$$

$$\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B} + \eta\delta\boldsymbol{J}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}$$

$$\mu_0 \boldsymbol{J} = \nabla \times \boldsymbol{B}$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{V})$$

$$\frac{d}{dt}\left(\frac{P}{\rho^\gamma}\right) = 0$$

Kinetic Fast Ions

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{v}$$

$$\frac{d\boldsymbol{v}}{dt} = \frac{q_i}{m_i}(\boldsymbol{E} - \eta\delta\boldsymbol{J} + \boldsymbol{v} \times \boldsymbol{B})$$

δF Scheme

$$F = F_0(\mathcal{E}, \mu, p_\phi) + \delta F(t)$$

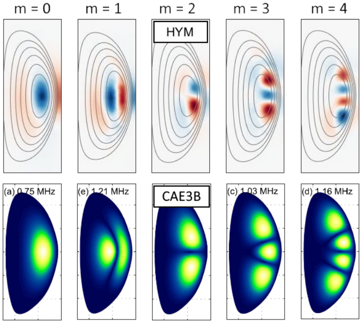
$$w \equiv \delta F / F$$

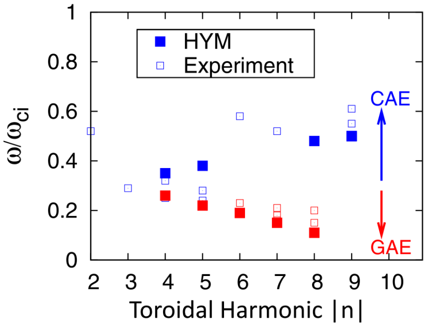
$$\frac{dw}{dt} = -(1 - w)\frac{d \ln F_0}{dt}$$

- ρ, $\boldsymbol{V}$, P are plasma mass density, velocity, and pressure
- n_b, $\boldsymbol{J_b}$ are beam ion density and current ($n_b \ll n_e$, though $\boldsymbol{J_b} \approx \boldsymbol{J_{th}}$)

Realistic Simulations Motivate Development of Theory

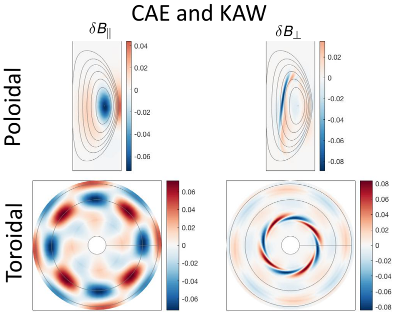
- HYM simulations accurately model CAEs/GAEs in NSTX(-U) experiments and recover eigenmodes from spectral codes
- Theory is needed to interpret simulation results and improve understanding to develop predictive capability

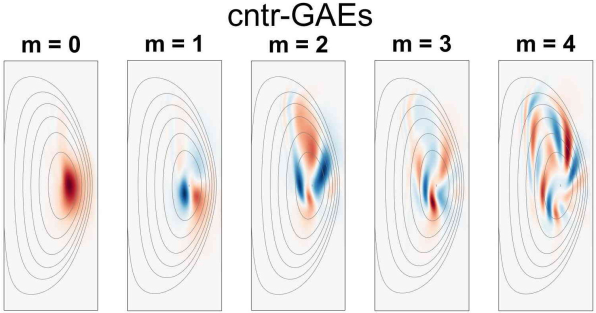




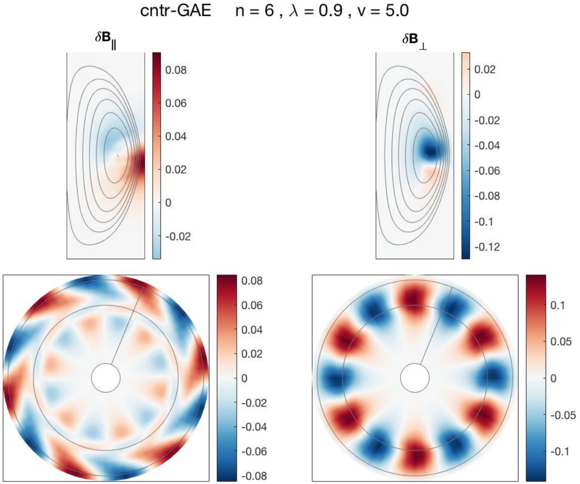
HYM Provides Input for Energy Transport Calculations

