



Overview of Bayesian plasma diagnostic modelling

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Content

- Introduction
- Bayesian modelling within the Minerva framework
- Example applications
- Summary



Plasma diagnostic modelling in nuclear fusion experiments

- Key parameters:
 - Particle densities: n_e, n_i
 - Particle temperatures: T_e, T_i
 - Plasma radiation, Z_{eff} , etc.
- Diagnostic observations of various physical processes:
 - Thomson scattering $\leftarrow n_e, T_e$
 - Interferometry $\leftarrow n_e$
 - Beam emission spectroscopy $\leftarrow n_e, T_e, n_i, T_i$
 - Passive spectroscopy (bremsstrahlung, charge exchange) $\leftarrow T_i, Z_{\text{eff}}$
 - Soft X-rays, bolometry, etc.



Plasma diagnostic modelling in nuclear fusion experiments

- Data analysis: constructing an inverse function f^{-1} for an individual diagnostic (inverse problem)

$$D \xrightarrow{f^{-1}} H$$

D : Thomson scattering spectra, interference patterns, etc.

H : n_e, T_e, T_i , etc.



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 - Model selection: avoiding over- or under-fitting
 - Additional practical concerns: maintenance, debugging, etc.



Bayesian modelling within the Minerva framework

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 - The model predicts observations given specific parameters.



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 - The state of knowledge of model parameters can be explicitly represented as a probability distribution both prior to and posterior to observations, allowing uncertainties to be quantified without any loss of information.



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- Bayes' theorem: the posterior distribution for any combination of observations
- Graphical model: unfolding the complexity of the model in which *everything* is declared.
 - Model assumptions (regularization), systematic parameters, and other aspects are clearly defined in the model and easily accessible through graphical representation, facilitating maintenance, debugging, reproducibility, etc.

J. Svensson, A. Werner et al. International Symposium on Intelligent Signal Processing-WISP (2007)



Bayesian modelling within the Minerva framework

- Defining the prior state of the system: $P(H)$ ($H: n_e, T_e$, etc.)
 - Declaring model assumptions based on underlying physics and/or ad-hoc regularisation.



Bayesian modelling within the Minerva framework

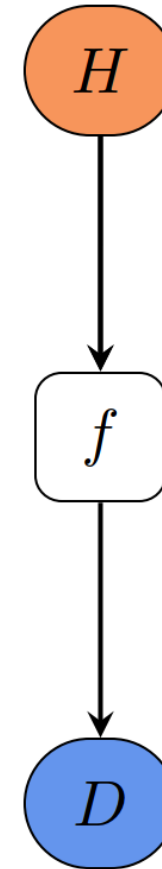
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 - Declaring model assumptions based on underlying physics and/or ad-hoc regularisation.
- The predictive distribution over observations $P(D|H)$ captures the physical processes occurring during experiments, defined by the forward model $f(H)$.
- Once the observations are available, we can update $P(H)$ to the posterior state $P(H|D)$ by Bayes' theorem:

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)}$$





Bayesian modelling within the Minerva framework

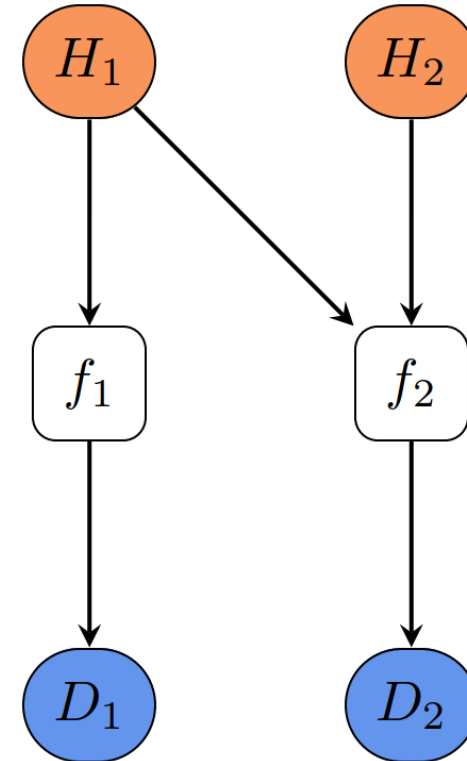
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Bayesian modelling within the Minerva framework

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- Example case: two unknown parameters H_1 and H_2 and two different observations D_1 and D_2 . D_1 depends on H_1 and D_2 depends on H_1 and H_2 :

$$\begin{aligned} P(H_1, H_2 | D_1, D_2) &= \frac{P(D_1, D_2 | H_1, H_2) P(H_1, H_2)}{P(D_1, D_2)} \\ &= \frac{P(D_1 | H_1) P(D_2 | H_1, H_2) P(H_1) P(H_2)}{P(D_1) P(D_2)} \end{aligned}$$



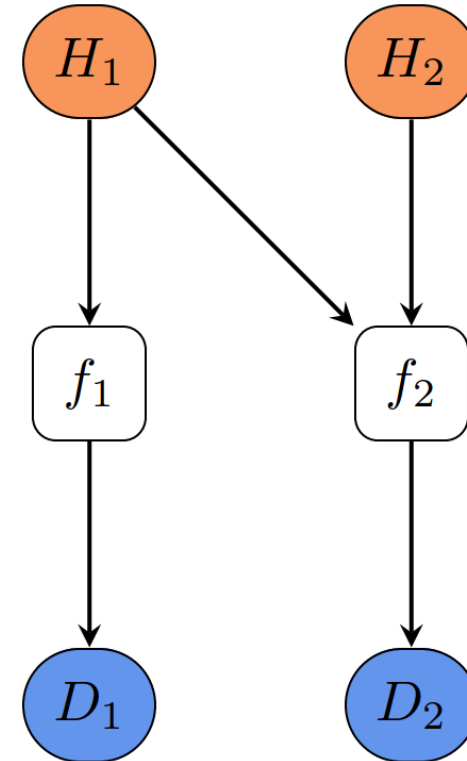


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- Can be generalised for any number of parameters and observations.



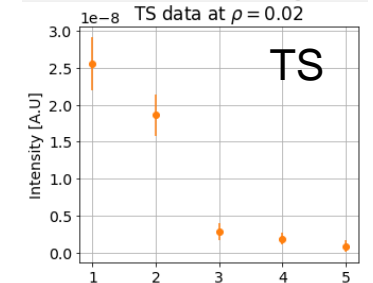
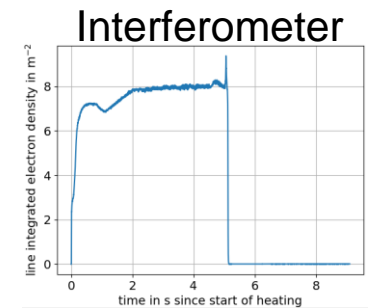
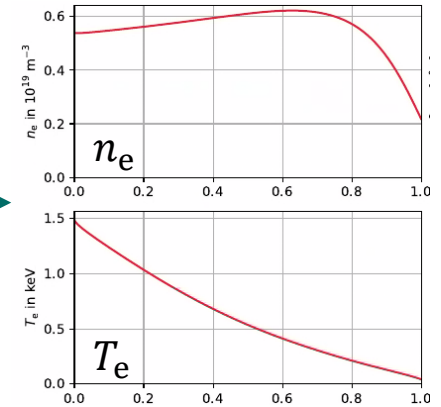
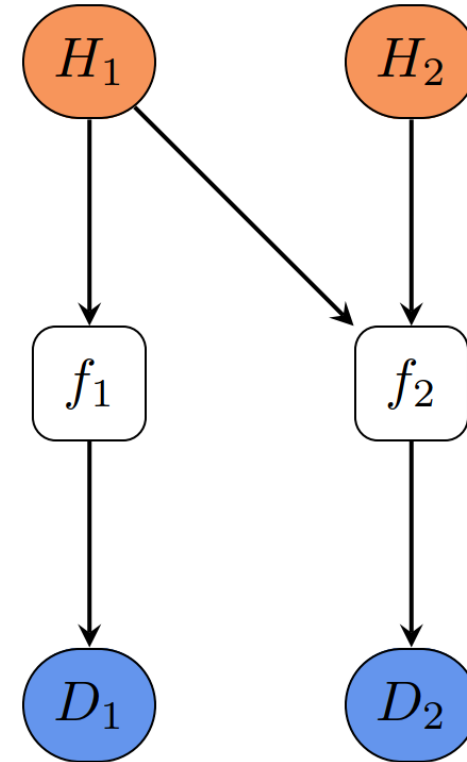


