Exploring the impact of 3D fields, beta limits and non-resonant modes with nonlinear MHD

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NSTX-U meeting
(February 28th, 2022)

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PRINCETON PLASMA PHYSICS LABORATORY

- Challenges associated with modelling 3D fields
- Assessing reduced models as tools for efficient calculation of 3D equilibria
- Exploring β-limits in stellarators with M3D-C[1]
- Examining the role of non-resonant modes in the formation of 3D fields
- Outlook: Summary and on-going work

- **Challenges associated with modelling 3D fields**
- *Assessing reduced models as tools for efficient calculation of 3D equilibria*
- *Exploring β-limits in stellarators with M3D-C^1*
- *Examining the role of non-resonant modes in the formation of 3D fields*
- *Outlook: Summary and on-going work*

3D magnetic fields are fundamentally different

- Breaking axisymmetry ($\partial_\phi \nrightarrow 0$) fundamentally changes the properties of magnetic fields:

Non-integrability of the magnetic field line Hamiltonian → continuously nested flux surfaces no longer guaranteed.

- 3D magnetic fields admit additional structures (e.g., islands and chaos) that can be leveraged to improve fusion plasma performance. But they can be challenging to model and understand.

For (much) more theoretical detail:

An Introduction to Stellarators
From magnetic fields to symmetries and optimization

Lise-Marie Imbert-Gérard, Elizabeth J. Paul, Adelle M. Wright

A self-contained introduction covering the basic theoretical building blocks for modelling 3D magnetic fields, with applications to fusion device optimisation and design.

- Early version available on arxiv [1].
- Coming in book form soon(-ish)!

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[1] Imbert-Gerard, Paul & Wright (2019+)

Modelling 3D magnetic fields can be challenging

Approach 1: Time-evolution models (i.e., initial-value methods)

+1 More complicated models circumvent challenges associated with solving $\mathbf{J} \times \mathbf{B} = \nabla p$ in 3D (see below).

-1 Comparatively slow and expensive to evaluate. E.g., 3D tokamak simulation with M3D-C^1 ~10^5 CPU hrs/run.

- Tools to rapidly calculate 3D fields are needed for optimisation and reconstruction.

Approach 2: Equilibrium models ($\mathbf{J} \times \mathbf{B} = \nabla p$)

+1 Comparatively fast and cheap to evaluate. E.g., Non-axisymmetric VMEC equilibrium ~1-10 CPU hrs.

-1 Pfirsch-Schlüter currents are unbounded when $\nabla \mathbf{p} \neq \mathbf{0}$ on rational surfaces → unphysical.

- How should realistic (e.g., smooth) pressure profiles be represented in these models?

-1 Dynamical accessibility of solutions is not guaranteed.

If an equilibrium code predicts a finite-β equilibrium with chaotic fields and magnetic islands:

- Can the plasma actually reach this state with heating?
- What happens if the system crosses a stability boundary?

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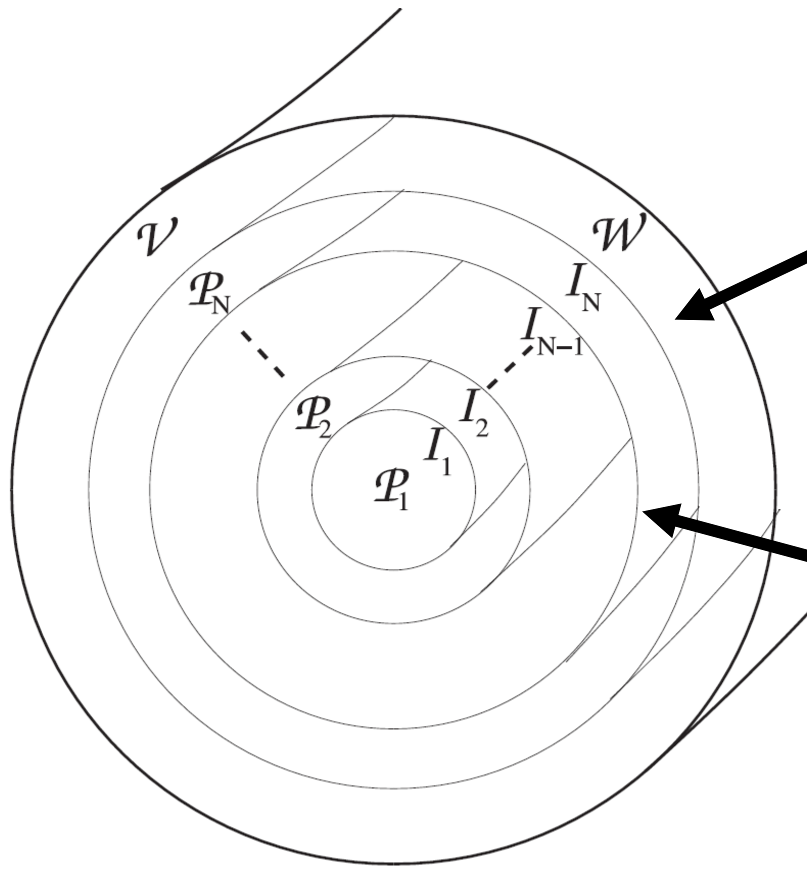
- The plasma discretised into N volumes and the MRxMHD energy functional, F, is minimised subject to a finite set of constraints:

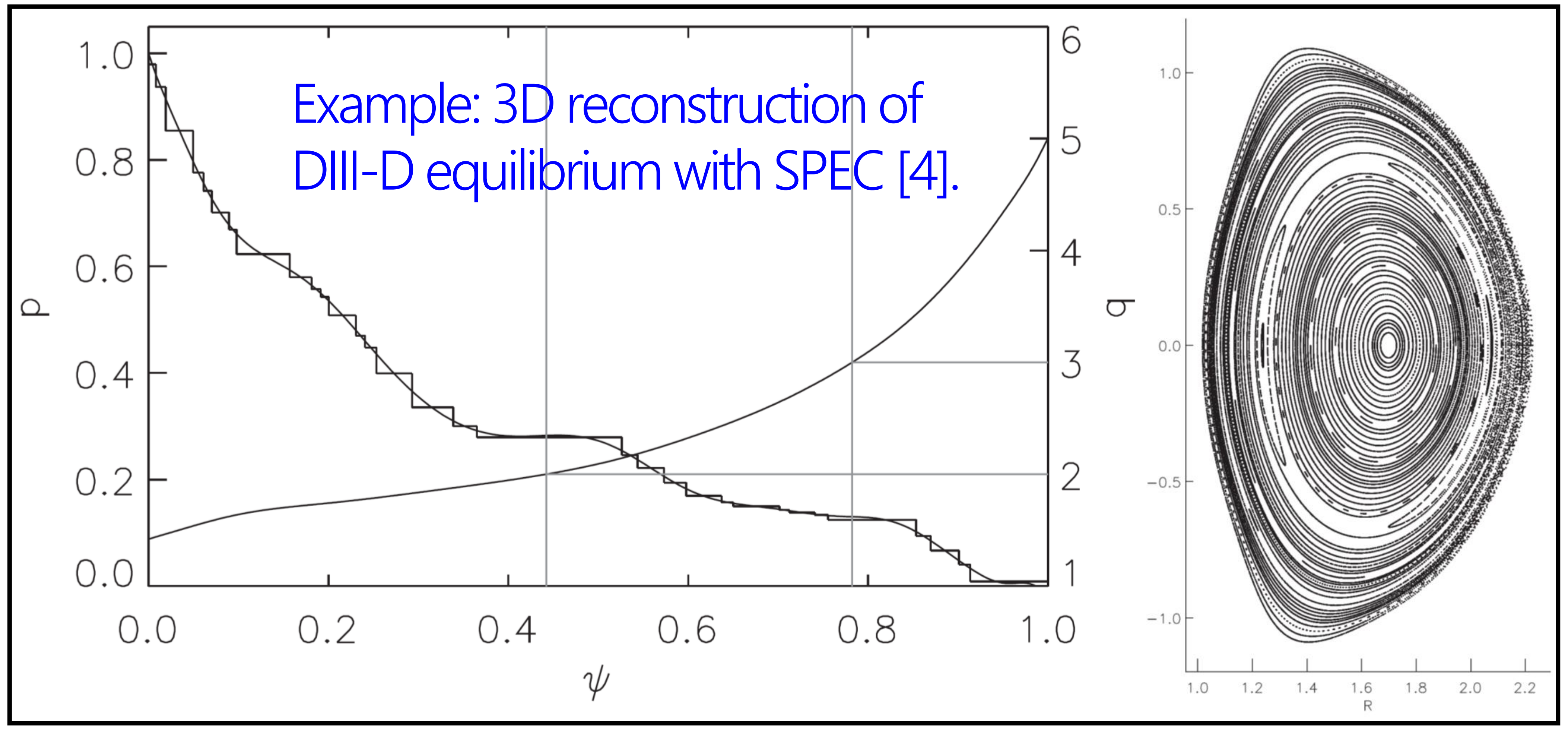
$$F = \sum_{i}^{N} \left[\underbrace{\int_{V_i} \left(\frac{p_i}{\gamma - 1} + \frac{B^2}{2} \right) dv}_{\text{Potential energy (in each } i\text{).}} - \underbrace{\frac{\mu_i}{2} \left(K_i - K_{0,i} \right)}_{\text{Helicity constraint (in each } i\text{).}} \right]$$



Taylor-relaxed volumes:
$\nabla \times \mathbf{B} = \mu_i \mathbf{B}$,
$\nabla p_i = 0$.

Current sheet interfaces:
$[\![p + B^2/2]\!] = 0$,
$\mathbf{B} \cdot \hat{\mathbf{n}} = 0$.

- Pressure profiles are approximated by piece-wise constant (i.e., stepped) function → $\nabla p = 0$ on rational surfaces.
- MRxMHD is the theoretical basis of the Stepped Pressure Equilibrium Code (SPEC) [4].



? Under what conditions:

- Are MRxMHD/SPEC solutions dynamically accessible?
- Is SPEC a viable tool for modelling plasma response to RMPs?

[2] Hole et al., Journal of Plasma Physics 72.6 (2006); [3] Dewar et al., Journal of Plasma Physics 81.6 (2015); [4] Hudson et al., Physics of Plasmas 19.11 (2012).

Assessing SPEC as a tool for rapidly calculating 3D response fields

Motivation:

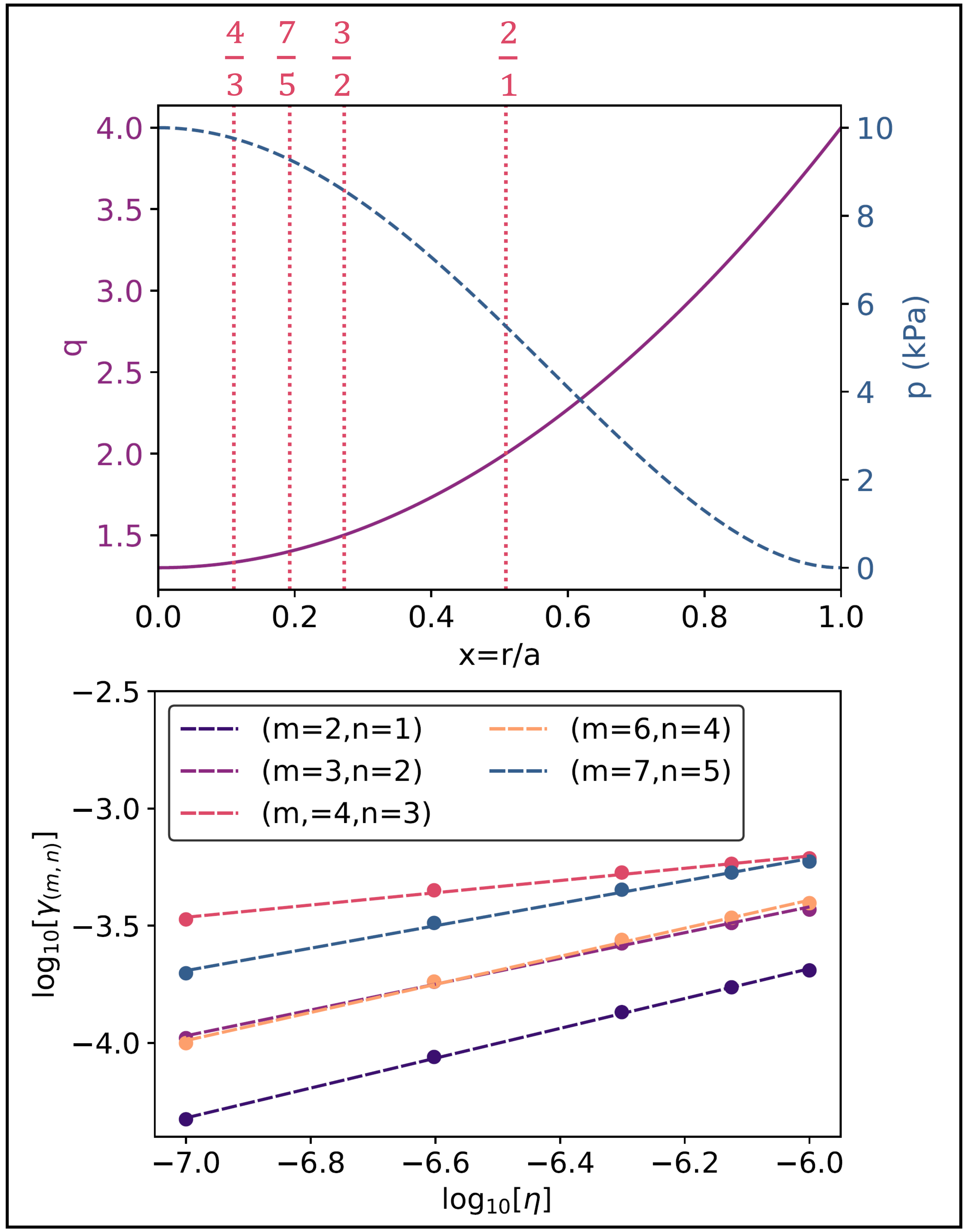
Can SPEC be used to model the nonlinear + non-ideal plasma response to RMPs in realistic geometries? [5]

Equilibrium characteristics:

- $a/R_0 = 0.1$, $\beta = 0.82\%$ and $1.3 \leq q \leq 4$.
- Unstable to (2,1), (3,2), (6,4), (7,5) tearing modes and (4,3) interchange.

M3D-C[1] (reference solutions):

- Solves extended-MHD model.
- Visco-resistive single fluid MHD with simple anisotropic heat transport (finite $\kappa_{\parallel}/\kappa_{\perp}$).
- Vacuum $(m = 2, n = 1)$ RMP field applied. (Possible due to absence of mode coupling in cylindrical geometry).



[5] Wright, Kim, Ferraro & Hudson (under review, JPP)

Survey of M3D-C¹ simulations shows distinct types of plasma response

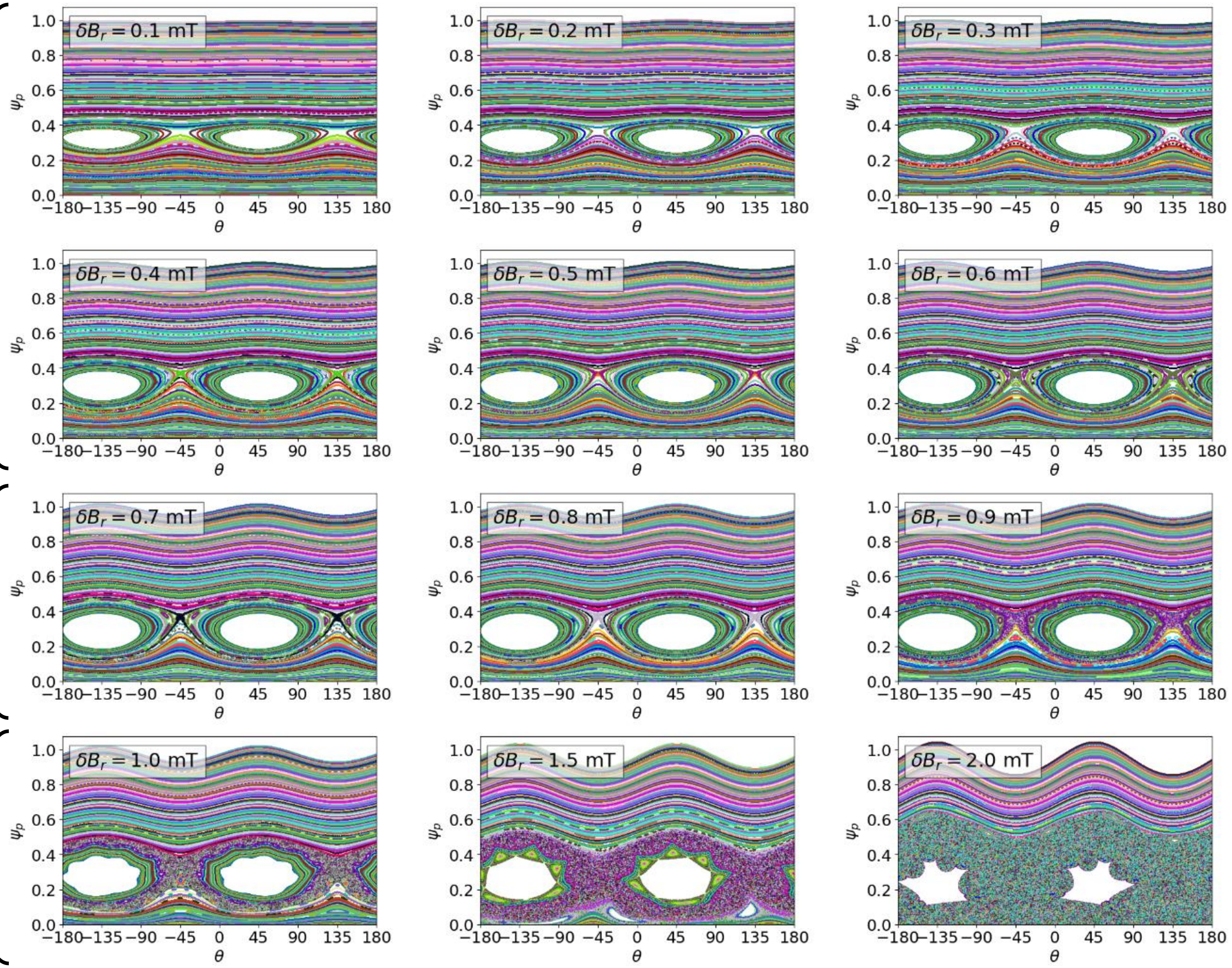
Reference solutions are calculated with M3D-C¹: A vacuum $(m = 2, n = 1)$ RMP field applied (δB_r).

