Validation of model for interaction between fast ion and neoclassical tearing mode in NSTX

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Outline

- Introduction

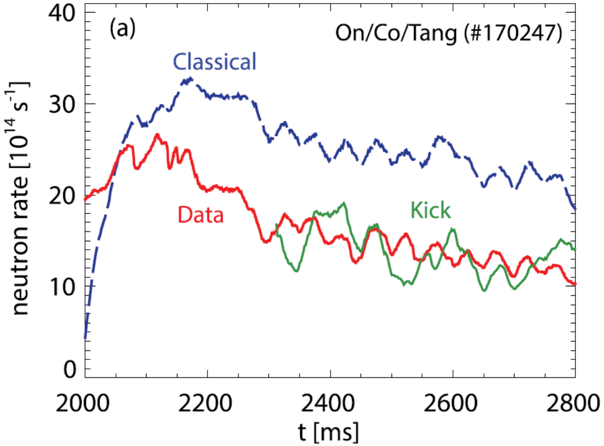
- Fast ion / NTM interaction analysis
  - Validation of kick model: Fast ion transport simulation
  - Validation of NTM stability model using kick model parameters as input

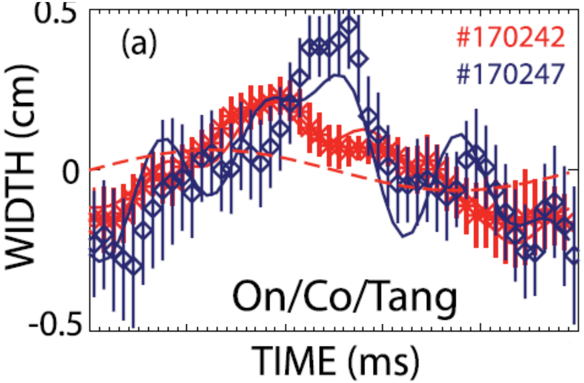
- Numerical experiments
  - Scan of relative phase of NTM to core kink
  - Dependence of energy exchange to mode combination
  - Scan of mode amplitude and orbit stochasticization threshold

- Conclusion
- Extended future work

Do fast ions interact with NTM as they do with AE?

- Fast ions interact with Alfvén eigenmodes (AEs) [1]
- Fast ions “seemingly” interact with neoclassical tearing modes (NTMs)
  - NTMs cause fast ion transport
  - Model validated qualitatively [2] and quantitatively [3]
  - Analytical model for stability cannot be validated [3]
  - NTM chirp is correlated with fast ion activity [4]



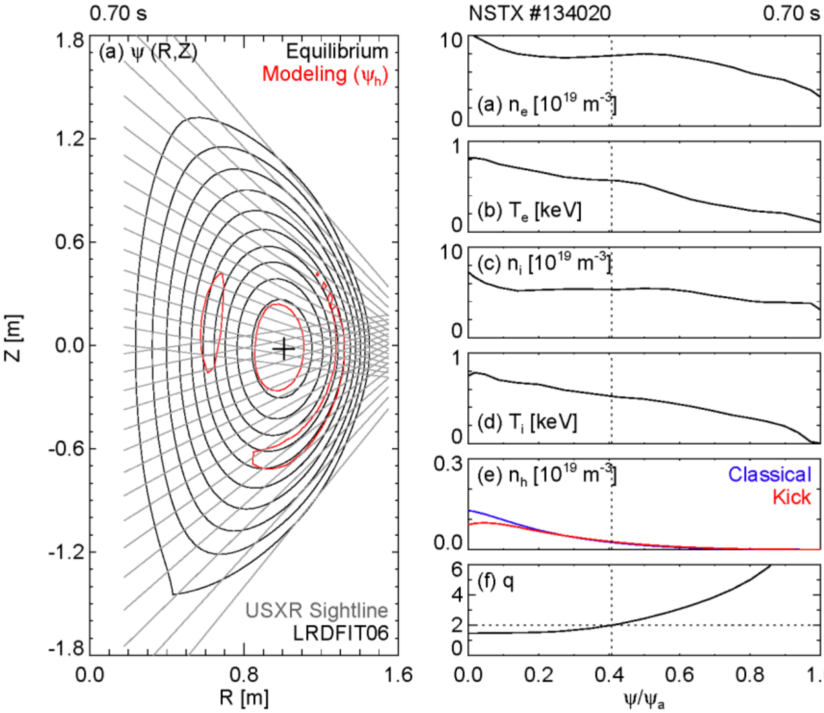


[1] Podestà *et al*., Plasma Phys. Control. Fusion **59** 095008 (2017)
[2] Zweben *et al*., Nucl. Fusion **39** 1097 (1999)
[3] Heidbrink *et al*., Nucl. Fusion **58** 082027 (2018)
[4] Fredrickson, Phys. Plasmas **9** 548 (2002)

NTM stability model with fast ion is validated utilizing kick model

- Do fast ions affect NTM stability?
  - Analytical model in Rutherford equation
  - Requires input of fast ion and NTM parameters

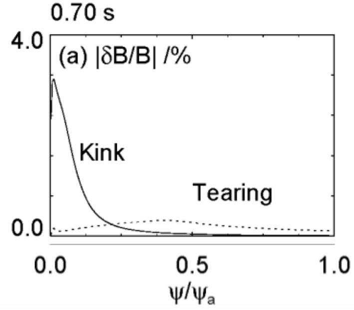
- Kick model [1] can provide necessary data
  - Thermal ion profiles
  - Fast ion profiles (replaces measurement)
  - Equilibrium profiles
  - NSTX 2/1 NTM discharge is analyzed [2]

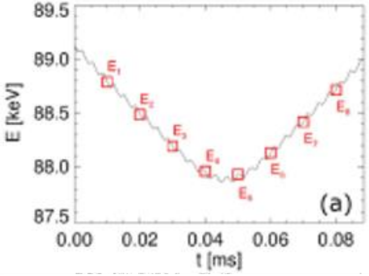


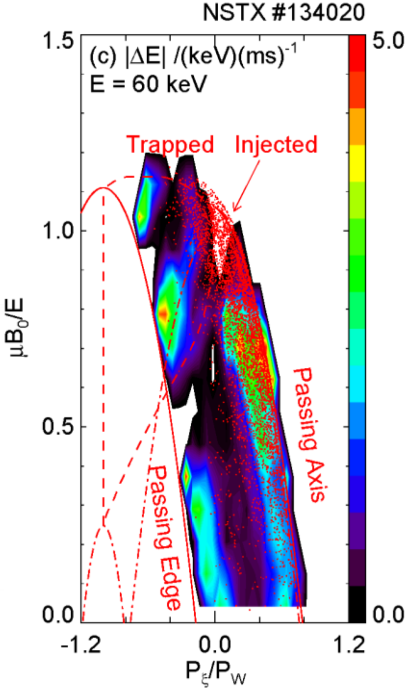
[1] Podestà *et al*., Plasma Phys. Control. Fusion **56** 055063 (2014)
[2] La Haye *et al*., Phys. Plasmas **19** 062506 (2012)

Kick model calculates transport with wave-particle interaction [1]

- ORBIT computes wave-particle interaction [2]
  - Test fast ion response to NTM is followed
  - Measured island parameters are input
- TRANSP computes transport [3]
  - Probability of fast ion response is input
  - NUBEAM [4] computes fast ion response
  - Kick (classical) with (without) NTM
  - All parameters in are calculated self-consistently





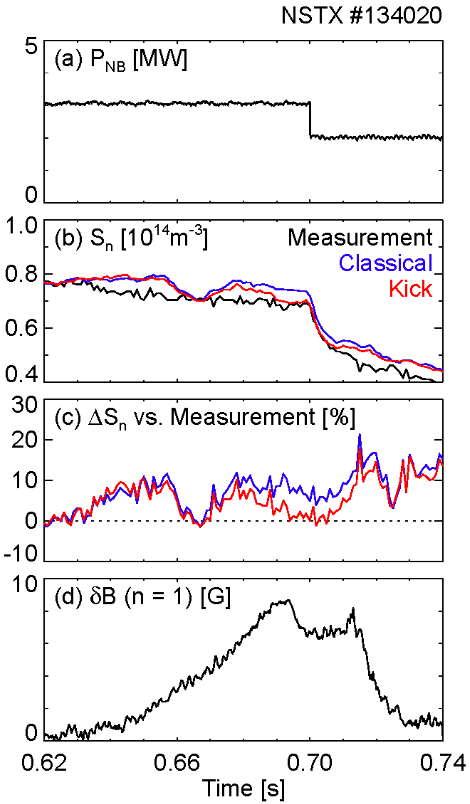


[1] Podesta *et al*., Plasma Phys. Control. Fusion **59** 095008 (2017)
[2] White and Chance, Phys. Fluids **27** 2455 (1984)
[3] Hawryluk, Physics Plasmas Close to Thermonuclear Conditions 19 (1981)
[4] Goldston *et al*., J. Comput. Phys. **43** 61 (1981)

Fast ion transport by NTM is modeled successfully by kick model

- Neutron rate is measured during NSTX NTM discharge [1]
  - Utilizing scintillators calibrated by fission chambers [2]