- Theories of CAE/GAE-induced electron energy transport require assumptions about the mode properties (frequency, amplitude, polarization, structure, *etc.*)
- HYM simulations generate realistic mode structures, beyond ideal MHD

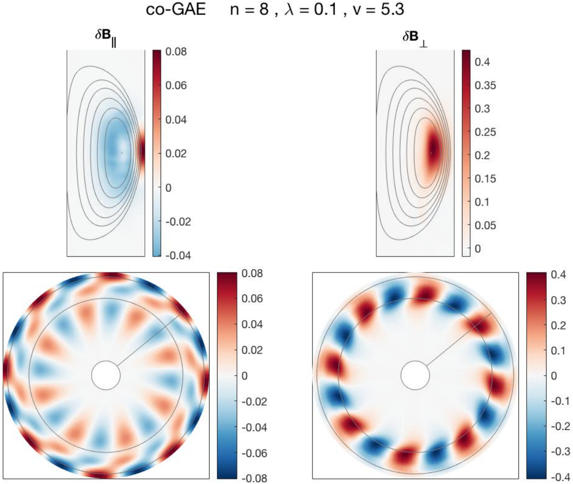




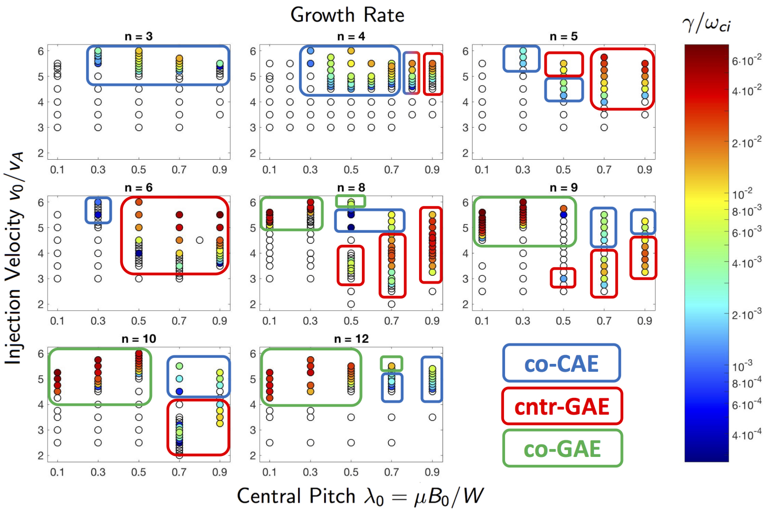
cntr-GAE Mode Structure from HYM



co-GAE Mode Structure from HYM



Linear Simulation Stability Results



Theory Applied to Understand Experiments and Simulations

- Local linear growth rate derived for realistic NBI distribution
  - Uncovers new instability regime – necessary to explain GAE excitation in NSTX-U
- Goal: simple expressions for fast ion drive depending on
  1. fast ion distribution parameters $(\lambda_0, v_0/v_A)$
  2. mode parameters $(\omega/\omega_{ci}, k_{\|}/k_{\perp})$
- Approach: restrict to 2D velocity space to avoid assumptions about equilibrium profiles, mode structure, particle orbits, *etc.*
  - Does not include contribution from $\partial f_0/\partial p_\phi$
- Provides upper bound on net growth rate, since neglecting bulk damping sources
  - Reminder: $\gamma_{\text{net}} = \gamma_{\text{EP}} - \gamma_{\text{th,damp}}$