$0.1\,\mathrm{mT} \leq \delta B_r \leq 2\,\mathrm{mT}$ @ $t = 5000\,\tau_A$
$(\delta B_r / B_t \in [10^{-4}, 2 \cdot 10^{-3}])$

$0.1\,\mathrm{mT} \leq \delta B_r \leq 0.5\,\mathrm{mT}$:
Saturated (2,1) island at $q = 2$ resonant surface.

$0.6\,\mathrm{mT} \leq \delta B_r \leq 0.9\,\mathrm{mT}$:
Formation of secondary island chains and break up of separatrix.

$\delta B_r \geq 1\,\mathrm{mT}$:
Remnants of the (2,1) island are embedded in a sea of chaos.



Parameters:
$S = 8 \cdot 10^6$
$\eta = 2.74\,\Omega \cdot \mathrm{m}$
$P_m = 1$
$\kappa_\parallel / \kappa_\perp = 10^6$
$\chi_\perp = 2.2\,\mathrm{m}^2/\mathrm{s}$

New workflow developed for preparing SPEC input profiles

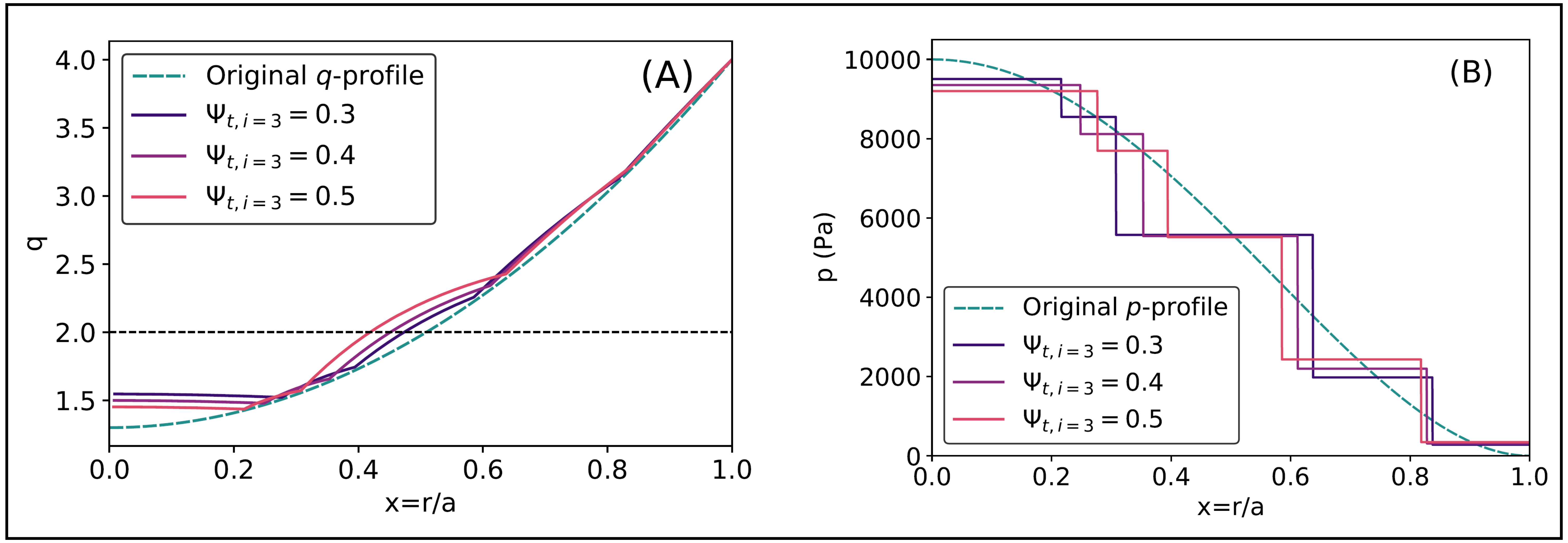
SPEC:

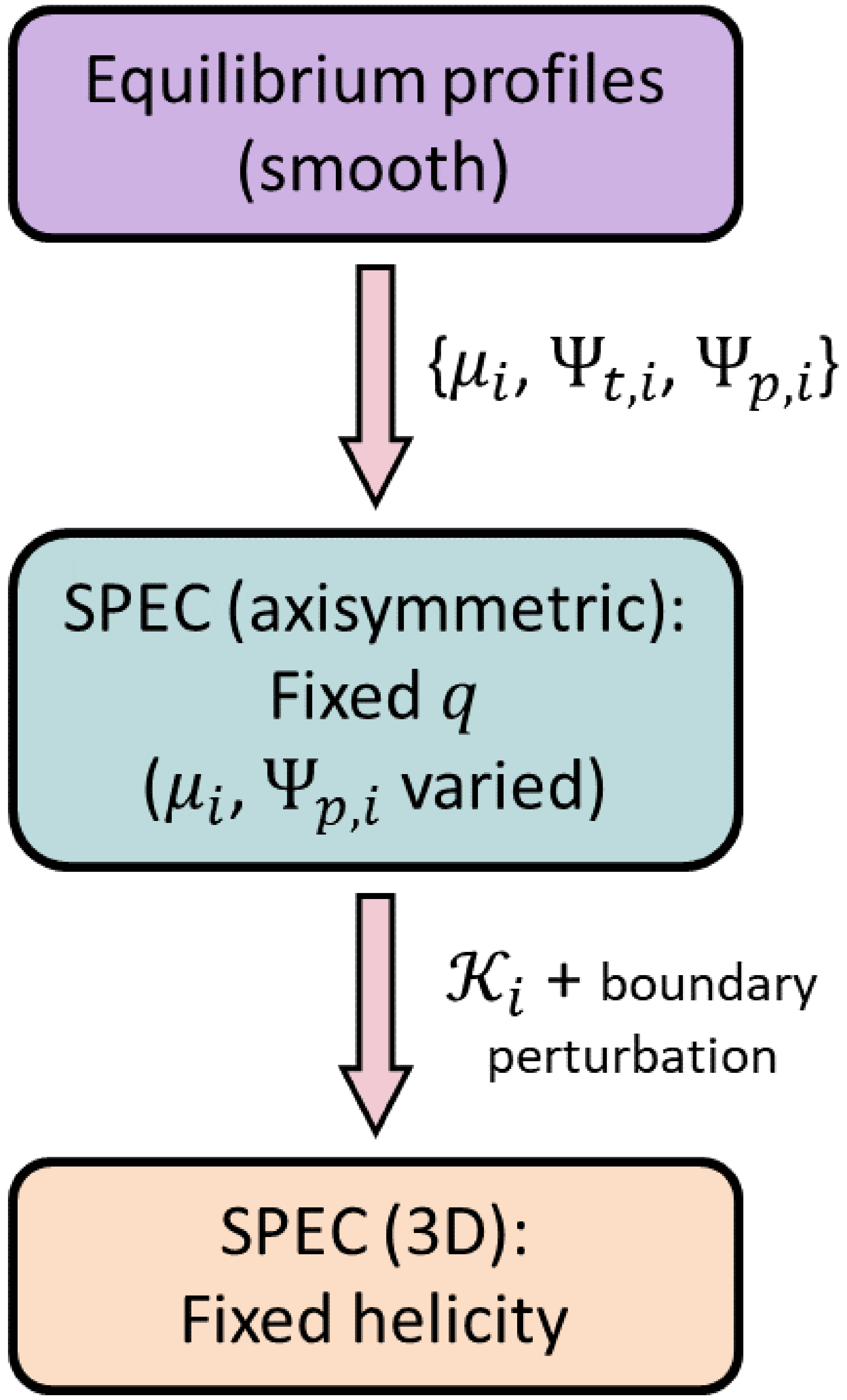
- Solves MRxMHD (equilibrium) model.
- Requires $\{p_i, \mu_i, \Psi_{t,i}, \Psi_{p,i}, \mathcal{K}_i\}$ to be specified to compute solutions.

? How should realistic (e.g., smooth) pressure profiles be represented in SPEC?

In this work:

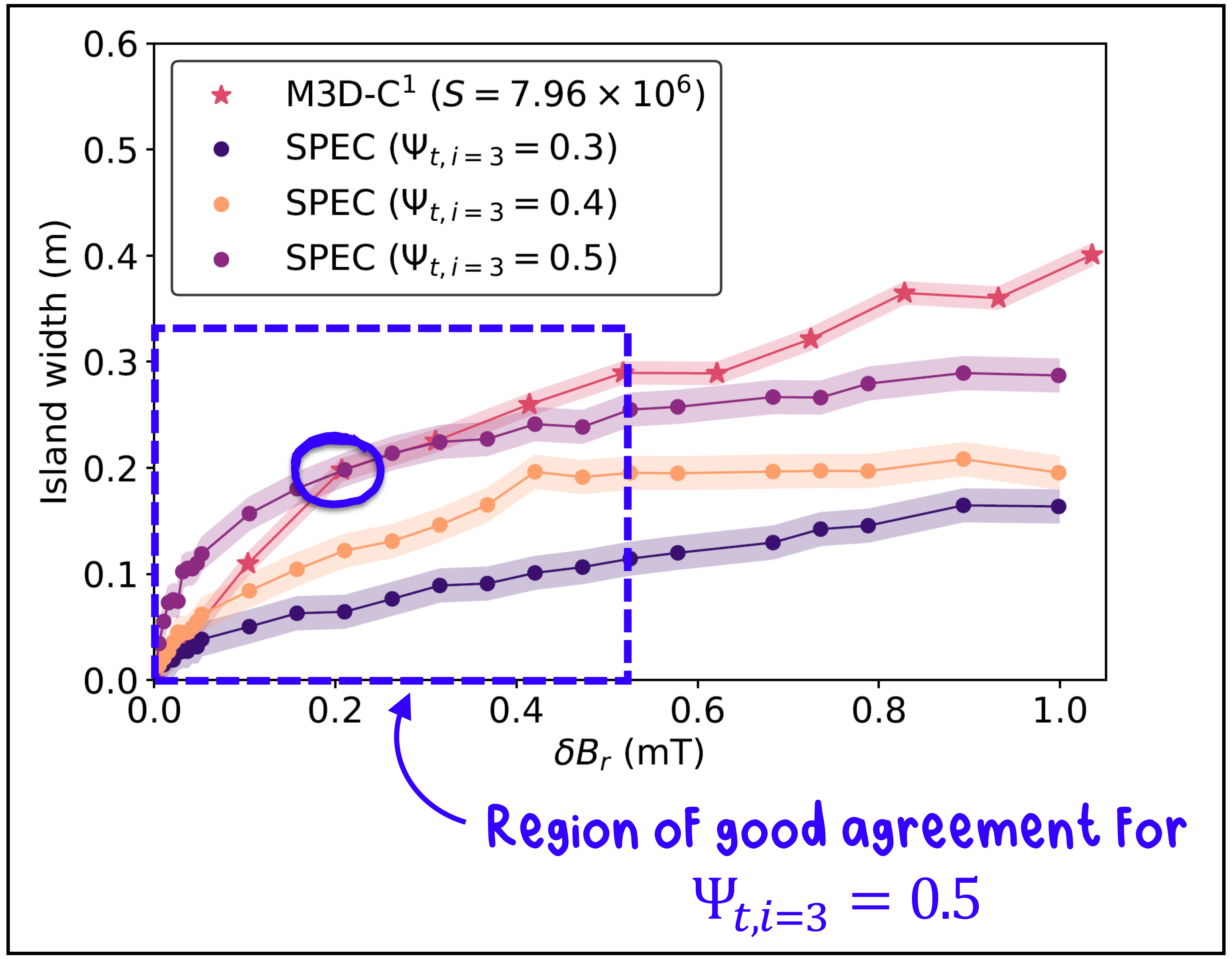
- The plasma is partitioned into $N_{vol} = 5$ volumes.
- We develop a workflow for discretization of smooth input profiles (here, analytic but could be taken from reconstructed profiles).

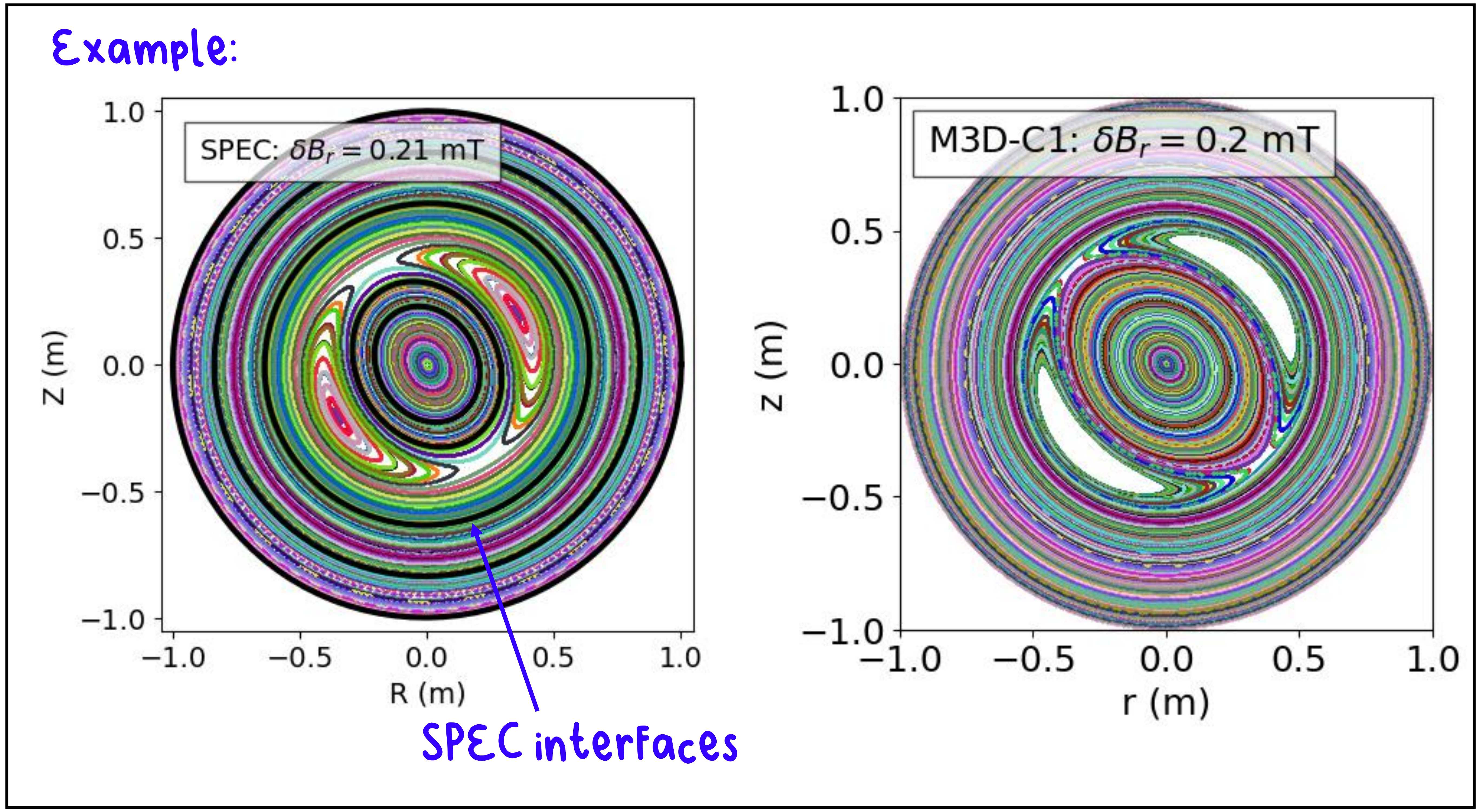




In the weakly nonlinear regime, SPEC can recover the M3D-C^1 RMP response

- The toroidal flux in the volume containing the $q = 2$ resonant surface, $\Psi_{t,i=3}$, has a significant effect on the island properties in SPEC.
- For suitably chosen value of $\Psi_{t,i=3}$, SPEC shows good agreement with M3D-C^1 in the weakly nonlinear regime, i.e., when RMP response → saturated $(2,1)$ island and separatrix remains intact.