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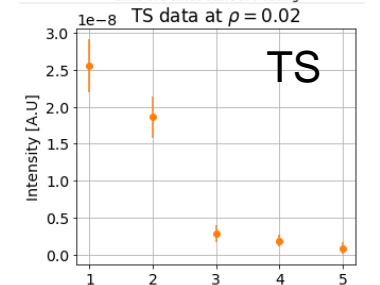
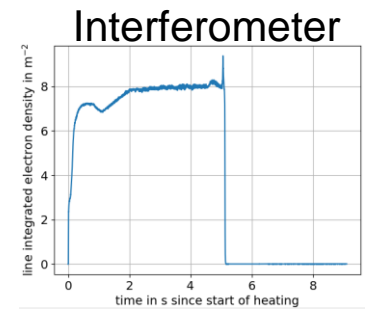
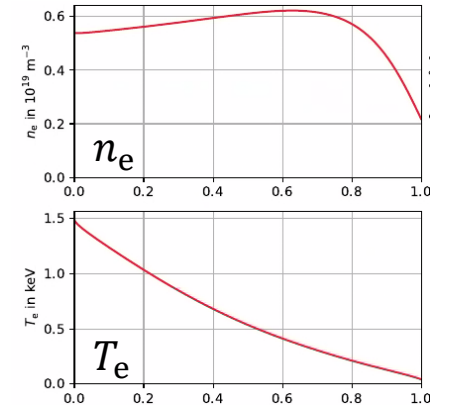
$$P(H_1, H_2 | D_1, D_2) = \frac{P(D_1, D_2 | H_1, H_2) P(H_1, H_2)}{P(D_1, D_2)}$$
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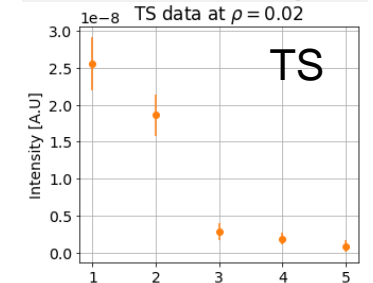
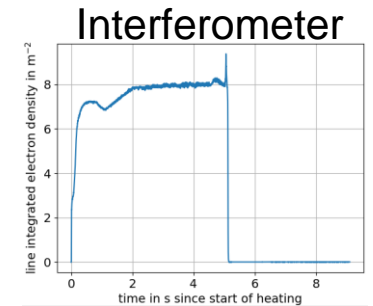
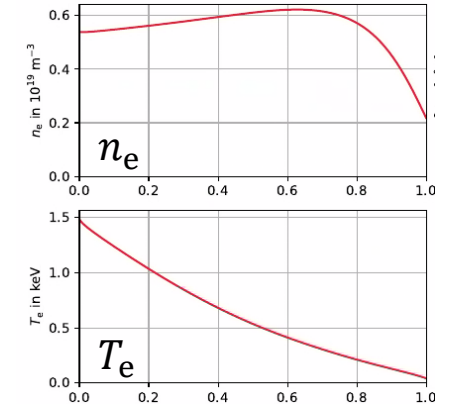
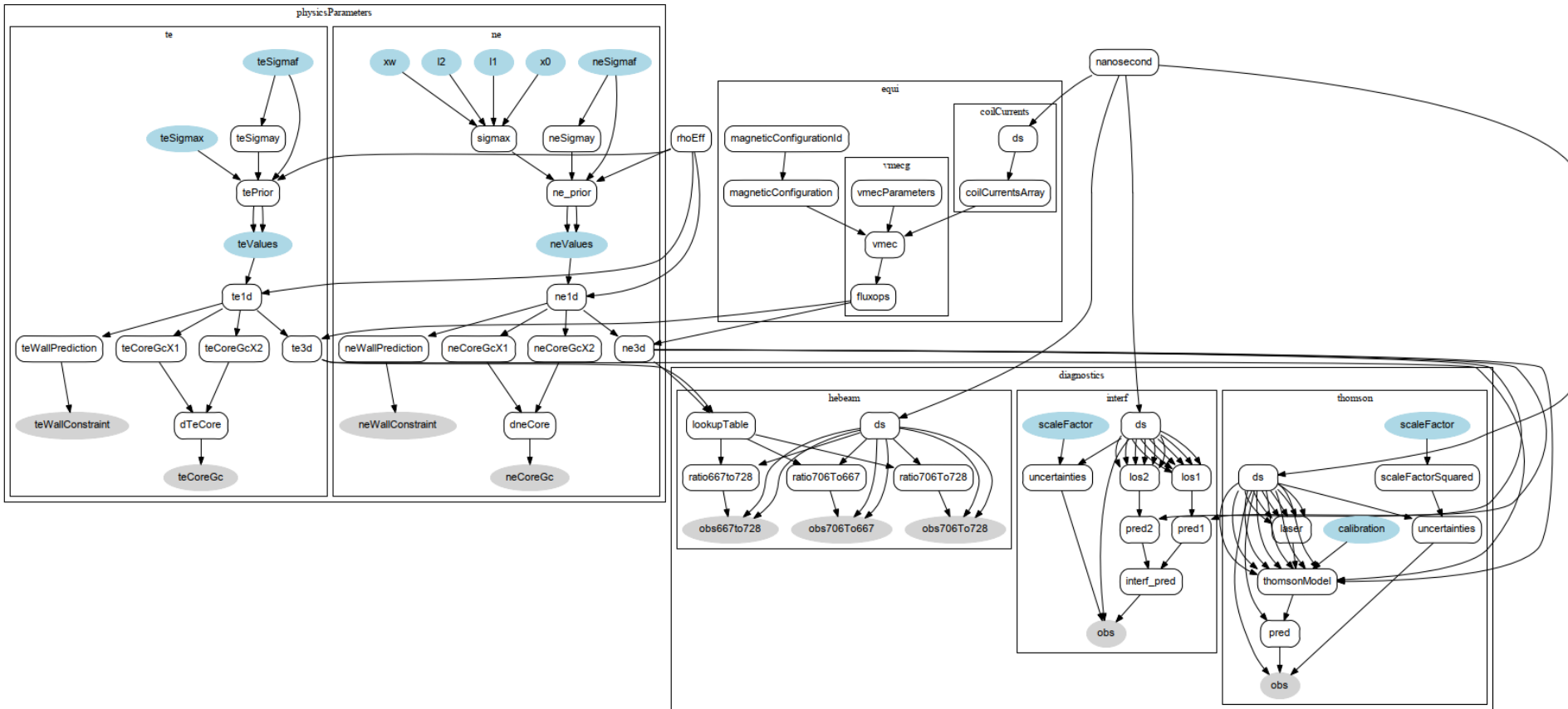