- Neutron rate simulated by kick model agrees with the measurement
  - In comparison, classical model overestimates neutron rate
  - No free parameters are introduced
  - This result gives confidence to using kick model output for the stability analysis



[1] La Haye *et al*., Phys. Plasmas **19** 062506 (2012)
[2] Roquemore *et al*., presented in 24th SOFE SP1-39 (2011)

Analytical model is introduced for NTM stability with fast ions

- Generalized Rutherford equation (GRE) governs NTM stability [1]
- Fast ions generate different currents depending on island versus orbit size ratio
  - When island is as large as orbit, parallel current is generated [2]
  - Otherwise, orbit averaging causes uncompensated cross field current [3]

$$\frac{1}{k_3}\frac{\tau_R}{r^2}\frac{dw}{dt} = \Delta'_{m,n}(w) + k_1\left[\frac{16J_{BS}}{s\langle J\rangle}\frac{w}{w^2+w_d^2} - Nw\right] - k_2\left[\varepsilon^{3/2}\frac{\rho_{\theta i}^2}{w^2} - \frac{L_{n_i}}{L_{n_h}}\frac{n_h}{n_i}\right]\frac{\beta_\theta}{w}\left(\frac{L_q}{L_p}\right)^2 - k_4\frac{\beta_\theta\varepsilon^2}{rw}\frac{L_q^2}{|L_p|}\frac{q^2-1}{q^2}$$

Classical [4]

Bootstrap [4]

Polarization [5]

Curvature [6]

Parallel current [2]

Uncompensated cross field current [3]

[1] Poli *et al.*, Nucl. Fusion **58** 016007 (2018)
[2] Hegna and Bhattacharjee, Phys. Rev. Lett. **63** 2056 (1989)
[3] Cai Nucl. Fusion **56** 126016 (2016)
[4] Fredrickson *et al.*, Phys. Plasmas **7** 4112 (2000)
[5] Gates *et al.*, Nucl. Fusion **37** 1593 (1997)
[6] Gorelenkov *et al.*, Phys. Plasmas **3** 3379 (1996)

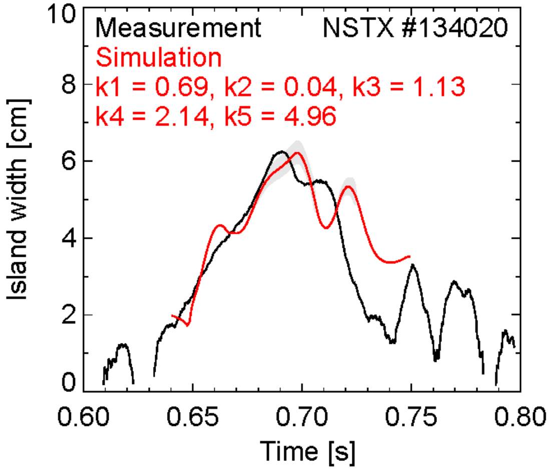
Fast ion term is essential for GRE modeling of island width

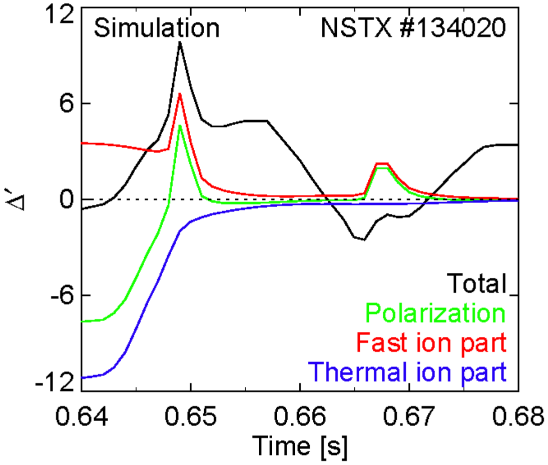
- Island width is measured by Mirnov coil and scaled by synthetic SXR diagnostic [1]
- Island width simulated by GRE with fast ion term agrees with the measurement
  - Free parameters are determined by numerical optimization
  - Fast ion term contribution is significant at island onset phase

$$\Delta'_{pol} = \left[\varepsilon^{3/2}\frac{\rho_{\theta i}^2}{w^2} - \frac{L_{n_i}}{L_{n_h}}\frac{n_h}{n_i}\right]\frac{\beta_\theta}{w}\left(\frac{L_q}{L_p}\right)^2$$

Thermal ion part
(Polarization current)

Fast ion part
(Uncompensated current)

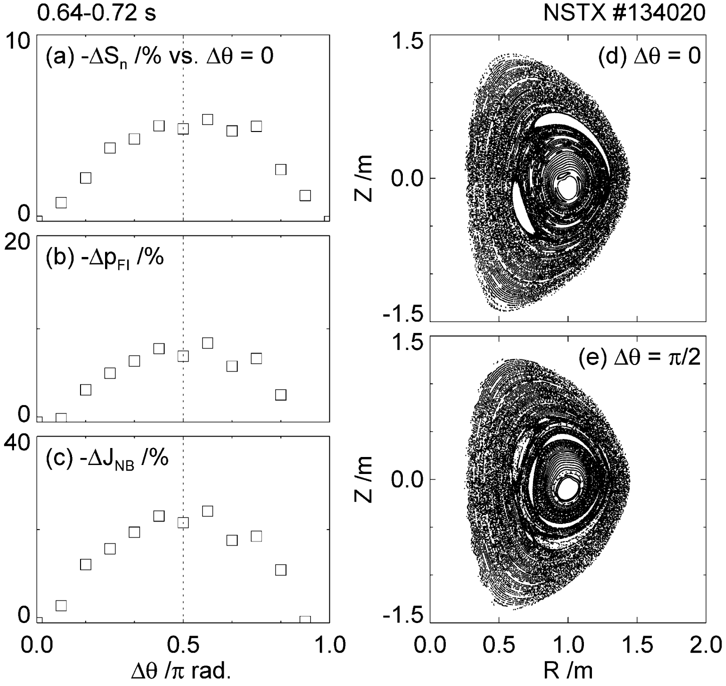




[1] Chang *et al*., Nucl. Fusion **34** 1309 (1994)

Relative phase of NTM and kink affects fast ion transport

- Core (1,1) kink accompanies NTM in NSTX [1]
  - Non-resonant with $q_{\text{min}} \approx 1.2$
  - Coupled to NTM (relative phase fixed at zero)
- Relative phase affects fast ion transport [2]
  - Transport channel may be formed†
  - Fast ions being displaced by kink, then NTM
- Kink and NTM may be coupled via fast ions



† Also suggested for the case of kink / AE coupling [3]

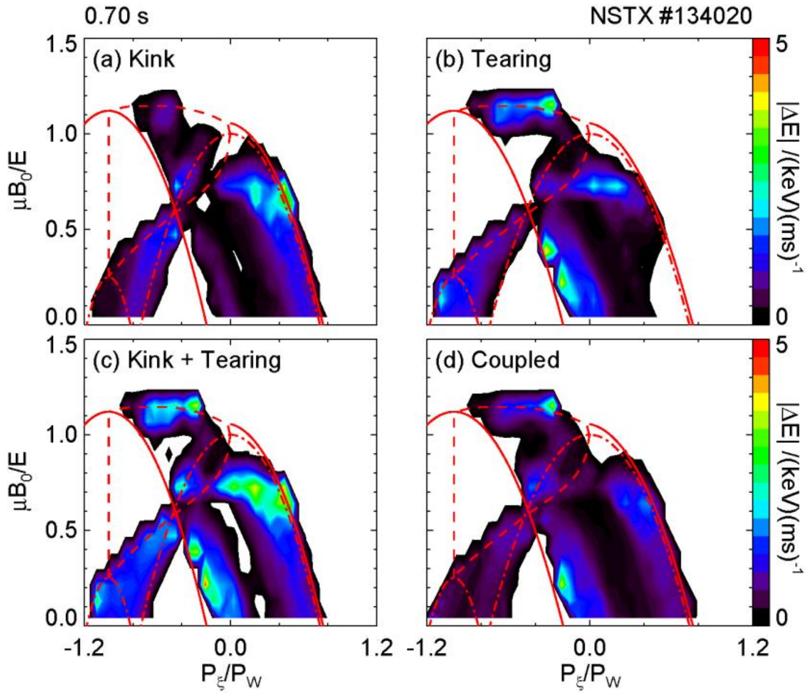
[1] Gerhardt *et al*., Nucl. Fusion **51** 073031 (2011)

[2] Yang *et al*., Plasma Phys. Control. Fusion **63** 045003 (2021)

[3] Duong *et al*., Nucl. Fusion **33** 749 (1993)

