# Ideal MHD limited electron temperature in (spherical) tokamaks

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(virtual)

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# Outline

I. Introduction

II. A Typical Case

III. A Family of Equilibria with Differing $\beta$ Values

IV. Apply Heating to a Stable Equilibrium

V. A Few Other Cases

VI. Comments and Summary

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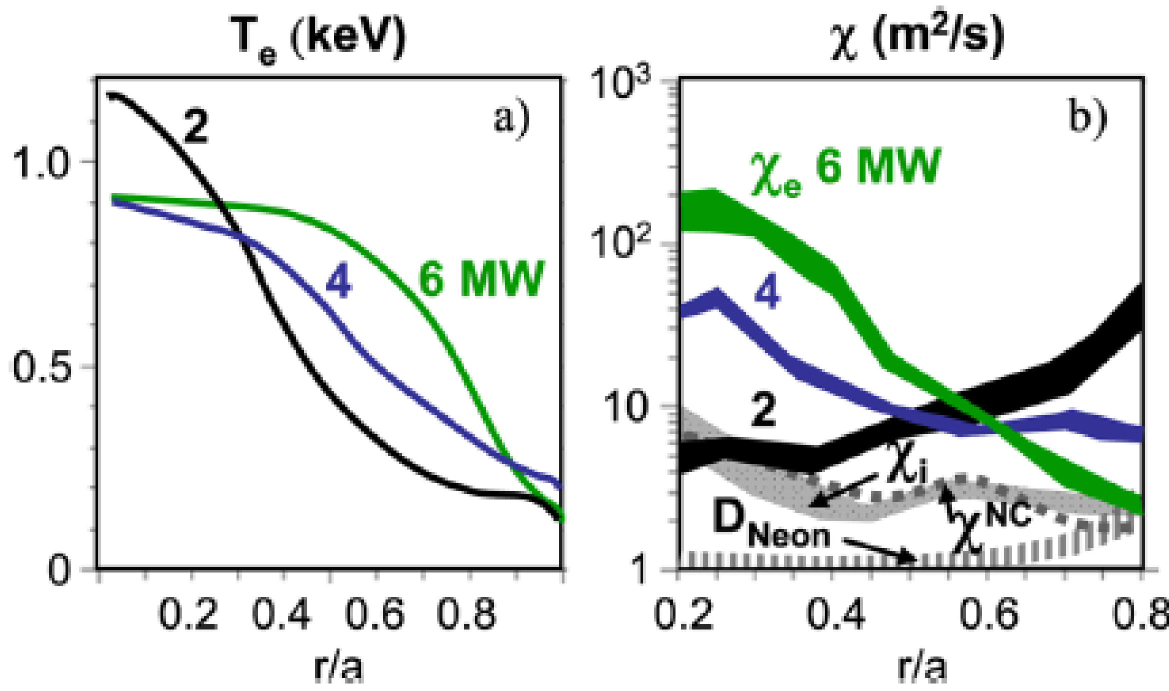
# From "soft beta limits" to "temperature flattening"

- The original motivation of this work was to better understand pressure-driven instabilities that do not cause disruptions in NSTX and other STs and tokamaks
- Examples of these are sawteeth and ELMs.
- But are there others, and if so: How do they saturate?
- Many NSTX discharges are predicted to be unstable to internal ideal MHD modes
- In performing many long-time nonlinear 3D MHD simulations, we saw that a common saturation mechanism for these internal ideal MHD modes is a flattening of the central electron temperature profile
- Could this "Ideal MHD" phenomena be responsible for the observed temperature flattening and associated thermal transport in NSTX?




Stutman, et al. PRL (2009)

- This large increase in the central $\chi_e$ with $\beta$ has not been convincingly explained by micro-instabilities or energetic-particle driven transport

# Theory Basis

"Infernal modes[1]" are localized global internal ideal MHD instabilities that can occur in low shear regions at $\beta$-values well below the ballooning limit.

A recent paper by Boozer[2] shows that ideal MHD instabilities can lead to magnetic surface breakup, even for an arbitrarily small resistivity.

This opens up the possibility that surfaces can be destroyed in the vicinity of large pressure gradients, and that anomalous transport could occur by way of parallel diffusion in the resulting stochastic magnetic fields

We investigate this with the 3D MHD code M3D-*C*[1]

[1]Manickam, J., Pomphrey, N., Todd, A., "Ideal MHD stability properties of pressure driven modes in low shear tokamak" Nuclear Fusion (1987)
[2]Boozer, A., "The Rapid destruction of toroidal magnetic surfaces", Physics of Plasmas (2022)

# 3D Extended MHD Equations in M3D-$C^1$

$$\frac{\partial n}{\partial t}+\nabla \bullet (n\mathbf{V})=\nabla \bullet D_n \nabla n+S_n$$

$$\frac{\partial \mathbf{A}}{\partial t}=-\mathbf{E}-\nabla\Phi, \quad \mathbf{B}=\nabla\times\mathbf{A}, \quad \mathbf{J}=\nabla\times\mathbf{B}, \quad \nabla_\perp \bullet \frac{1}{R^2}\nabla\Phi=-\nabla_\perp \bullet \frac{1}{R^2}\mathbf{E}$$

$$nM_i\left(\frac{\partial \mathbf{V}}{\partial t}+\mathbf{V}\bullet\nabla\mathbf{V}\right)+\nabla p=\mathbf{J}\times\mathbf{B}-\nabla\bullet\mathbf{\Pi}_i+\mathbf{S}_m$$

$$\mathbf{E}+\mathbf{V}\times\mathbf{B}=\eta\mathbf{J}+\mathbf{S}_{CD}$$

$$\frac{3}{2}\left[\frac{\partial p_e}{\partial t}+\nabla\bullet\left(p_e\mathbf{V}\right)\right]=-p_e\nabla\bullet\mathbf{V}+\eta J^2-\nabla\bullet\mathbf{q}_e+Q_\Delta+S_{eE}$$

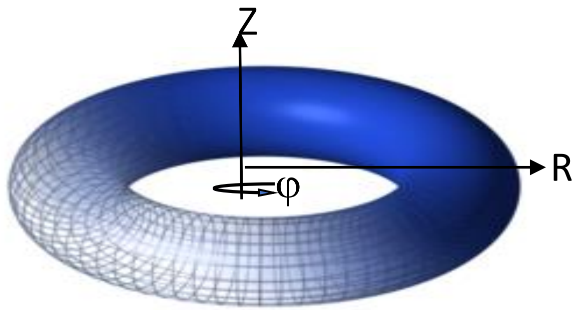
$$\frac{3}{2}\left[\frac{\partial p_i}{\partial t}+\nabla\bullet\left(p_i\mathbf{V}\right)\right]=-p_i\nabla\bullet\mathbf{V}-\mathbf{\Pi}_i:\nabla\mathbf{V}-\nabla\bullet\mathbf{q}_i-Q_\Delta+S_{iE}$$

$$\mathbf{\Pi}_i=-\mu\left[\nabla\mathbf{V}+\nabla\mathbf{V}^\dagger\right] \qquad Q_\Delta=3m_e(p_i-p_e)/\left(M_i\tau_e\right)$$

$$\mathbf{q}_{e,i}=-\kappa_{e,i}\nabla T_{e,i}-\kappa_{\parallel e,i}\nabla_\parallel T_{e,i}$$

Loop voltage at boundary, $V_L$, adjusted to keep $I_P$ fixed.

Sources: $S_n, S_m, S_{CD}, S_{eE}, S_{iE}$ Transport Coefs: $D_n, \mu, \eta, \kappa_e, \kappa_i, \kappa_{\parallel e}, \kappa_{\parallel i}$



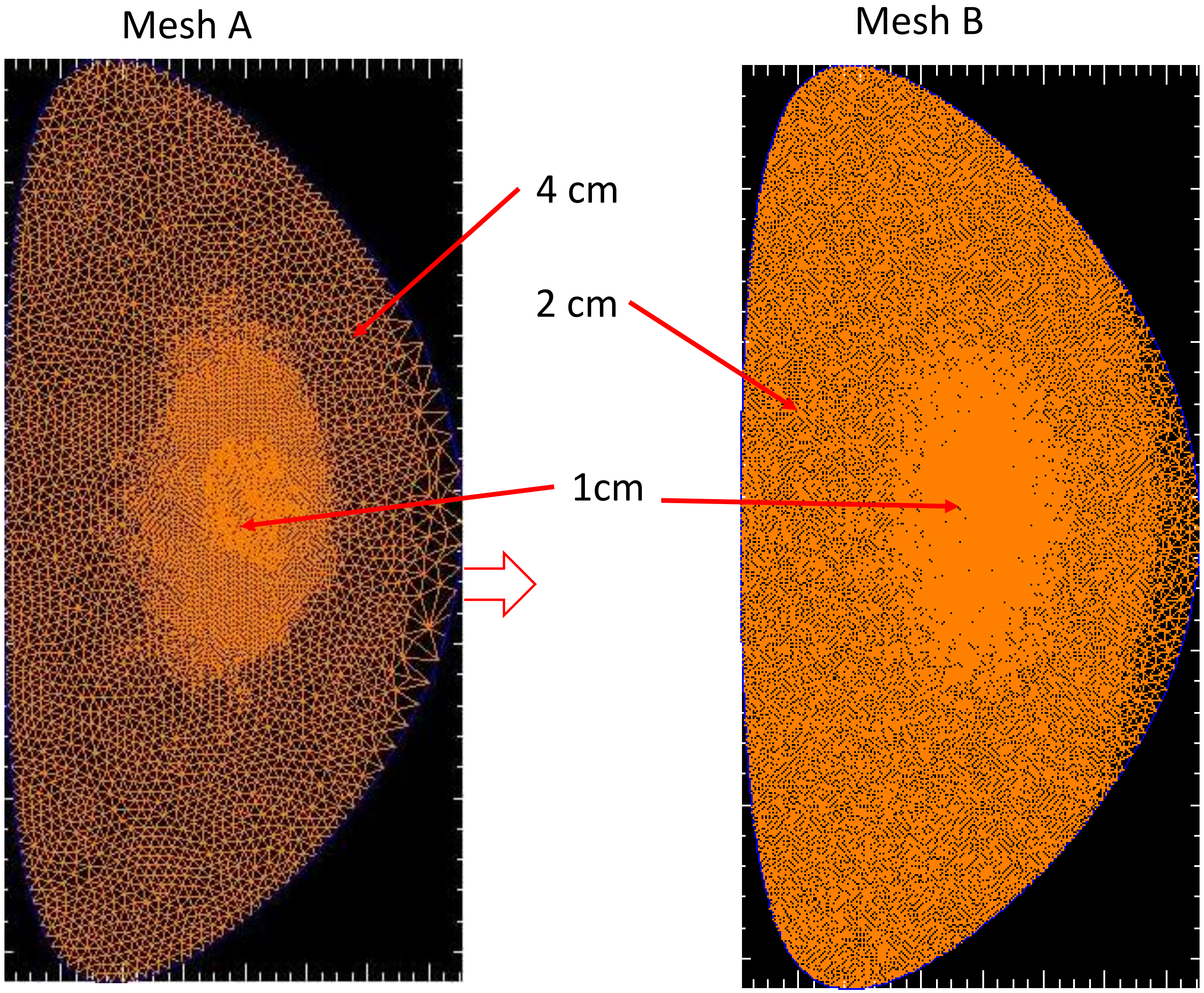

These are the equations used in this study. Many other options available: Radiation, pellet ablation, conducting wall, reduced MHD, 2-fluid MHD, -K

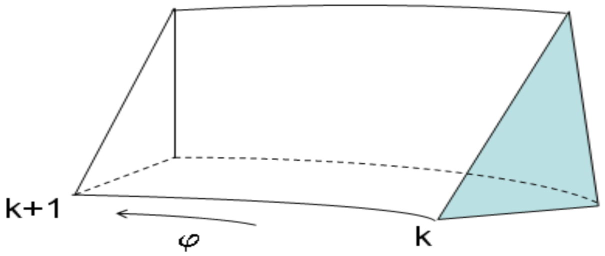
Some of the results presented assumed $p_e = p_i$ for simplicity.

Code can be run in 2D or 3D

(2D should give very similar results as TRANSP with same sources and transport coefs.)

# Unstructured Meshes used in this study




- Triangular prism mesh is structured in toroidal direction, unstructured in poloidal plane.
- Use a high-resolution mesh so calculation can go to high $S = 10^8$
- Variable size, highest resolution in center where S is largest
- Perform same calculation on 2 meshes for convergence study.
- High order $C^1$ finite elements error $\sim h^5$ in (R,Z) plane

24 poloidal planes represented by Hermite cubic elements, error $\sim h^4$
Limited 48 plane runs done for convergence studies

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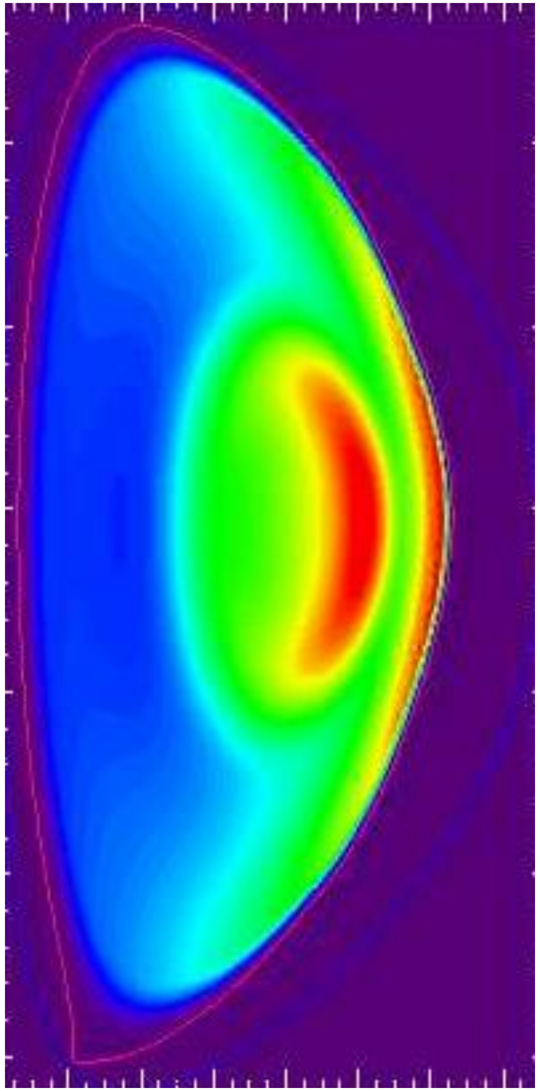
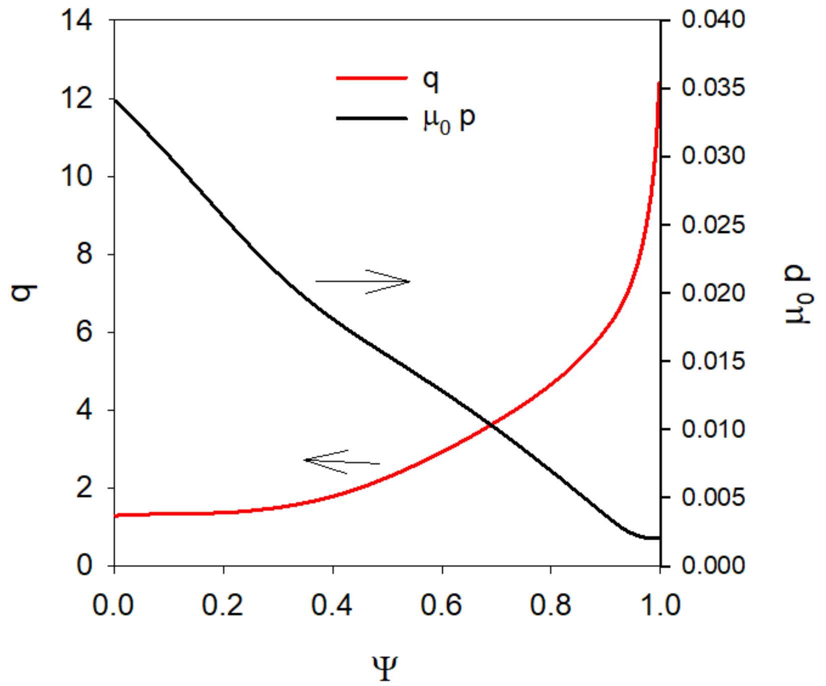
# Consider a typical reconstructed NSTX equilibrium