Application: profile diagnostics for n_e and T_e





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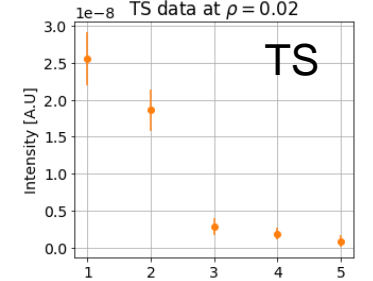
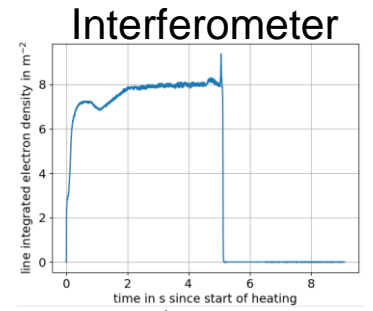
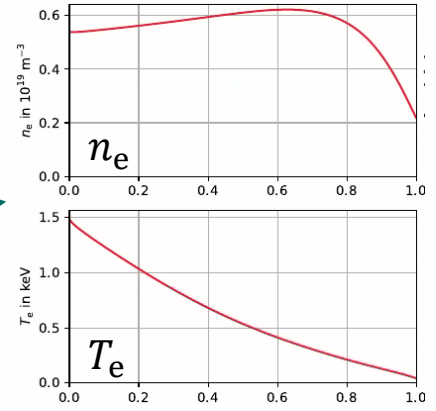
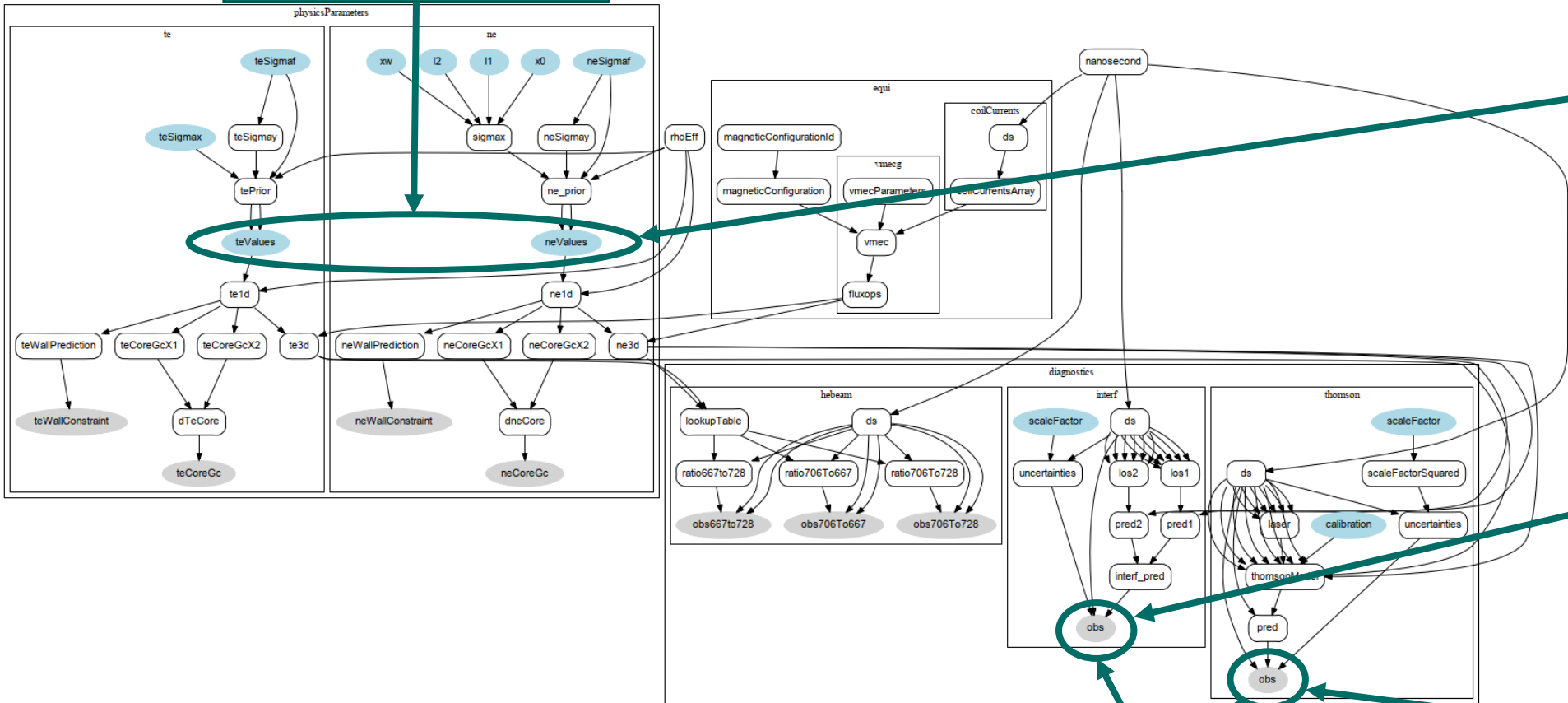
S Kwak et al, 2020 Nucl. Fusion **60** 046009

S Kwak et al Bayesian modelling of multiple plasma diagnostics at Wendelstein 7-X., to be published



Application: profile diagnostics for n_e and T_e

Plasma parameters (n_e, T_e)