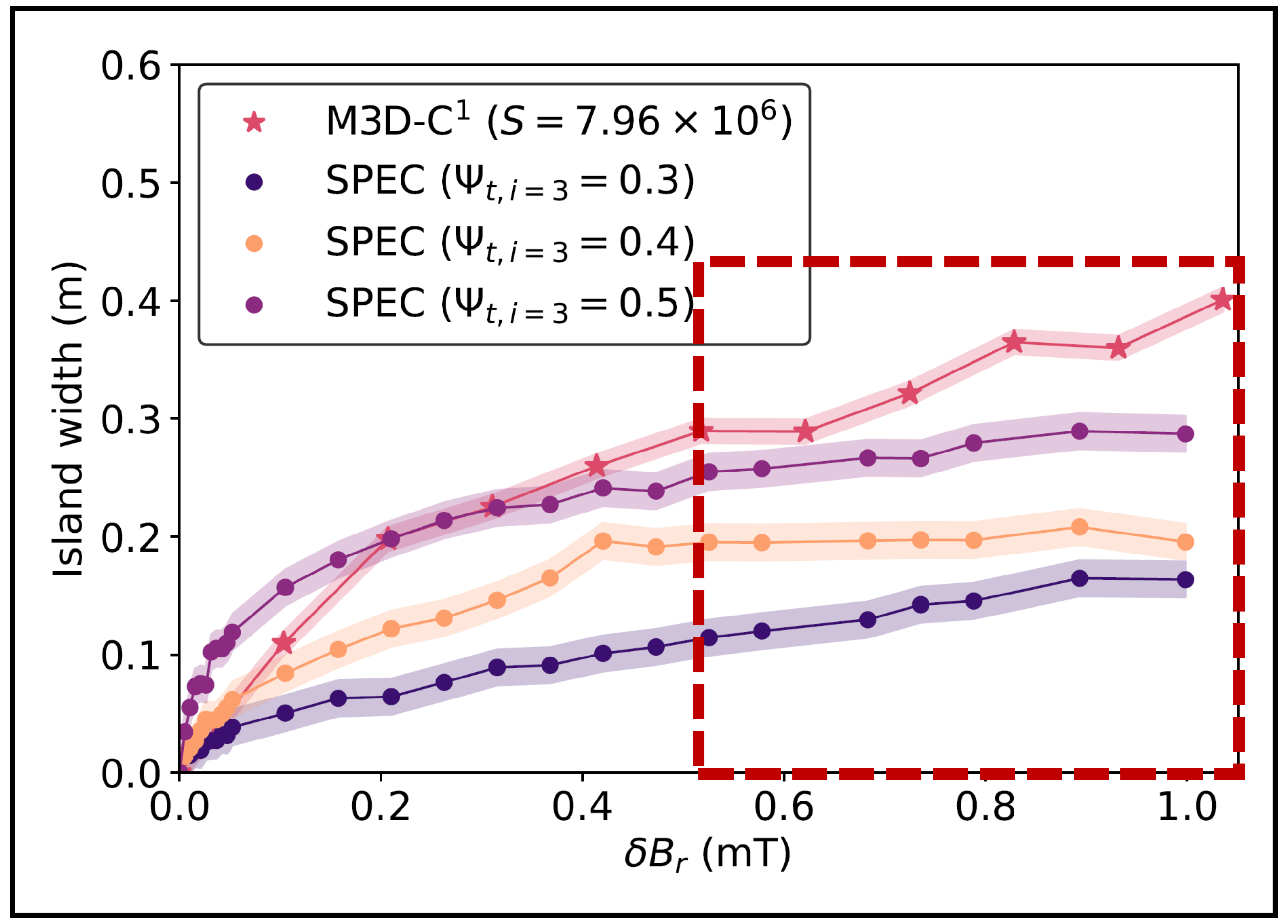


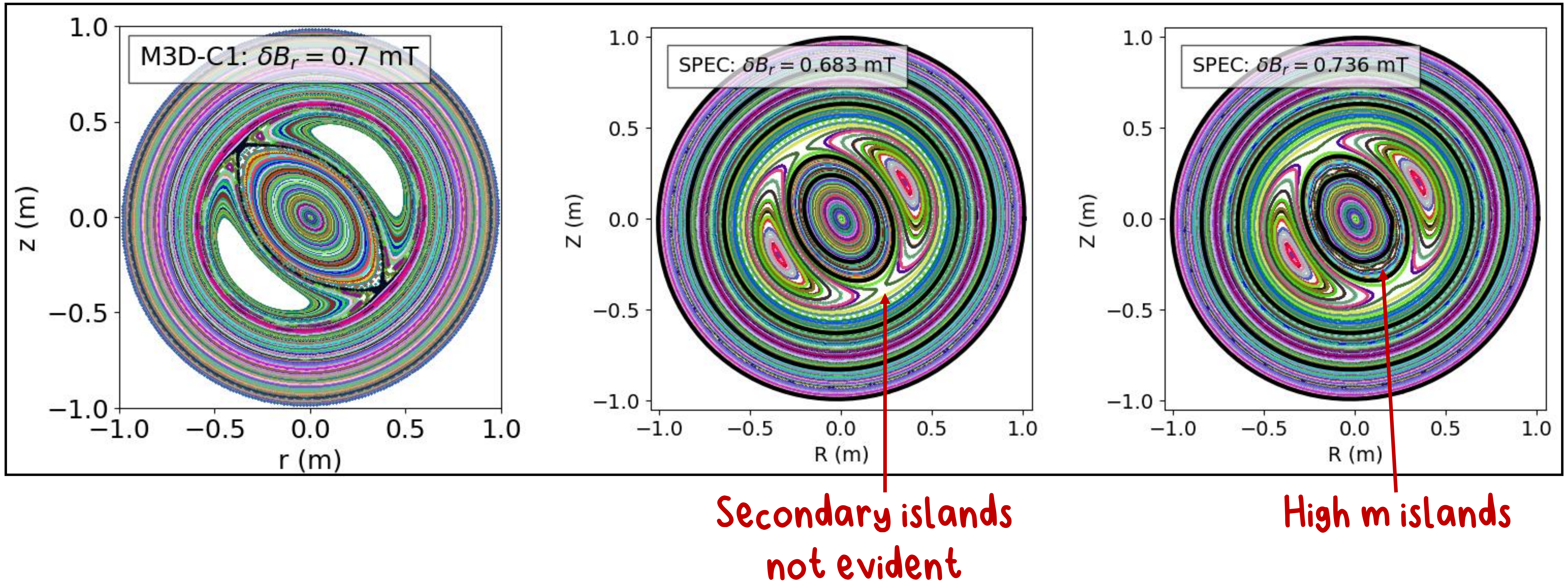


SPEC and M3D-C1 responses differ in when 3D field strength is increased

For larger RMPs amplitudes ($\delta B_r > 0.5\,\mathrm{mT}$), the plasma response calculated by SPEC and M3D-C^1 differ:

- The transition occurs where M3D-C^1 shows break up of the separatrix.

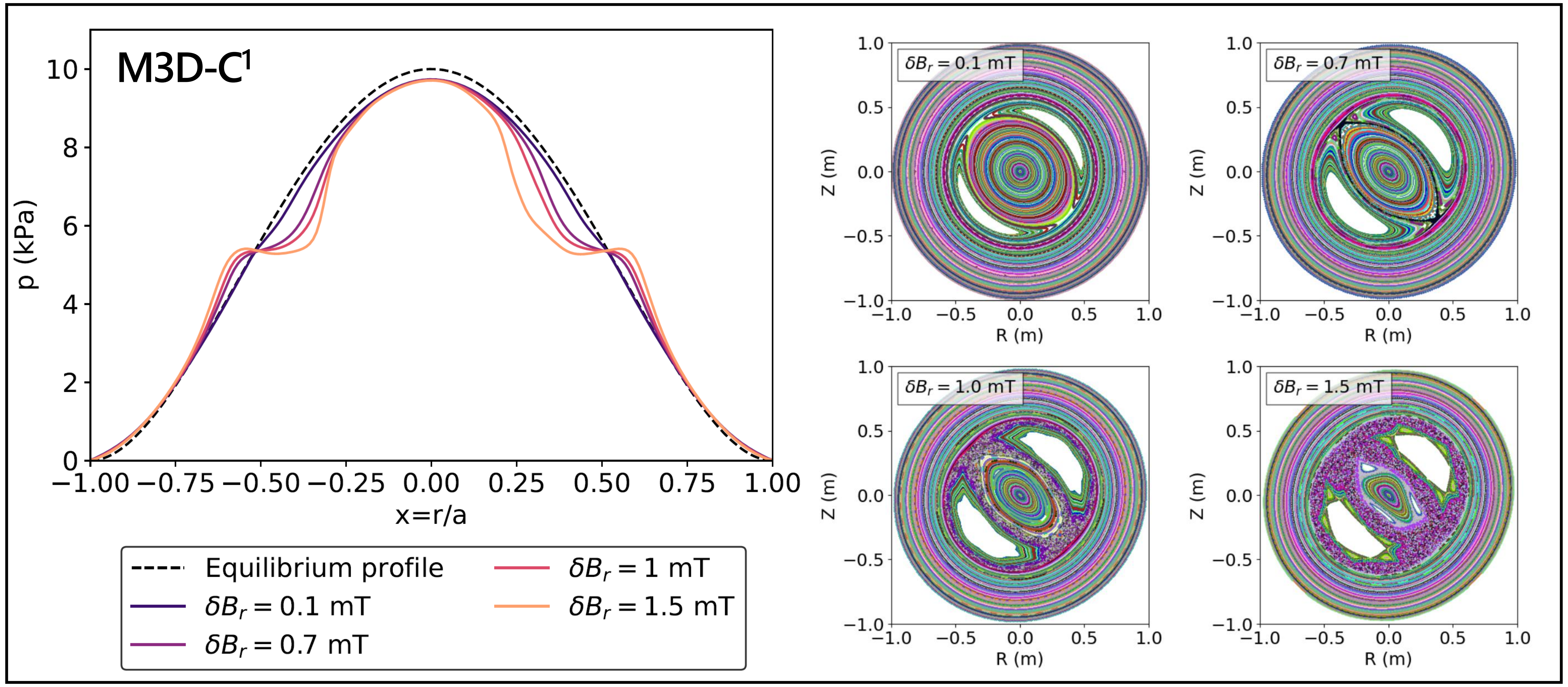




? Does this critical threshold hold in general?

? If so, what are the implications for the validity/applicability of the MRxMHD model?

? What is the role of finite $\kappa_{\|}/\kappa_{\perp}$ is determining the applicability of MRxMHD/SPEC to model 3D field responses?



- M3D-C[1] profiles show significant pressure gradients, even when there is significant volume of chaotic fields.
- By contrast, the MRxMHD model requires $\nabla p = 0$ in regions of chaotic fields.

Next steps (w/ P. Kim):

- Exploring SPEC as a tool for modelling internal relaxation events (e.g., helical core states) on NSTX-U.

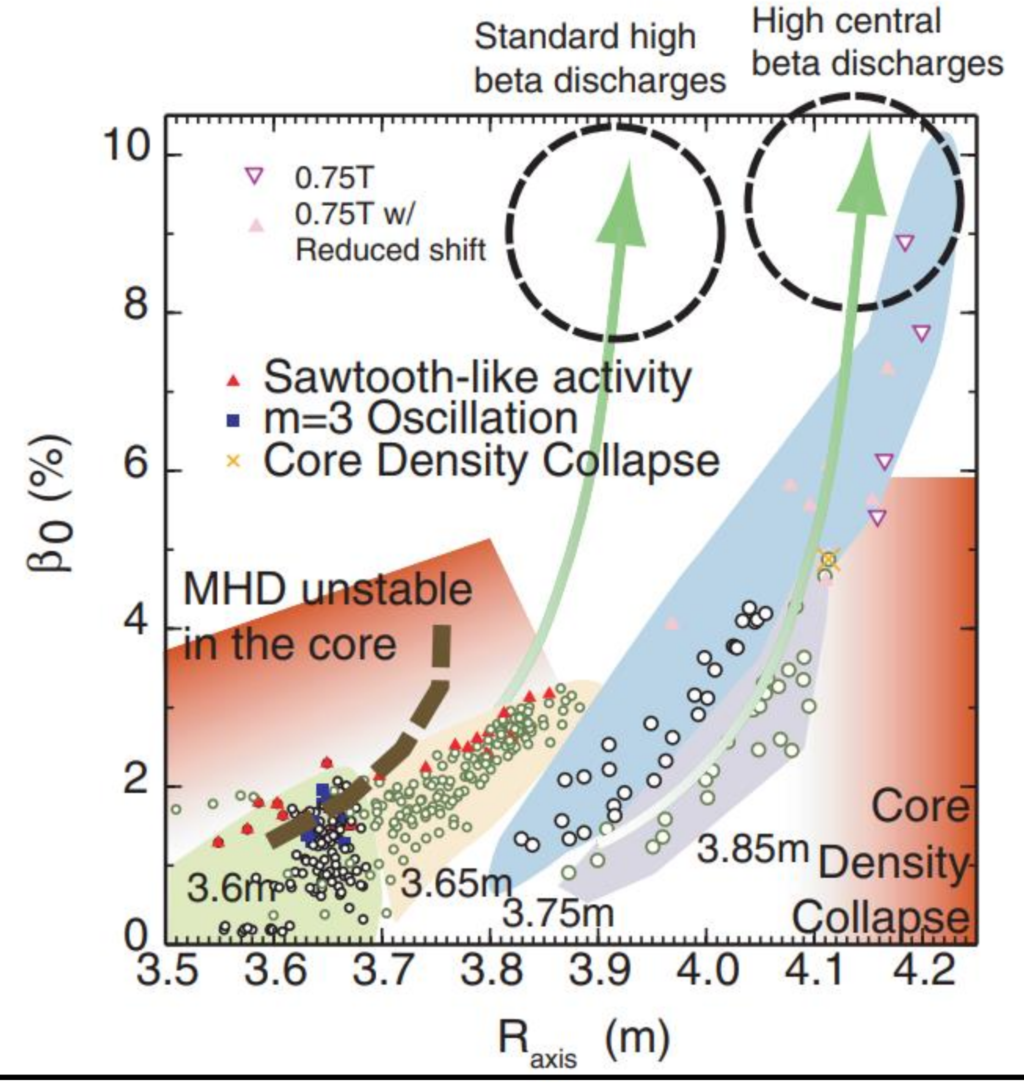
? Do MRxMHD interfaces coincide with internal transport barriers, as observed in experiment?

Clarifying the role of 3D effects is critical for determining when instabilities are benign or become disruptive.

Like tokamaks, stellarators can be susceptible to (sometimes disruptive) pressure- and current-driven instabilities:

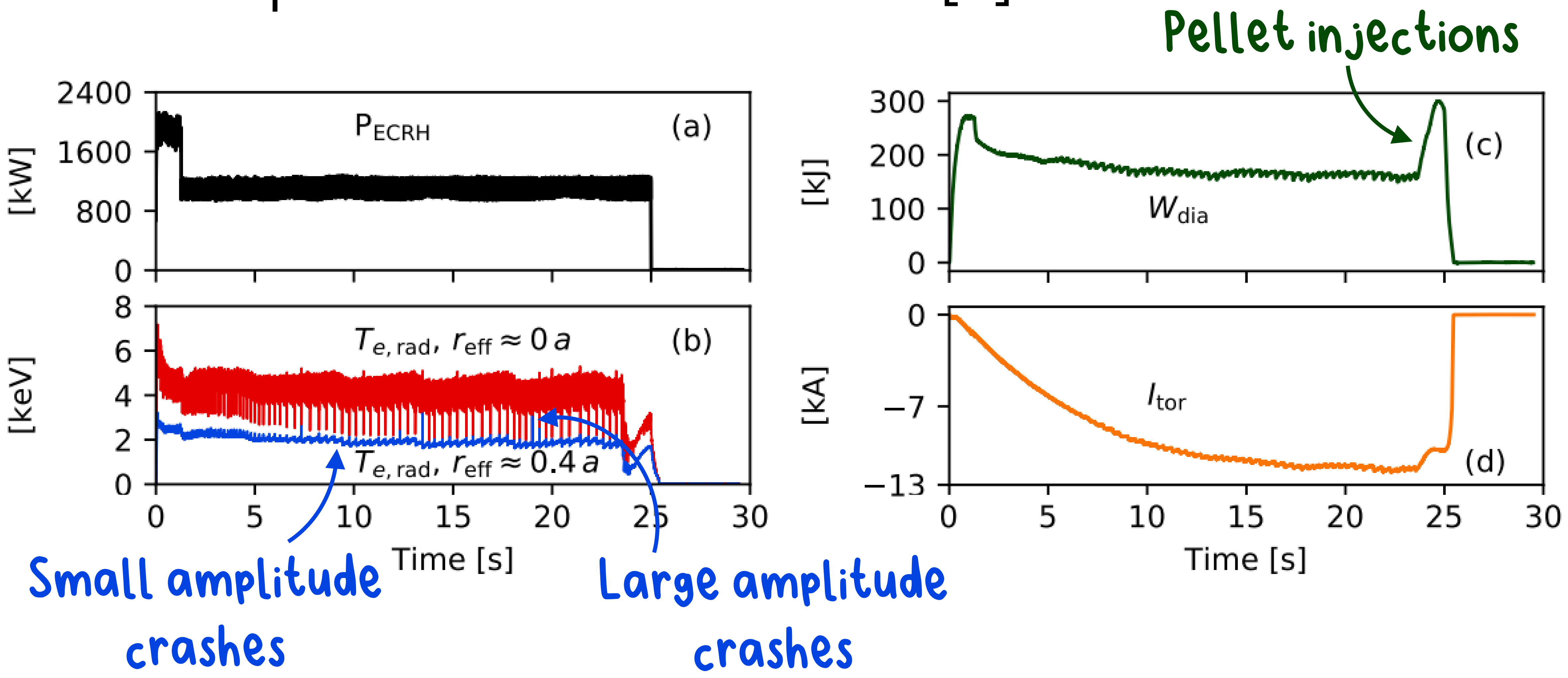
Pressure-driven MHD activity (LHD)

Benign MHD activity and disruptive core density collapse (CDC) events have been observed [6].



Current-driven MHD activity (W7-X)

Sawtooth-like crashes observed during current-drive experiments with ECRH [7].



[6] Ohdachi et al. FEC NIFS--890 (2008); [7] Zanini et al. Nuclear Fusion 60.10 (2020).

M3D-C1 successfully extended to stellarator geometry

M3D-C1 recently extended to accommodate strongly shaped, non-axisymmetric computational domains [8]:

- **Fixed boundary:** req. boundary shape specification
- **Free-boundary:** req. boundary shape specification + MGRID (vacuum) or FIELDLINES ($\beta > 0$)

With M3D-C1, we now have first-of-a-kind capability to explore nonlinear MHD in stellarators.

With this, we can now examine (important) questions that could not be addressed previously:

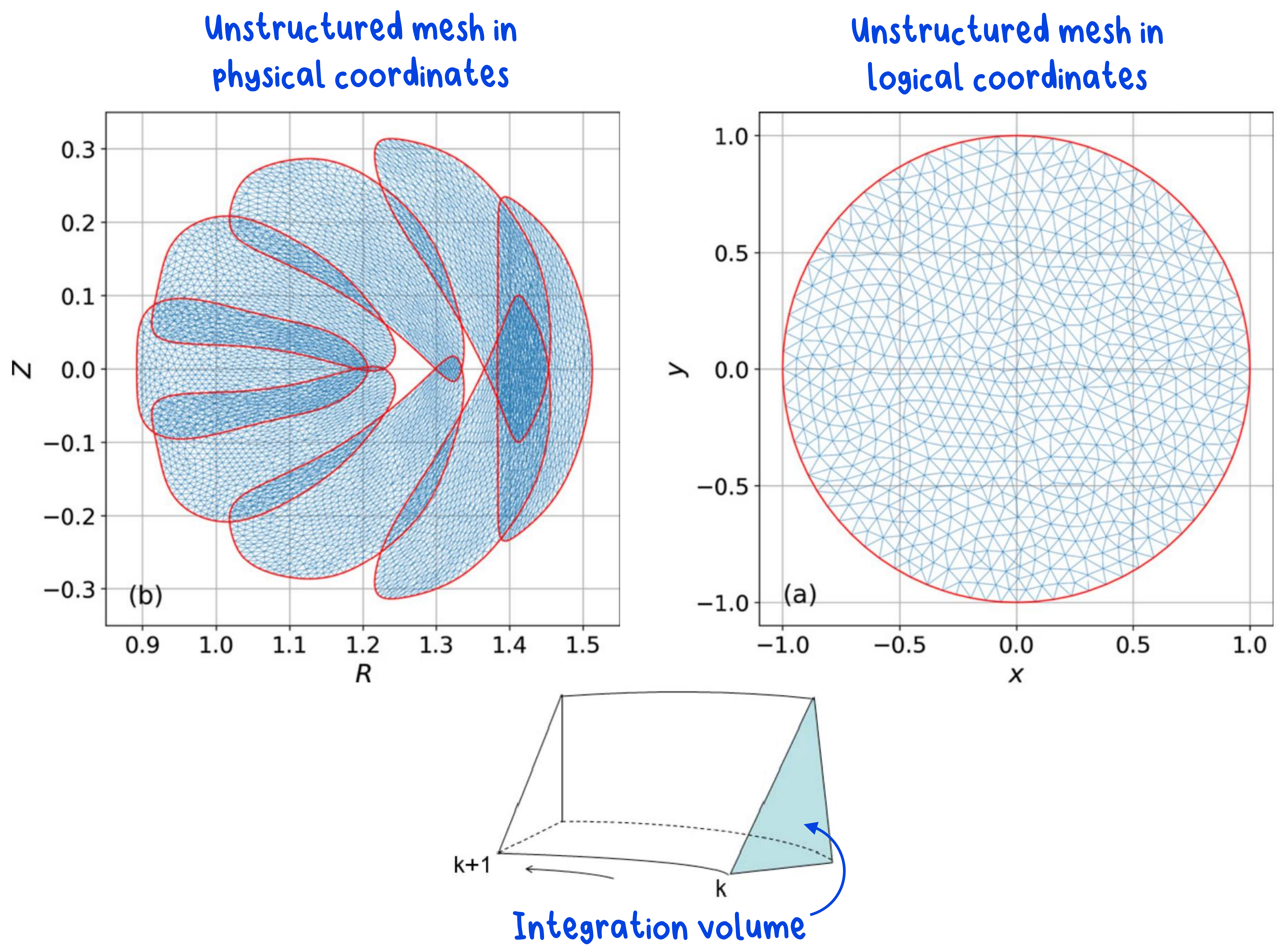
- Evolution of pressure profiles for self-consistent equilibria, including for non-integrable fields.
- Examine dynamical accessibility of 3D equilibria (integrable and non-integrable).
- Determine nonlinear stability.