NSTX Shot 124379 @640 ms

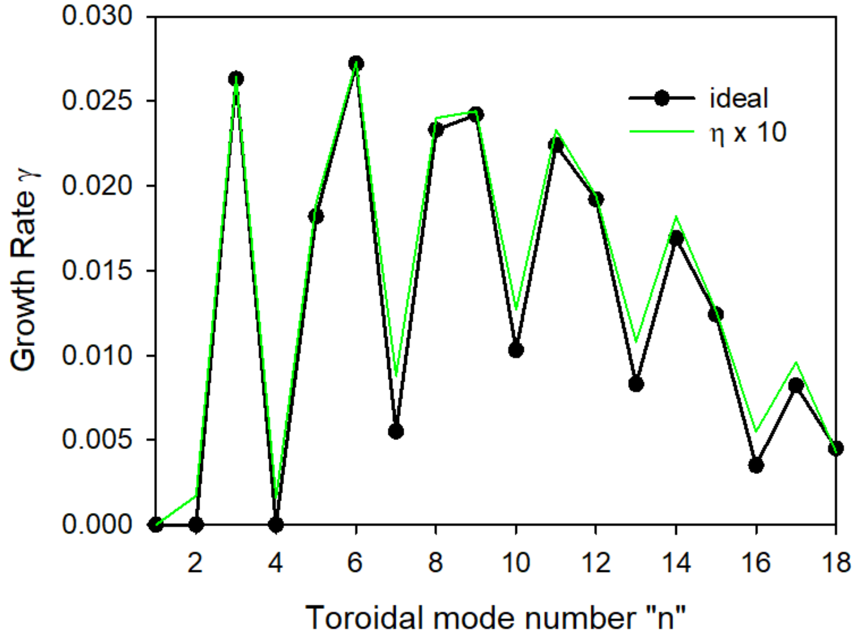
$\beta$ = 6.8%   $\beta_N$ = 3.9   $I_P$ = 990 kA   $RB_T$ = 0.418 T-m   q(0) = 1.29

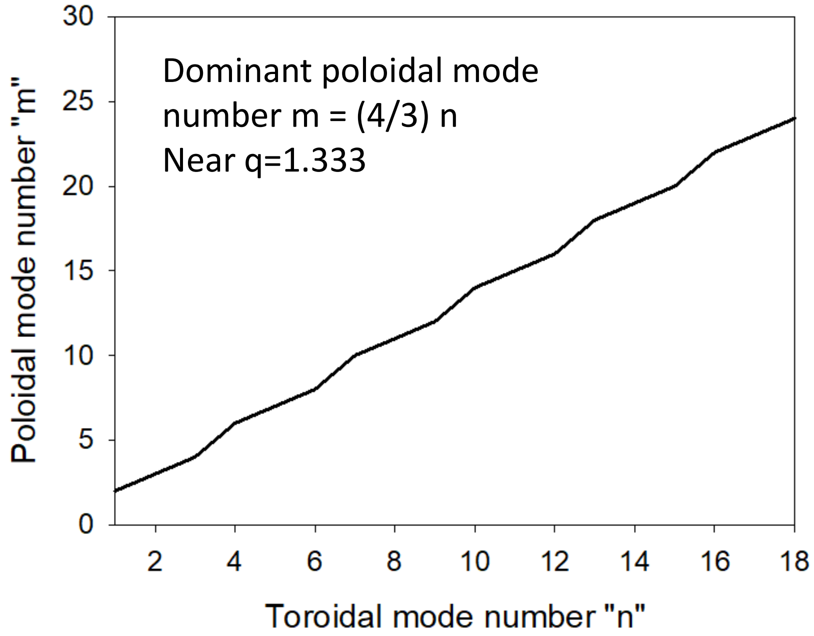


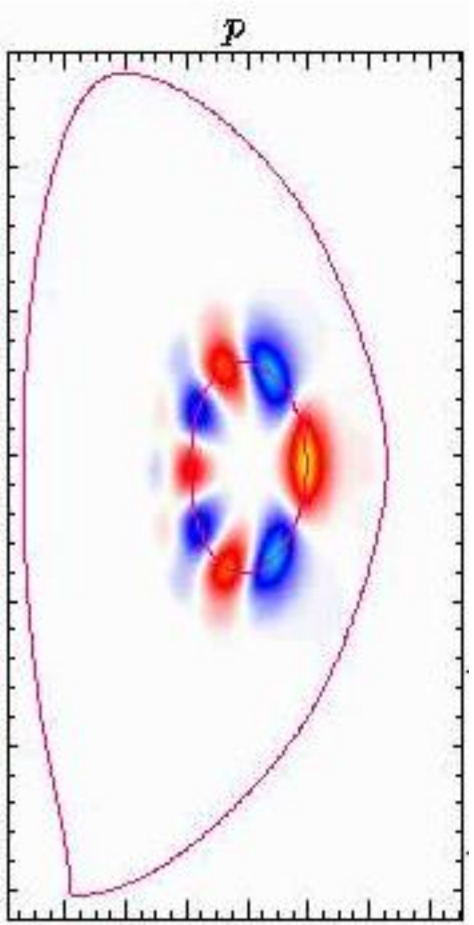

Central temperature :
Te = 916 eV

Spitzer resistivity gives
S = 5 x $10^7$ (at center_

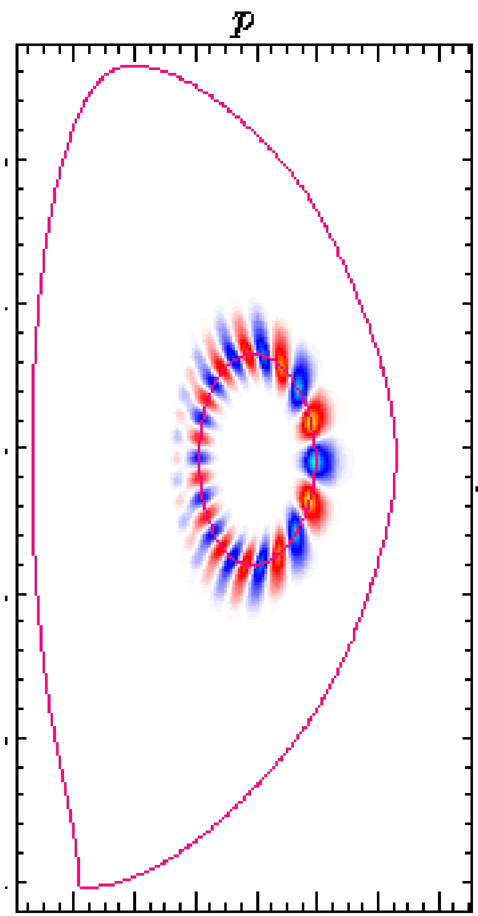
# geqdsk equilibrium linearly unstable to many ideal modes



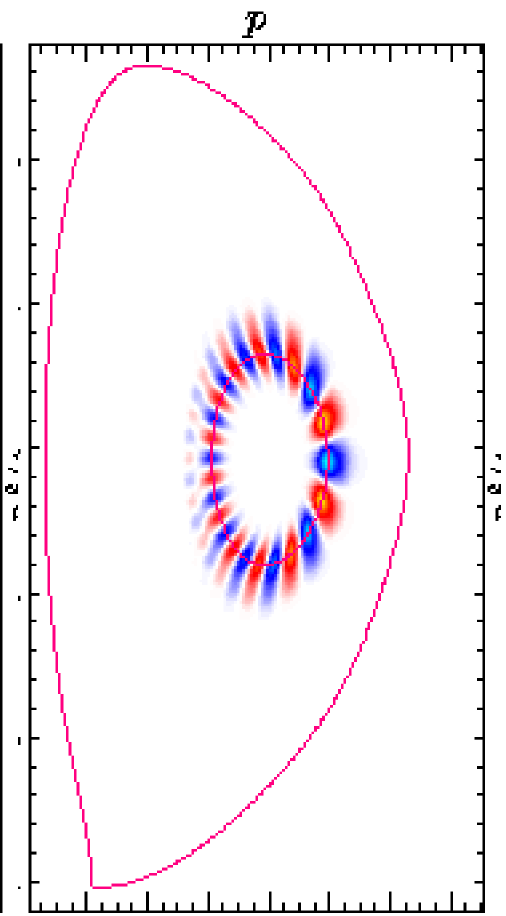



n = 3



n = 9



n = 15

These modes where the growth rate is an oscillatory function of n have been referred to as “infernal modes[1]”

NSTX Shot 124379 @640 ms

[1]Manickam, et al., NF (1987)

# Nonlinear Development of surfaces and temperature

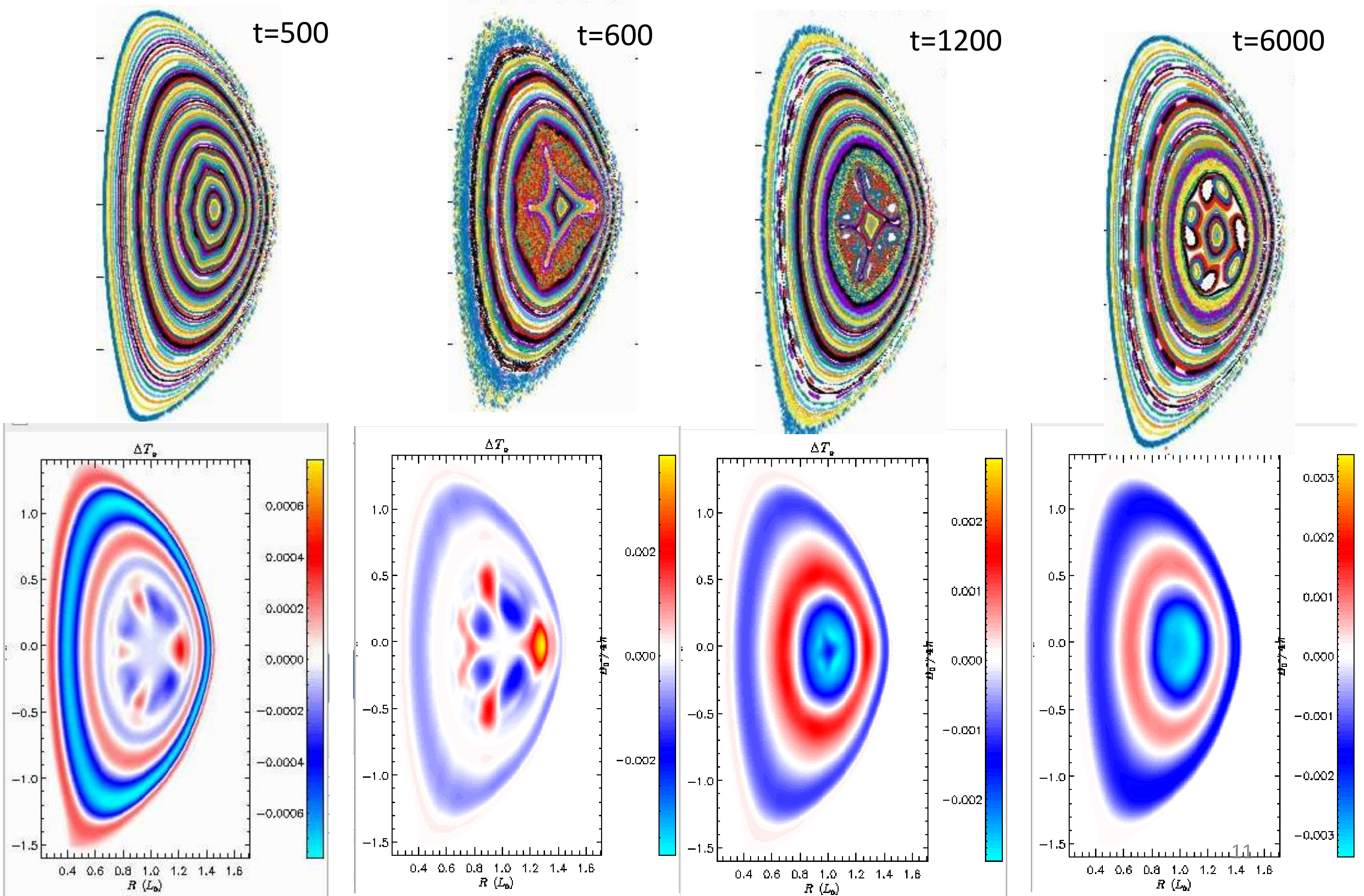
We have run this case non-linearly up to 6000 $\tau_A$ with no sources and small transport coefficients