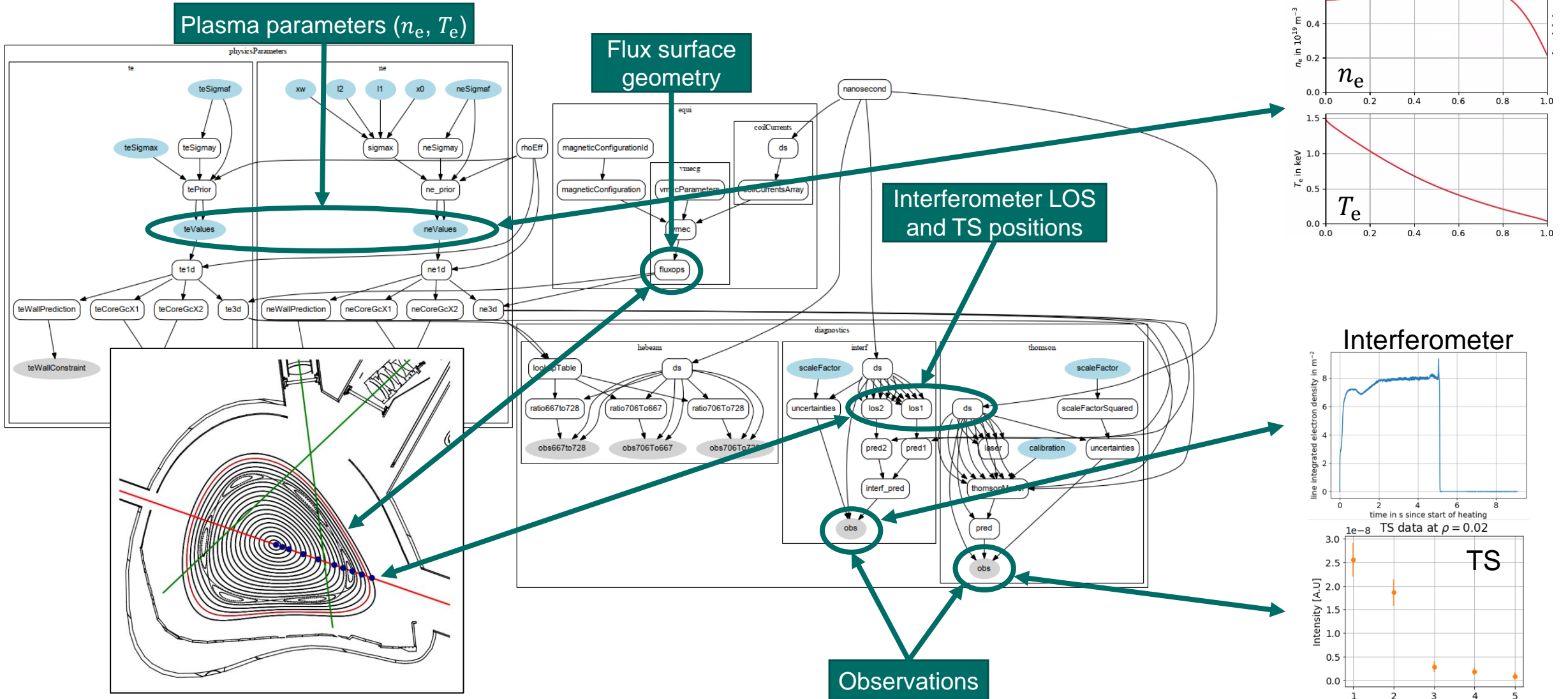


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Observations



Application: profile diagnostics for n_e and T_e





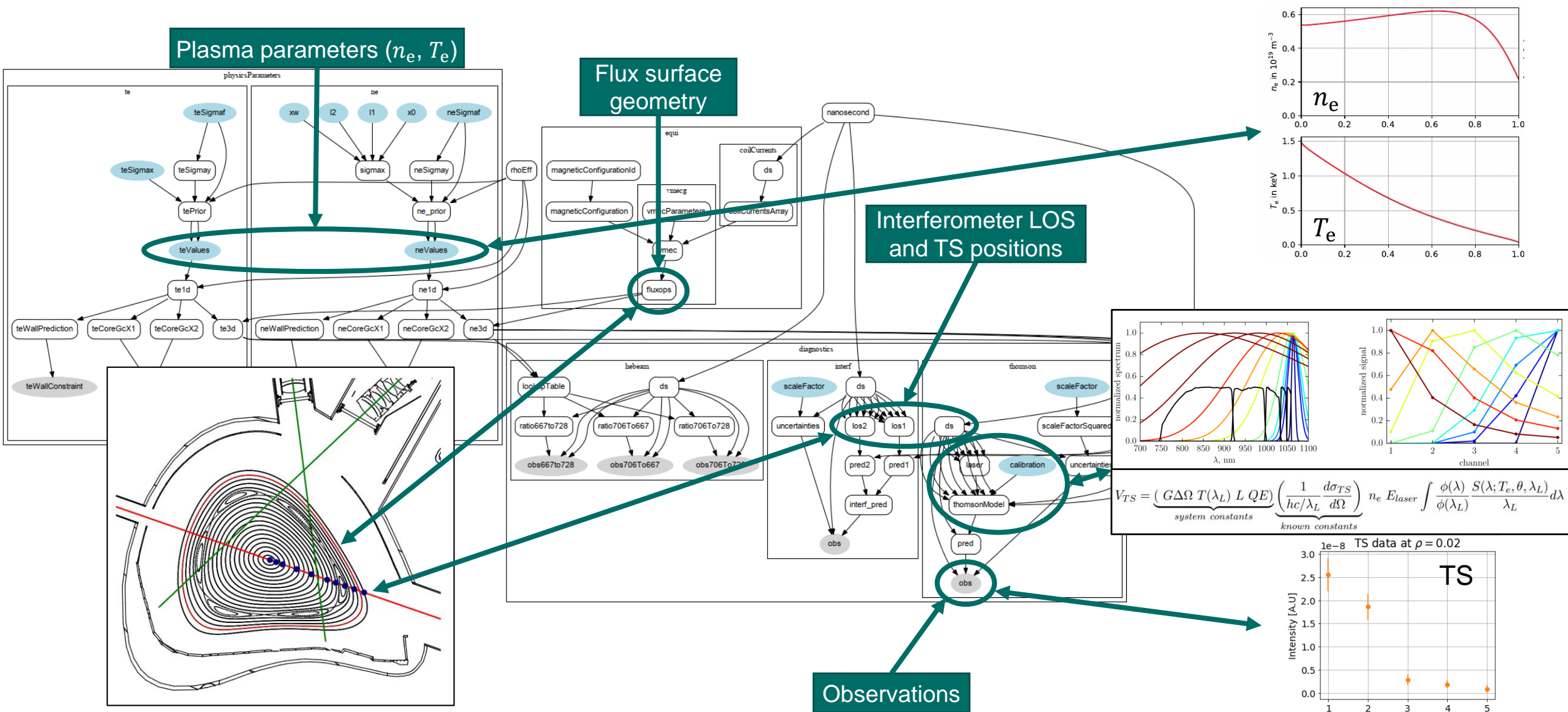
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Plasma parameters (n_e, T_e)