In principle, the existing suite of M3D-C1 capabilities can be used directly to model additional phenomena including pellet injection, resistive wall physics and transport in 3D fields.

[8] Zhou et al. Nuclear Fusion 61.8 (2021).

M3D-C^1-S (method)

- Model equations expanded in cylindrical (R, φ, Z) coordinates.
- Numerical integration performed in 'logical' (x, y, z) coordinates by mapping finite elements from $(R, \varphi, Z) \rightarrow (x, y, z)$.



M3D-C^1 model (single-fluid and two-fluid)

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{u}) = 0$$

$$nm_i\left(\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u}\right) = \vec{J}\times\vec{B} - \nabla p - \nabla\cdot\Pi + \vec{F}$$

$$\frac{\partial p}{\partial t} + \vec{u}\cdot\nabla p + \Gamma p \nabla\cdot\vec{u} = (\Gamma-1)\left[Q - \nabla\cdot\vec{q} + \eta J^2 - \vec{u}\cdot\vec{F} - \Pi:\nabla u\right] + \frac{1}{ne}\vec{J}\cdot\left(\frac{\nabla n}{n}p_e - \nabla p_e\right) + (\Gamma-1)\Pi_e:\nabla\left(\frac{1}{ne}\vec{J}\right)$$

$$\frac{\partial p_e}{\partial t} + \vec{u}\cdot\nabla p_e + \Gamma p_e \nabla\cdot\vec{u} = (\Gamma-1)\left[Q_e - \vec{q}_e + \eta J^2 - \vec{u}\cdot\vec{F}_e - \Pi_e:\nabla u\right] + \frac{1}{ne}\vec{J}\cdot\left(\frac{\nabla n}{n}p_e - \nabla p_e\right) + (\Gamma-1)\left[\Pi_e:\nabla\left(\frac{1}{ne}\vec{J}\right) + \frac{1}{ne}\vec{J}\cdot\vec{F}_e\right]$$

$$\vec{E} = -\vec{u}\times\vec{B} + \eta\vec{J} + \frac{1}{ne}\left(\vec{J}\times\vec{B} - \nabla p_e - \nabla\cdot\Pi_e + \vec{F}_e\right)$$

$$\vec{J} = \frac{1}{\mu_0}\nabla\times\vec{B}$$

$$\frac{\partial\vec{B}}{\partial t} = -\nabla\times\vec{E}$$

$$\vec{q}_{(i,e)} = -\kappa^{\perp}_{(i,e)}\nabla T_{(i,e)} - \kappa^{\parallel}_{(i,e)}\frac{\vec{B}\vec{B}\cdot\nabla T_{(i,e)}}{B^2}$$

The Large Helical Device (LHD) (1998-) is an NFP=10 heliotron (continuous helical winding) that has been used to explore a wide range of 3D physics, including at high-β [9].

Nominal Machine Specification of the LHD*

Major radius	3.9 m	
Minor radius of helical coil	0.975 m	
Minor radius of plasma	0.5 to 0.65 m	
Magnetic field	3 T at $R = 3.9$ m	(2.96 T at $R = 3.6$ m)
Magnetic energy	0.9 GJ	(0.77 GJ)
Coil temperature	4.4 K	(3.5 K)
Heating power		
ECRH	10 MW	(2.5 MW)
ICRH	3 MW	(3.0 MW)
NBI	15 MW	(23 MW)
Steady state (ICRH + ECRH)	3 MW	(1.7 MW)

TABLE III

Plasma Parameters Achieved

	High $n\tau_E T$	High T_i	High β	Long Pulse	High n_{e0}	High p_0
$T_e(0)$ (keV)	0.47	3.8	0.43	1.0	0.22	0.8
$T_i(0)$ (keV)	0.47	5.6	—	1.1	0.22	0.8
$n_e(0)$ (10^{19} m^{-3})	50.0	1.6	2.3	0.57	125	61.0
$\bar{n}_e$ (10^{19} m^{-3})	25.8	1.3	2.3	0.4	64.0	23.4
τ_E (s)	0.22	0.046	0.008	0.082	0.039	0.076
$\langle\beta\rangle$ (%)	0.74	0.80	5.1	0.027	1.43	1.18
W_p (kJ)	740	879	91	21	838	1181
P_{abs} (MW)	3.3	19.1	11.9	0.33	21.4	15.6
R_{ax} (m)	3.8	3.6	3.6	3.67–3.7	3.95	3.8
B (T)	2.763	2.9	0.425	2.75	2.506	2.763
Remarks	$n\tau_E T = 5.2 \times 10^{19}$ m^{-3} s keV			54 min, 48 s $P_{in} = 0.49$ MW	IDB	IDB

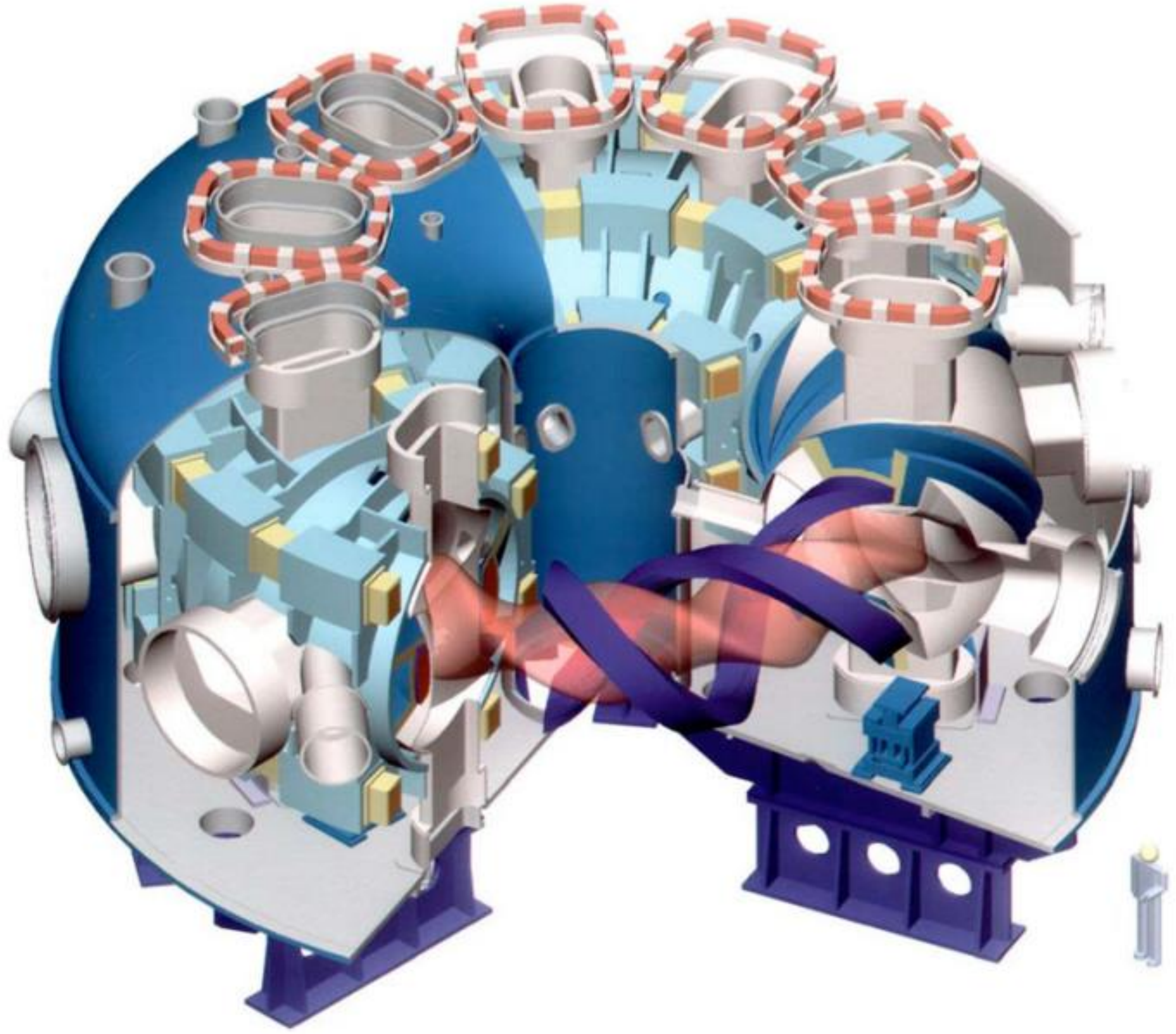
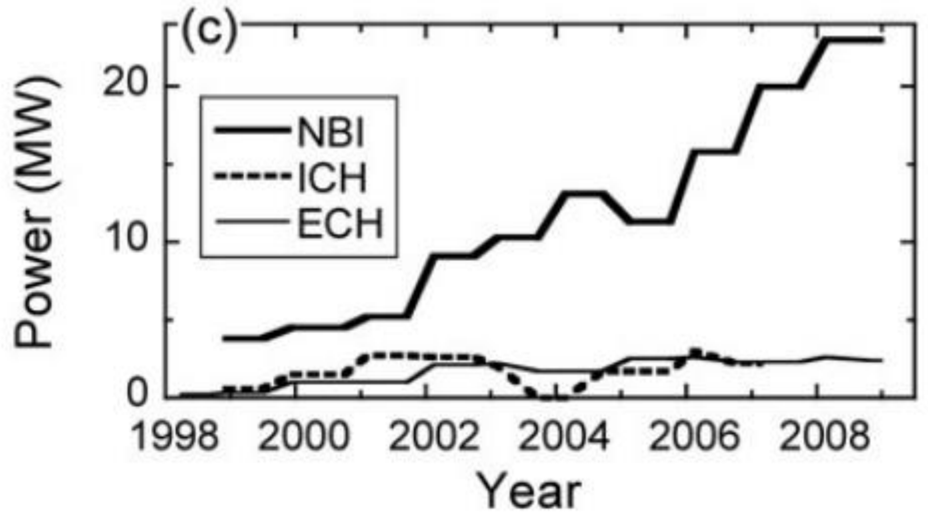


Fig. 2. A bird's-eye view of the LHD.



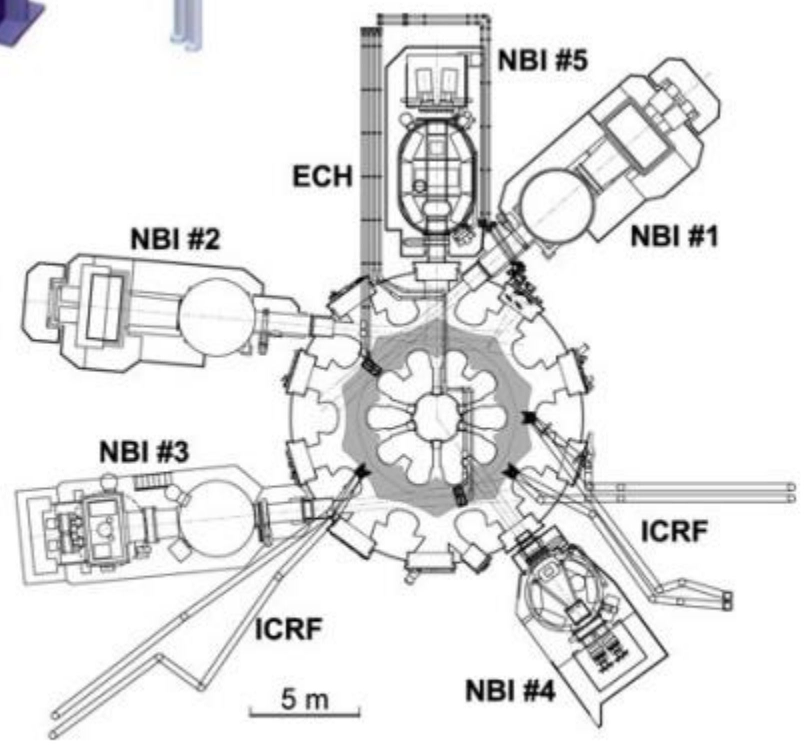


Fig. 7. Arrangement of heating devices. Four beam lines are in operation and the fifth beam will be available in 2010. ECRH and ICRH are injected from four and six ports, respectively.

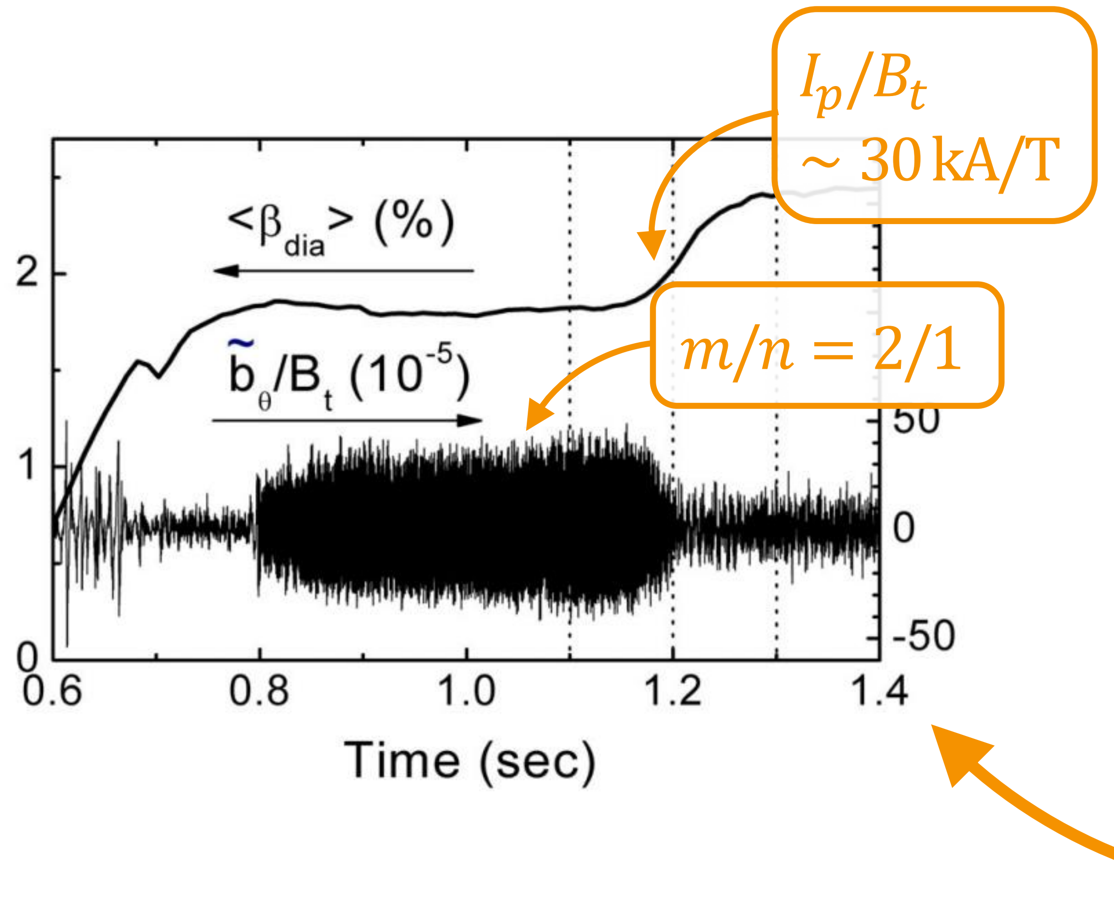
[9] Komori et al., Fusion Science and Technology 58.1 (2010).