Energy exchange depends on mode combination

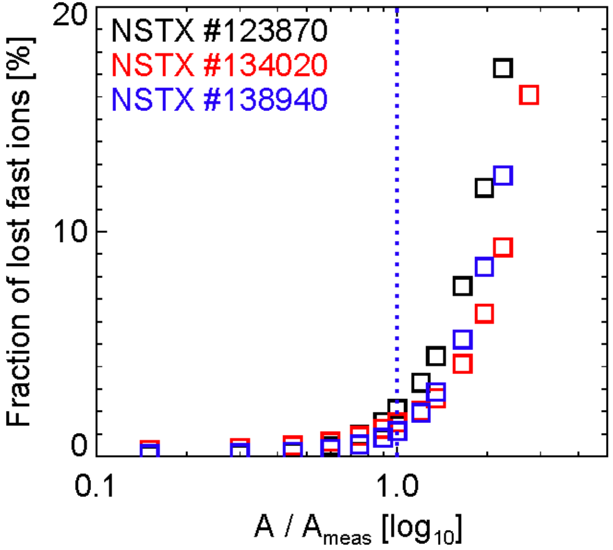
- Fast ion ΔE comes from MHD modes
  - No other energy source / sink
- Mode combination affects ΔE structure [1]
  - Also seen in DIII-D [2]
  - Kink (NTM) affects ΔE by NTM (kink)
  - Kink and NTM are synergistic when interacting with fast ions
- Kink and NTM may be affecting each other via fast ions



[1] Yang *et al*., Plasma Phys. Control. Fusion **63** 045003 (2021)
[2] Liu *et al*., Nucl. Fusion **60** 112009 (2020)

Saturated NTM island width is orbit stochasticization threshold

- Fast ion orbits turn stochastic when $w > w_{\text{thres}}$ [1]
  - Threshold w_{thres} is found by numerical scan
- NTM island width saturates at the threshold [2]
  - Unlike in DIII-D when no kink mode is present [3]
  - Orbit becomes stochastic by overlapping phase space islands from kink and NTM
- Fast ions and/or kink may be suppressing NTM

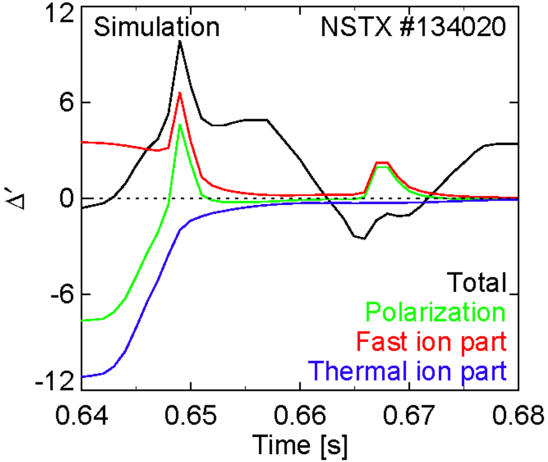


[1] Heidbrink and White, Phys. Plasmas **27** 030901 (2020)
[2] Yang *et al*., Plasma Phys. Control. Fusion **63** 045003 (2021)
[3] Bardóczi *et al*., Plasma Phys. Control. Fusion **61** 055012 (2019)

No evidence rejects possibility that fast ions affect NTM stability

Conclusion

- Fast ion term is essential for GRE modeling of NTM island width
- Kick model is used for necessary input to NTM stability analysis
  - All parameters are calculated self-consistently
  - Successful fast ion transport simulation validates the procedure
- Numerical experiments support GRE modeling result
  - NTM and kink may be coupled via fast ion population
  - NTM and kink may be affecting each other via fast ions
  - NTM may be suppressed as orbits become stochastic

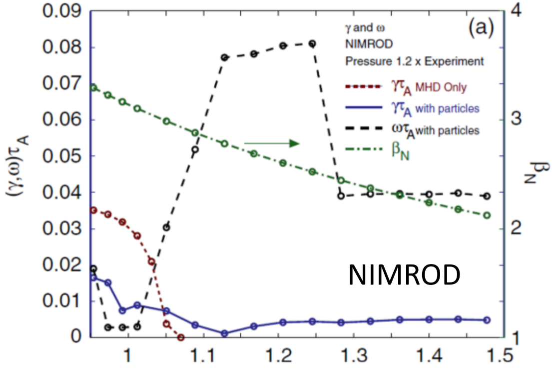


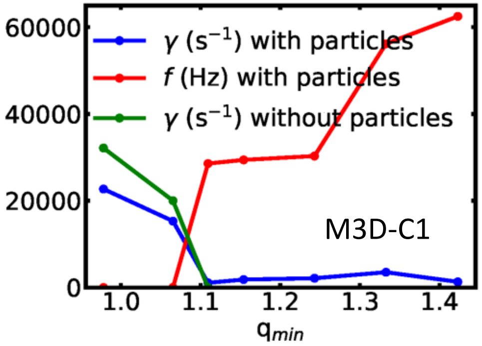
Dedicated experiment will follow to confirm the assertion

- M3D-C1 [1] with fast ions will support analytical model
  - Preliminary result reproduces NIMROD result [2]
  - Can input realistic fast ion distribution from kick model

- Analysis procedure will be improved
  - ASCOT [3] will help validate use of ORBIT in NSTX/U
  - Free parameter will be determined from a database

- Dedicated experiments will follow in NSTX-U
  - Contributions to MAST-U and DIII-D are also considered





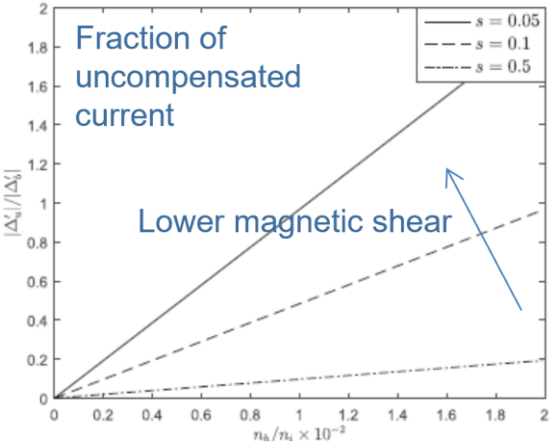
[1] Breslau *et al*., Phys. Plasmas **16** 092503 (2009)
[2] Brennan *et al*., Nucl. Fusion **52** 033004 (2012)
[3] Varje *et al*., arXiv https://arxiv.org/abs/1908.02482 (2019)

Future Work

Further experimental study will help answer bigger question

- Quantitative prediction capability for EP interaction with NTM is not yet achieved
  - EP transport model with NTM can predict with some accuracy
  - NTM stability model with EP model is being developed
  - EP effect may become more significant at low magnetic shear plasmas [1]

- Controlled experiments provide necessary data points
  - Similar work has been done for other topics
  - Impact of rotation and EP on IWM stability (NSTX) [2]
  - Impact of rotation on NTM stability (DIII-D) [3]



Fraction of uncompensated current

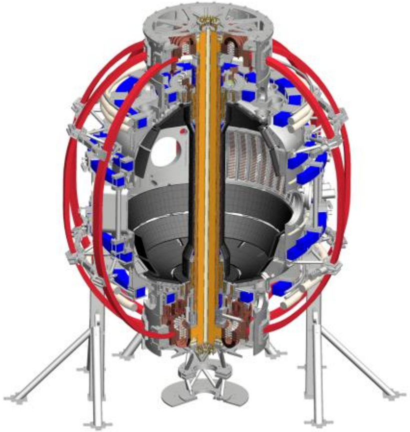
Lower magnetic shear

Fast ion density fraction

[1] Cai, Nucl. Fusion **56** 126016 (2016)
[2] Menard *et al*., Phys. Rev. Lett. **113** 255002 (2014)
[3] Buttery *et al*., Phys. Plasmas **15** 056115 (2008)

Summary of research plans by year [1]

- Years 2022-3 for advancement and testing of reduced energetic particle models
  - Interpretation of fast ion phase space dependence on NTM†
  - Development of predictive capability for NTM† stability
- Years 2024-5 for integration of stability and transport models
  - Predictive model for interaction between fast ion and NTM† within TRANSP
- Key diagnostic and modeling tools
  - Diagnostic: Magnetics, SXR‡, FIDA
  - Modeling: LRDFIT, ORBIT/ASCOT, M3D-C1



† As well as kink mode
‡ As well as BES and/or reflectometry
[1] In line with Kaye *et al*., NSTX-U Five Year Plan, EP-1 (2020)