Flux surface geometry

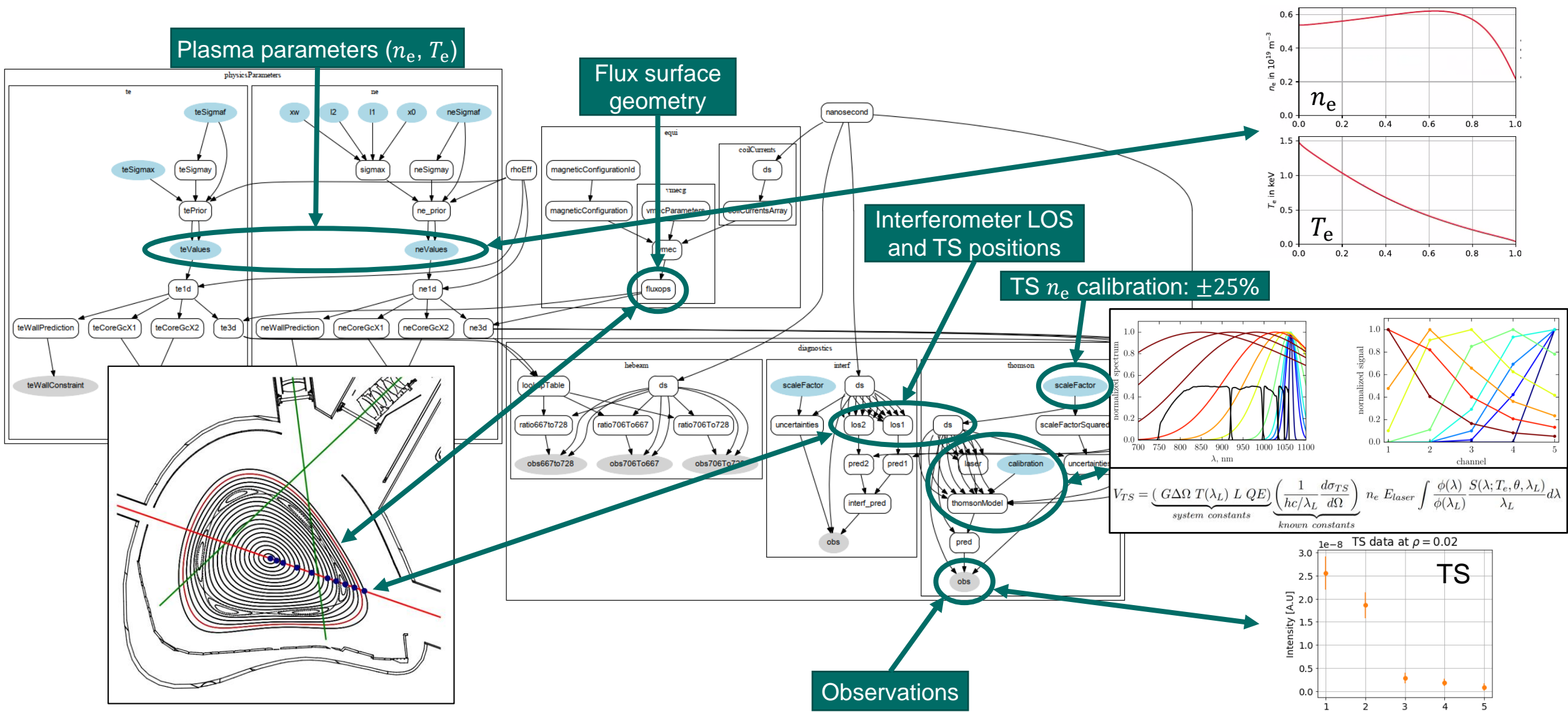
Interferometer LOS and TS positions

Observations



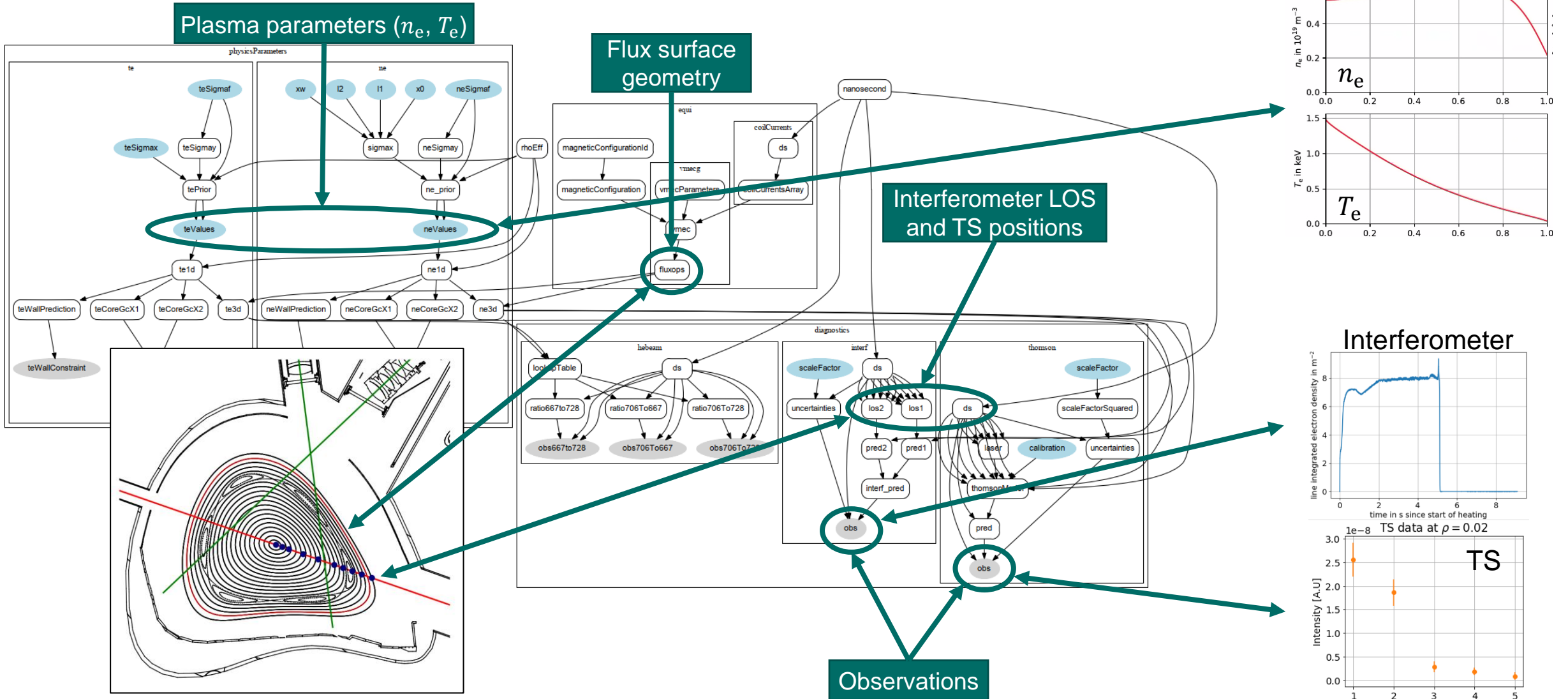


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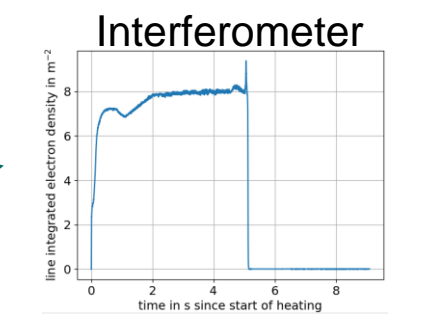