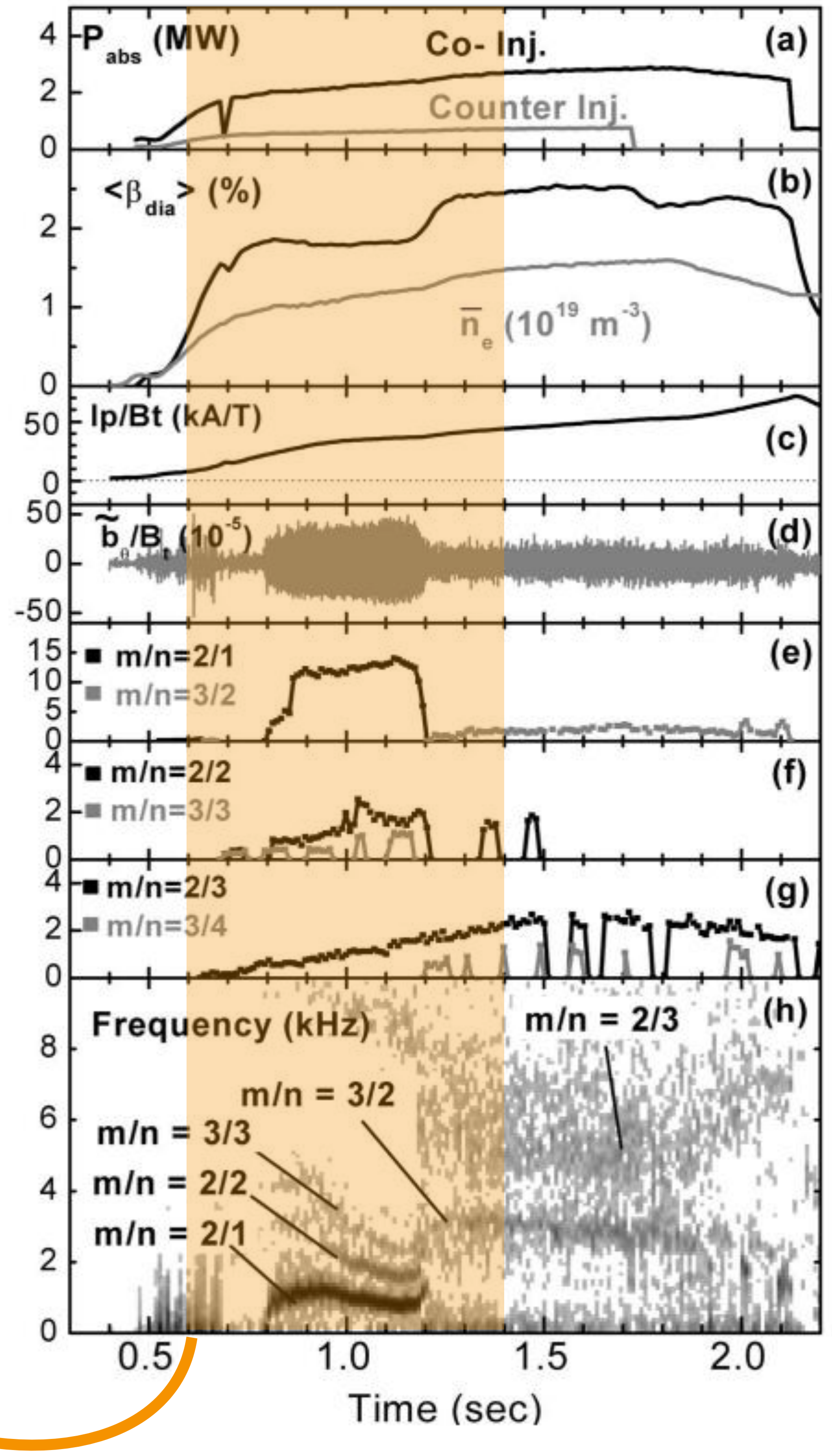
- In LHD, a range of MHD activity has been observed:

  Experimental evidence of both "soft" linear stability limits and "hard" β-limits (core density collapse).

Core-MHD activity (benign) [10]

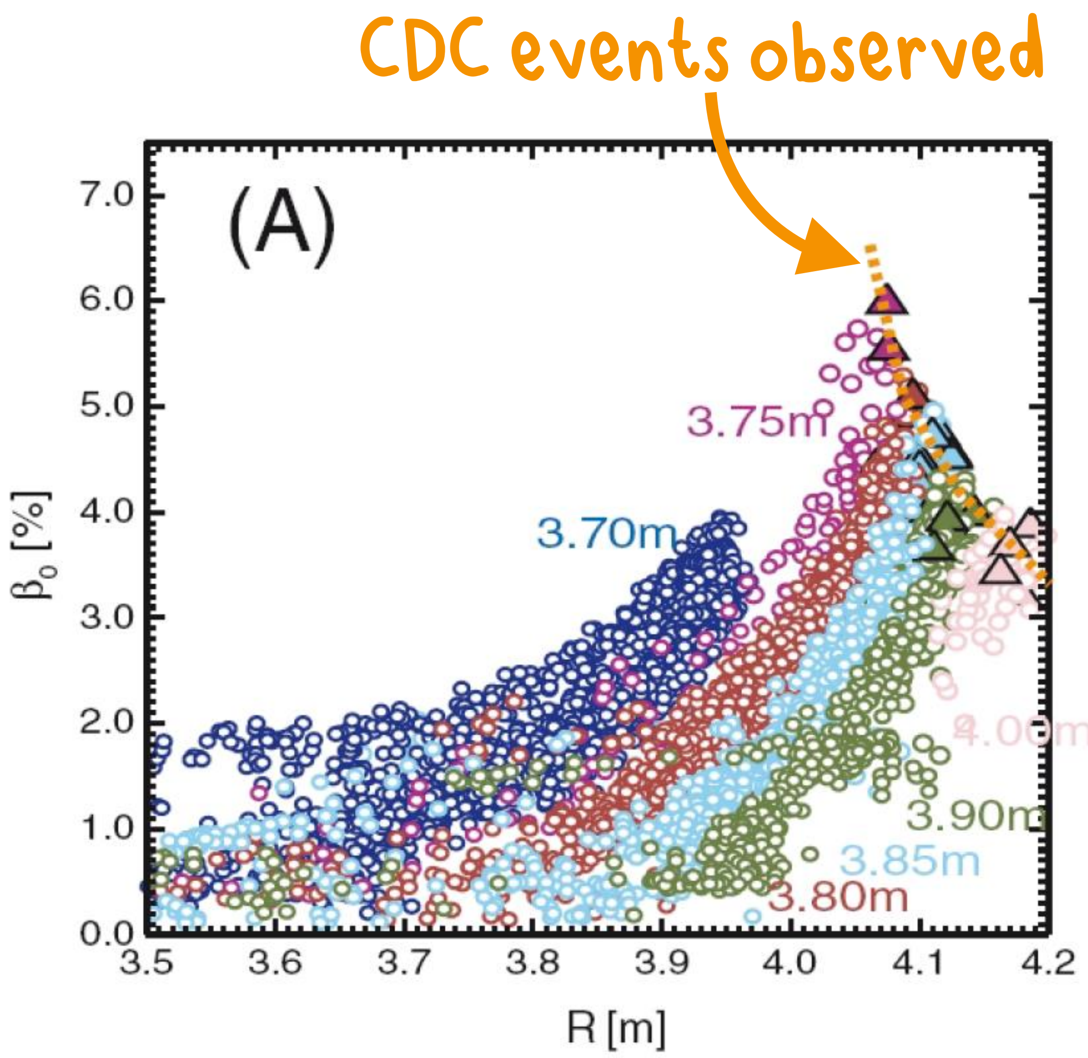
Dominant (2,1) core mode does not lead to collapse in β. Disappears abruptly when $q = 2$ resonant surface vanishes due to increase in plasma current.

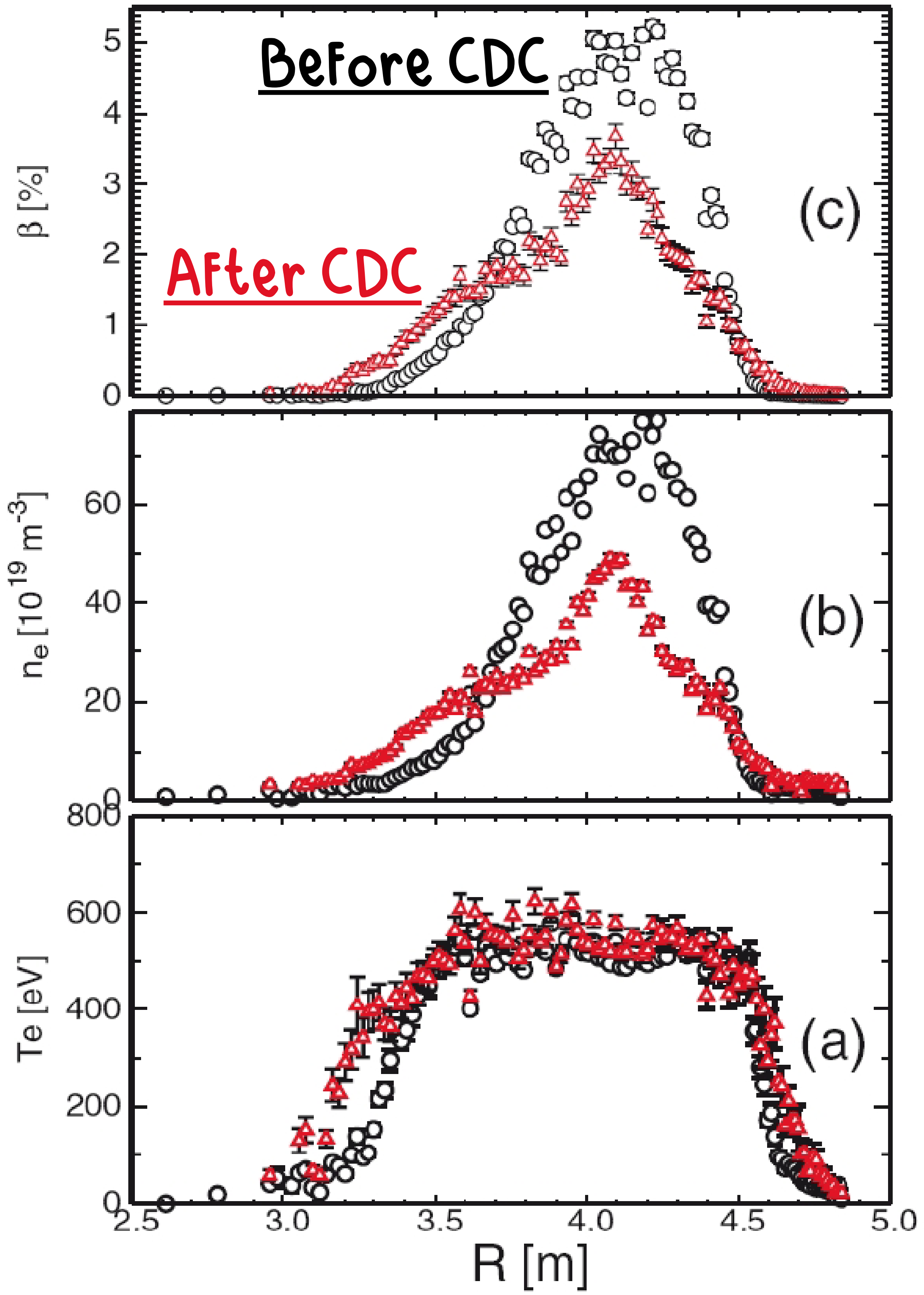




Core density collapse [11]

Increase in ion saturation currents at the divertor plate → deconfinement not profile redistribution event.





[10] Sakakibara et al., Plasma and Fusion Research 1 (2006); [11] Ohdachi et al., Contributions to Plasma Physics 50.6-7 (2010)

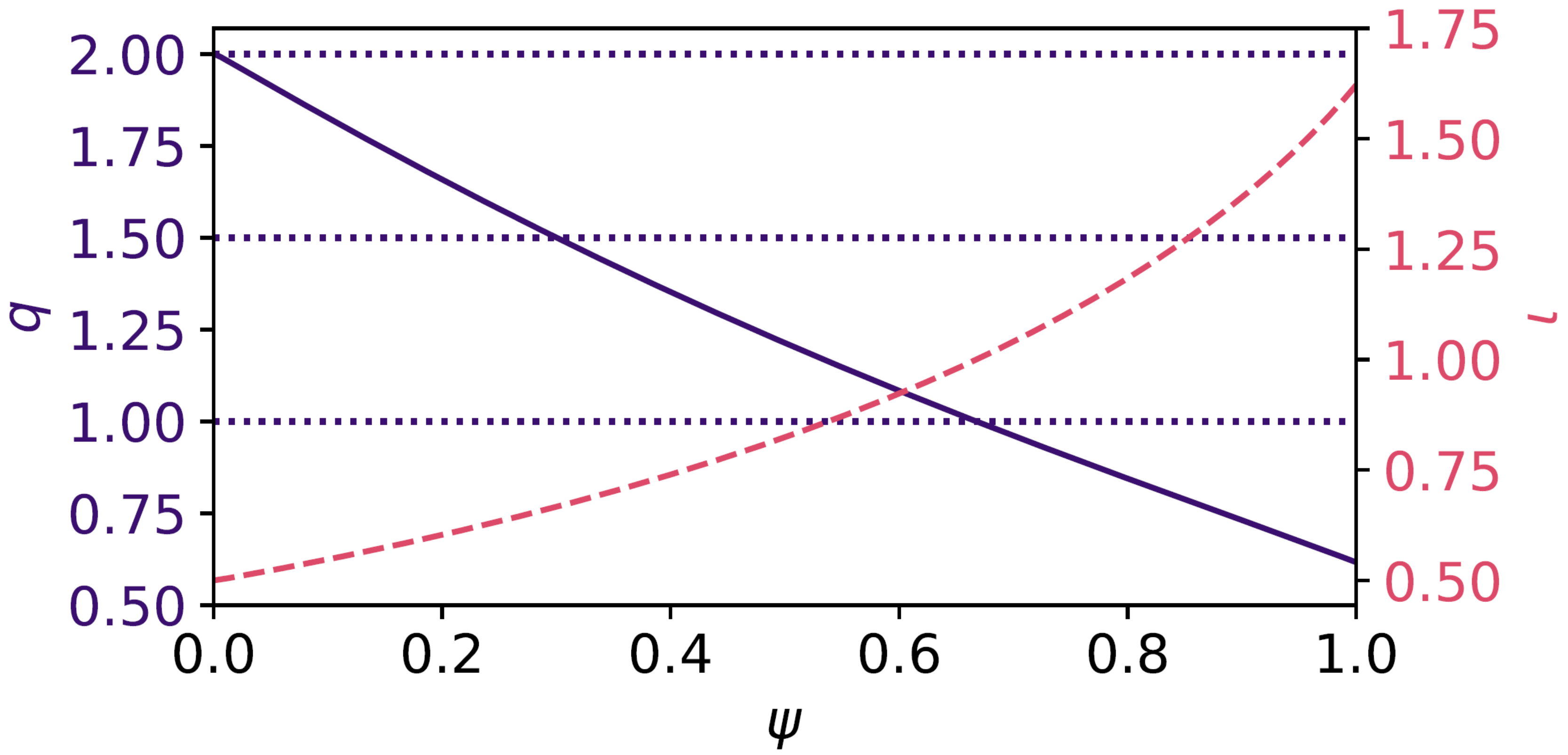
Using M3D-C[1], we apply heating to vacuum field calculated from LHD coils:

- Free boundary: MGRID (Biot-Savart) + boundary shape.

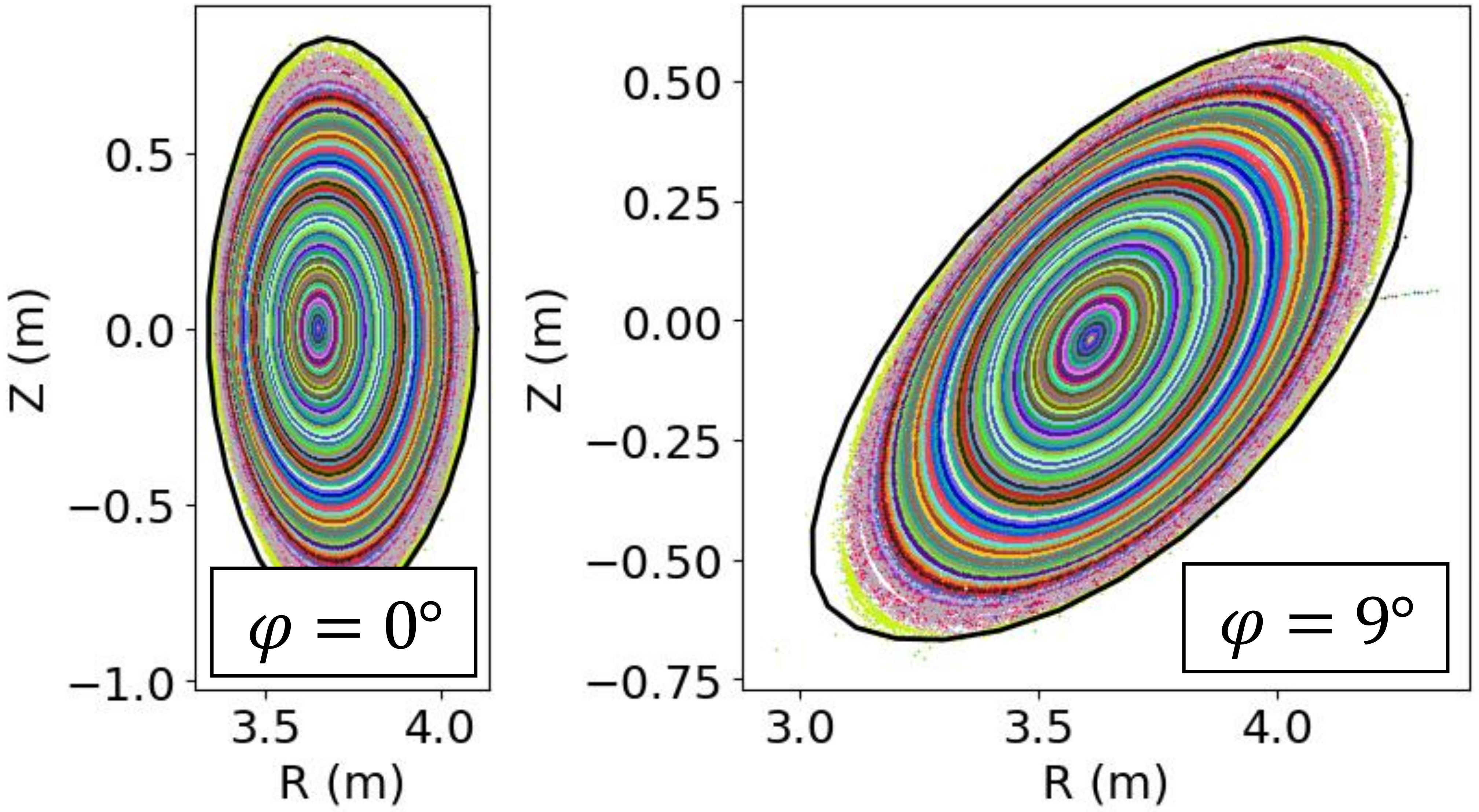
Vacuum plasma parameters:

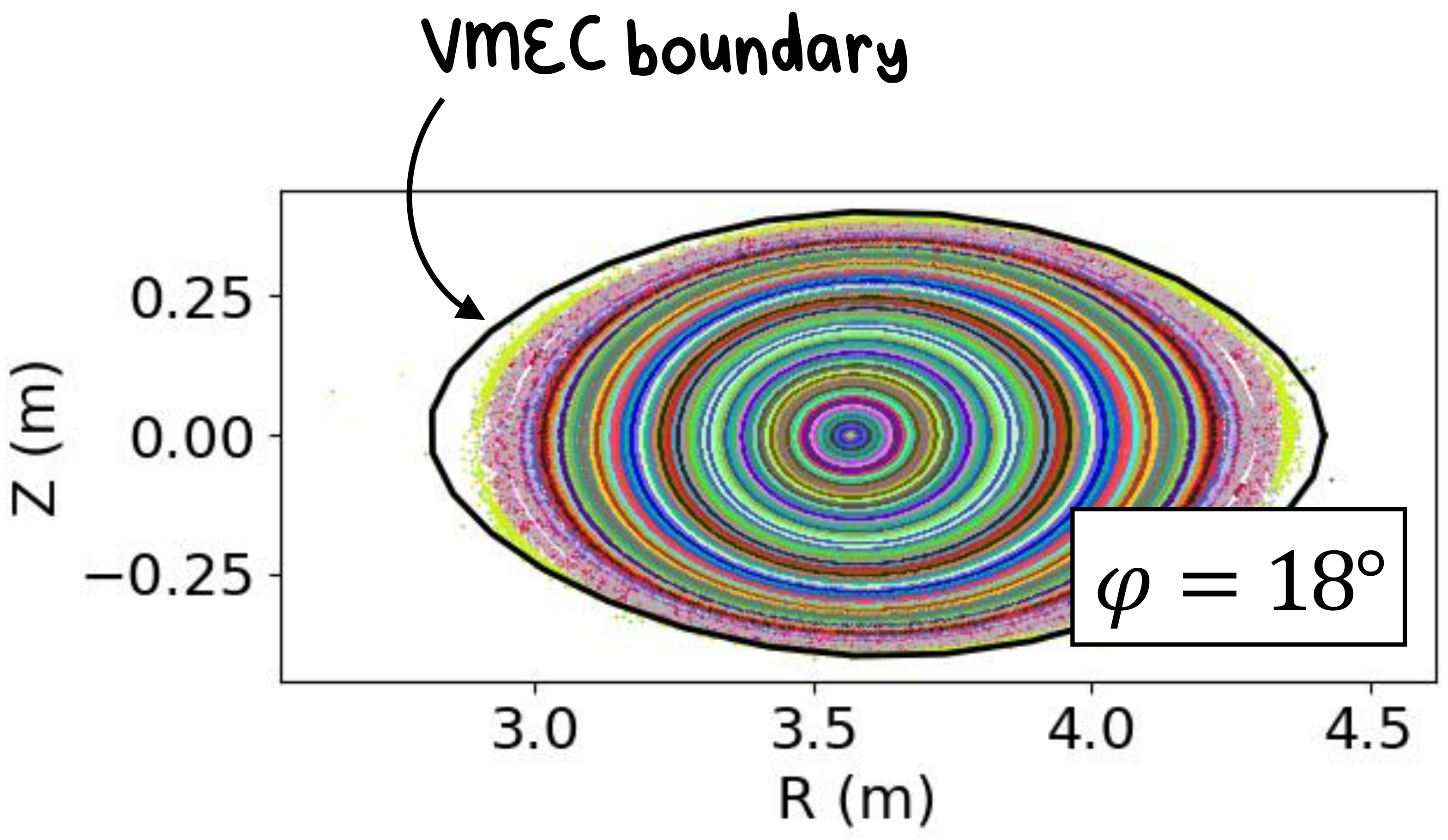
$$\begin{cases} A_p = 6.6 \\ V_p = 22.3\ \mathrm{m}^3 \\ R_0 = 3.66\ \mathrm{m} \\ a = 0.56\ \mathrm{m} \end{cases}$$

Vacuum rotational transform:

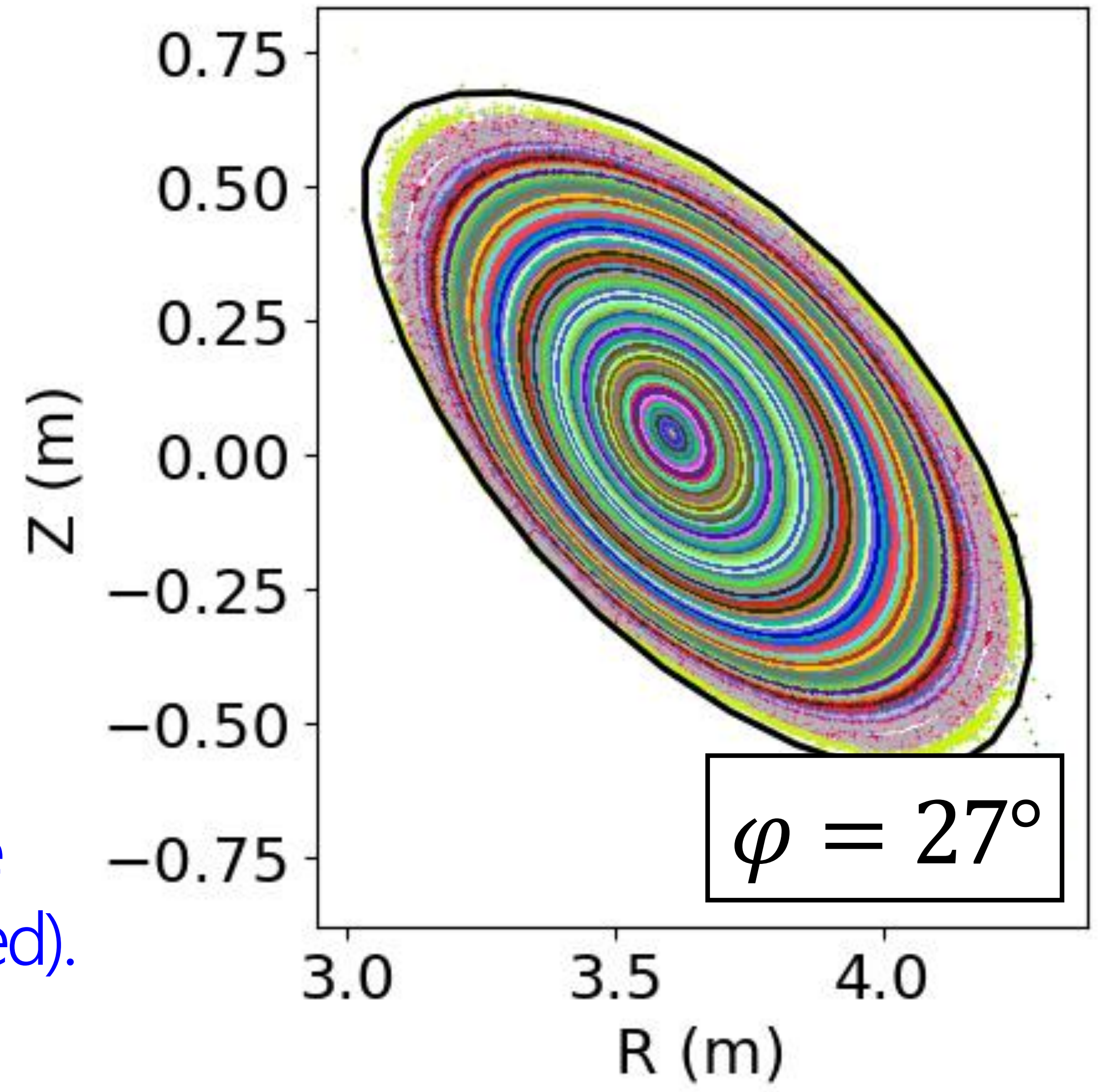


Vacuum magnetic field:





Note: M3D-C[1] does not impose constraints on the boundary shape of the plasma (e.g., no LCFS assumed).

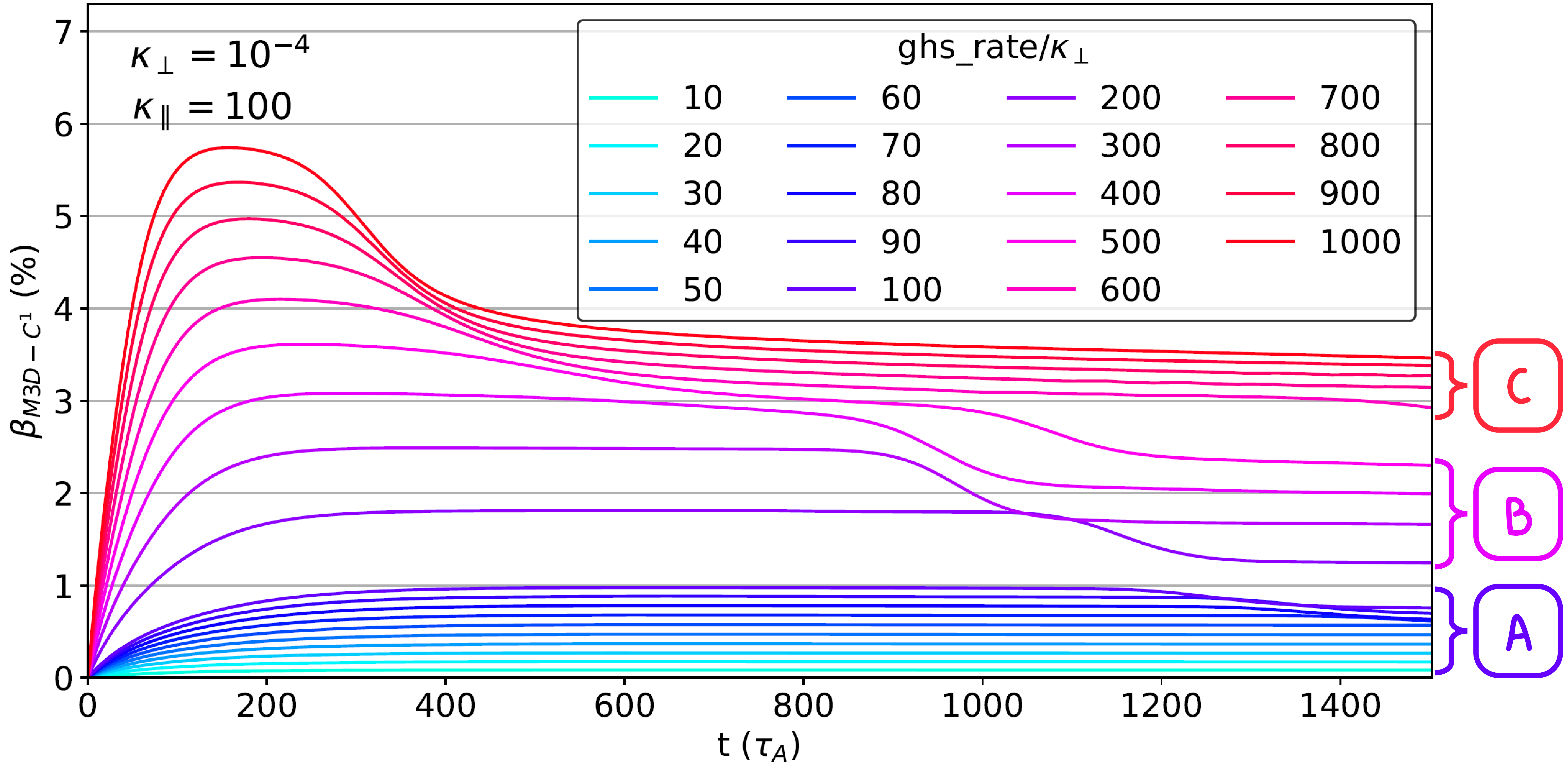


Effect of increased heating power on β

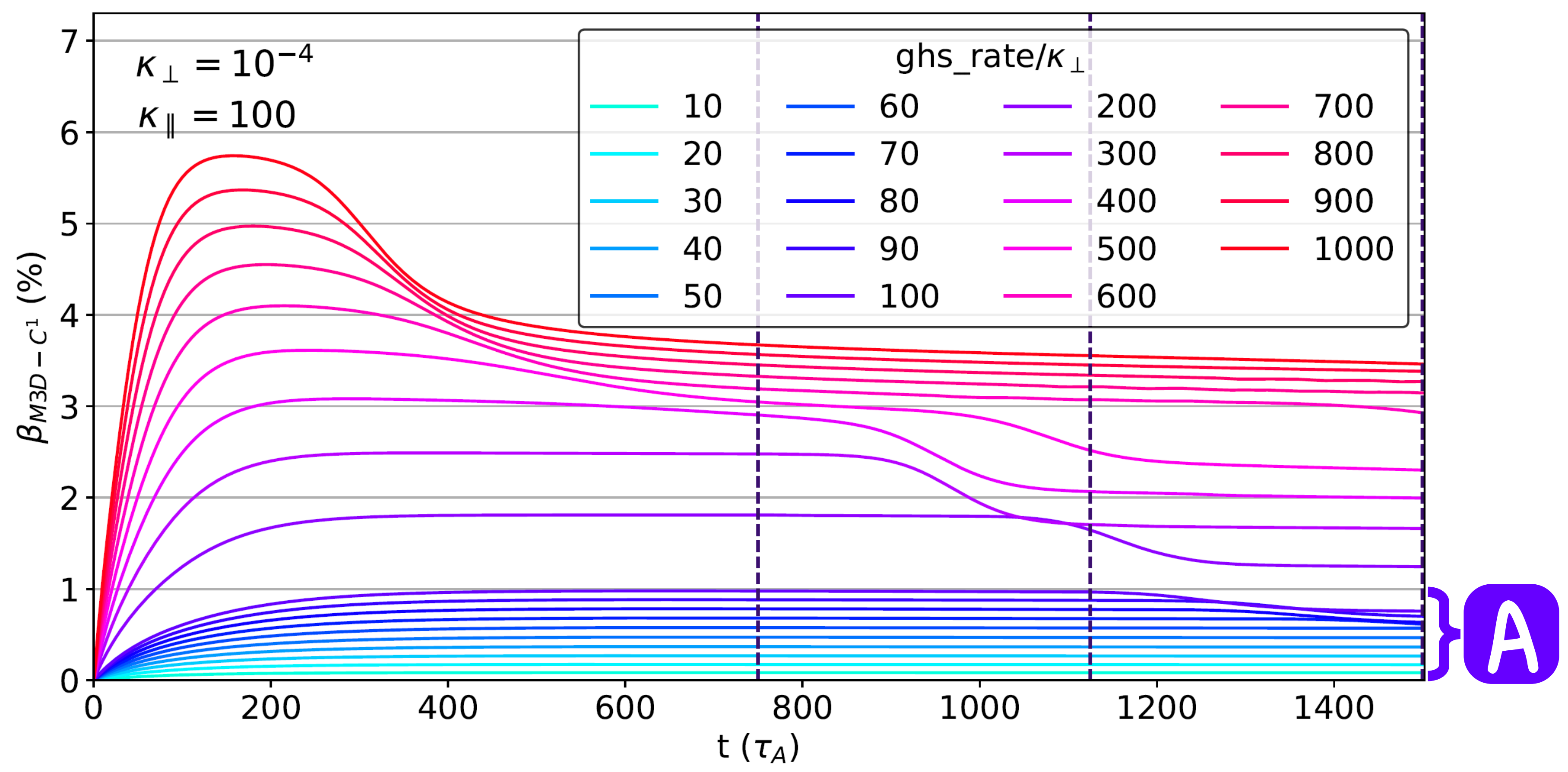
For fixed $\chi_\perp$ and $\kappa_\parallel/\kappa_\perp$, we examine the effect of increased heating power on β:

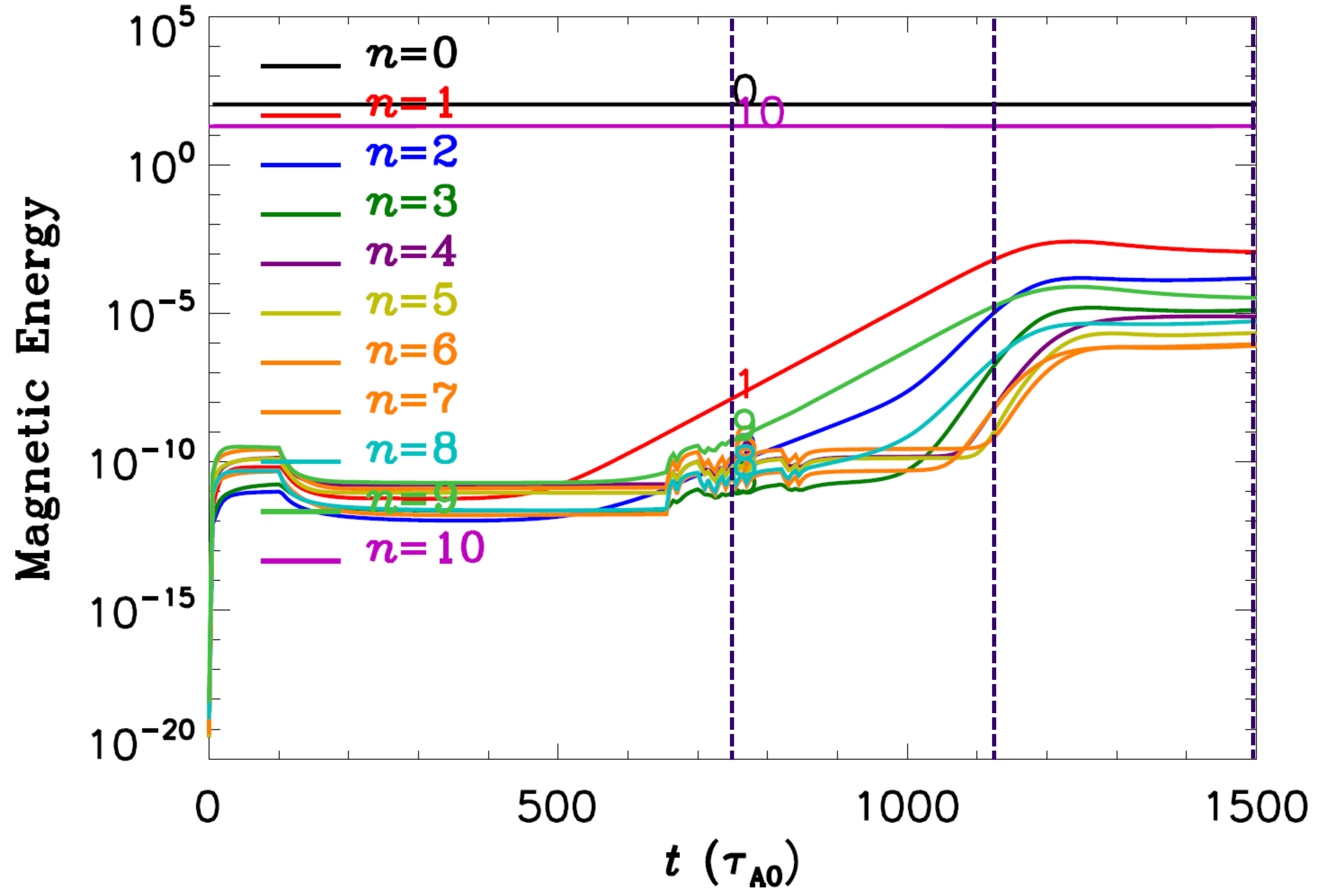
Parameters: $S = 8 \cdot 10^7$, $\nu = 5 \cdot 10^{-3}$, $\kappa_\parallel/\kappa_\perp = 10^6$, $\kappa_\perp = 10^{-4} \rightarrow \chi_\perp = 220\ \mathrm{m}^2/\mathrm{s}$