Growth Rate Calculated for Anisotropic Beam Distribution

- For a beam-like distribution, with $x \equiv v_\perp^2/v^2$, $u \equiv v^2/v_0^2 = v_{\|,\mathrm{res}}^2/v_0^2(1-x)$

$$\frac{\gamma}{\omega_{ci}} \propto -\sum_\ell \int_0^{1-v_{\|,\mathrm{res}}^2/v_0^2} \underbrace{\frac{x}{(1-x)^2} \overbrace{\mathscr{J}_\ell^m\left(\frac{k_\perp v_{\|,\mathrm{res}}}{\omega_{ci}}\sqrt{\frac{x}{1-x}}\right)}^{\text{FLR terms}} \overbrace{\frac{e^{-(x-\lambda_0\langle\bar{\omega}_{ci}\rangle)^2/\Delta\lambda^2\langle\bar{\omega}_{ci}\rangle^2}}{1+(4u)^{3/2}}}^{\text{fast ion distribution}}}_{\text{non-negative}} \times$$

$$\Bigg[\underbrace{\frac{2}{\Delta\lambda^2\langle\bar{\omega}_{ci}\rangle^2}\left(x-\lambda_0\langle\bar{\omega}_{ci}\rangle\right)\left(\frac{\ell}{\bar{\omega}}-x\right)}_{\substack{\partial f_b/\partial\lambda \text{ drive/damping} \\ (\text{has sign of } x-\lambda_0\langle\bar{\omega}_{ci}\rangle)}} + \underbrace{\frac{3}{2}\left(1-\frac{1}{1+(4u)^{3/2}}\right)}_{\substack{\partial f_b/\partial\mathcal{E} \text{ damping} \\ (\text{negligible for } \ell\neq 0)}}\Bigg] dx$$

- Can integrate numerically, but further analytic progress requires approximation

Approximations Necessary to Derive Instability Conditions

1. Growth rate dominated by anisotropy for $\ell \neq 0$ resonances $\left(\frac{\omega_{ci}}{\omega} \frac{\partial f_0}{\partial \lambda} \gg \frac{v}{2} \frac{\partial f_0}{\partial v} \right)$
2. "Wide beam approximation" for $\Delta x \approx 0.3$ $\left(\frac{d}{dx} e^{-(x-x_0)^2/\Delta x^2} \approx -2(x - x_0)/\Delta x^2 \right)$
3. Small (or large) $k_\perp \rho_{\perp b}$ expansion of finite Larmor radius Bessel function terms
4. Neglect slowing down velocity dependence (weak dependence)

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Define $\eta = v_{\|,\mathrm{res}}^2 / v_0^2$, then for $k_\perp \rho_{\perp b} \lesssim 1$ and $\ell = 1$, the growth rate is proportional to

$$\gamma \mathrel{\underset{\sim}{\propto}} -\int_0^{1-\eta} \frac{x(x-x_0)}{(1-x)^2} dx > 0 \longrightarrow x_0 > \frac{1-\eta^2+2\eta\log\eta}{1-\eta+\eta\log\eta}$$

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$$\gamma \mathrel{\underset{\sim}{\propto}} -\int_0^{1-\eta} \frac{x(x-x_0)}{(1-x)^2}dx > 0 \longrightarrow x_0 > \frac{1-\eta^2+2\eta\log\eta}{1-\eta+\eta\log\eta} \underbrace{\approx 1-\eta^{2/3}}_{1\%\text{ error for } 0<\eta<1}$$

$$\implies v_0 < \frac{v_{\|,\text{res}}}{(1-x_0)^{3/4}}$$

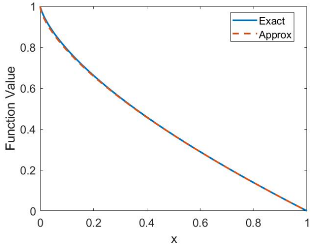
Serendipitous Approximations

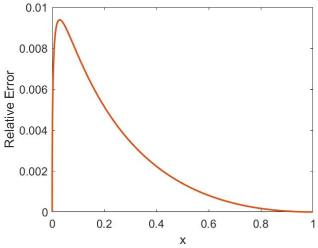
- Where did this approximation come from?

$$f(x) = \frac{1 - x^2 + 2x \log x}{1 - x + x \log x} \approx 1 - x^{2/3}$$

Accurate on $0 < x < 1$ to within 1%!