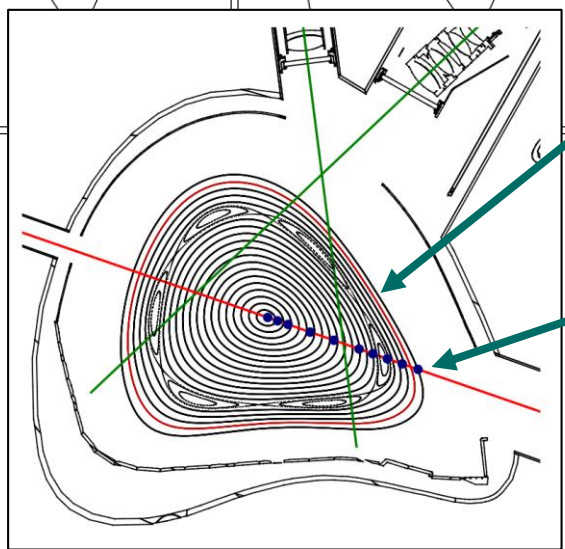
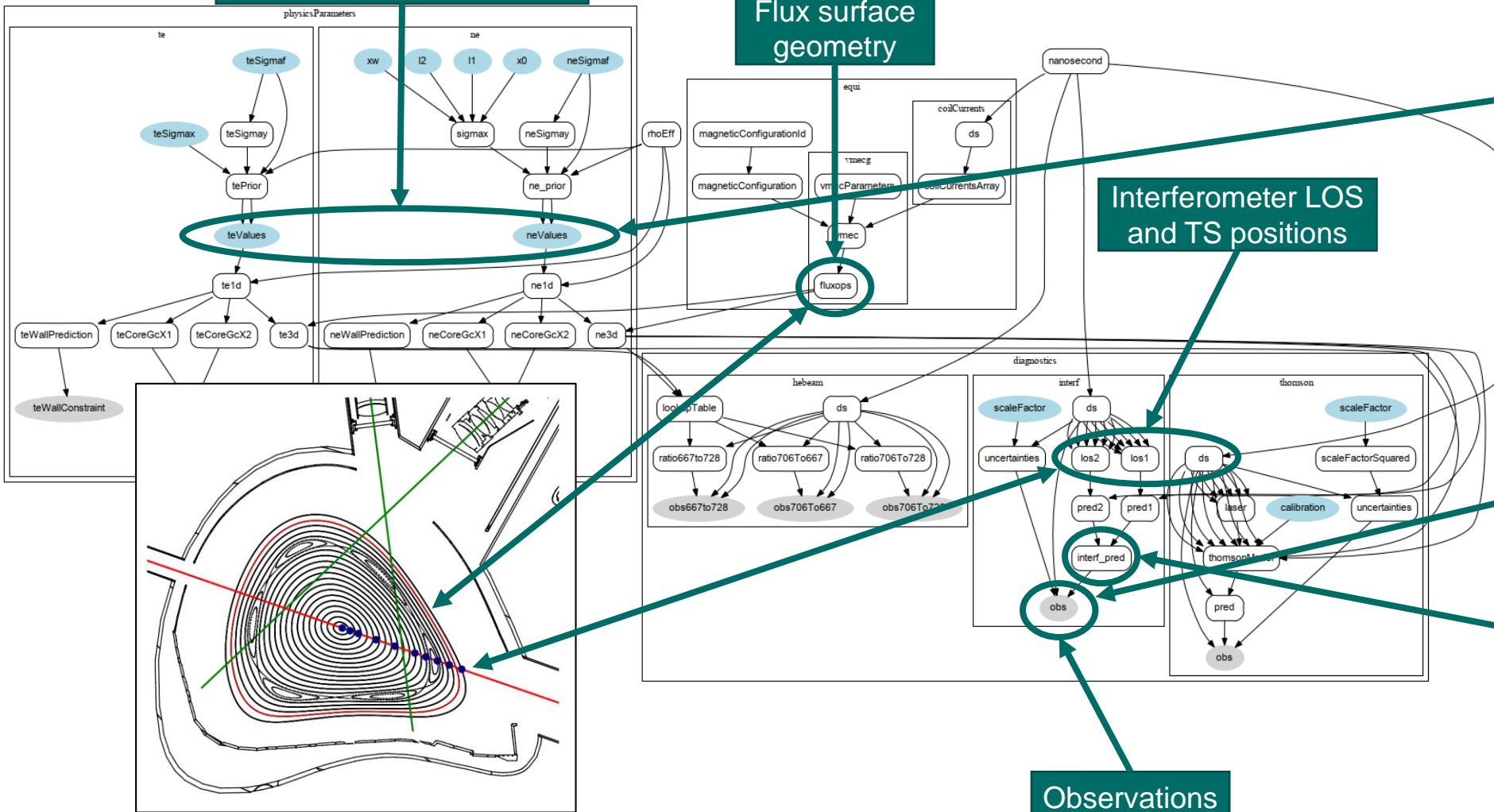
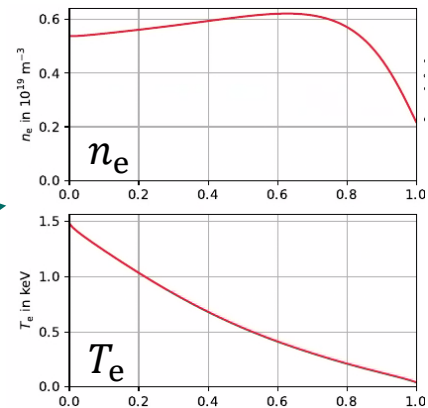


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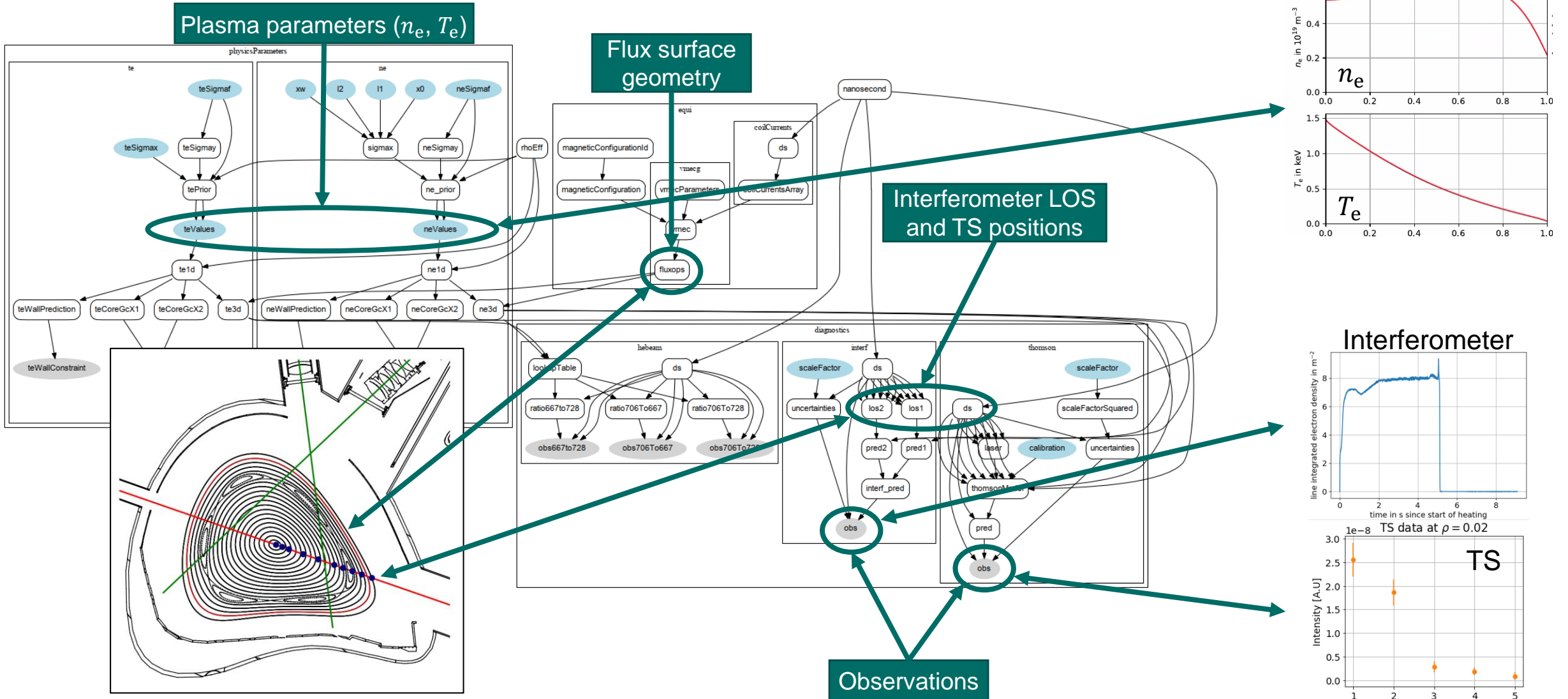