Poincare Plots →

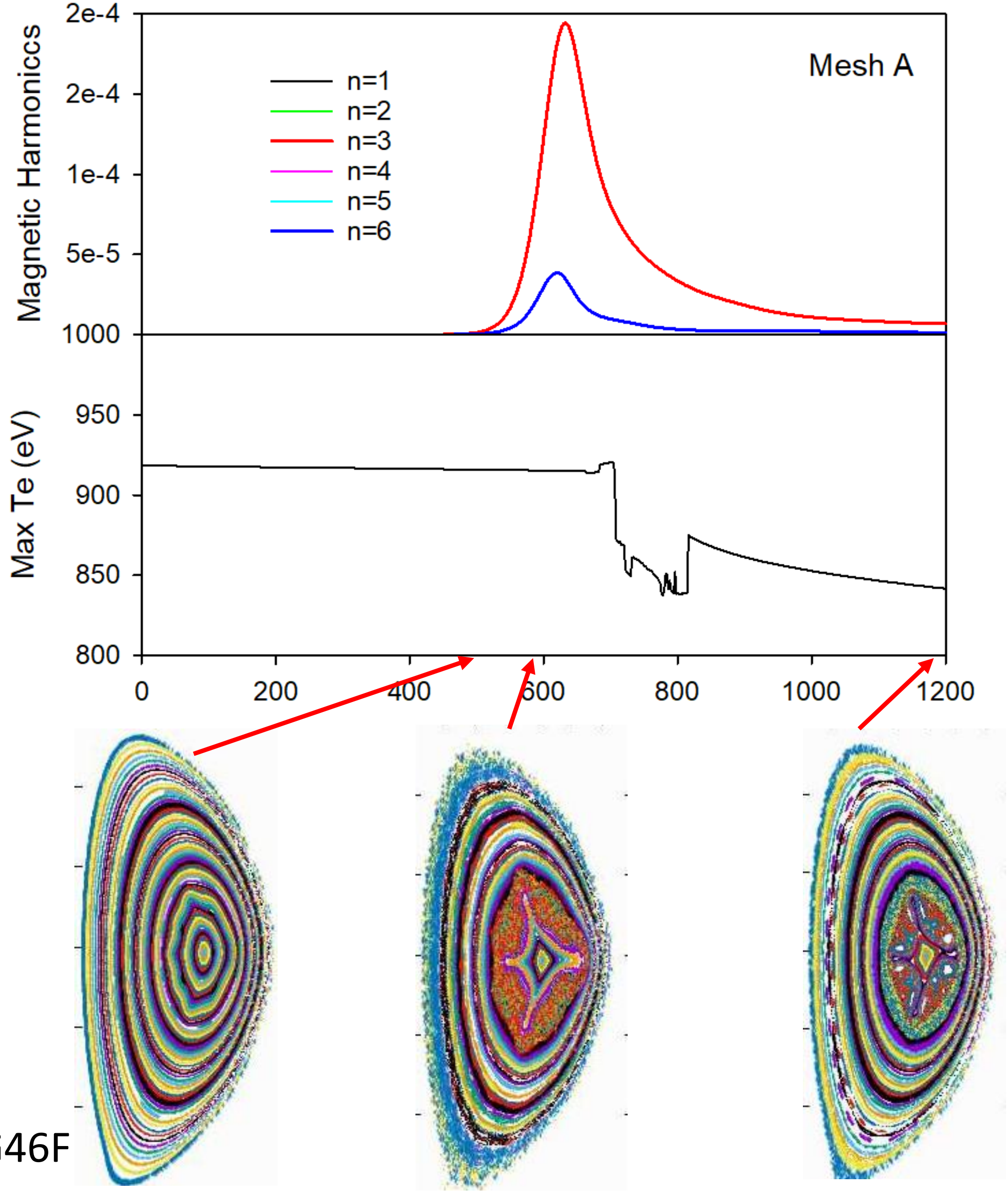
Change in Temperature from time t=0 →

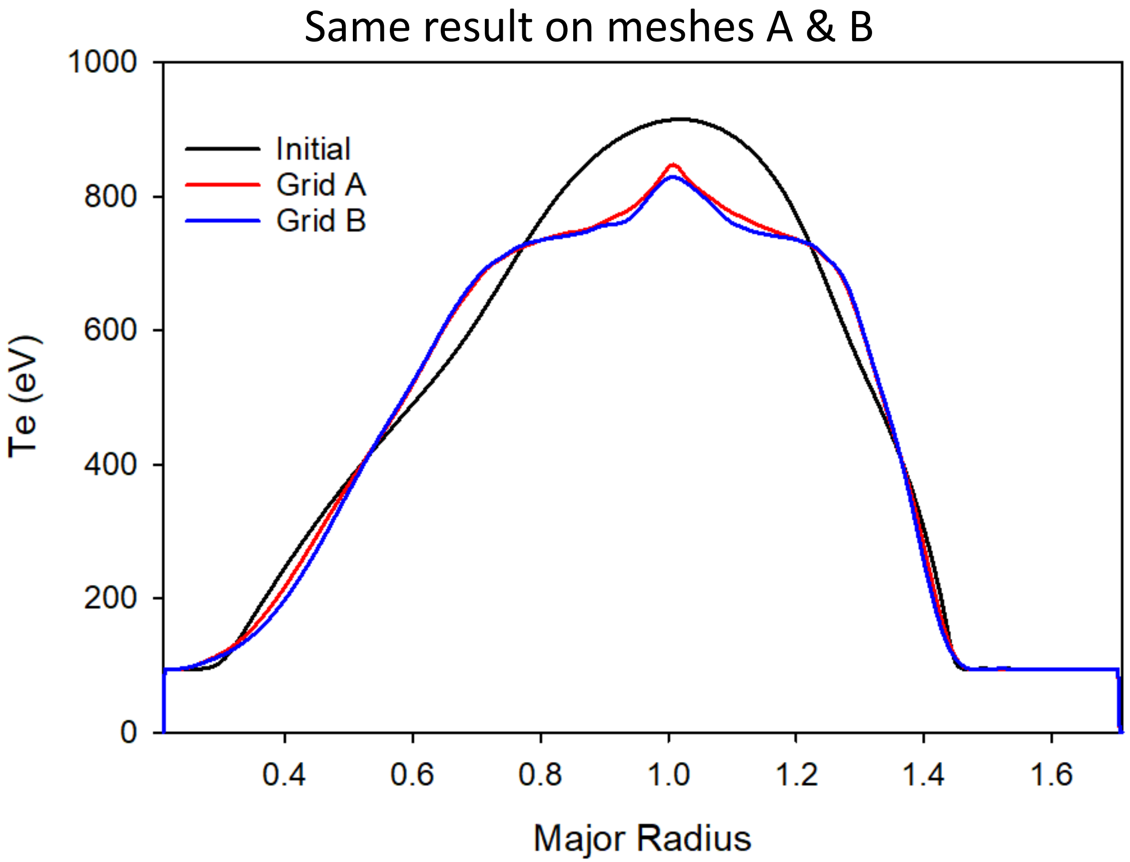
(t = 6000 corresponds to 2.75 ms)

G46F



# Summary of 0 < t < 1200 $\tau_A$

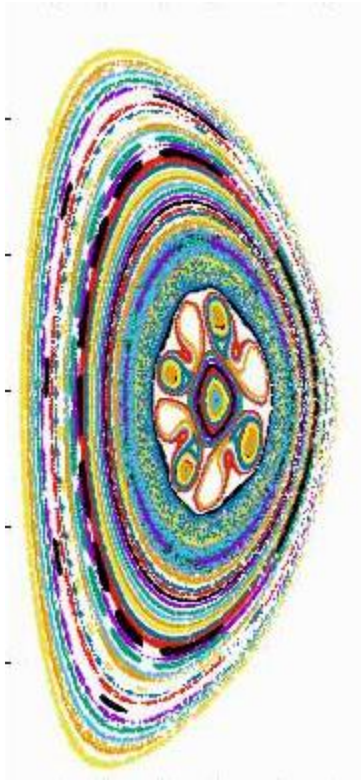


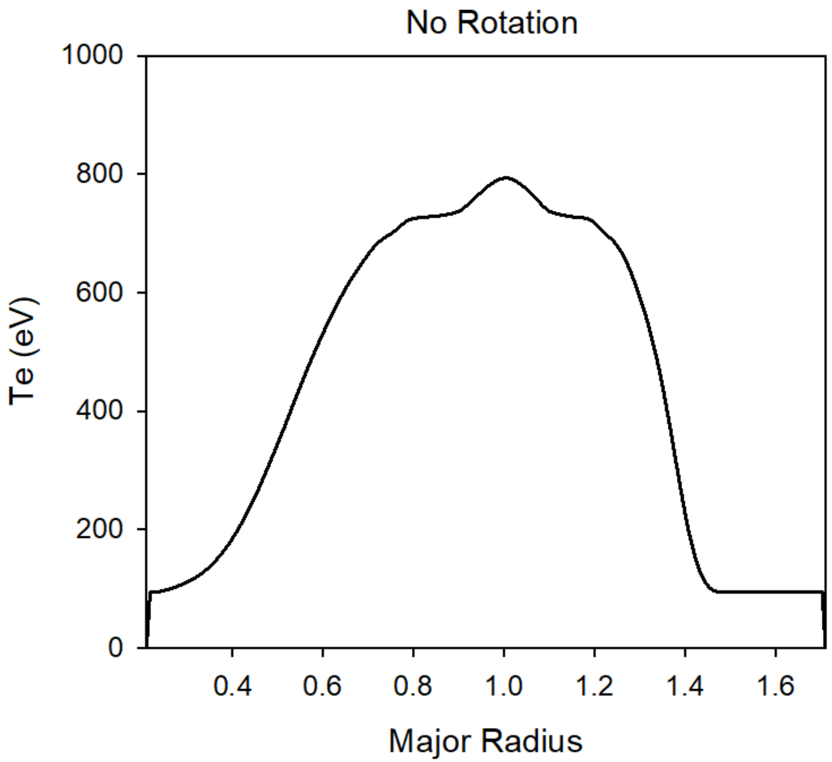



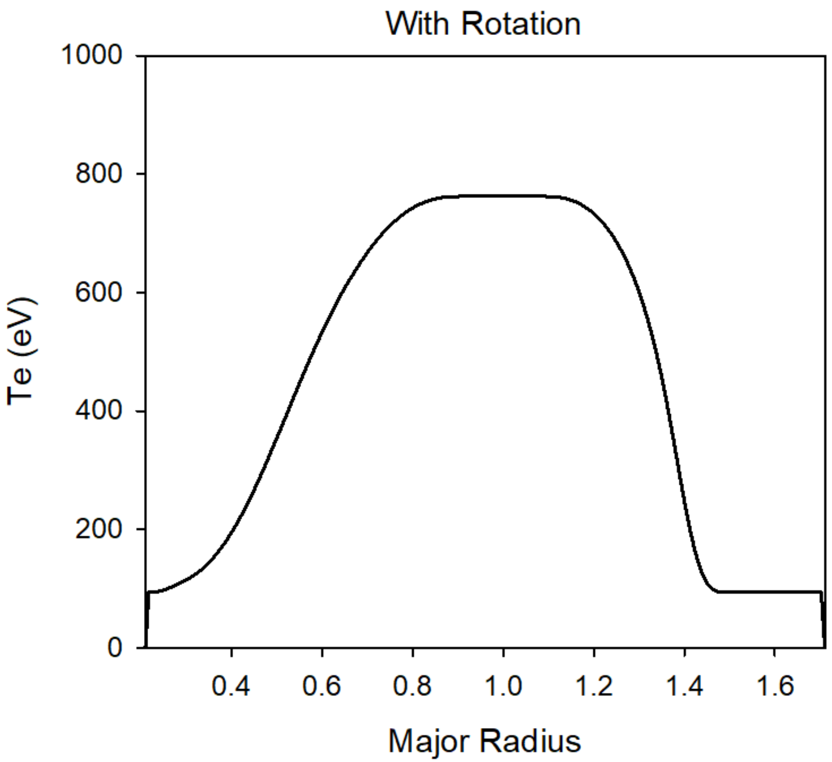

Unstable (4,3) mode grows up, breaks magnetic surfaces near and interior to rational surface, causing central temperature and pressure to decrease, stabilizing plasma

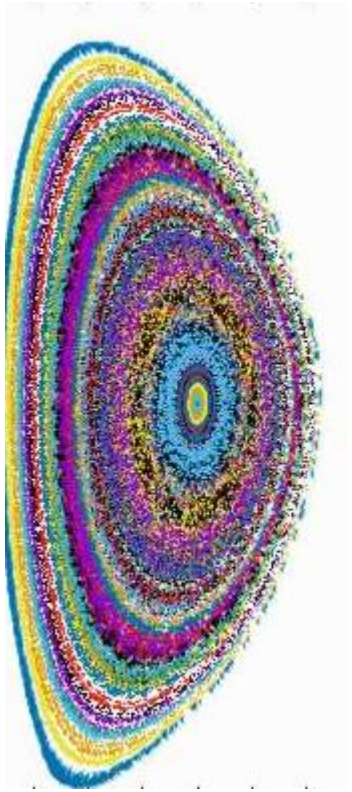
$D_n, \mu, \kappa_e, \kappa_i = 10^{-6} \quad \kappa_{\|e}, \kappa_{\|i} = 10 \quad \eta =$ Spitzer

# With sheared rotation (25 kH in center) results are qualitatively similar





Including sheared rotation smooths final temperature profile (shown at t=3200 $\tau_A$ or 1.90 ms)

G46F - G46F-R

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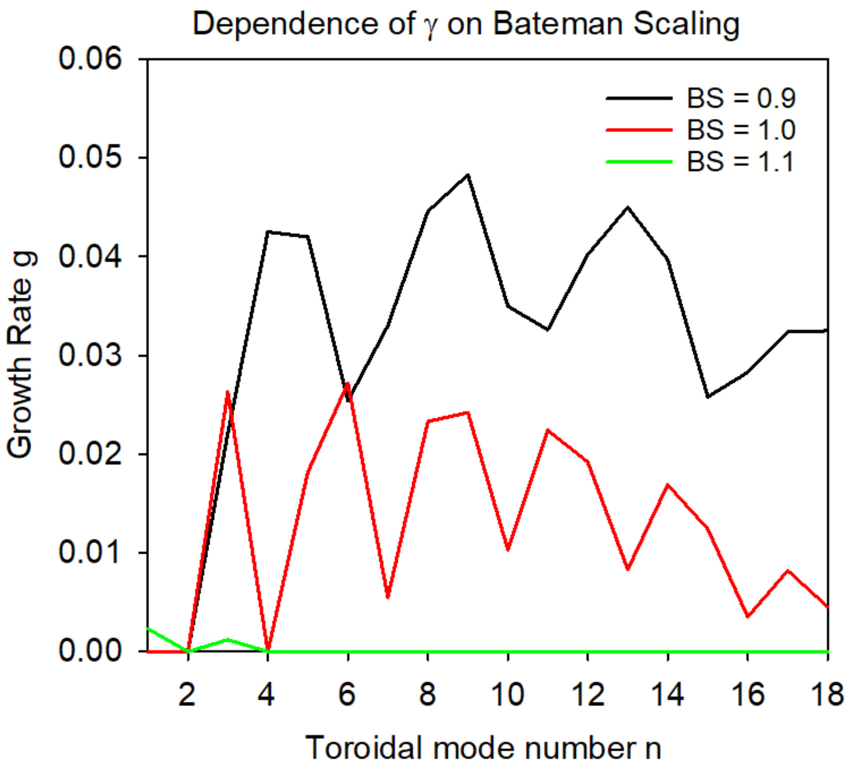
# Generate a family of 3 equilibrium by Bateman scaling

A Bateman scaling factor of 0.9 (10% weaker toroidal field) produces a more unstable equilibrium with q(0) = 1.2 and $\beta$ = 8.2 %

BS = 1.1 (stronger TF) is almost stable to all modes q(0)= 1.4, $\beta$ = 5.8 %

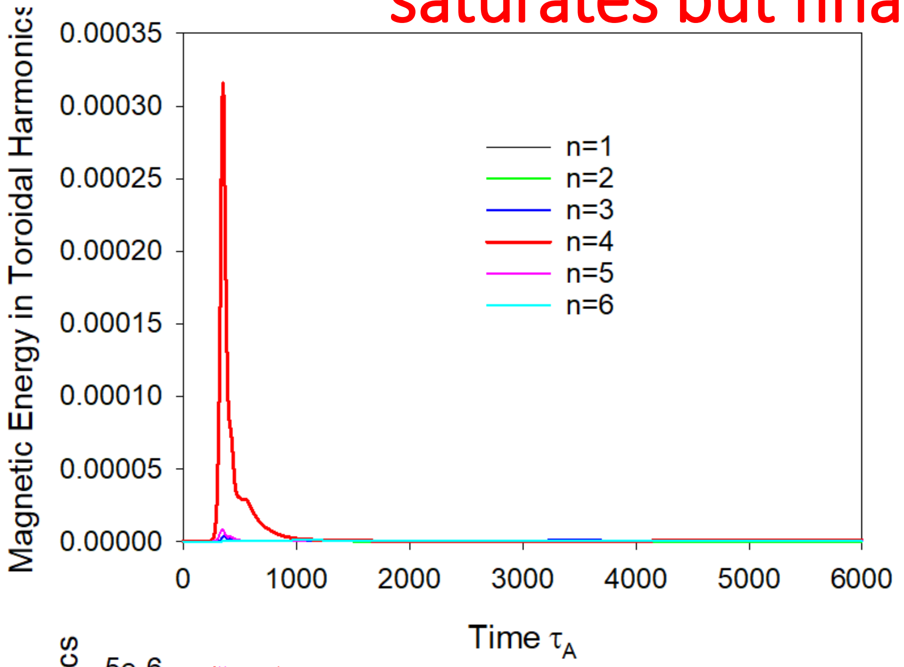
Shown on right are the linear stability properties of the 3 equilibrium