Development of synthetic soft x-ray fluctuation diagnostics

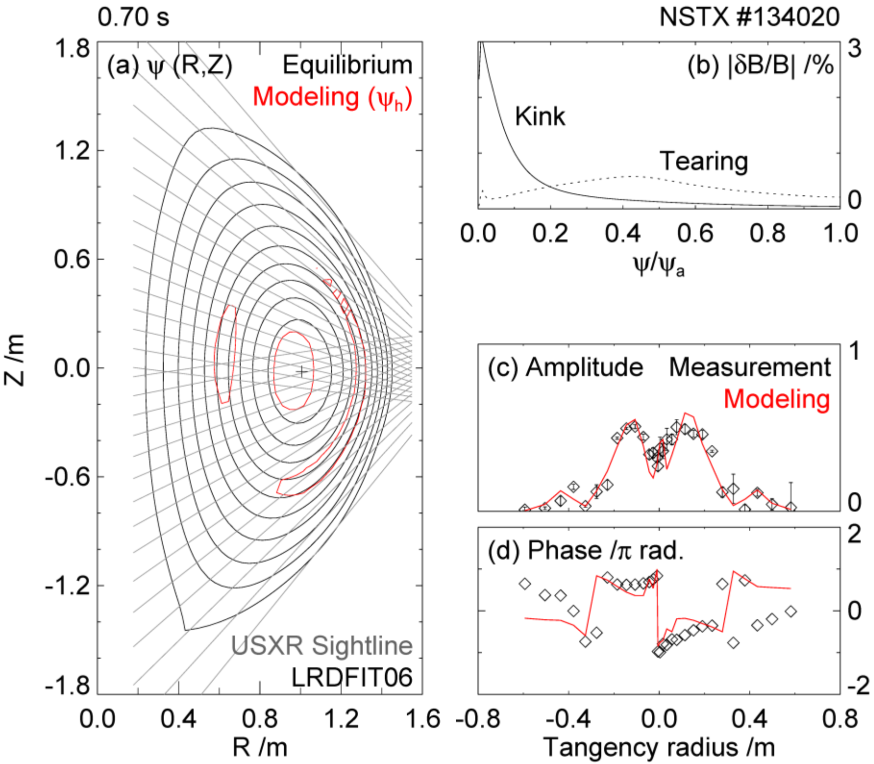
- Overview & flow chart

Appendix

- Diagnostic setup
  - Pinhole diode array for soft x-ray radiometry
  - Signal processing
  - Equilibrium and forward-modeling
  - Fit results
- Analysis of fit quality
  - Spatial resolution of diagnostic
  - Sensitivity study: Need for determination of island location initial guess

Synthetic diagnostic utilizes analytic model and SXR fluctuation

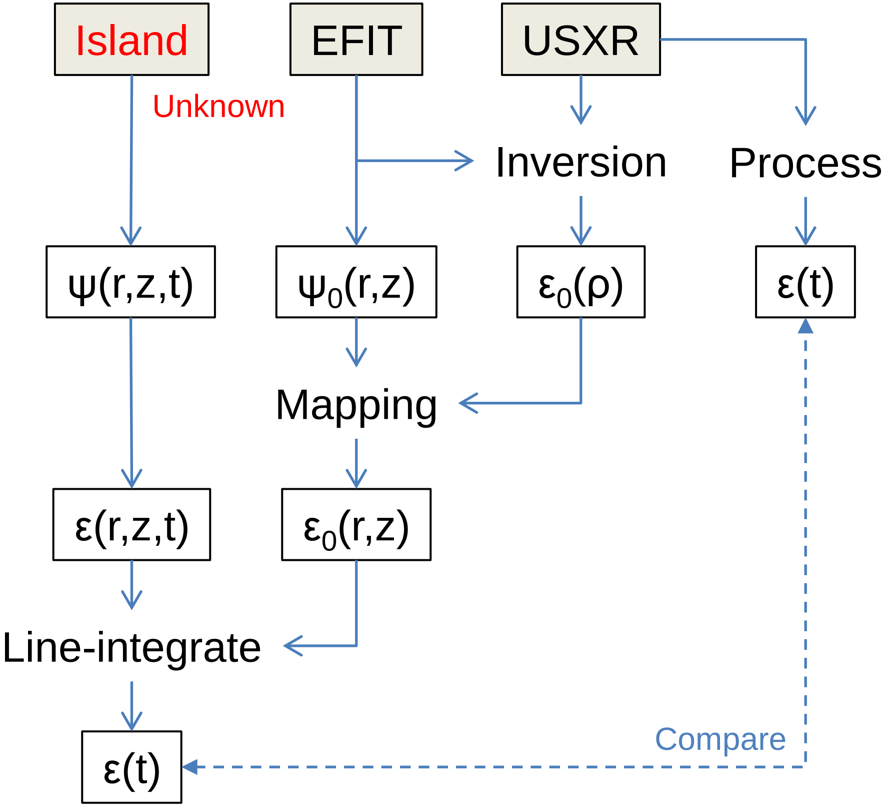
- SXR fluctuation phase jumps at modes
  - Three π-jumps are observed typically
  - One for kink, two for tearing [1]
- Overview of synthetic diagnostics
  - Analytic model for mode structures [1]
  - Mirnov coil, CHERS [2], SXR [3]
  - Automation of interactive analysis code by Eric Fredrickson [1]



[1] Fredrickson *et al*., Rev. Sci. Instrum. **59** 1797 (1988)
[2] Bell *et al*., Phys. Plasmas **17** 082507 (2010)
[3] Stutman *et al*., Rev. Sci. Instrum. **74** 1982 (2003)

Flowchart shows algorithm of synthetic diagnostics

- Tomography of perturbed emissivity [1]
  - Not alike equilibrium emissivity
  - Inaccurate for multiple modes [2]
- Synthetic diagnostic scheme [3]
  - Automated human intuition part
  - Numerical optimization [4]



Island

EFIT

USXR

Unknown

Inversion

Process

$\psi(r,z,t)$

$\psi_0(r,z)$

$\varepsilon_0(\rho)$

$\varepsilon(t)$

Mapping

$\varepsilon(r,z,t)$

$\varepsilon_0(r,z)$

Line-integrate

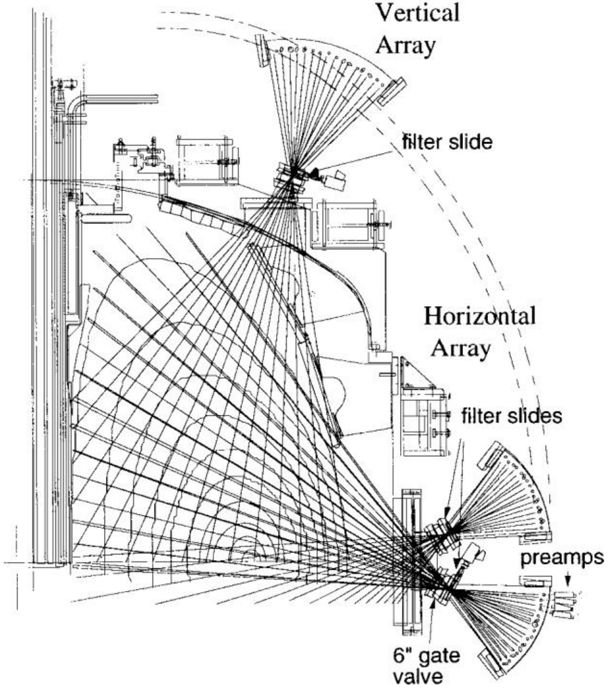
$\varepsilon(t)$

Compare

[1] Nagayama Jpn. J. Appl. Phys. **20** L779 (1981)
[2] Nagayama Phys. Plasmas **3** 2681 (1996)
[3] Fredrickson *et al*., Rev. Sci. Instrum. **59** 1797 (1988)
[4] Levenberg, Quart. Appl. Math. **2** 164 (1944)

Synthetic diagnostic utilizes already installed SXR radiometry

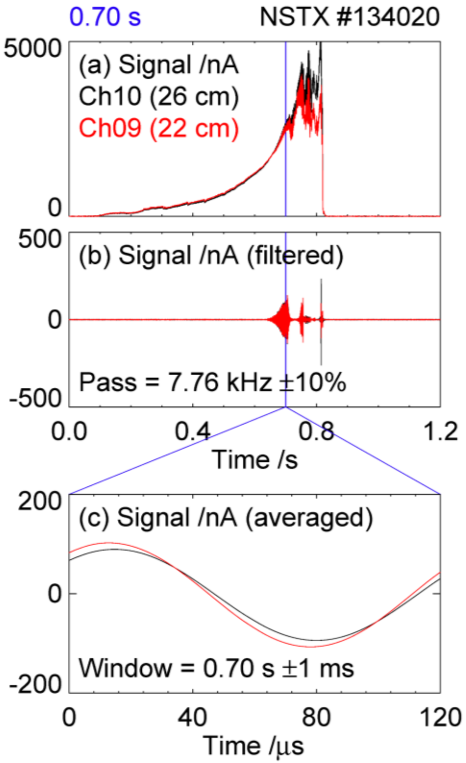
- Filtered pinhole diodes (Be 5 μm) [1]
  - Sampled at 5 MHz
  - USXR range (10 – 300 Å)
  - Edge resolution† < 6 cm
  - Core resolution† > 1 cm
- Utilizes horizontal array (both angles)
  - Since extra constraints are useful
  - Added measurements at $\pi/2$



[1] Stutman *et al.*, Rev. Sci. Instrum. **74** 1982 (2003)