$$\frac{\lambda e^2}{4\pi^2 \epsilon_0 m_e} \int n_e(l) dl$$

Observations

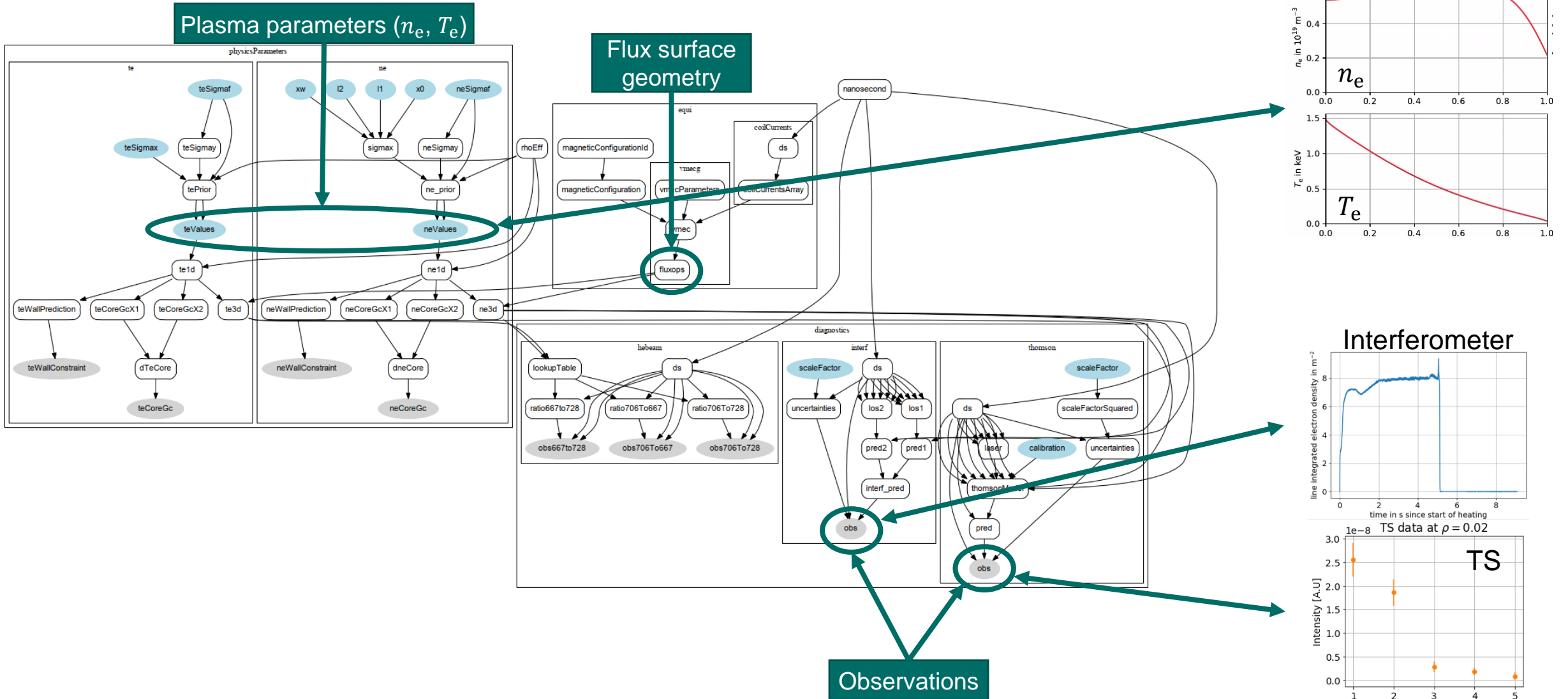


Application: profile diagnostics for n_e and T_e





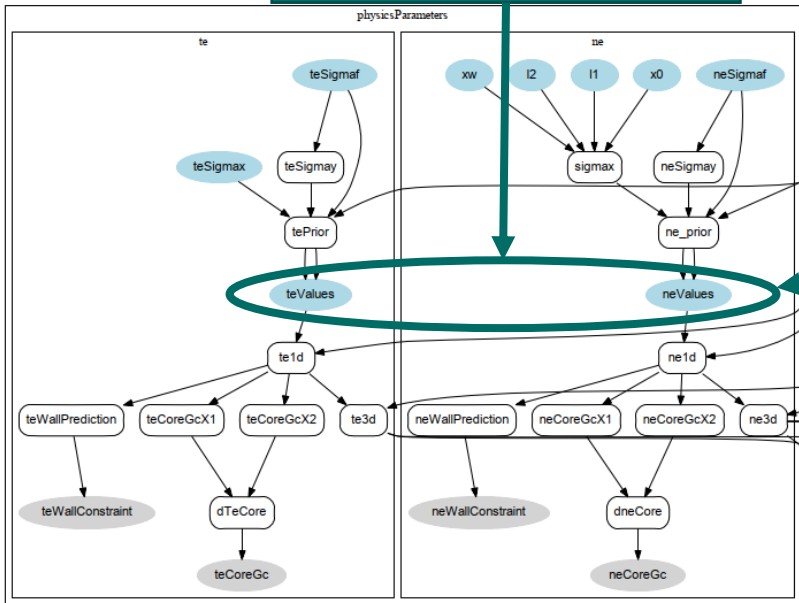
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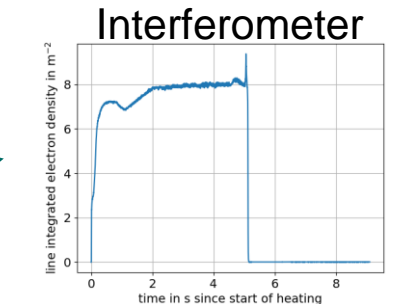
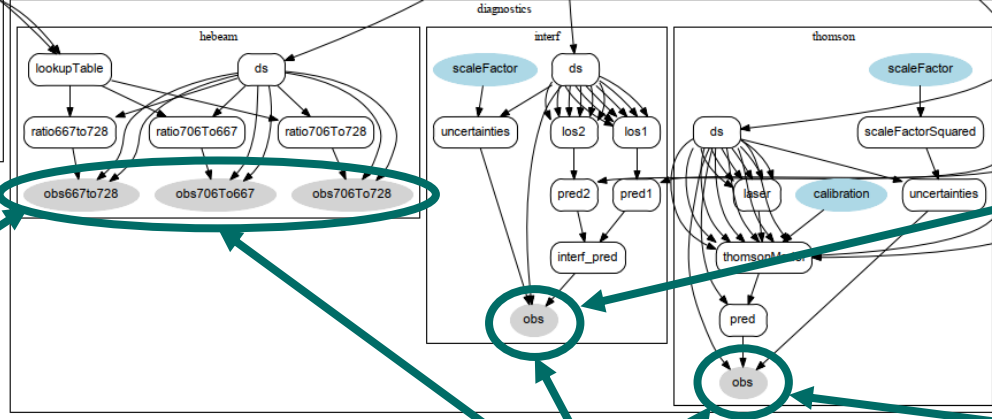
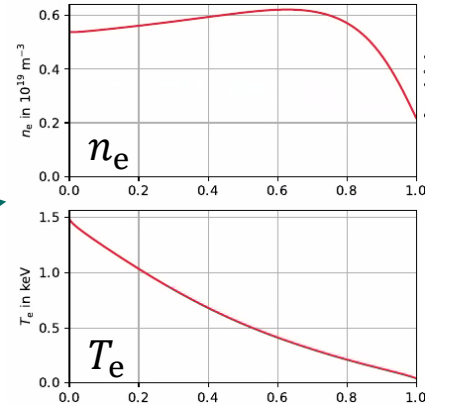
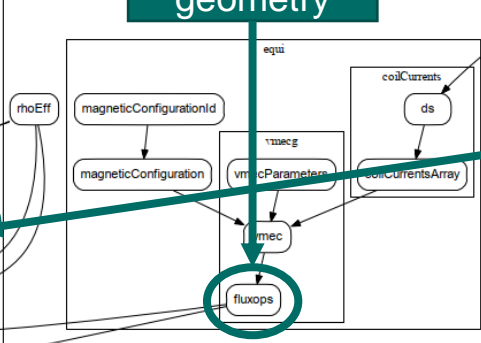