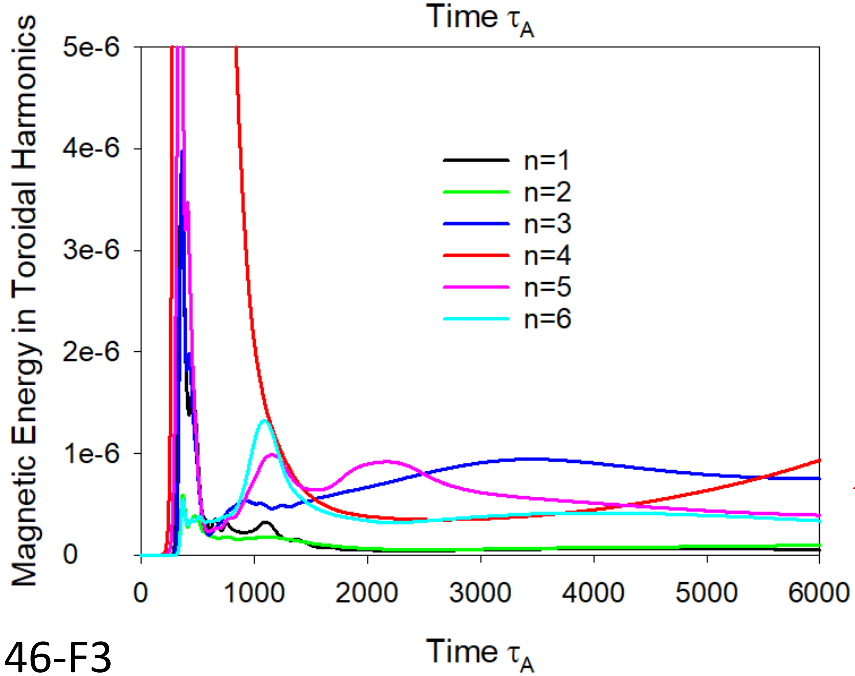


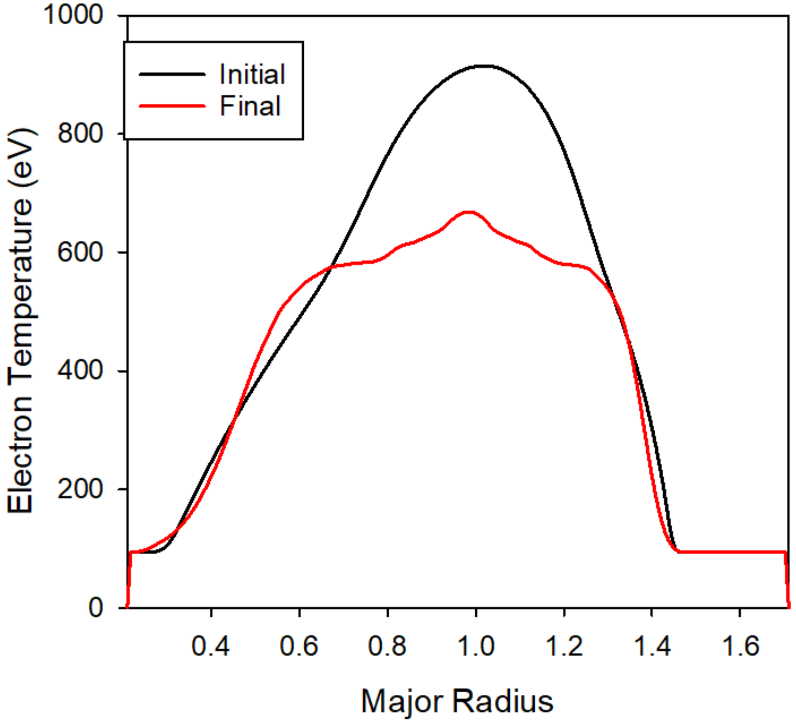

Next, evolve the (more unstable) BS=0.9 equilibrium nonlinearly

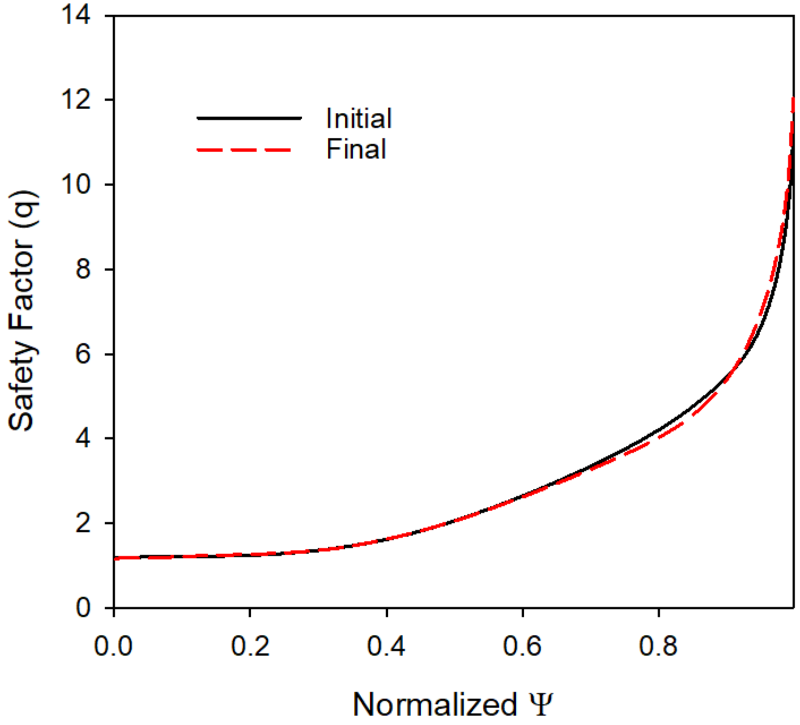
Bateman scaling keeps the current density fixed (P' and FF') but varies the toroidal field to generate a family of equilibrium from a given geqdsk file

# Bateman scaled equilibrium with BS=0.9, q(0) = 1.2, β=8.2% also saturates but final state has multiple n modes (5,4) most unstable




Note:

- n =3,4,5,6 toroidal harmonics all comparable at end
- q-profiles does not change during evolution
- Axis temperature greatly reduced during evolution

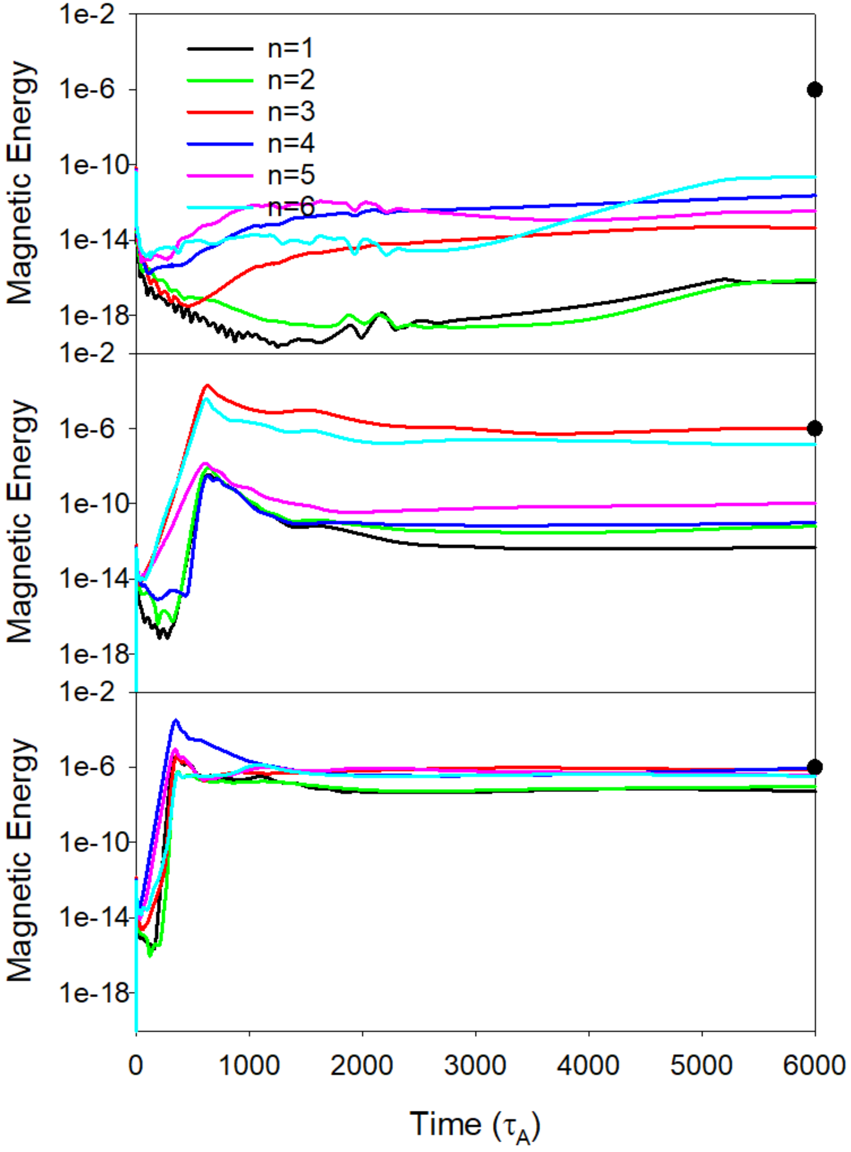
G46-F3

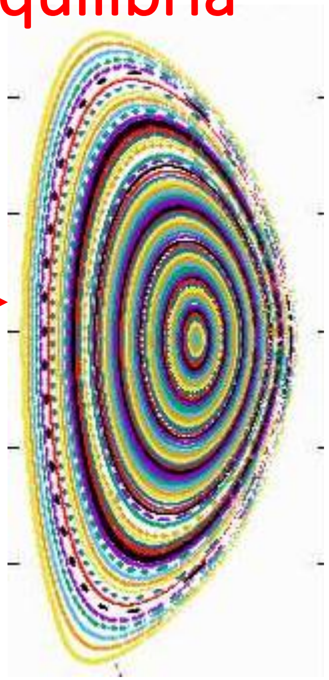
# Comparison of the time evolution of the 3 scaled equilibria