Perturbed emissivity is extracted from SXR measurement

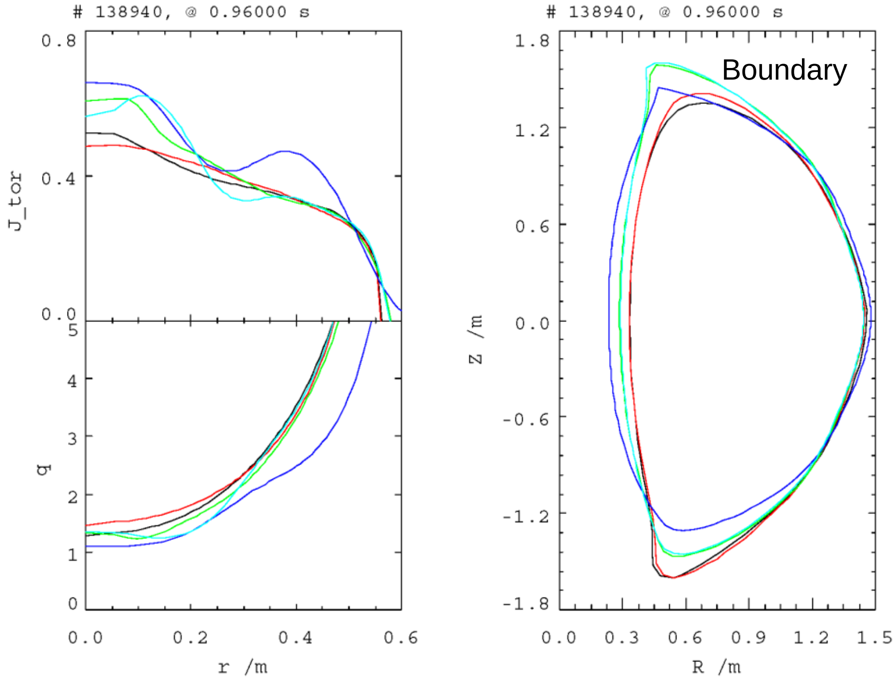
- Numerical band pass filter is applied
  - Pass band set at mode frequency ± 10%
- Numerical periodic averaging is applied
  - Find zero-crossings and accumulate
  - For further reduction of measurement noise



Equilibrium reconstruction by LRDFIT06 is utilized

- Forward modeling requires good *q*
- LRDFIT is useful for MSE + EFIT
  - Boundary should agree with EFIT
  - 06 and 12 meet criteria
  - 06 has smoother profiles
  - 06 is considered more reliable [1]

EFIT01	Magnetics (MD)
EFIT02	MD + Kinetic
LRDFIT06	MD + MSE + T_e
LRDFIT09	MD + MSE + T_e + V_ϕ
LRDFIT12	MD + MSE + T_e + P_{th}



[1] Podestà, private conversation with Menard

Perturbed emissivity is forward-modeled using analytic model [1]

- Kink mode displacement

$$\xi = \delta/[1+(r/r_k)^p]\cos(m\theta_k - n\phi + \omega t)$$

- Tearing mode perturbed helical flux

$$\delta\psi_{m,n} = w^2(16r_s/sB_\theta)\cos(m\theta_s - n\phi + \omega t)$$

- Seven parameters marked in red are used in multi-curve fitting [2]

- Equilibrium emissivity profile† is used to convert perturbed fields to perturbed emissivity

† Measured emissivity minus perturbed emissivity, then inversion

[1] Fredrickson *et al*., Rev. Sci. Instrum. **59** 1797 (1988)

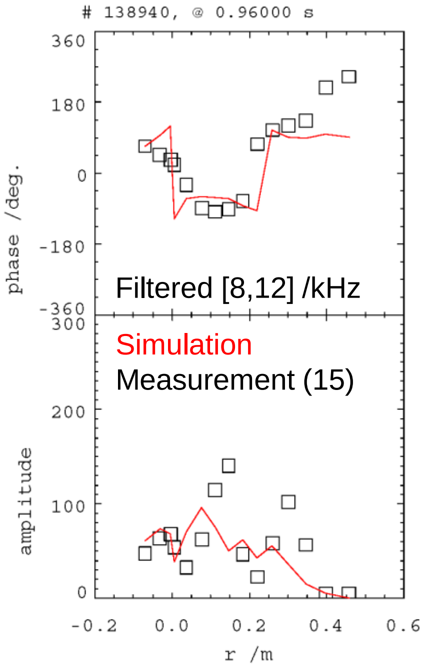
[2] Levenberg, Quart. Appl. Math. **2** 164 (1944)

Sample synthetic diagnostic result shows 2/1 islands

- Amplitudes & phases are constraints
  - Added measurements at $\pi/2$

- Unknowns are seven mode parameters
- Sample result is shown

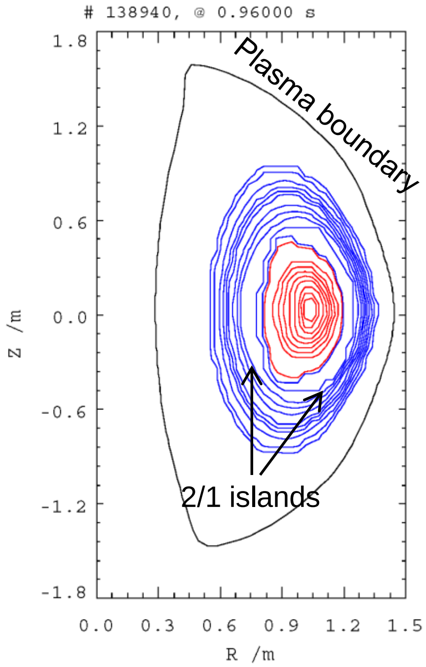
TM	Location	24.5	±	0.2	cm
	Width	8.0	±	1.4	cm
	Phase	-4.0	±	17.6	°
Kink	Location	10.4	±	3.8	cm
	Amplitude	2.8	±	1.9	cm
	Power	5.3	±	4.7	
	Rel. Phase	4.1	±	17.6	°



Filtered [8,12] /kHz

Simulation

Measurement (15)



Plasma boundary

2/1 islands

Colored: USXR coverage

Island width is calculated from Mirnov coil, EFIT, and SXR data

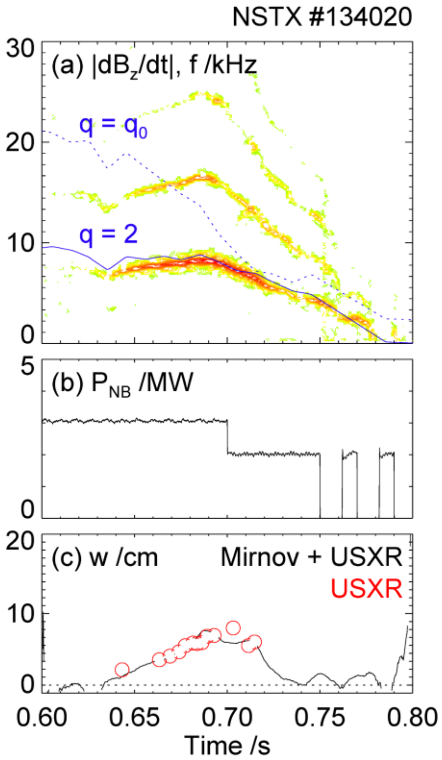
- Time evolution of mode amplitudes is required for kick model
- Mirnov coil signal b_θ is used to compute $w(t)$ [1]:

$$w^2 = g(rb_r q/mB_\theta q')$$

where perturbed radial field is approximated by [2]:

$$b_r \approx (1/2)(r_w/r)^{m+1} b_\theta$$

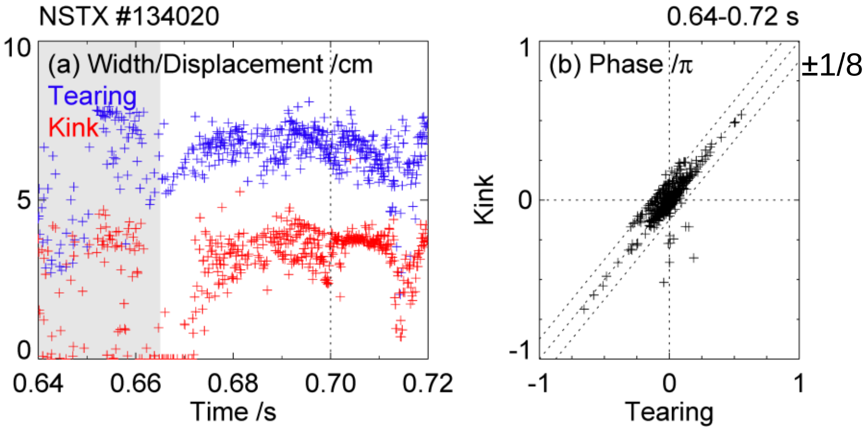
- Constant g is found by scaling with SXR results
- Captures island smaller than SXR spatial resolution



[1] Chang *et al*., NF **34** 1309 (1994)
[2] La Haye *et al*., PoP **7** 3349 (2000)