[For reference: When $\chi_\perp = 22\ \mathrm{m}^2/\mathrm{s}$, 20MW → ghs_rate/$\kappa_\perp$=0.45]

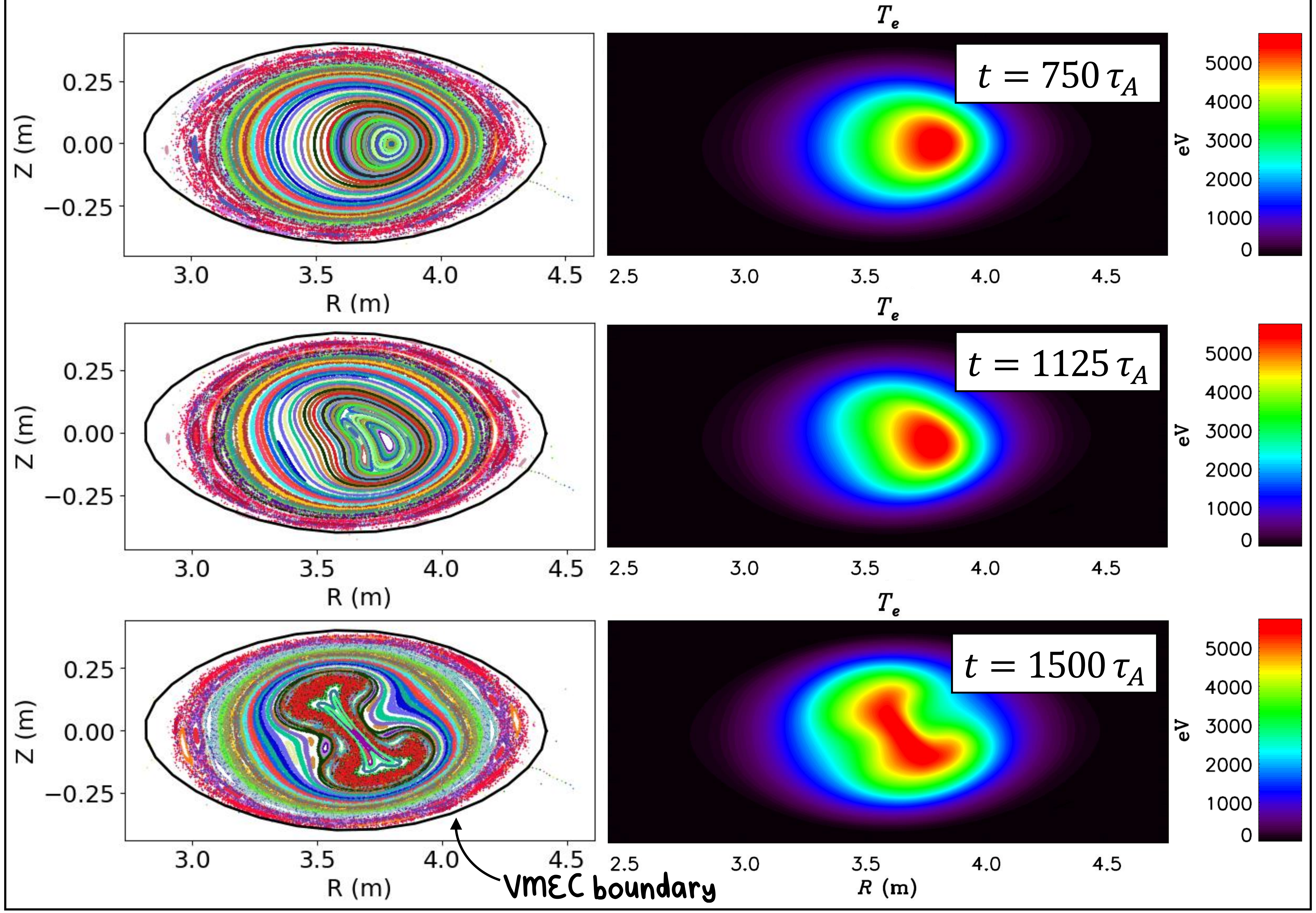


Core MHD activity associated with m/n=2/1 resonance

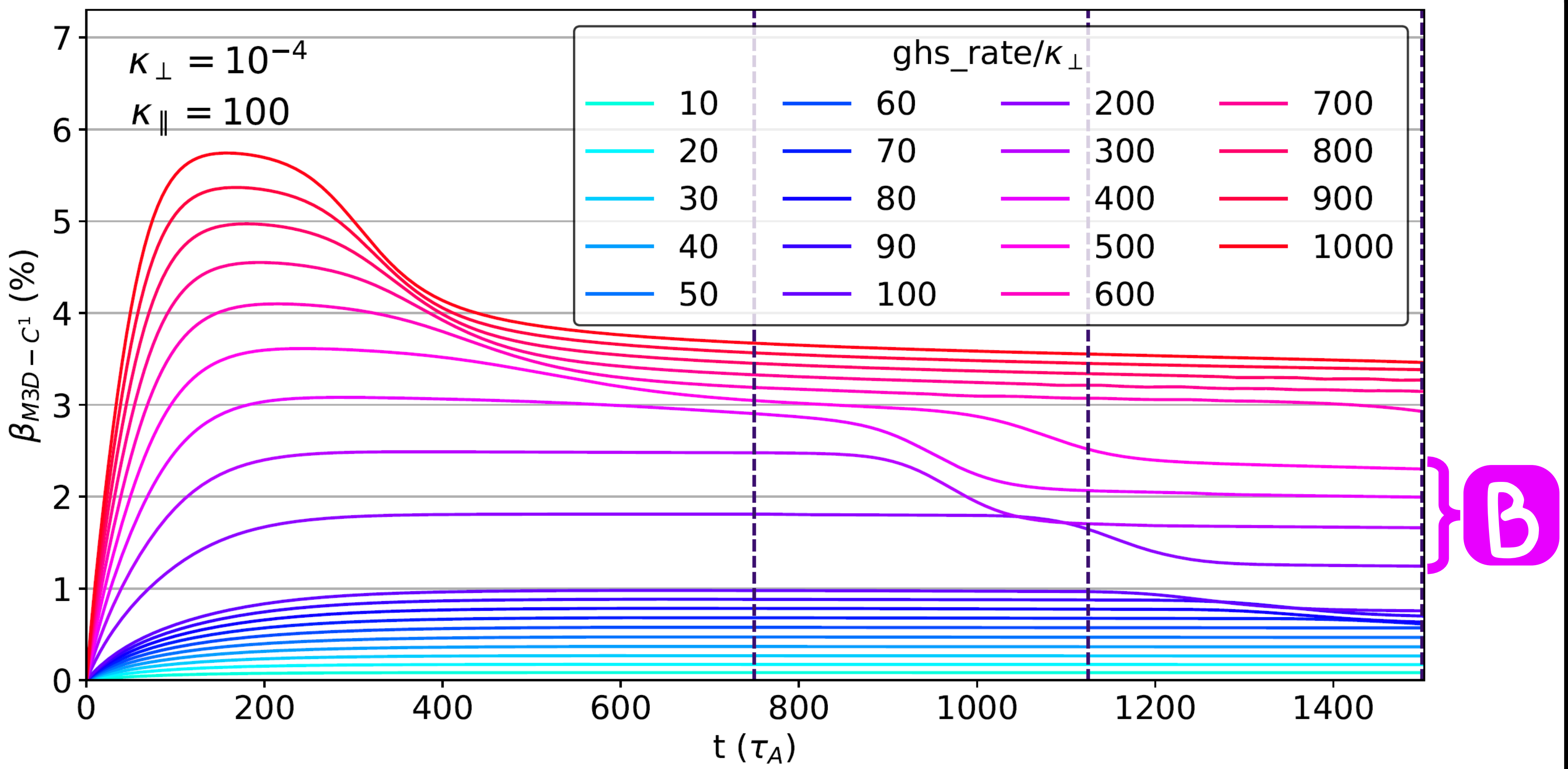


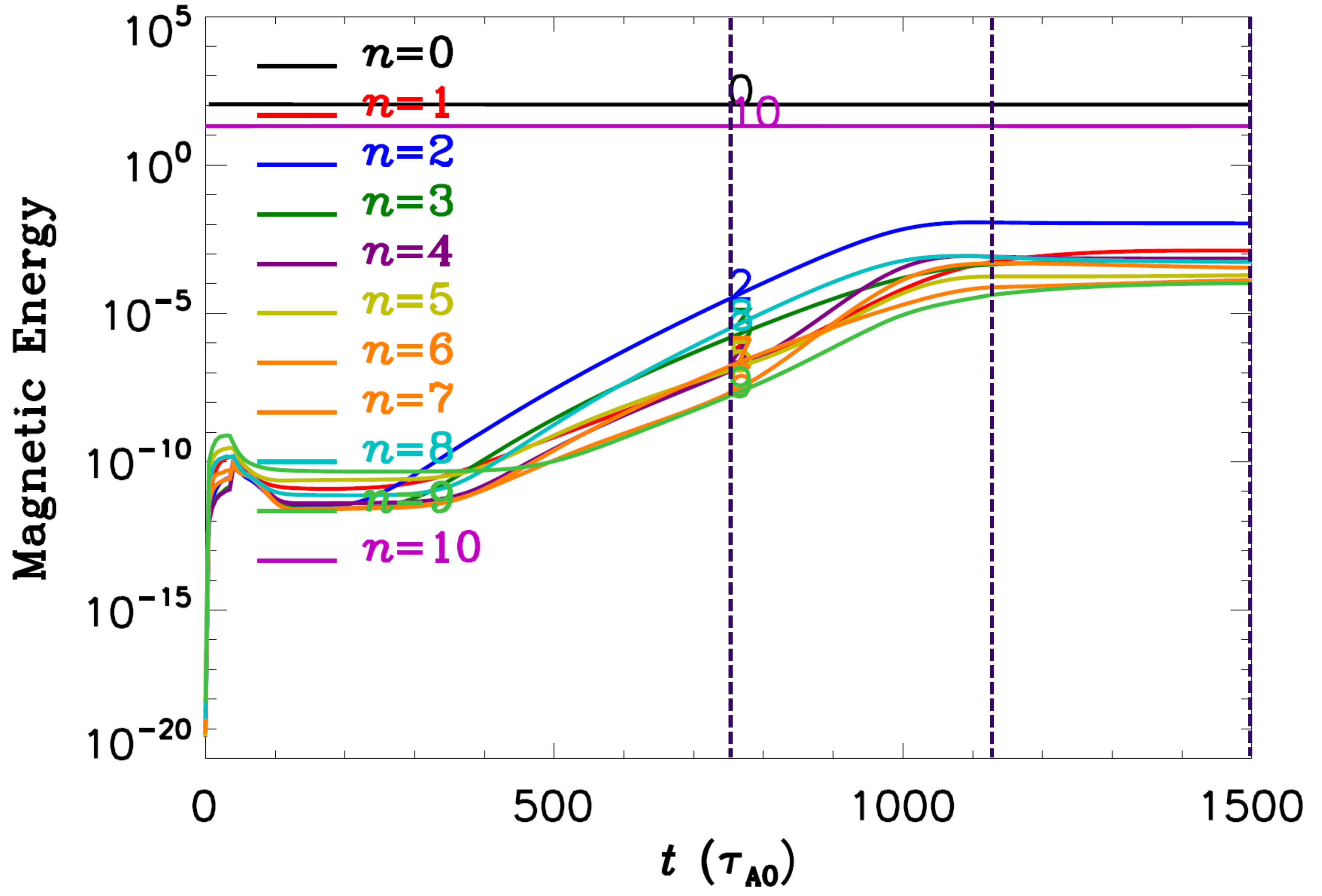


Example: ghs_rate/$\kappa_\perp$=100

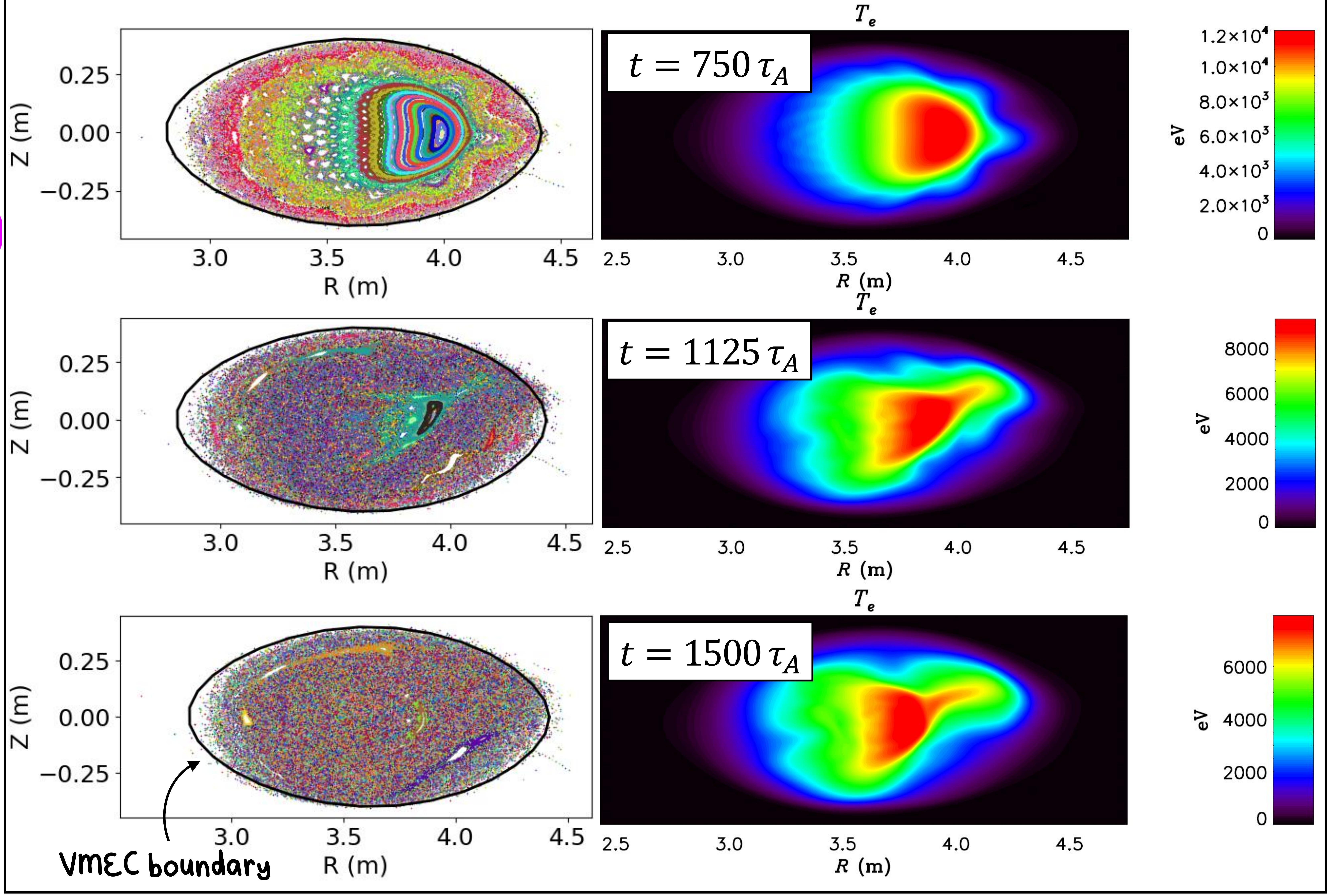


Rapid field stochasticisation with heating-driven high m mode activity



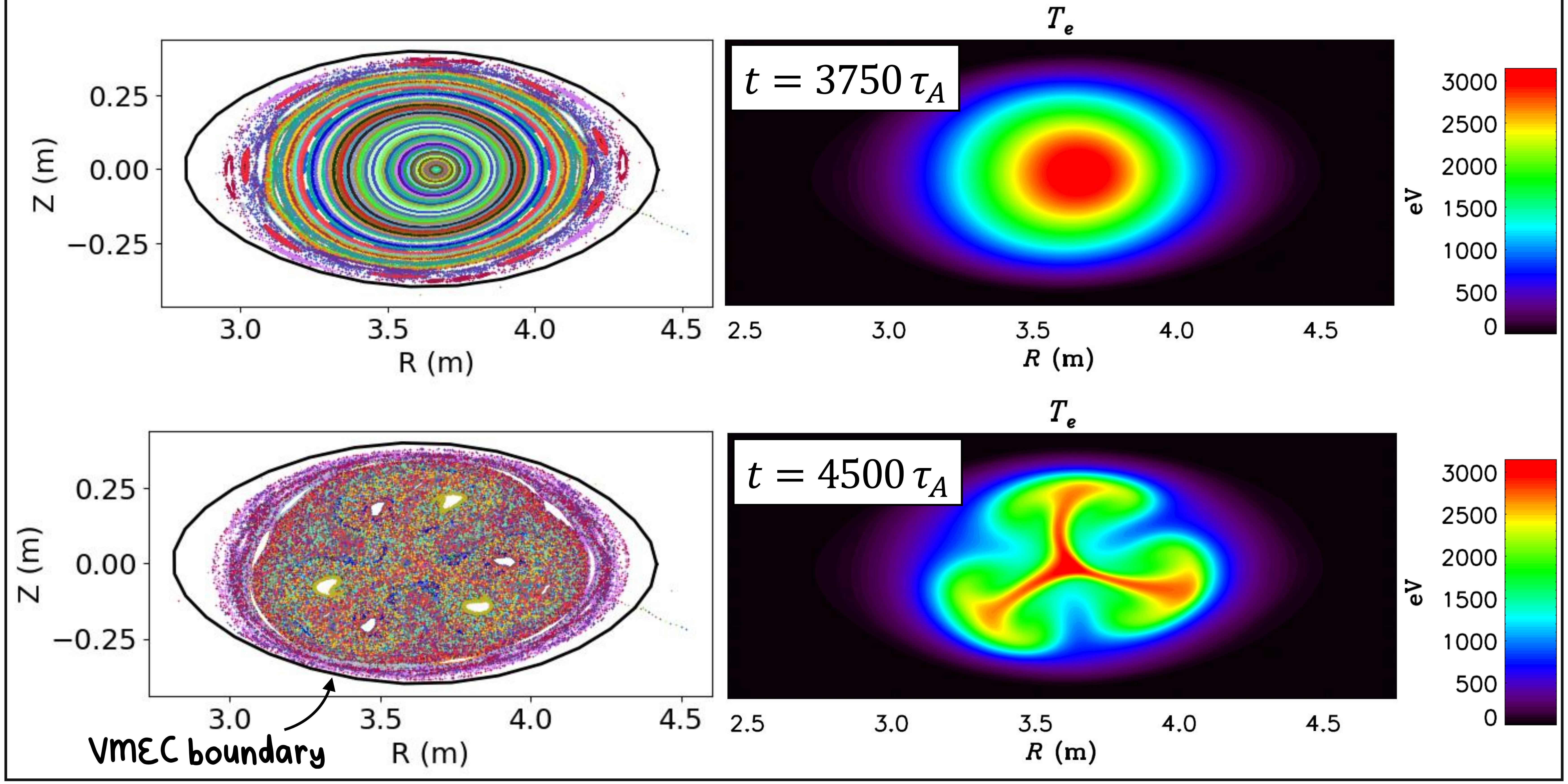


Example: ghs_rate/$\kappa_\perp$=500



? How does transport in 3D fields affect the MHD dynamics and nonlinear stability characteristics?