BS=1.1
β=5.8%

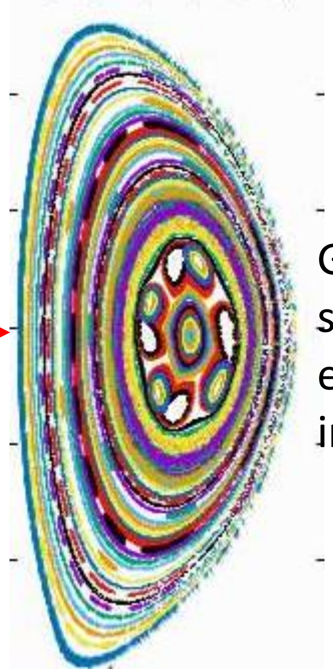
BS=1.0
β=6.8%

BS=0.9
β=8.2%

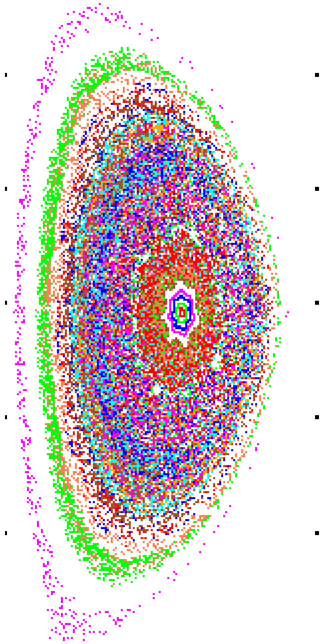


Good surfaces everywhere



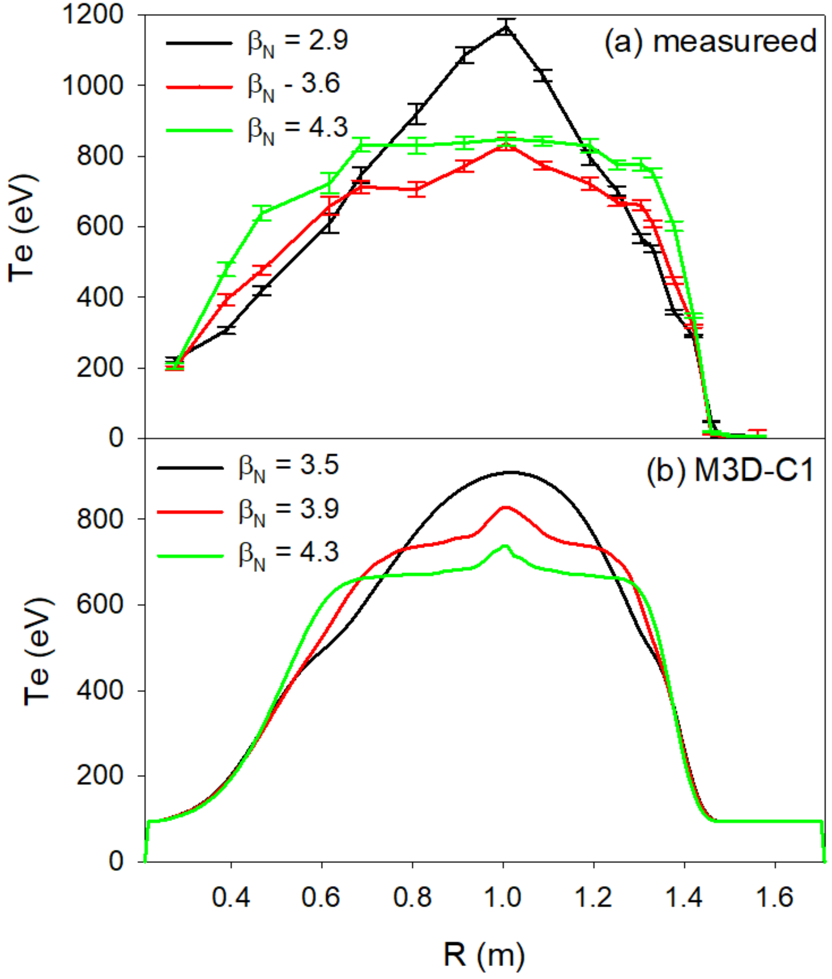
Good surfaces except n=3 in center



Poor surfaces with multiple n-modes

G46F4 - G46F – G46F3

# Comparison of 3 Stutman shots and 3 BS equilibria

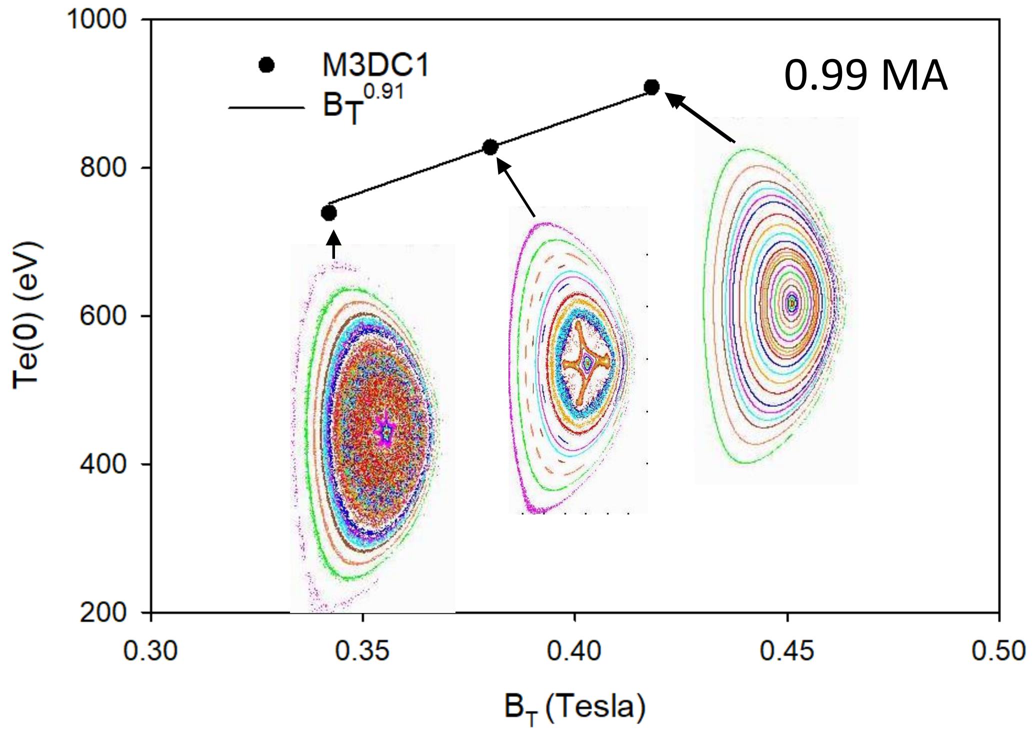



- Top is the 3 shots analyzed in the Stutman PRL
- Bottom are the 3 Bateman-scaled equilibria nonlinearly evolved with M3D-C1
- These were not the same shots, but the trends are similar
- Te most peaked at low $\beta_N$. Increasing $\beta_N$ results in broader profiles

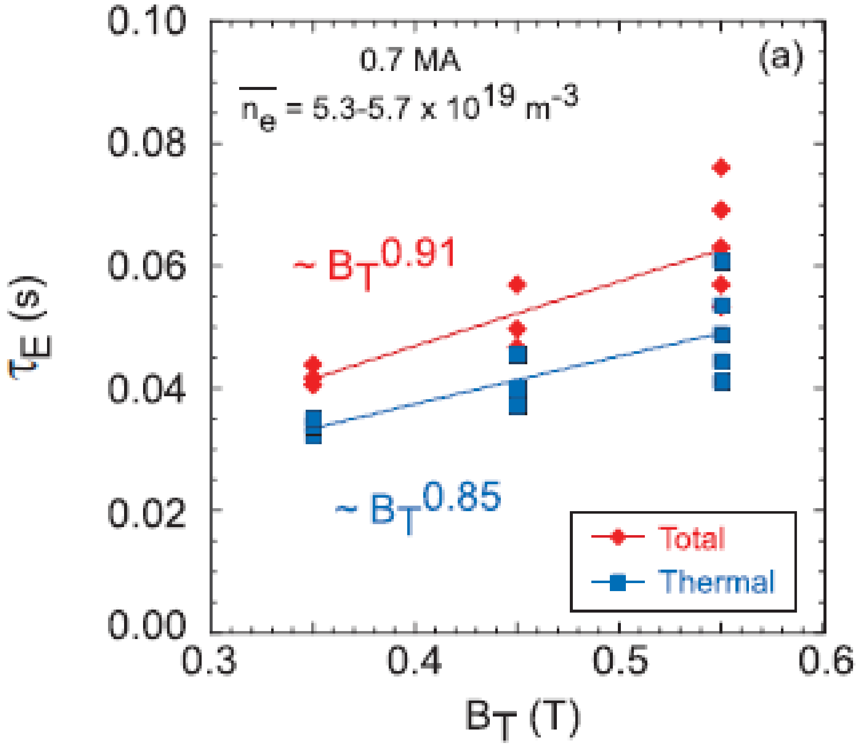
Stutman, et al. PRL (2009)

G46F4 - G46F – G46F3

# M3D-$C^1$ shows similar scaling with $B_T$ as experiment




Kaye, et al, PRL (2007)




Note: Plot on left is Te(0). On the right is $\tau_E$

"Some of the discharges in this study did exhibit both low amplitude low-n MHD activity as well as the fast ion driven Alfven eigenmode (AE) activity,"



G46F4 - G46F – G46F3

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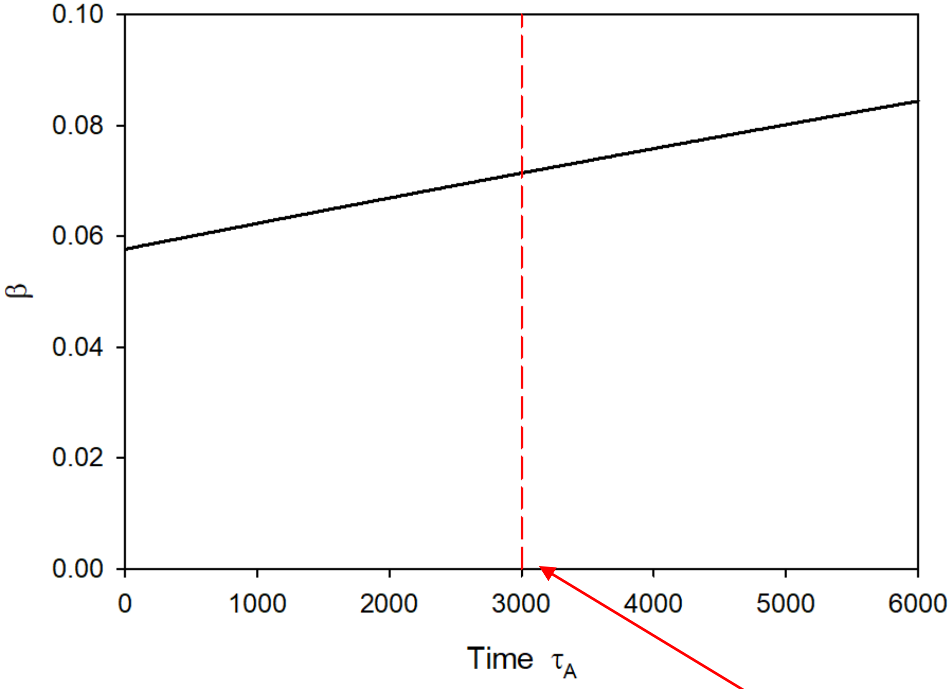
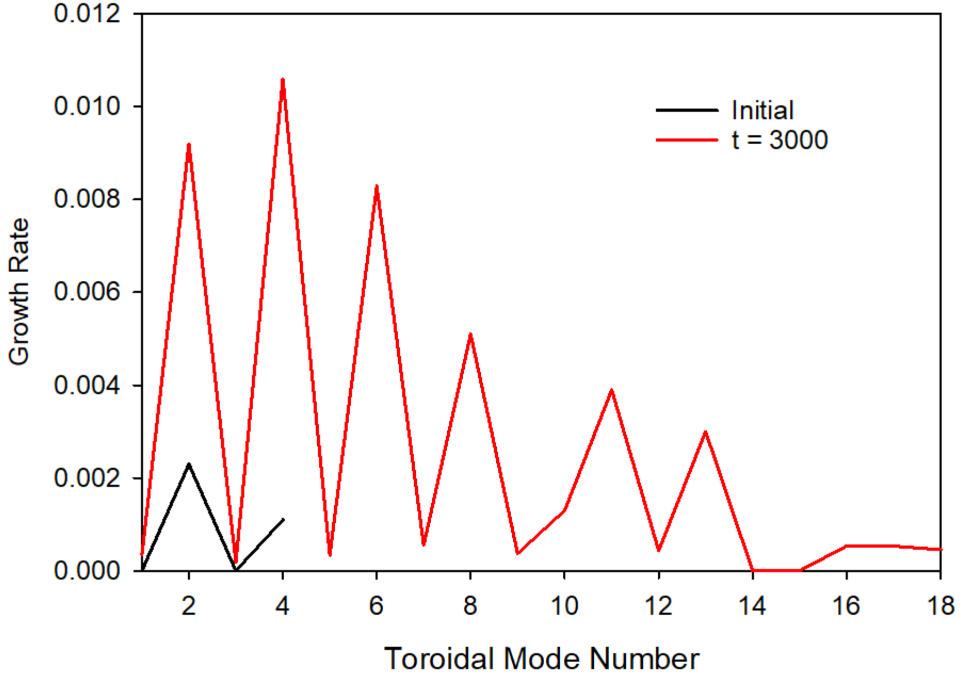
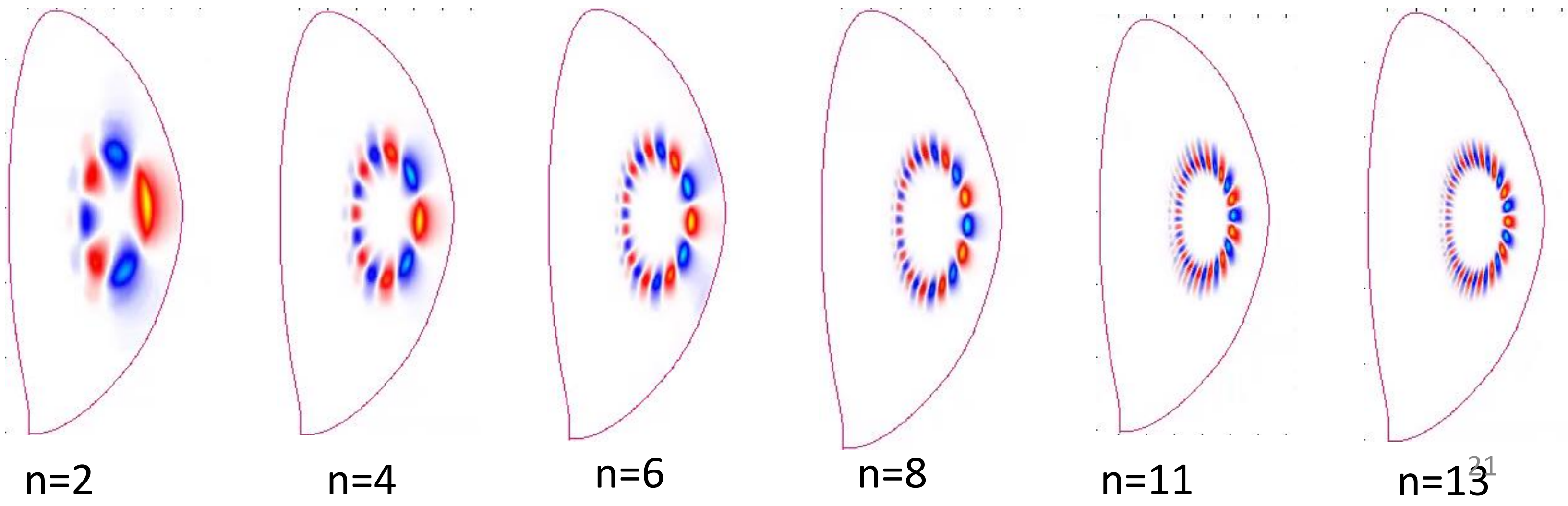
# More realistic:  Start with stable equilibrium and apply heating power: First in 2D



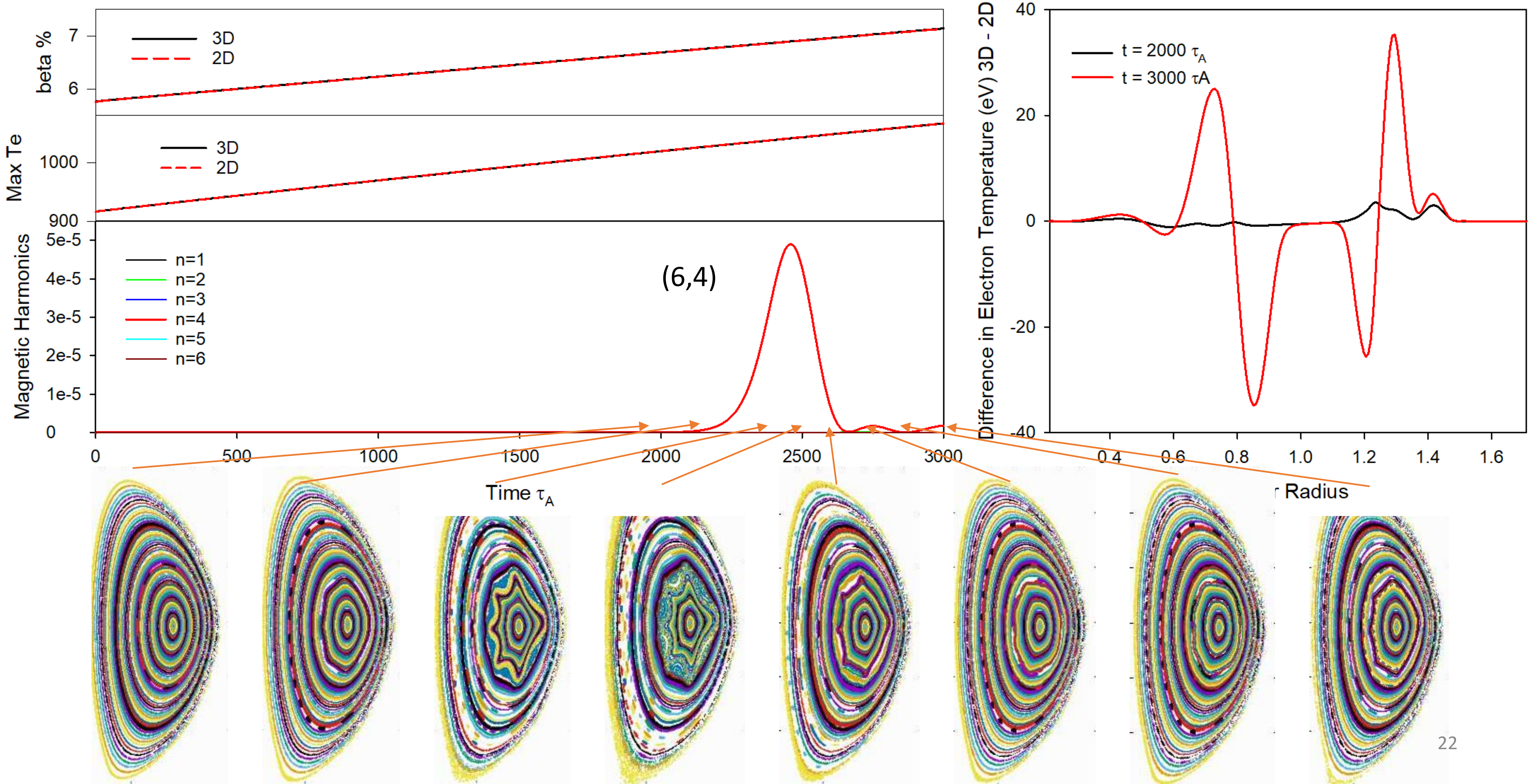

- Start with stable Bateman scaled equilibrium with β = 5.8%
- Run in 2D with heating source, increasing β to 8% at t=6000
- Linear analysis shows intermediate equilibrium unstable to many modes (shown on right)
- Now repeat with 3D run.  Do these saturate nonlinearly?