- Assume $f(x) \approx 1 - x^p$
  - Preserves smoothness, convexity, and monotonicity
  - $f(0) = 1$ and $f^{(n)}(0) \to (-1)^n \infty$ for $0 < p < 1$
  - $f(1) = 0$ and match $f'(1) = -p$
- Correct boundary behavior + sufficiently smooth function $\longrightarrow$ accurate global approximation
  - Same procedure used many times in this work



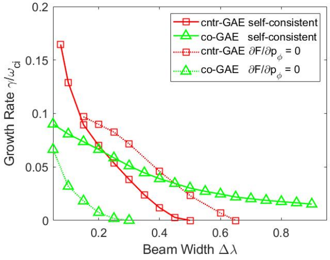


Gradients in p_ϕ Destabilize Co-Propagating Modes

- Local theory analysis neglected $\partial f_0/\partial p_\phi$
- Effect can be determined heuristically by comparing to nonlocal theory[13]

$$\gamma \propto \int d\Gamma \left[\left(\frac{\ell}{\bar{\omega}} - \lambda \right) \frac{\partial f_0}{\partial \lambda} + \mathcal{E} \frac{\partial f_0}{\partial \mathcal{E}} + \frac{n}{\bar{\omega}} \frac{\mathcal{E}}{\omega_{ci}} \frac{\partial f_0}{\partial p_\phi} \right]$$

- For non-hollow distributions, $\partial f_0/\partial p_\phi > 0$ $\rightarrow$ sign of n determines contribution
  - co-modes are **driven**, cntr-modes are **damped**
- HYM simulations that artificially remove $\partial f_0/\partial p_\phi$ contribution confirm its effect for co- vs cntr-GAEs



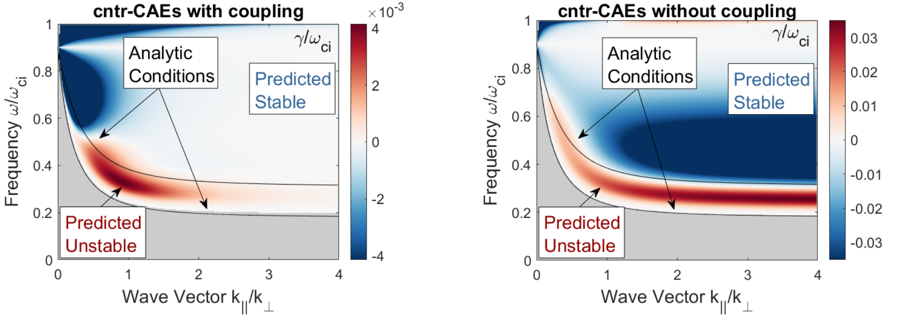
[13] A.N. Kaufman *et al.* Phys. Fluids **15**, 1063 (1972)

CAE/GAE Coupling Can Alter Most Unstable Modes

- Including two fluid effects, dispersions are coupled, modifying the polarization

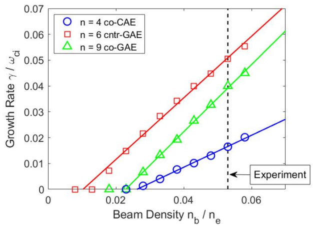
$$\left[1-\frac{k_{\parallel}^2 v_A^2}{\omega^2}\left(1-\frac{\omega^2}{\omega_{ci}^2}\right)\right]\left[1-\frac{k^2 v_A^2}{\omega^2}\left(1-\frac{\omega^2}{\omega_{ci}^2}\right)\right]=\frac{\omega^2}{\omega_{ci}^2}$$