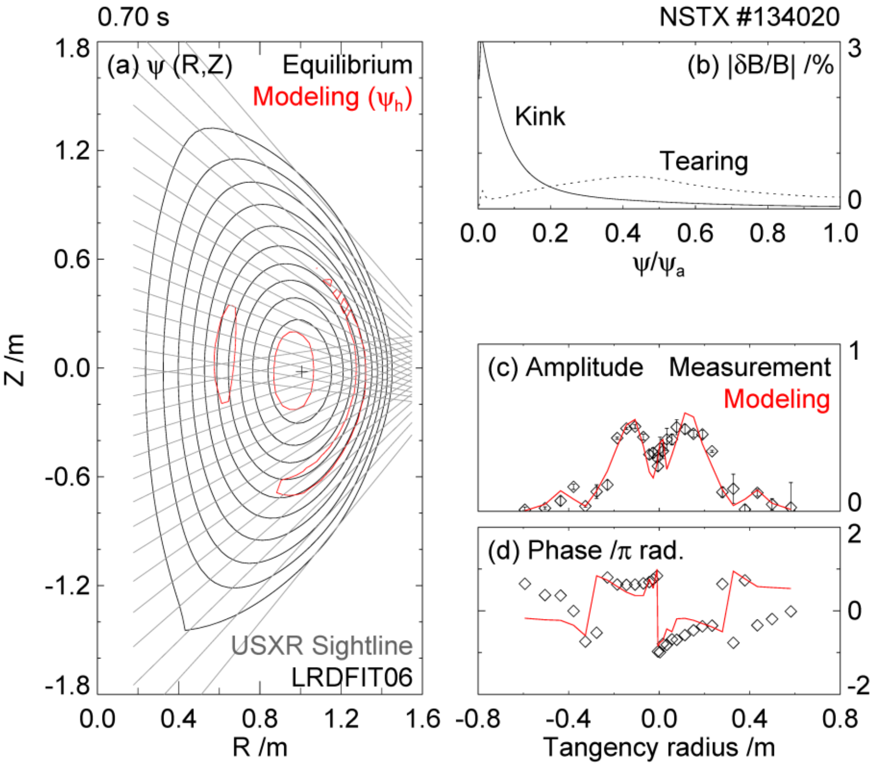
Diagnostic provides relative phase of kink and tearing modes

- More constraints, better fit accuracy
- Relative amplitude and phase are fit
  - Amplitude ratio is rather constant
  - Relative phase is fixed at zero

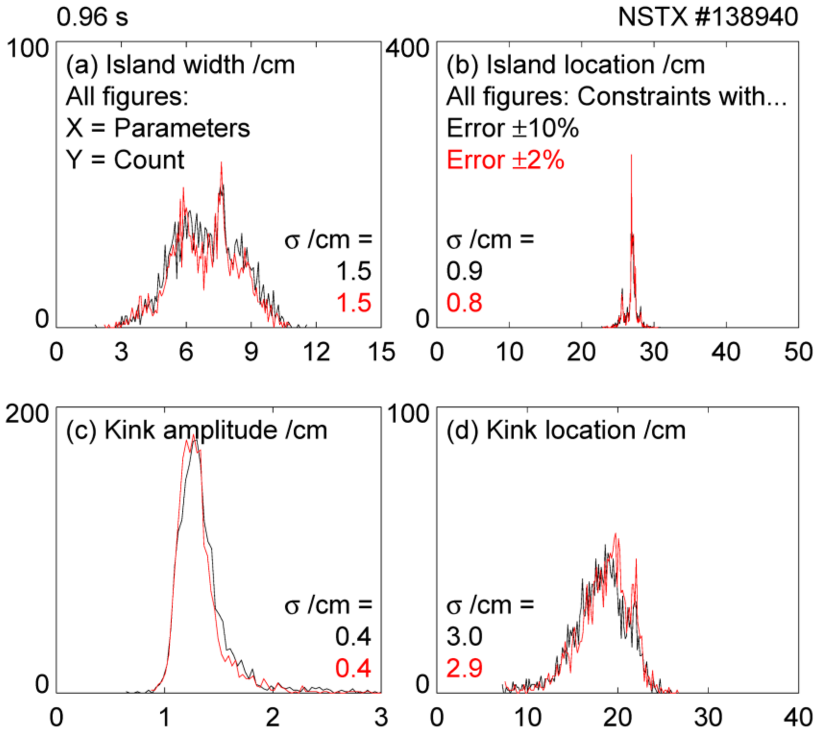


$\sigma_{XY}/\sigma_X\sigma_Y = 0.65$

$\sigma_{XY}/\sigma_X\sigma_Y = 0.83$



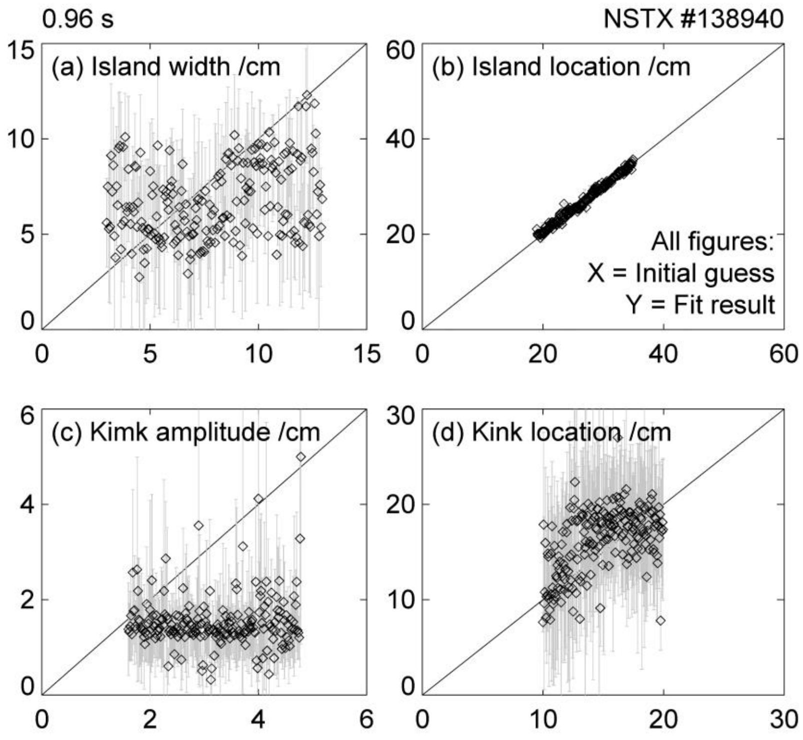
Fit uncertainties are comparable to SXR resolution



- Island width fit uncertainty is
  - At a = 0.26 m, 3.9 cm†
  - At a = 0.25 m, ±1.4 cm from analysis
- White noise makes small difference
  - Real level is at 1.5%‡
  - Does not impact much when rose to typical 10%

† NSTX #138940, 0.96 s
‡ NSTX #138940, chord 9, after filtering.
Background at 0 – 0.08 s, signal at 0.96 s

Fit of island location is sensitive to initial guess



- Initial guess is needed for fit
  - Scanned range of initial guesses
  - Other parameters: Small correlation
  - Island location: Fit result has linear correlation with initial guess
- Good initial guess is needed for r_{mode}

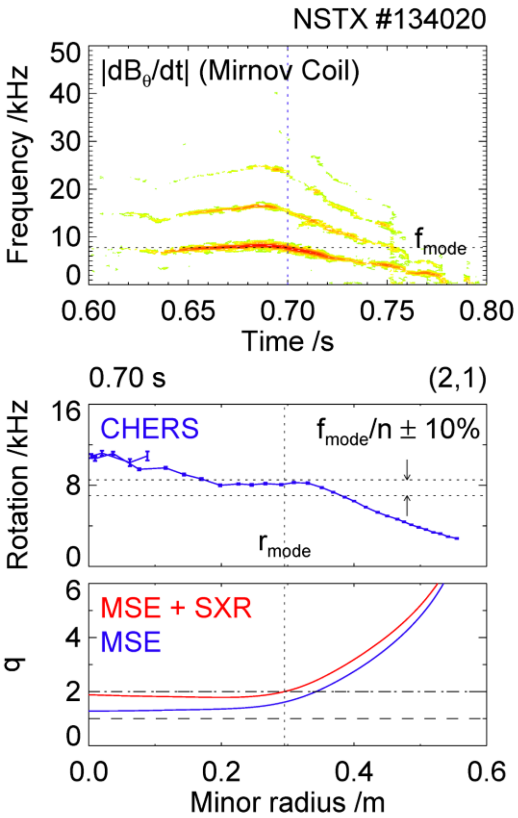
Measurements determine initial guess for island location

- Good initial guess for island location is needed

- Mirnov coil spectrum and CHERS are utilized
  - Mode frequency and n from Mirnov coil spectrum
  - Plasma rotation frequency profile from CHERS [1]
  - Island location is where†

$$f_{mode} = n f_{plasma}$$

- Island location is updated at each iteration of fit
  - Useful output of MHD (SXR) constrained q profile [2]



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† Allowed ±10% tolerance

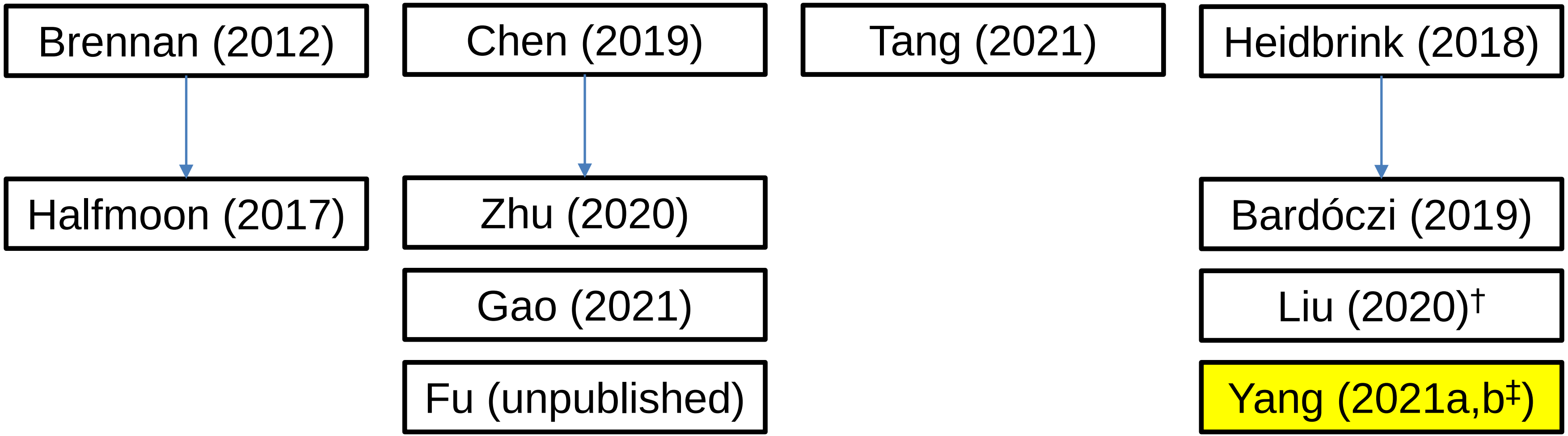
[1] Bell *et al*., Phys. Plasmas **17** 082507 (2010)

[2] Chang *et al*., Nucl. Fusion **34** 1309 (1994)

Backup Slides

This work is supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences under contract number DE-AC02-09CH11466.