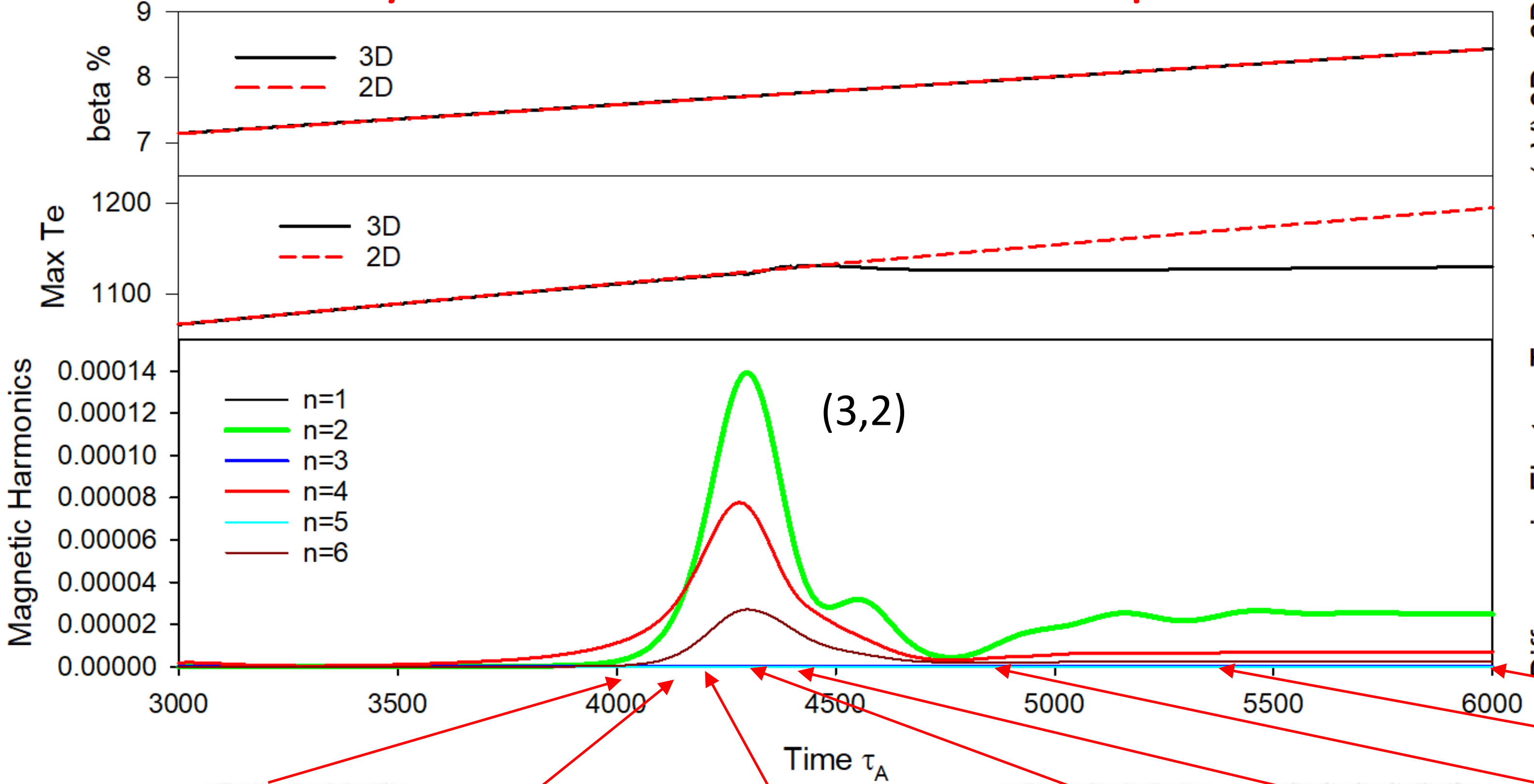

G46F4-H-2D

# Results (in 3D) will be shown in 2 phases
# In first phase, 0< t < 3000 $\tau_{A,}$, β increases to 7%

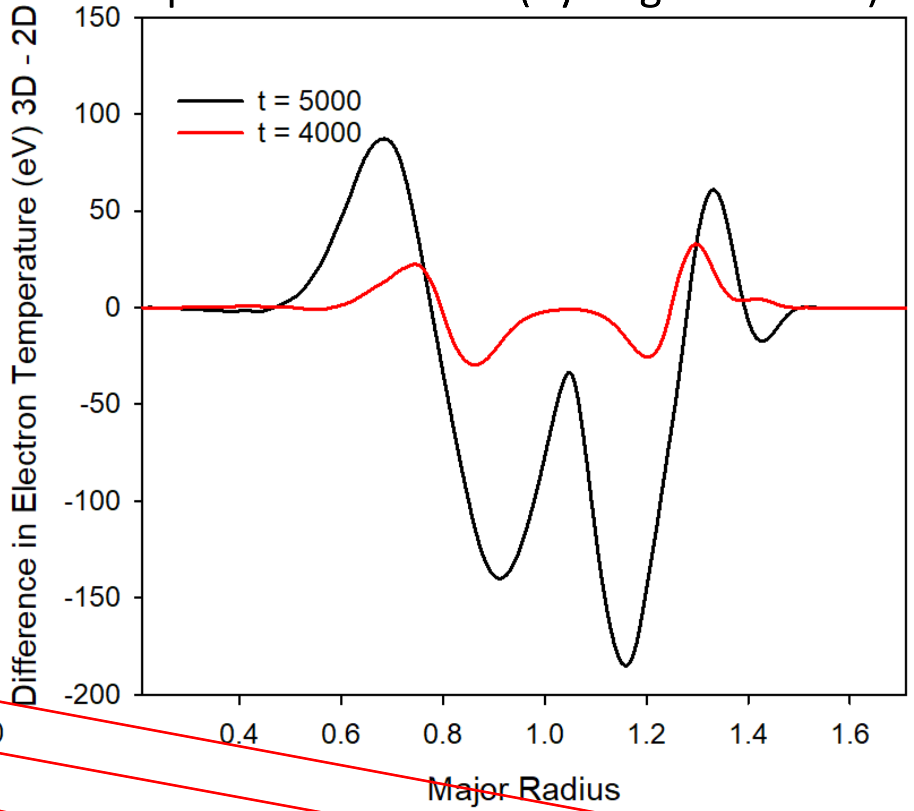
In first phase, only minor temperature transport near q=1.5

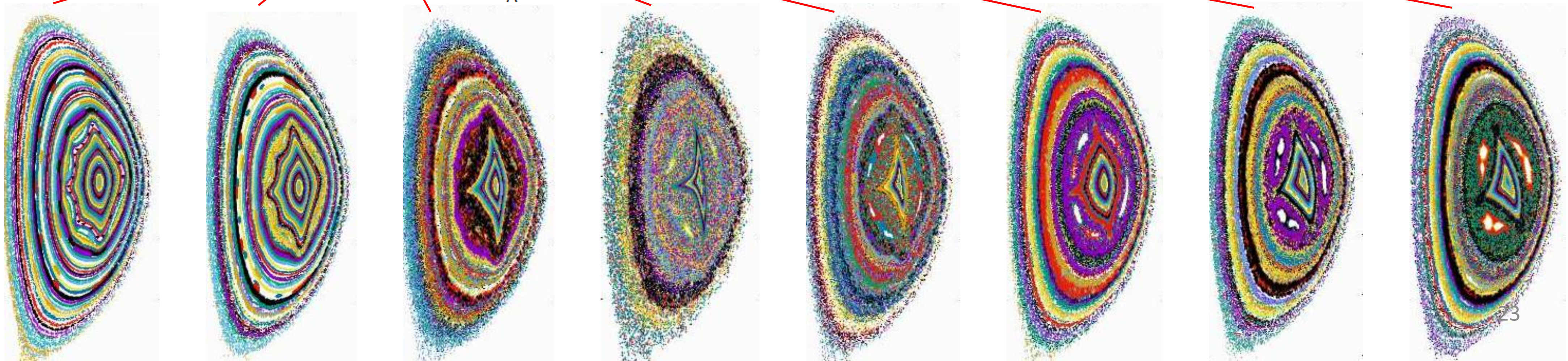


# In second phase, β increases to over 8%
# Summary of 3000 < t < 6000:  Note drop in 3D Te



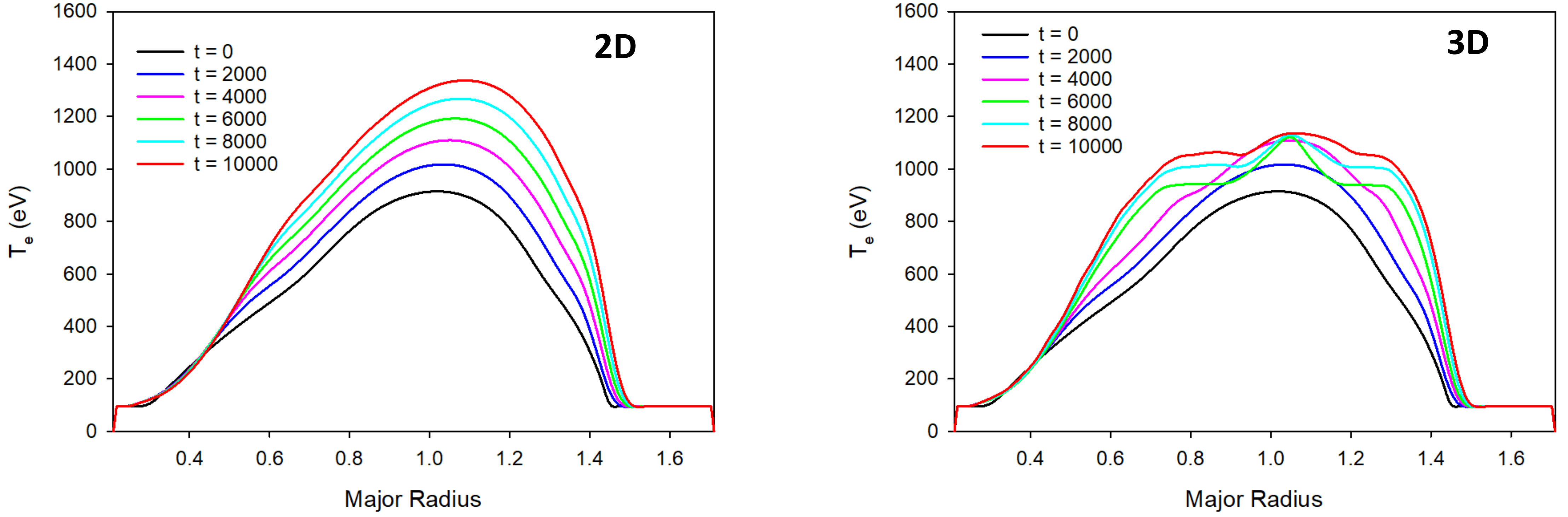

In this phase, all surfaces interior to q=1.5 are affected (by larger amount)



G46F4-H2
G46F4-H2-2D

# Summary of Temperature Profiles



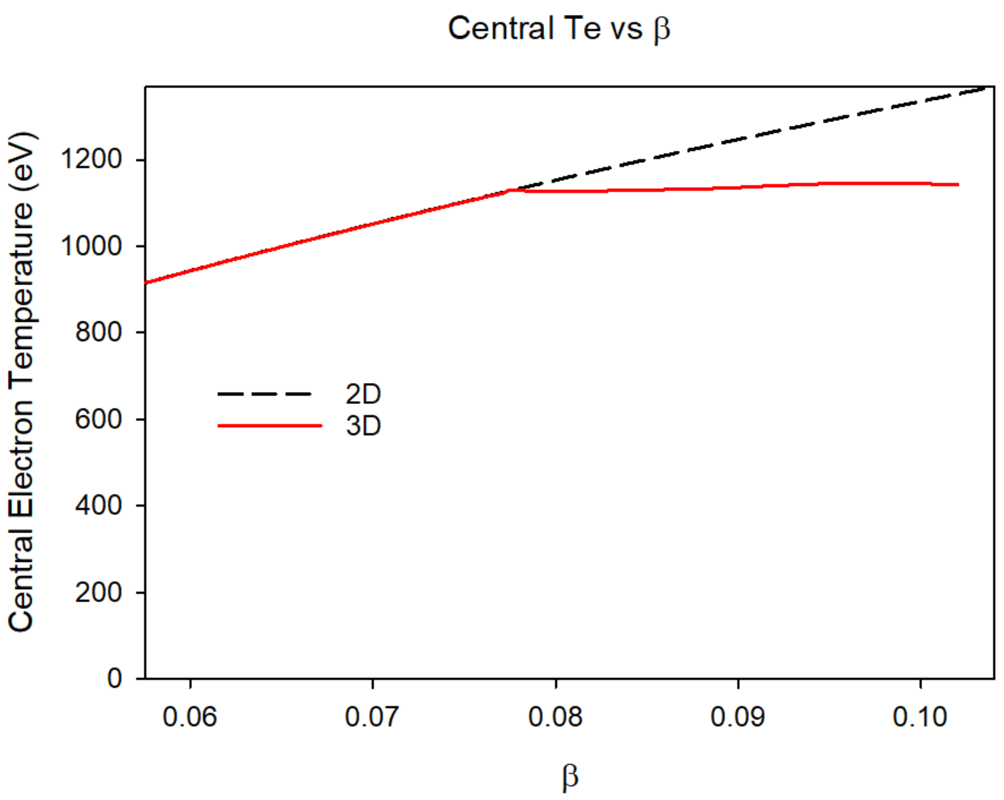

After t=4000, 3D central temperature no longer increases, but temperature profile broadens.

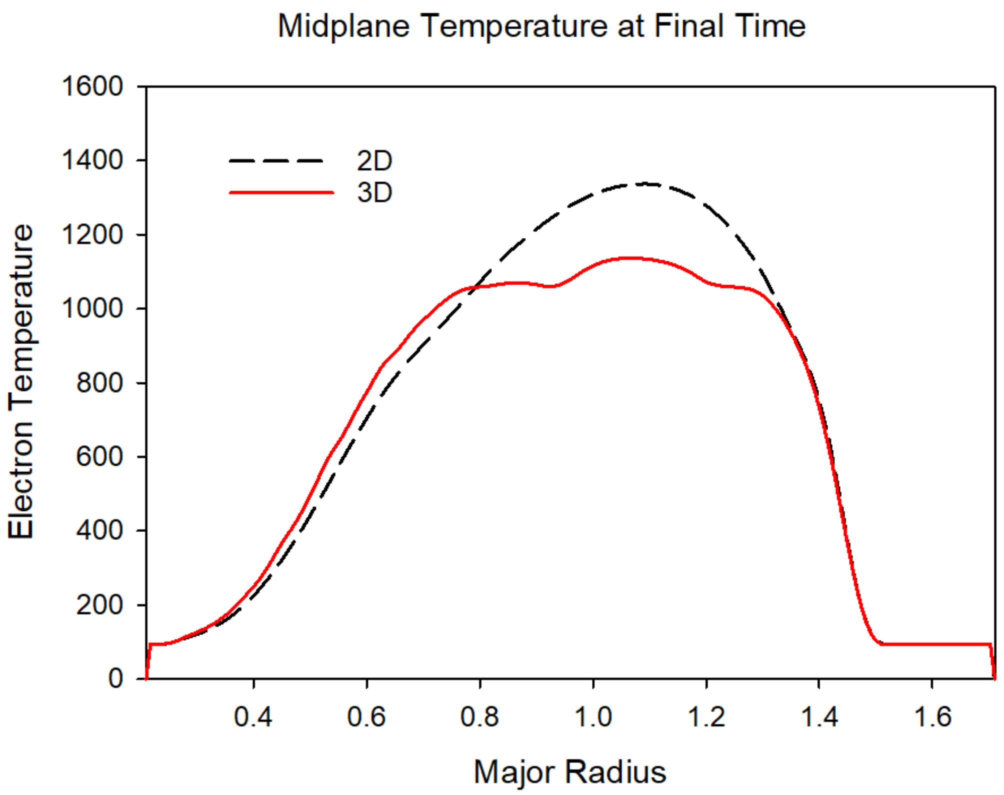
In 2D, central temperature continues to rise.