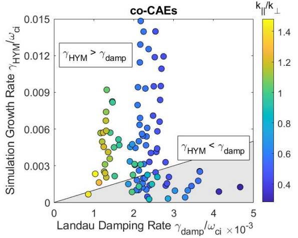
  - Changes growth rate, most unstable parts of spectrum
    - Most important for cntr-CAEs, also co-CAEs at smaller v_0/v_A
- $\ell = 0$ co-GAE can not exist without this coupling



Continuum/Radiative Damping is Dominant in Simulations

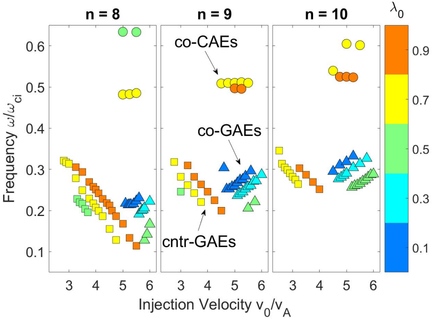
- Beam density scan in simulations shows $\gamma_{\text{damp}}/\gamma_{\text{drive}} \approx 20 - 60\%$
- Attributed to continuum/radiative damping since it is insensitive to viscosity and resistivity
- Electron damping (absent in simulations) calculated analytically for unstable modes
  - GAE electron damping rates are very small $\gamma_{\text{damp}}/\gamma_{\text{drive}} \sim 1\%$
  - CAE electron damping could be large enough to stabilize some modes near marginal stability





GAE Frequency Depends Strongly on v_0/v_A in Simulations

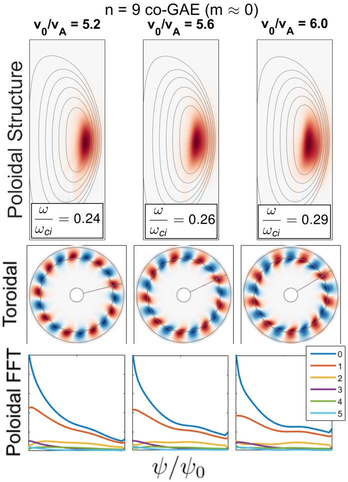
- GAE frequency changes dramatically with v_0/v_A for all n in linear HYM simulations: change of $20-50\%$ or $100-500$ kHz
- Change is continuous to at least $\Delta v_0 = 0.1 v_A$ resolution
  - Uncharacteristic of excitation of distinct MHD modes with discrete frequencies
- Sign of change in frequency is consistent with resonance condition



GAE Mode Structure Nearly Independent of Frequency

- Mode structure does not change qualitatively as frequency changes (mode numbers unchanged)
- Slight differences: peak location moves gradually inwards, mode becomes slightly elongated
- Frequency shifts $\approx 20\%$ from $\omega/\omega_{ci} = 0.24$ to 0.29 ($\Delta\omega = 125$ kHz) due to 15% change in v_0/v_A

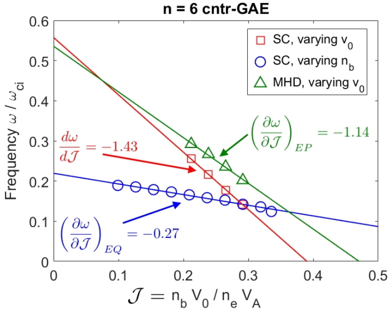
This is unusual behavior for MHD modes!