Map of previous works on fast ion contribution to NTM stability



Brennan (2012)
Halfmoon (2017)

NIMROD

EP drives FB/BAE
that transitions into NTM

Chen (2019)
Zhu (2020)
Gao (2021)
Fu (unpublished)

M3D-K

EP and NTM interacts
to drive FB(, BAE)

Tang (2021)

GTC

Qualitatively tested
parallel current theory

Heidbrink (2018)
Bardóczi (2019)
Liu (2020)†
Yang (2021a,b‡)

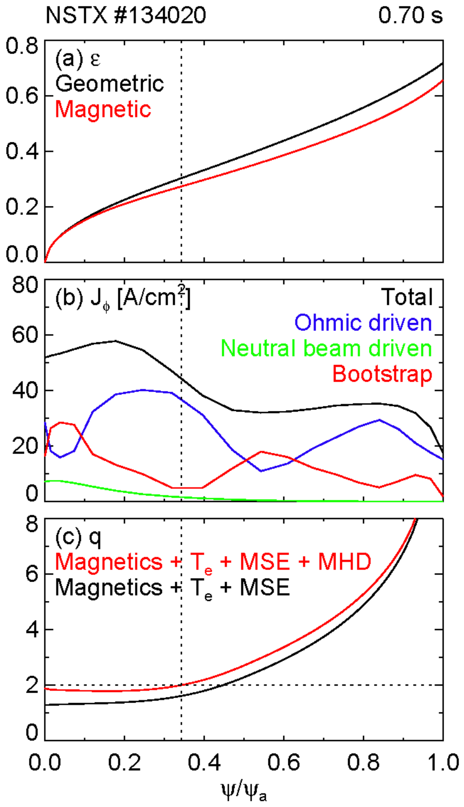
Kick model

Experimental
† Reports chirping
‡ Suggests use of M3D-C1-K

Premises for GRE modeling with fast ion term

- GRE takes input of both thermal and fast ion parameters
  - Integrated modeling provides self-consistent parameters
  - Main input is MSE-constrained *q* profile [1] with correction with NTM location
  - Bootstrap current is calculated by NCLASS module [2]
  - Magnetic inverse aspect ratio is used† [3]

$$\varepsilon_B \equiv \frac{B_{in} - B_{out}}{B_{in} + B_{out}}$$



† Contributes to small island effect: polarization and curvature terms

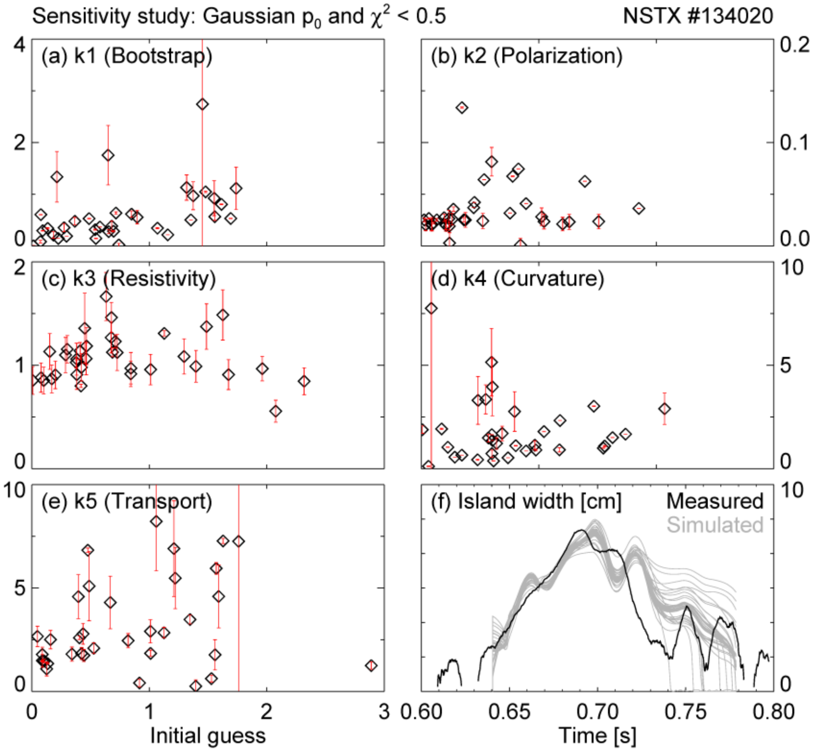
[1] Levinton and Yuh, Rev. Sci. Instrum. **79** 10F522 (2008)

[2] Houlberg *et al*., Phys. Plasmas **4** 3230 (1997)

[3] La Haye *et al*., Phys. Plasmas **19** 062506 (2012)

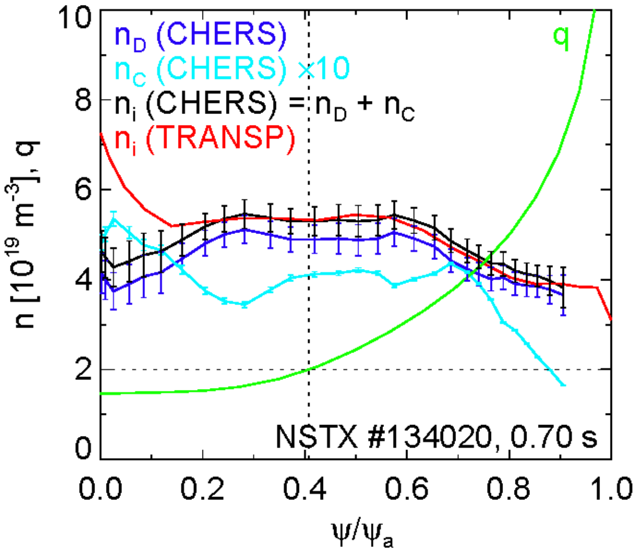
Sensitivity study shows our fit is mathematical optimum

- Fit might depend on initial guess
  - Assigned random initial guess
  - Gaussian distribution (positive side)
  - Finding if solution is mathematical optimum
- Flat response means fit is not sensitive
  - Selected only small χ^2 results
  - Flat for most parameters
  - Initial guess of k > 2 rarely survives
  - Large uncertainty for k_5
    - Electron transport wash-up effect



Thermal ion gradient scale length is cross checked

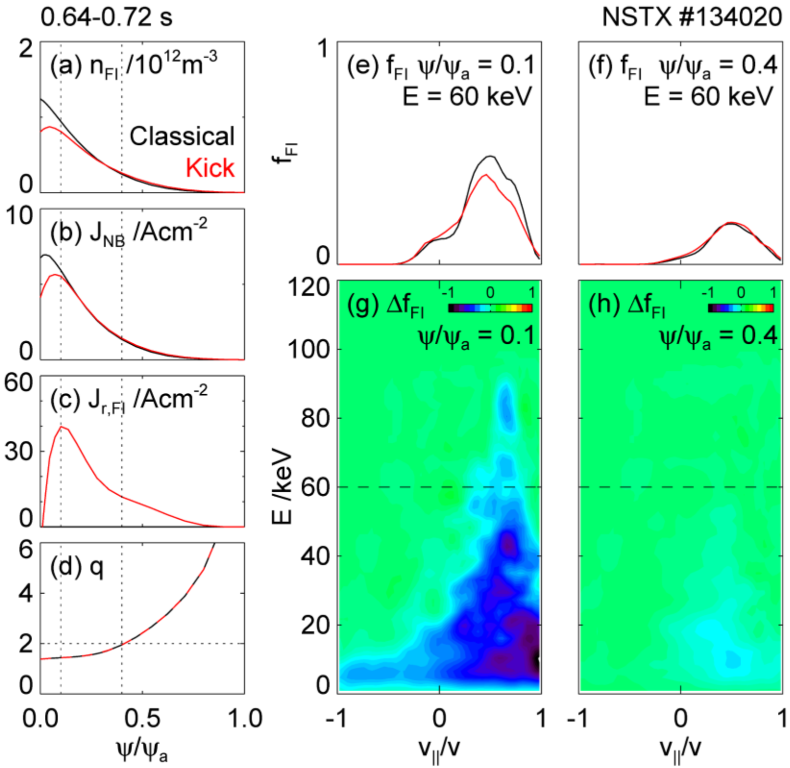
- EP contribution depends on $n_i/\nabla n_i$
  - Known to have large uncertainty
  - Due to "nonlinear" processing involving Z_{eff}
- TRANSP is used to cross check the data
  - Good agreement near $q = 2$
  - Meaning CHERS is consistent with other diagnostics such as TS
  - Divergence near the core can be explained
    - End of discharge C accumulation
    - Bump in electron density