G46F4-H2 G46F4-H2-2D

# Plot of Central Te vs β during heating sequence – 2D vs 3D




Central temperatures in 2D and 3D are the same for β < 8%.   For β > 8%, Te(0) does not increase with β



G46F4-H2     G46F4-H2-2D

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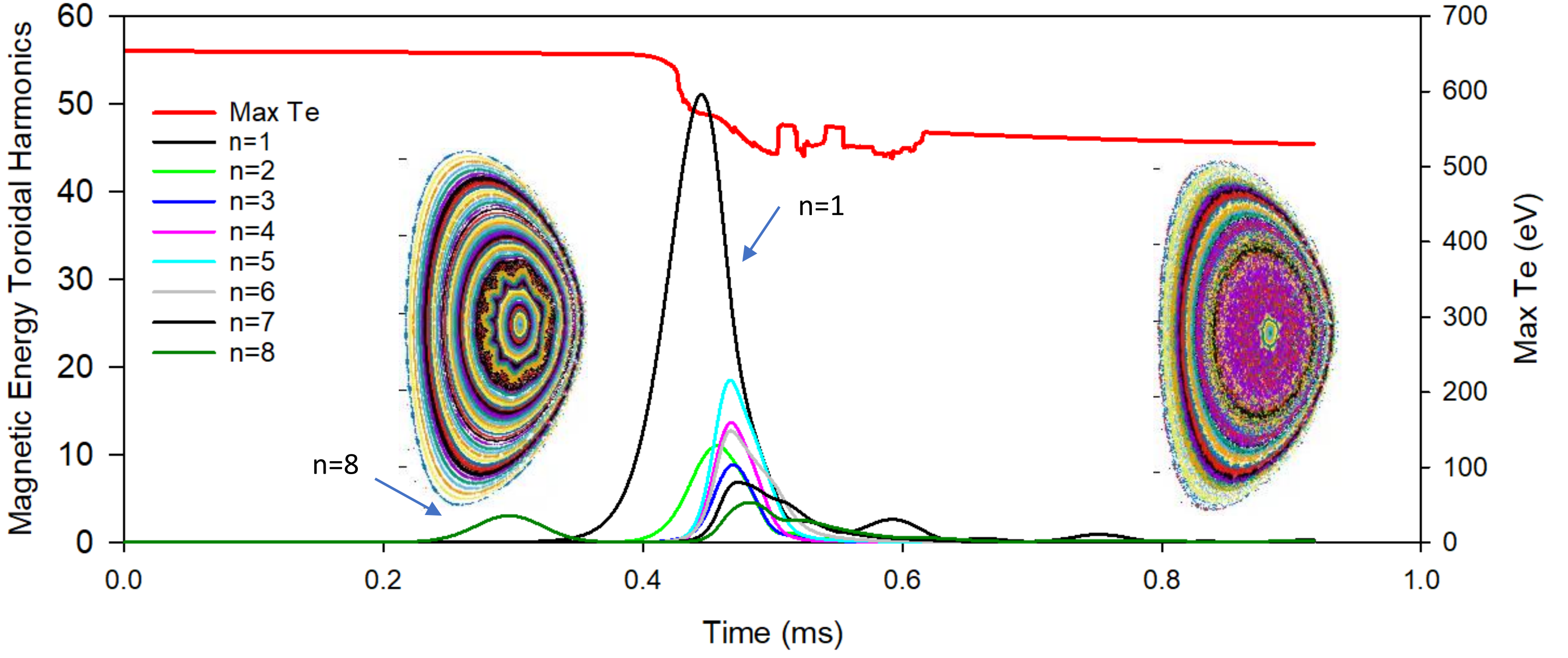
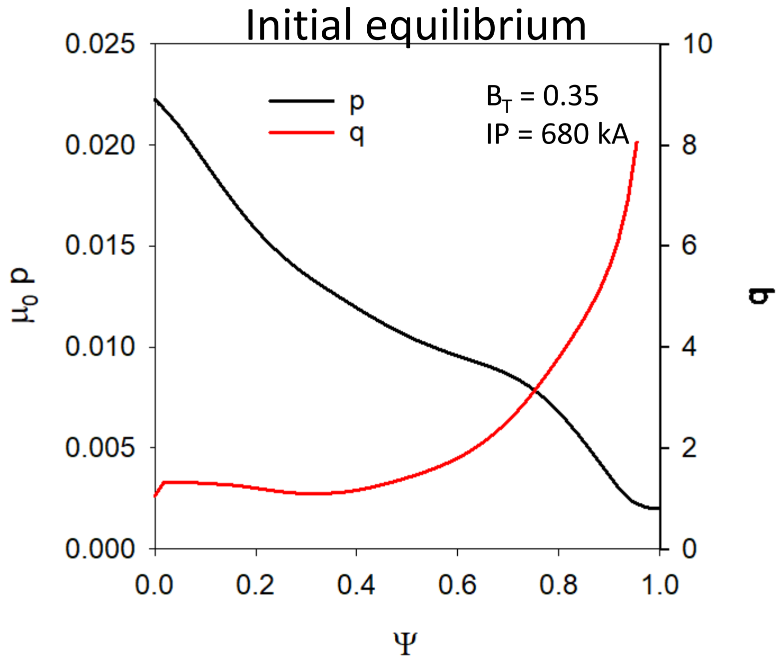
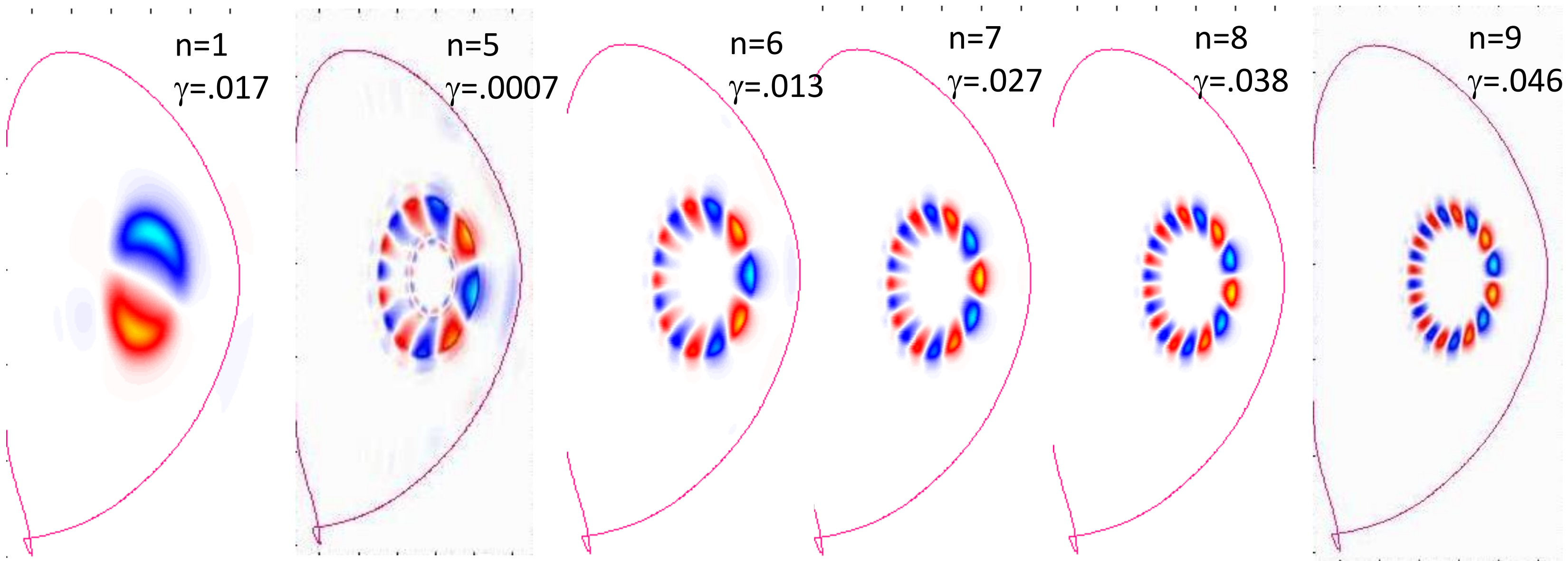
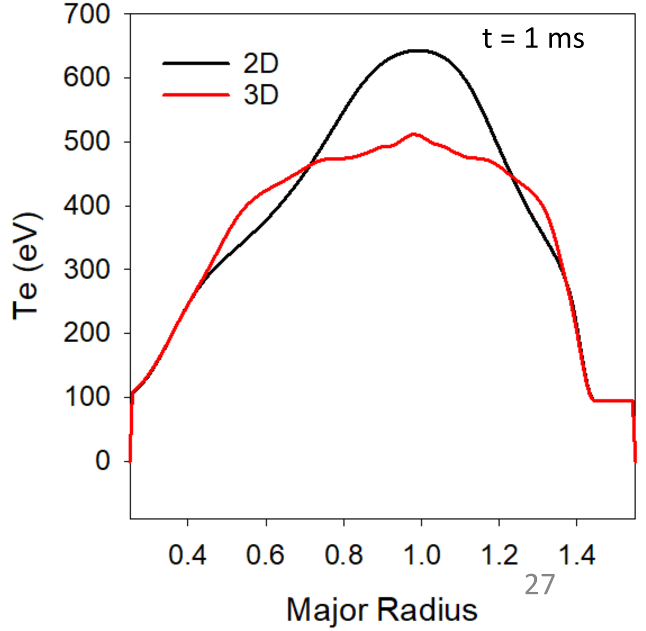
# Shot 121014@0.51 s (From S. Kaye series of $\tau_E$ scaling with $B_T$)

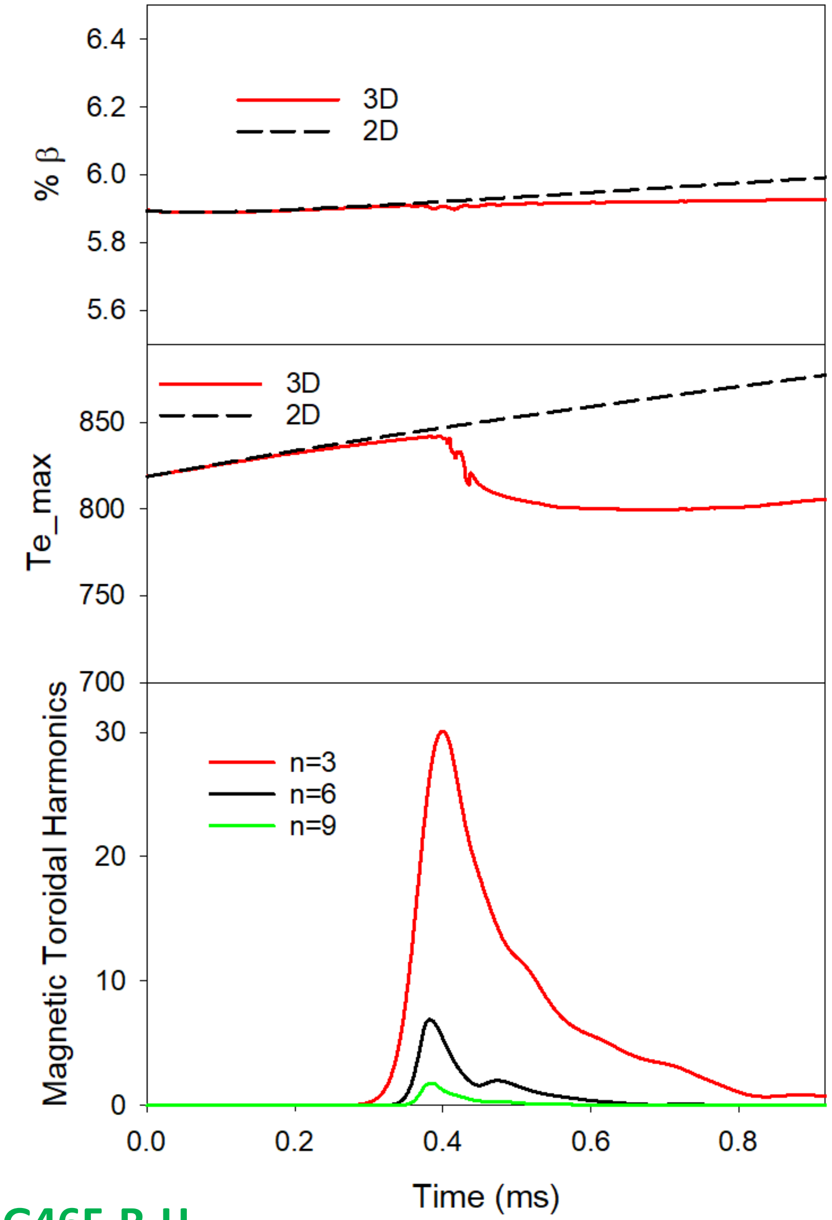


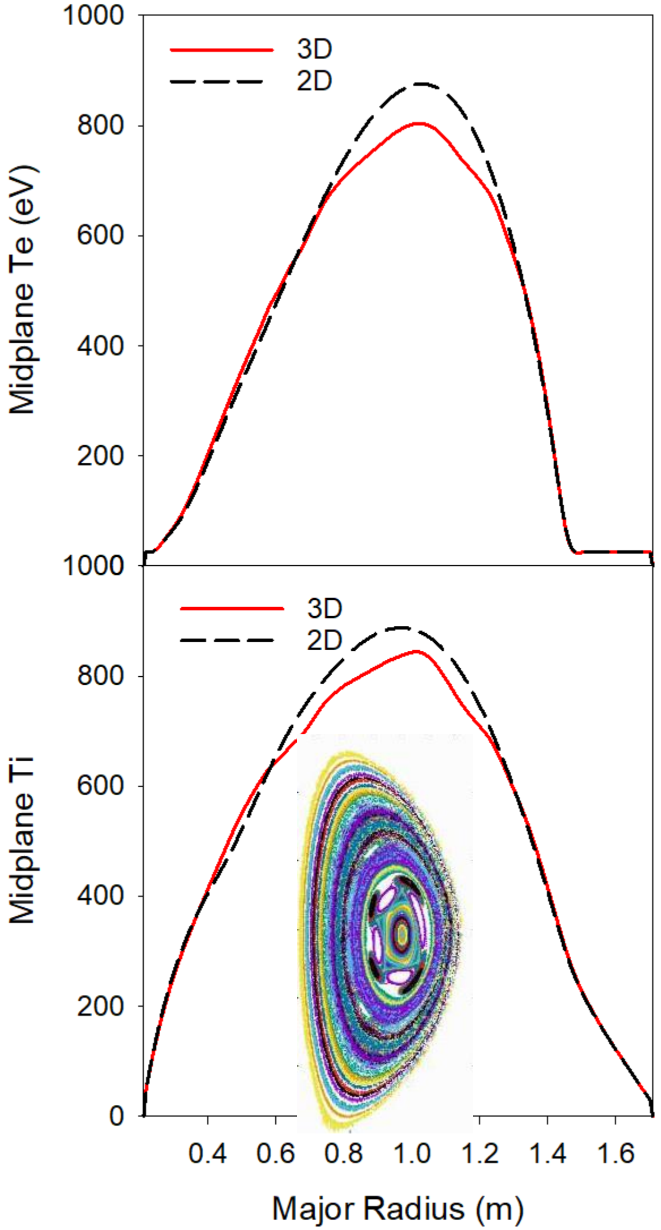



# Near stationary state sequence with rotation




- BS=1 case with low heating power and torque drive
- $\kappa_\perp$ = 1.e-5, $\kappa_{||}$ = 10 (Te only)
- Strengths of sources chosen to make 3D case approximately stationary
- Comparison of 2D and 3D case with same transport coefficients shows affects of 3D instability
- In 3D, β is slightly lower and Te(0) is significantly lower
- Te more affected than Ti



G46F-R-H

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# Does this occur in high-β discharges in conventional aspect ratio tokamaks?