[14] J.B. Lestz *et al.* Phys. Plasmas **25**, 042508 (2018)

Frequency Change Is Not Due to Equilibrium Changes

	Equilibrium Effect	Phase Space Effect	Total Effect
Equil:	Self-cons.	MHD only	Self-cons.
Vary:	n_b / n_e	v_0 / v_A	v_0 / v_A
Fix:	v_0 / v_A	n_b / n_e	n_b / n_e



Equil. and EP phase space effects are nearly linear

$$\frac{d\omega}{d\mathcal{J}} \approx \left(\frac{\partial\omega}{\partial\mathcal{J}}\right)_{EQ} + \left(\frac{\partial\omega}{\partial\mathcal{J}}\right)_{EP} = n_e v_A \left[\frac{1}{v_0}\frac{\partial\omega}{\partial n_b} + \frac{1}{n_b}\frac{\partial\omega}{\partial v_0}\right]$$

Simulations show that $\Delta\omega_{EP} \gg \Delta\omega_{EQ}$, which may indicate the existence of the first EPM driven by a cyclotron resonance: the energetic-particle-modified GAE

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	Equilibrium Effect	Phase Space Effect	Total Effect
Equil:	Self-cons.	MHD only	Self-cons.
Vary:	n_b/n_e	v_0/v_A	v_0/v_A
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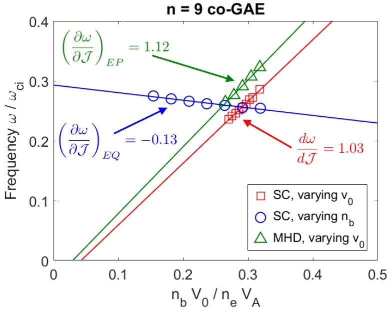
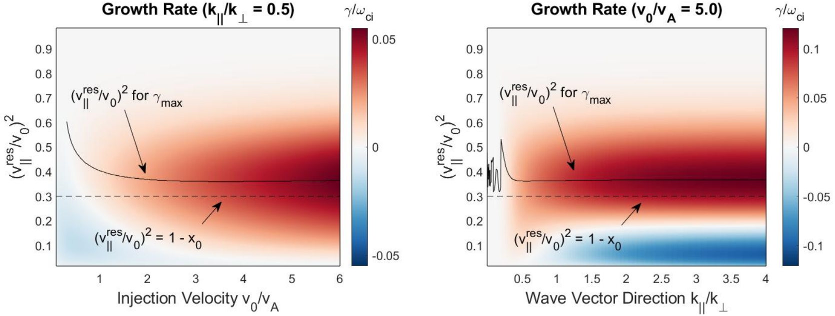
where $\mathcal{J} \equiv \dfrac{n_b}{n_e}\dfrac{v_0}{v_A}$



Growth Rate Calculation May Explain Frequency Change

- The GAE growth rate is maximized at a specific value $v_{\|,\text{res}}^2 / v_0^2 = 0.36 \equiv \eta_{\max}$
  - Independent of v_0 / v_A and $\left| k_\| / k_\perp \right|$
- The most unstable frequency changes with v_0 / v_A due to the resonance condition
  - $\omega = \langle \omega_{ci} \rangle + v_0 k_\| \sqrt{\eta_{\max}}$
- Future work: verify with non-perturbative calculation



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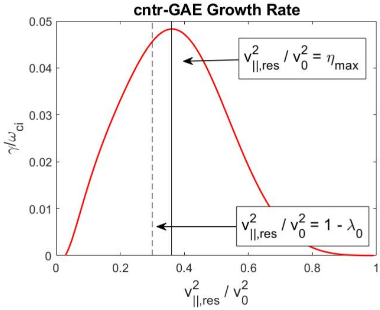
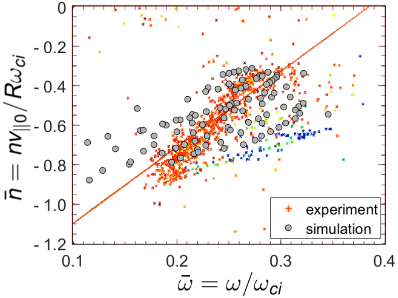
  - Independent of v_0 / v_A and $\left| k_\| / k_\perp \right|$
- The most unstable frequency changes with v_0 / v_A due to the resonance condition
  - Scaling: $\omega \approx \langle \omega_{ci} \rangle + \frac{n v_0}{R} \sqrt{\eta_{\max}}$
- NSTX observations of ω / ω_{ci} vs n are consistent with this trend[14]
  - Provides clues but not confirmation without careful measurements of mode structure
- TBD: verify with non-perturbative calculation