Fast ion transport causes neutron rate to drop

- Fast ion distribution is output
  - Core fast ions are depleted
  - Core current drive is reduced
  - Radial flow of fast ions is clear
  - Fast ion distribution at *q* = 2 surface is unchanged

- Consistent with previous slide
  - Neutrons originate mostly at core

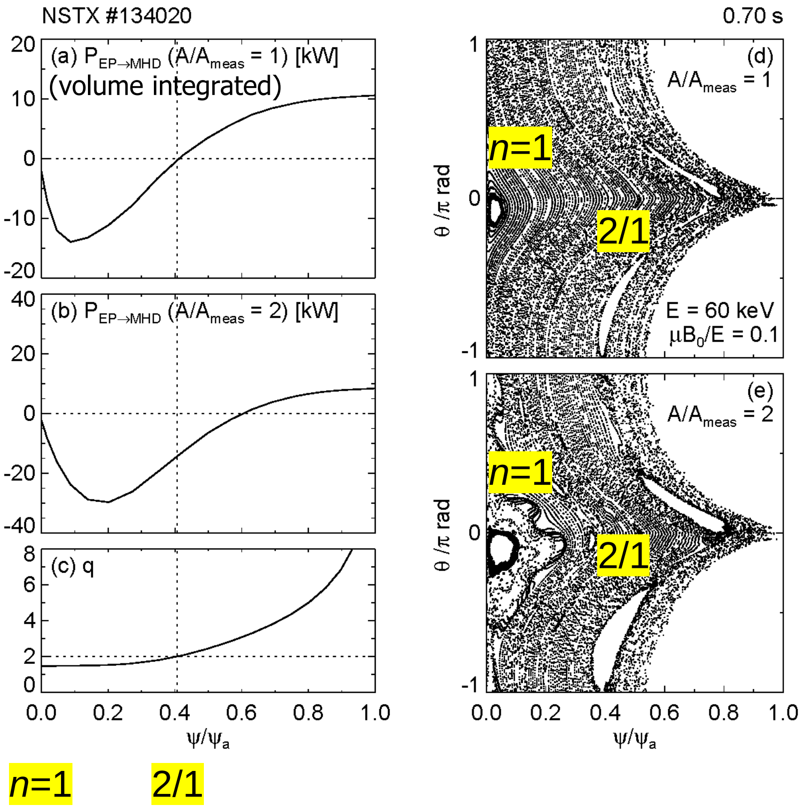


EP correction explains island saturation at orbit stochasticization

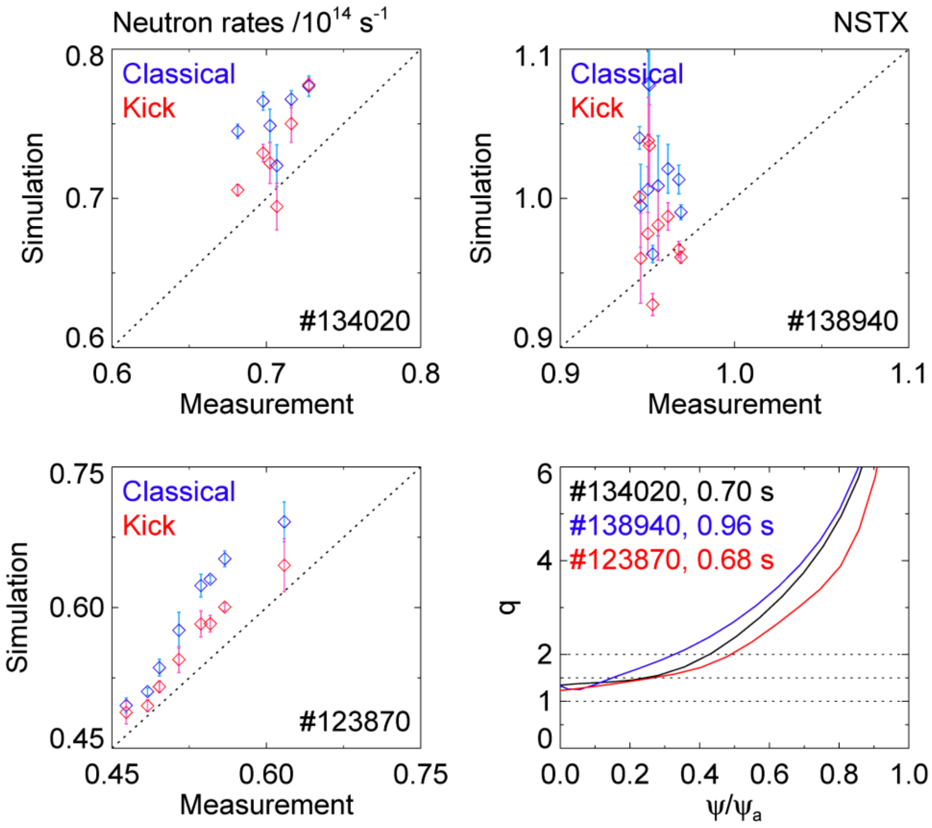
- Kick model: Energy exchange of EP/NTM
  - Convention: P < 0 = mode power loss
  - P < 0 at *q* = 2 when A > A_{meas}
- EP theory [1] offers interpretation
  - Orbits become stochastic
  - EP transport is enhanced
  - NTM drive is weakened
    - Manifests itself as mode losing energy

$$\Delta'_{pol} = -\varepsilon^{3/2}\frac{\rho_{\theta i}^2\beta_\theta}{w^3}\left(\frac{L_q}{L_p}\right)^2 + \frac{\beta_\theta}{w}\left(\frac{L_q}{L_p}\right)^2\left(\frac{L_{n_i}}{L_{n_h}}\frac{n_h}{n_i}\right)$$

[1] Cai, Nucl. Fusion **56** 126016 (2016)



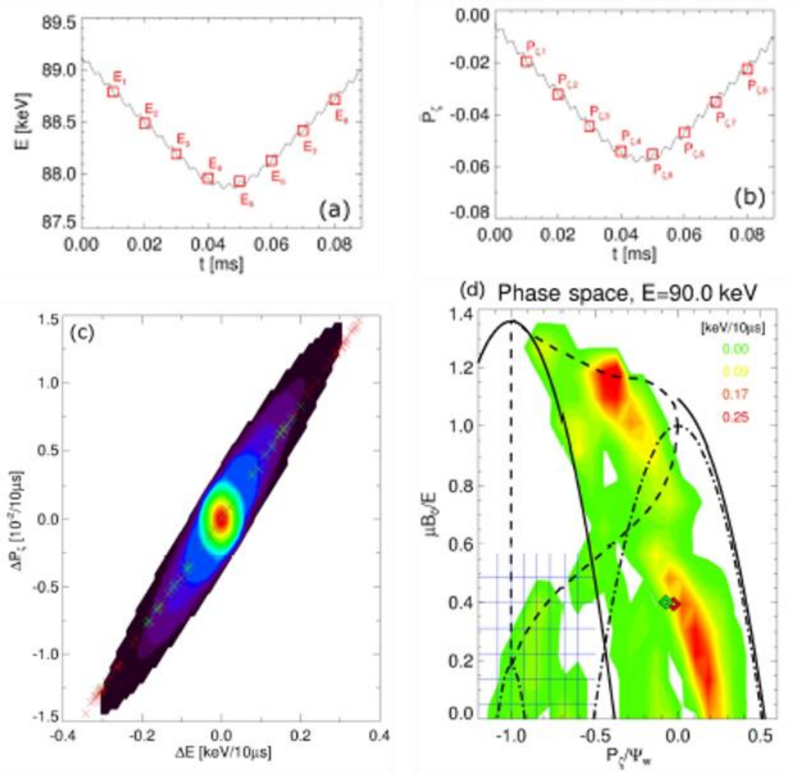
Kick model is valid for three NSTX discharges with different $q(\psi)$



Kick Model [1] Suggests to Include Instabilities in EP Calculations

- EP dynamics can be affected by δB
  - Perturbation sources: Ripple, MHD…
  - EP follows magnetic field lines, hence affected by such perturbations

- ORBIT [2] code is used for calculation
  - Follow test particles
  - Accumulate ΔE and ΔP_ξ to evaluate wave particle resonance
  - Produce kick probability matrix



Reproduced from Fig. A2 of [3]

[1] Podestà *et al.*, PPCF **56** 055063 (2014)
[2] White and Chance, PoF **27** 2455 (1984)
[3] Podestà *et al.*, PPCF **59** 095008 (2017)