- DIII-D, Turnbull, et al. Fusion Science and Tech. **48** (2005)
  - Infernal modes unambiguously identified in high $\beta_P$ discharges, inversely correlated with periods of improved confinement
- JET, Charlton, et al, Nucl Fusion **31** 1835 (1991)
  - Pellet fuelled shots with peaked pressure profiles terminated by an abrupt flattening of the temperature profile. (3,2) infernal mode when q(0) drops below 1.5
- TFTR, Chang, et al, Nucl. Fusion **34** 1209 (1995)
  - Supershot performance degradation in presence of (3,2) and (4,3) macroscopic modes
- JT-60, Ozeki, Nucl. Fusion 35 861 (1995)
  - In high $l_i$ plasmas with peaked pressure, the stability limit is determined by infernal modes in the low $q_0$ regime

# Summary

- NSTX equilibrium 124379 @640 ms found to be linearly unstable to ideal MHD modes that saturate at modest amplitude with small non-axisymmetric n=3 toroidal harmonic that flatten Te

- Higher beta equilibrium obtained by Bateman scaling are much more unstable linearly, and saturate with n=3,4,5,6 toroidal harmonics which lead to stochastic surfaces and more Te flattening

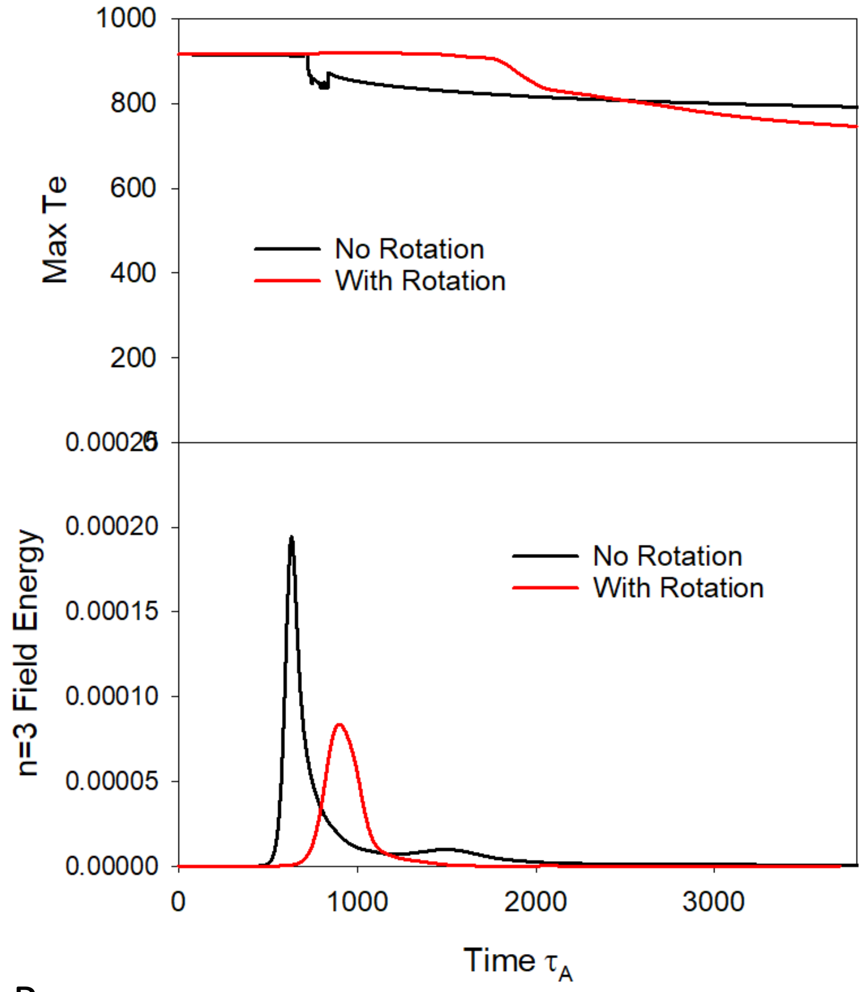
- Calculations performed with actual Spitzer resistivity, $S = 5 \times 10^7$ and with variable fine meshes

- More realistic calculations start with lower-beta (stable) equilibria, apply heating source to drive it through the beta limit. Stays at marginal stability in center by broadening the temperature profiles and distorting the flux surfaces..

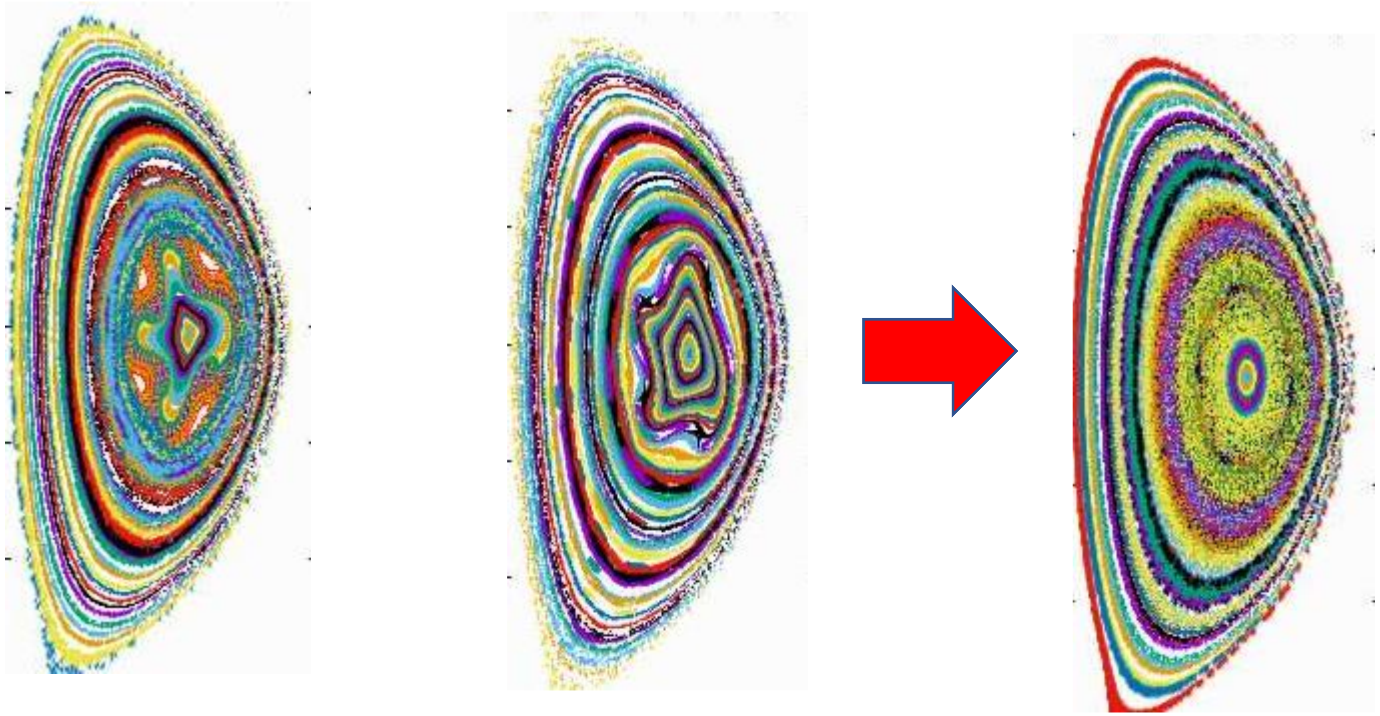
- Future:
  1. We are examining other high-$\beta$ NSTX equilibria to see how universal this effect is? Are most high $\beta$ equilibrium in a nonlinearly saturated state? Implications for transport analysis.
  2. Longer term: Role of energetic particles with M3D-$C^1$-K
  3. What does this say about preferred operational regimes in NSTX-U? Can we operate the machine in such a way as to minimize the deleterious effects of these modes?

# Extra VGs

# Effect of Sheared Rotation ~ 20 kHz in Center



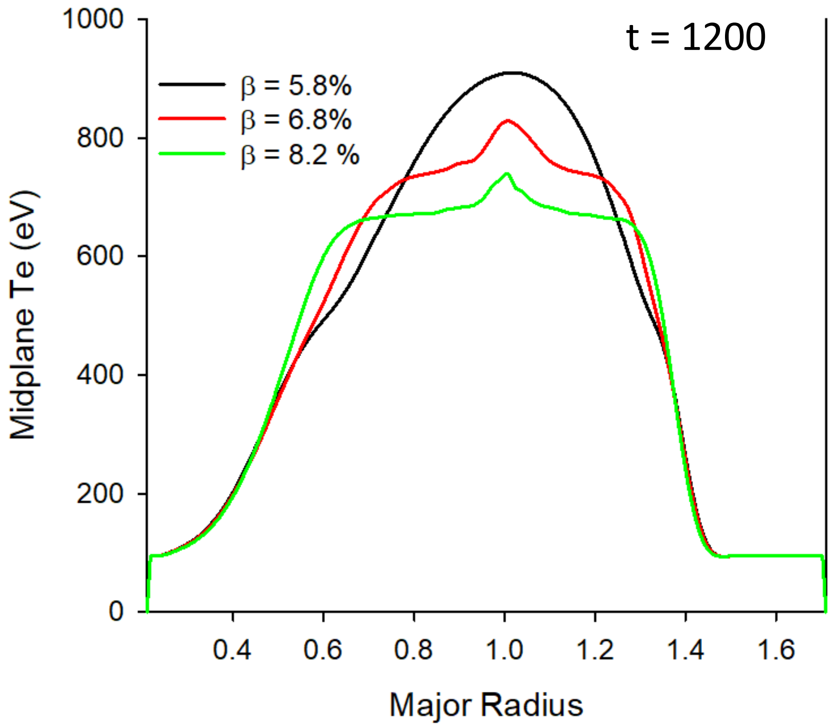

t = 800    t = 900    t = 3300

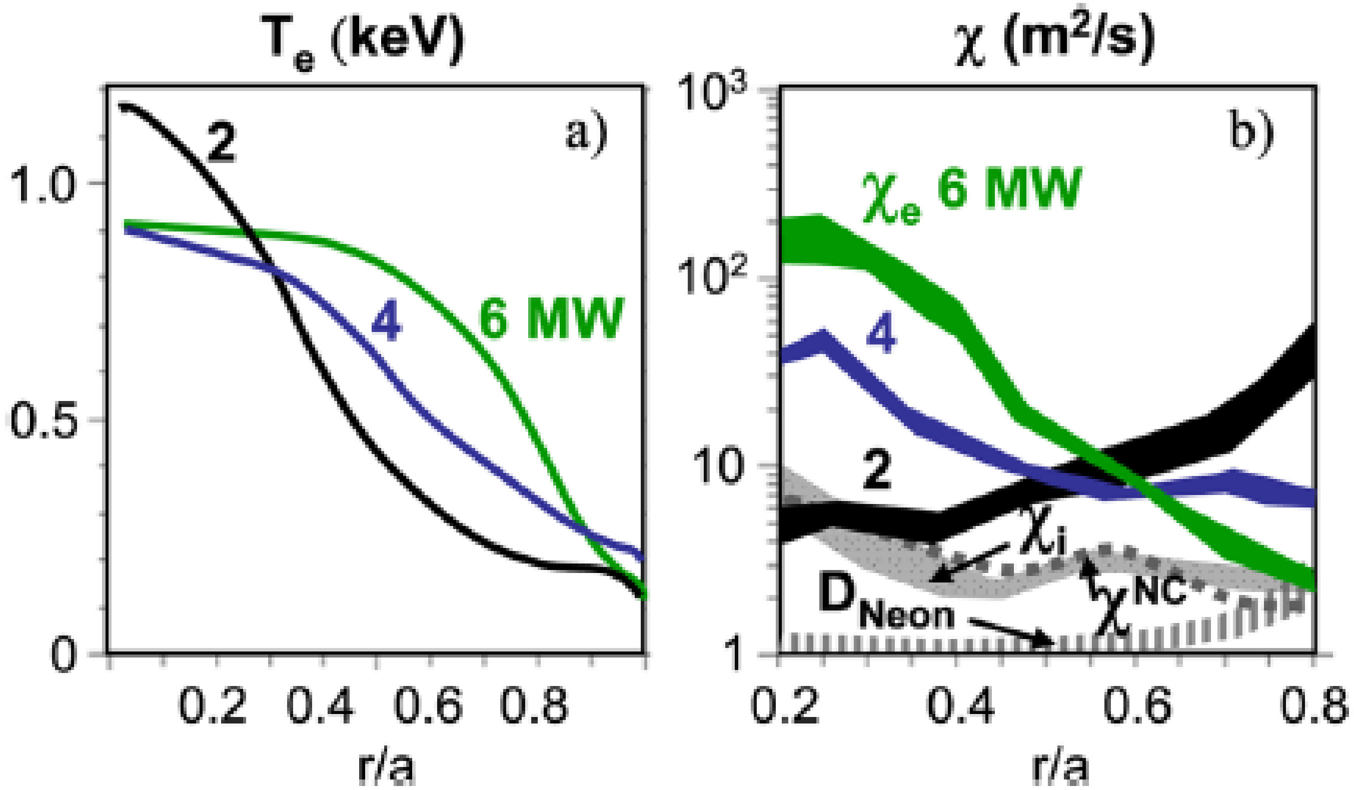


Results are similar, but instability growth rates are less and tend to symmetrize final configuration

# Trend is similar to experiments on NSTX

3 Bateman scaled NL runs




Stutman, et al. PRL (2009)

- M3D-C1: Central temperature decreases with β
- Exp data: Central transport increases with β

G46F4 - G46F – G46F3

# 133964: LRDFIT and TRANSP files have different profiles and stability properties