[15] S.X. Tang *et al.* 2017 TTF Meeting

Potential Experimental Support for EP-GAEs

- Experimental analysis of many NSTX discharges[15] shows cntr-GAE frequency *decrease* with increasing $|n|$
  - Opposite trend expected from dispersion $\omega = |k_{\parallel}| v_A \propto |n|$
  - Consistent with conclusion that resonant particles determine frequency
- Provides clues but not confirmation without careful measurements of m to determine mode structure

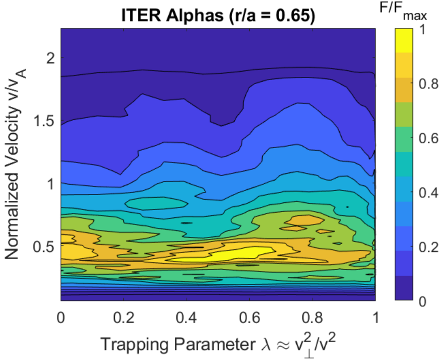


- Color: NSTX observations
- Gray: HYM simulations
- Normalized resonance condition: $\bar{n} \approx \bar{\omega} - 1$

[16] S.X. Tang *et al.* 2017 TTF Meeting

CAEs/GAEs May Be Present in Burning Plasmas

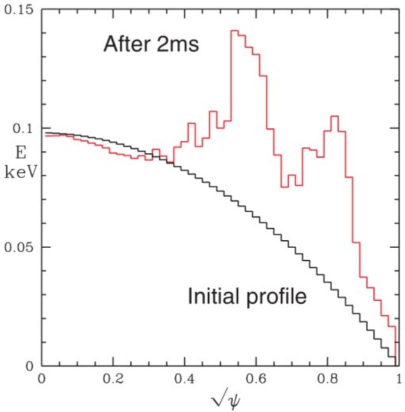
- ITER will have super-Alfvénic NBI and alpha particles $(v_0/v_A = 1.5 - 2)$
- Anisotropy of alphas near the edge could destabilize cntr-GAEs/CAEs
  - Similar to NSTX(-U) beam parameters
- ITER NBI distribution has $\lambda_0 = 0.3 - 0.8$ depending on radius
  - Could be either destabilizing or stabilizing (NSTX-U multi-beam suppression)



- **Open question**: if the modes are excited, will the anomalous electron transport also be present in ITER or is it unique to spherical tokamaks?

CAE/GAE-Induced Ion Heating Was Also Explored on NSTX

- Anomalously high $T_i > T_e$ was observed in some NBI-dominated NSTX discharges[16]
- Proof-of-principle stochastic heating of ions by CAEs shown in test particle simulations[17]
- Subsequent experimental analysis[18] found CAE to thermal ion power transfer to be insufficient to explain the surplus $T_i - T_e$
- Not yet fully resolved



[17] D.A. Gates *et al.* Phys. Rev. Lett. **87**, 205003 (2001)
[18] N.N. Gorelenkov *et al.* Nucl. Fusion **43**, 228 (2003)
[19] E.D. Fredrickson *et al.* Phys. Plasmas **9**, 2069 (2002)

Open Questions

1. Which transport mechanism is the dominant cause of anomalous flat T_e profiles?
2. Will CAEs/GAEs be unstable in ITER? Will they induce anomalous transport?
3. What is the dominant mechanism for co-CAE stabilization by tangential injection?
4. Can the analytic stability boundaries be generalized to $\omega \gg \omega_{ci}$ in order to interpret ion cyclotron emission (ICE)?