Example: ghs_rate/$\kappa_\perp$=400 and $\chi_\perp = 1.1\,\mathrm{m}^2/\mathrm{s}$



Next steps (in collaboration w/ Y. Suzuki):

- Work towards direct experimental comparison, including understanding LHD core density collapse events.

- *Challenges associated with modelling 3D fields*
- *Assessing reduced models as tools for efficient calculation of 3D equilibria*
- *Exploring β-limits in stellarators with M3D-C[1]*
- **Examining the role of non-resonant modes in the formation of 3D fields**
- *Outlook: Summary and on-going work*

- Under some conditions, NSTX discharges showed flattening of T_e profiles with increasing beam power [12], together with evidence of enhanced electron transport.
- Sawteeth have been excluded as possible MHD origins for this enhanced transport, since $q > 1$.
- Understanding the mechanisms by which stochastic (3D) fields form may improve understanding of these soft β-limits (c.f. on-going work by Jardin et al.).

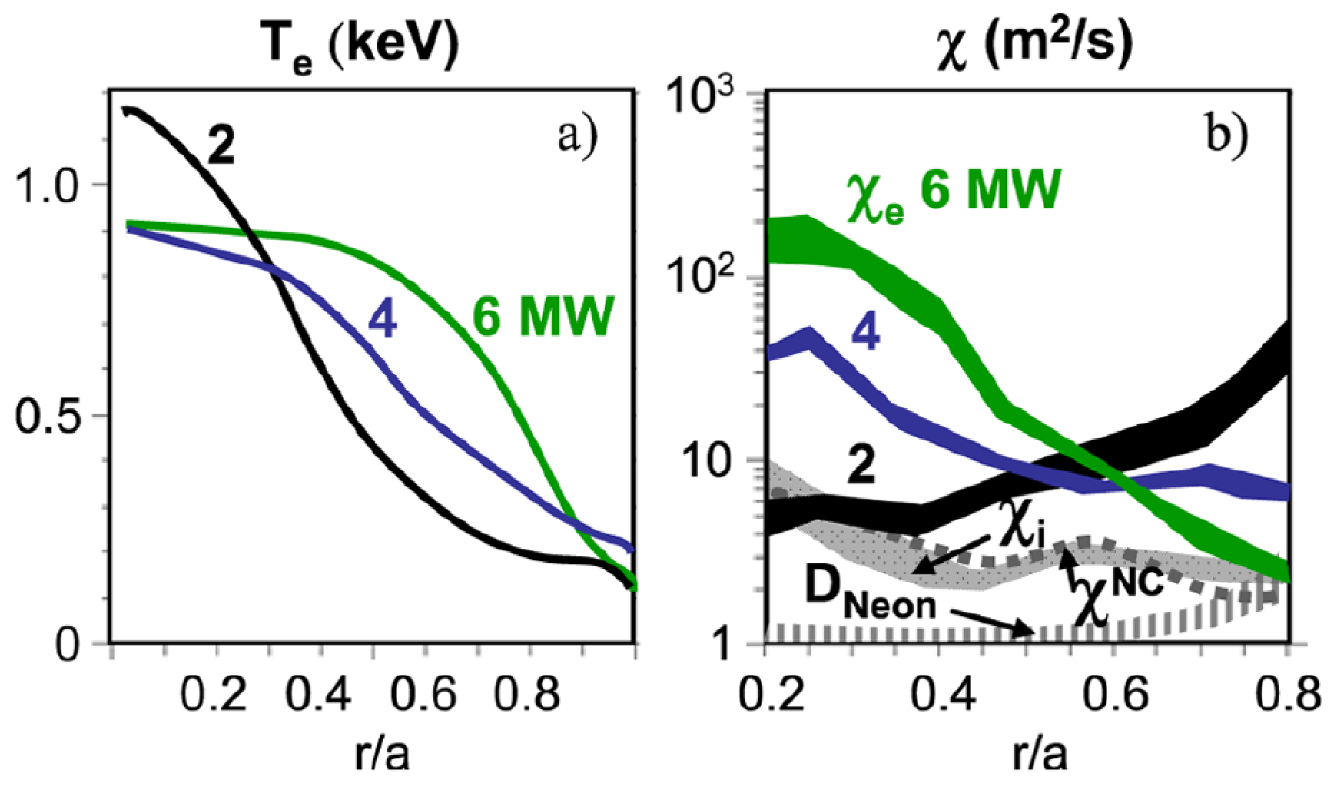


FIG. 1 (color online). (a) T_e profiles in 2, 4, and 6 MW NSTX H modes at $t \sim 0.4$ s. The measurement uncertainty is around 25 eV. (b) TRANSP computed χ_e in the same plasmas. Also shown are the χ_i and NCLASS ion thermal diffusivity for the 6 MW case and the measured neon diffusivity. The bands of values represent the 20 ms variability.

<u>Nonresonant pressure-driven modes (where $q \neq m/n$) :</u>

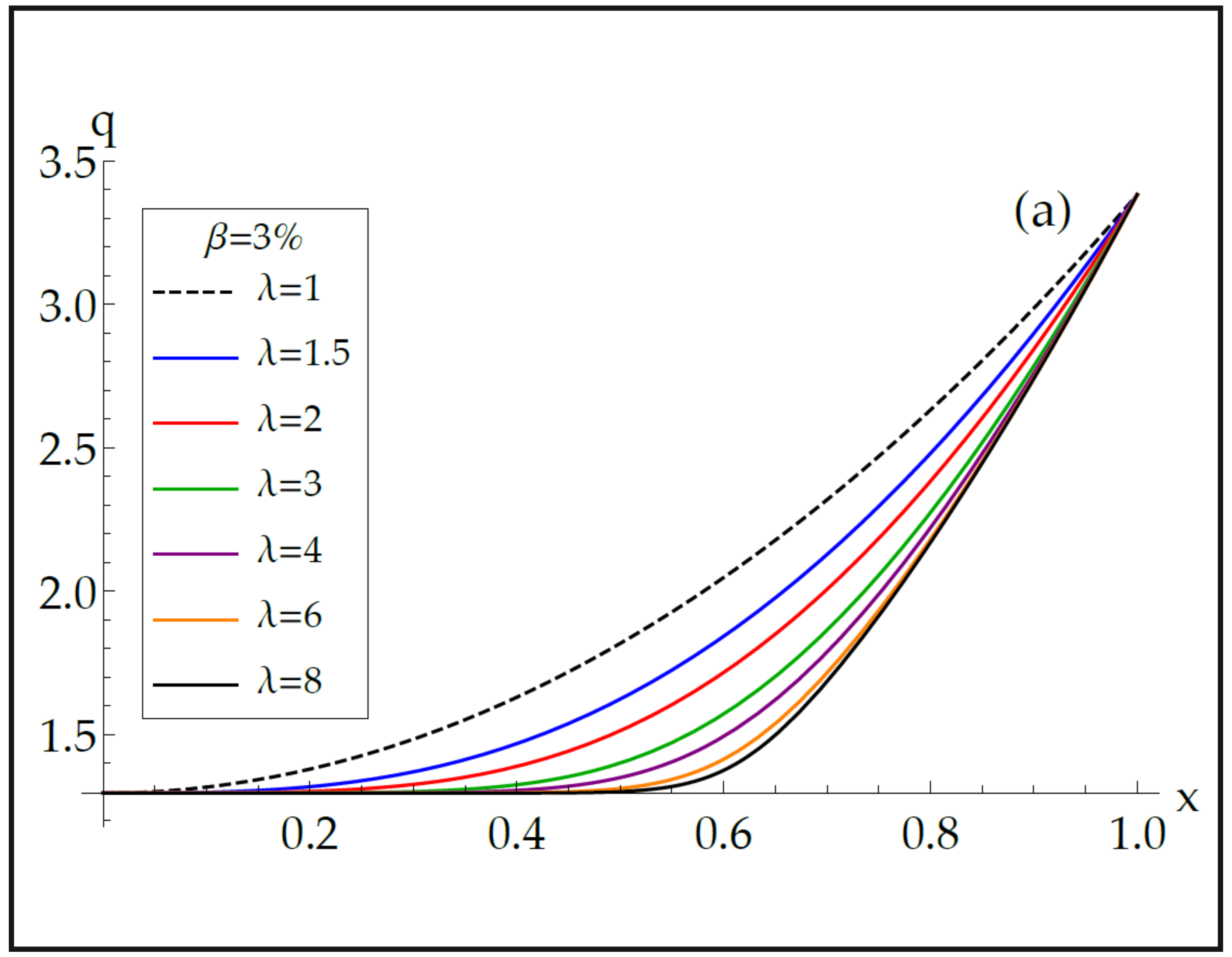
- Can appear when magnetic shear is reduced when $q > 1$, even in Mercier- and ballooning-stable equilibria.
- Have global mode structures, generating substantial plasma displacements that may lead to more efficient flattening of pressure gradients.

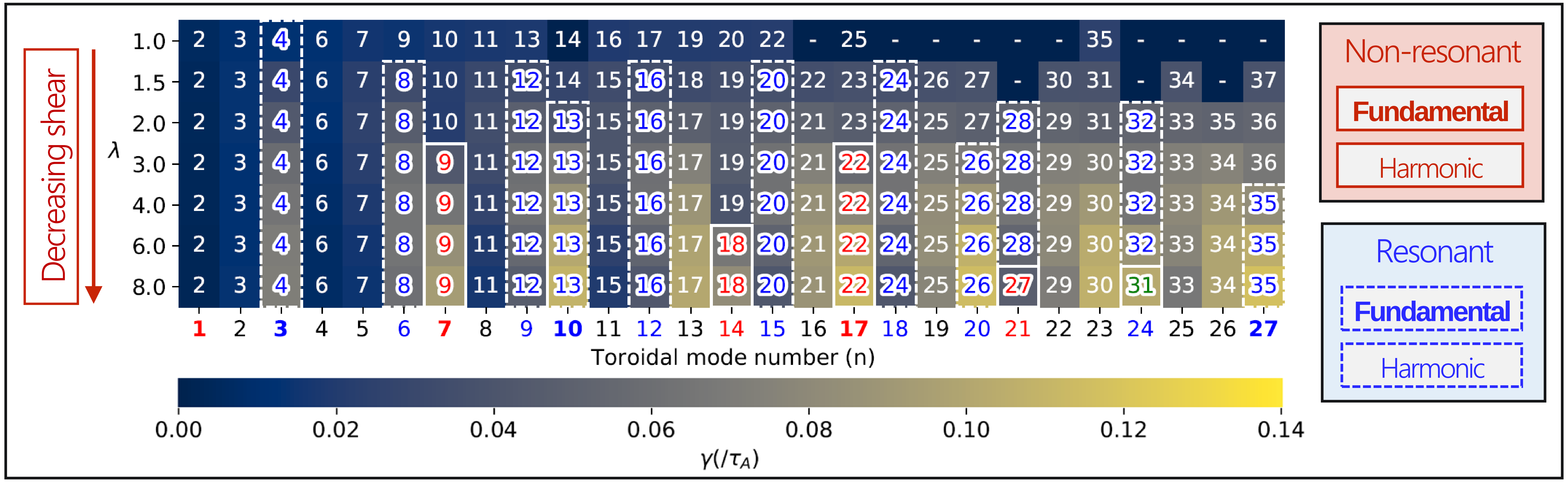
? What is the role of non-resonant modes in the formation of 3D fields?

[12] Stutman et al., Physical Review Letters 102.11 (2009).

Low shear equilibria can be associated with a characteristic spectrum of unstable modes

- Even without mode-coupling, low-shear linear MHD stability can be dominated by certain moderate/high-n modes that can be associated with a characteristic spectrum [13,14]:





? Can and how do nonlinear mode interactions affect the formation of 3D fields?

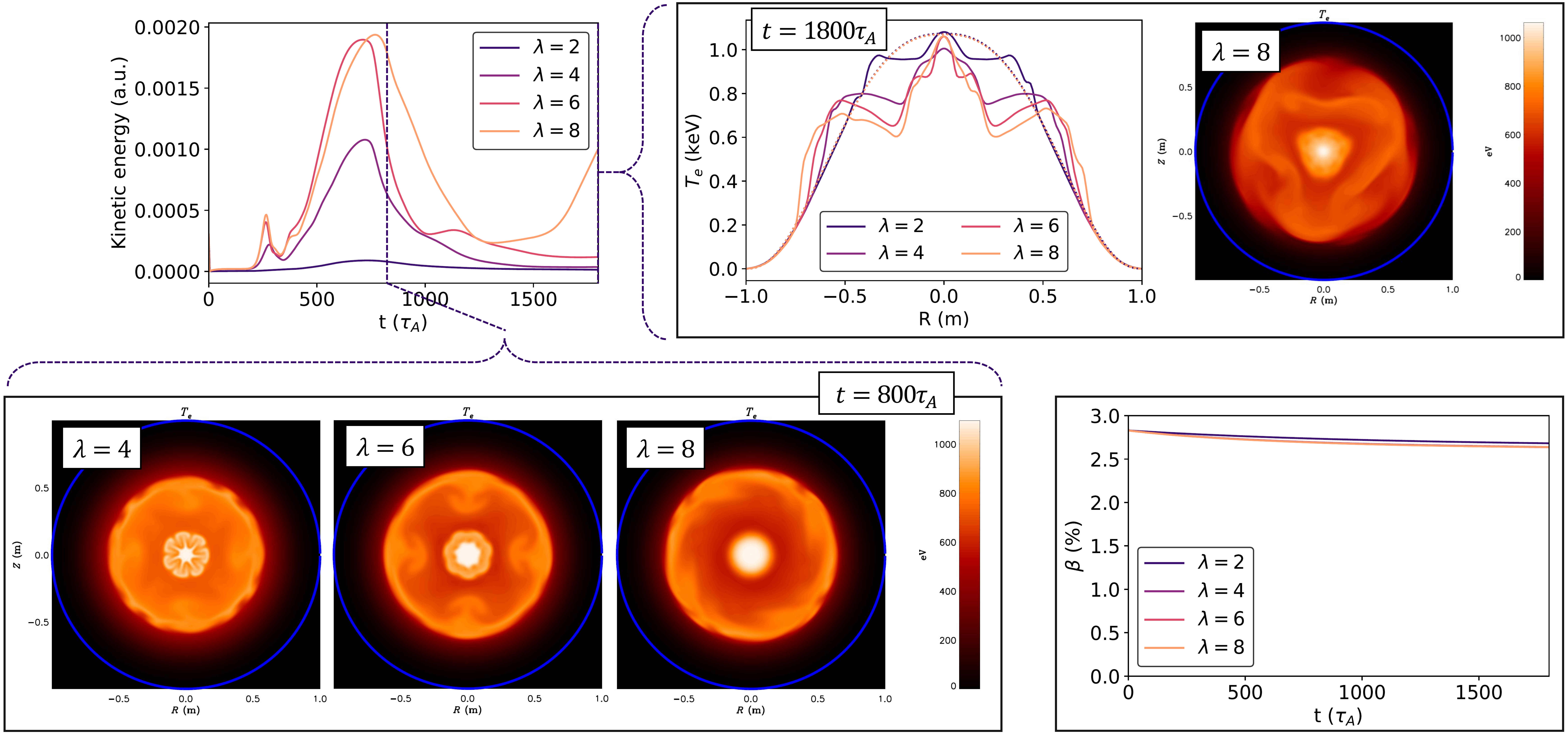
? What is the impact on transport in fields generated via these mechanisms?

[13] Wright & Ferraro, Physics of Plasmas 28, 072511 (2021).; [14] Wright et al., Physics of Plasmas 28, 012106 (2021).

Preliminary studies show significant Te evolution

Preliminary nonlinear simulations show significant evolution of the temperature profile (though no collapse in β):

? What are the implications for transport in these fields?



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Modelling 3D magnetic fields can be challenging:

- Tools to rapidly calculate 3D fields are needed for optimisation and reconstruction. However:
  - ? How should realistic (e.g., smooth) pressure profiles be represented in equilibrium models?
  - ? Can the plasma actually reach the predicted equilibrium with heating (i.e., dynamical accessibility)?

We have seen that:

+1 SPEC is viable as a tool for modelling the nonlinear + non-ideal plasma response to external 3D fields in the weakly nonlinear regime:

- >> Exploring SPEC as a tool for modelling internal relaxation events (e.g., helical core states) on NSTX-U.

+1 With $M3D\text{-}C^1$, we now have first-of-a-kind capability to explore nonlinear MHD in stellarators:

- >> Evolution of pressure profiles for self-consistent equilibria, including for non-integrable fields.
- >> Understanding core density collapse events in LHD experiments.

+1 Pressure-driven modes may lead to efficient formation of 3D fields in regions of low shear.

- >> What are the implications for transport in these fields?