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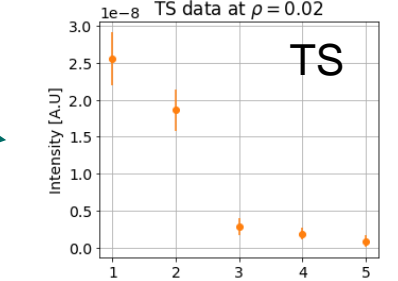
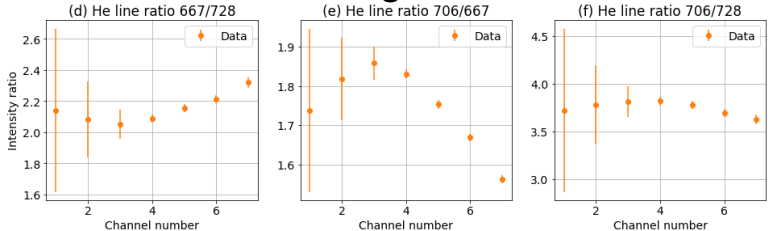
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Flux surface geometry



Helium beam diagnostics

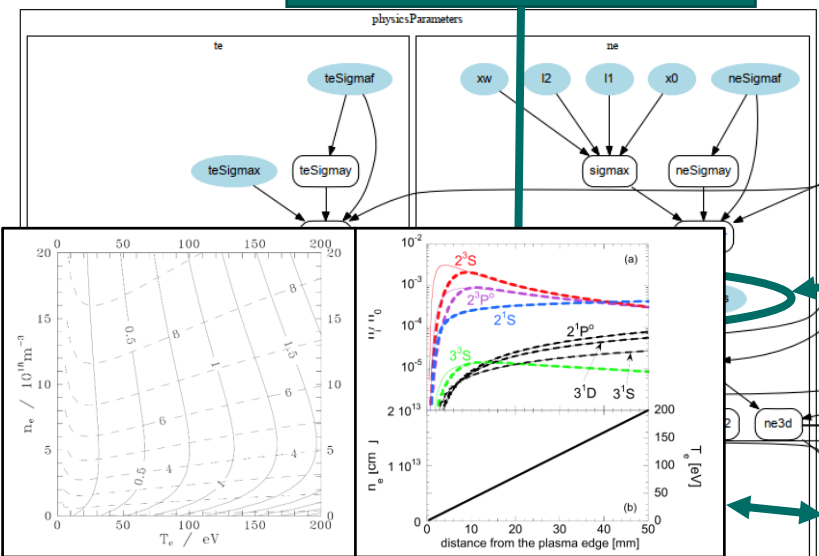


Observations

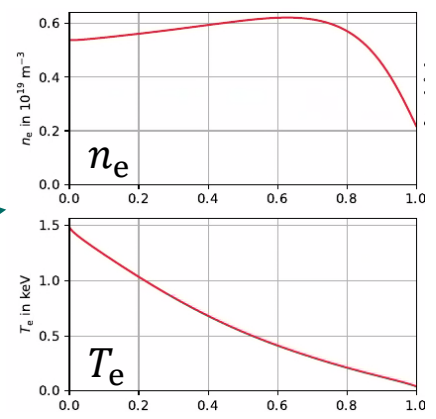
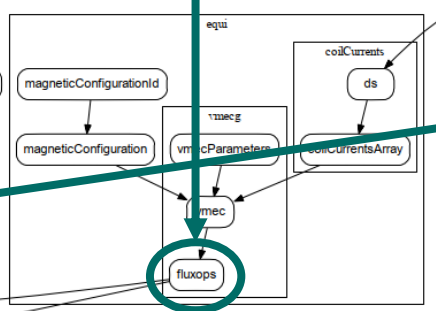


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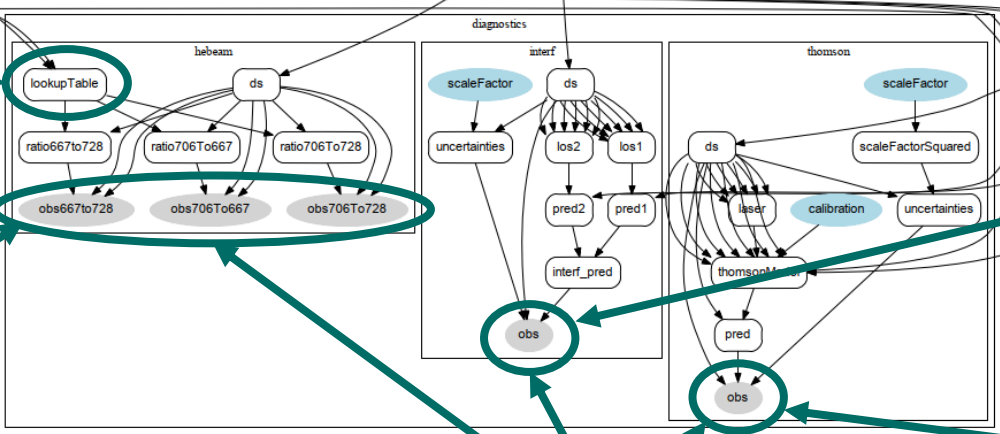
Plasma parameters (n_e, T_e)



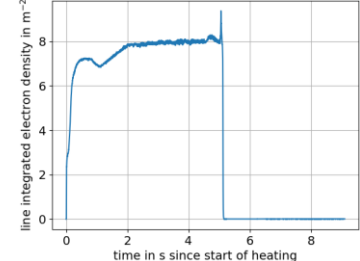
Flux surface geometry



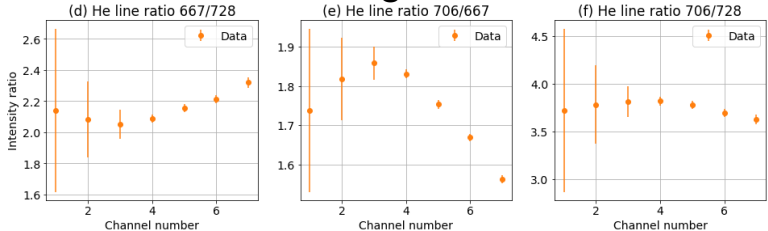
$$\frac{dN_i(z)}{dz} = \frac{1}{v} \sum_{j=1}^M \left[\sum_s n_s(z) a_{ij}^s (v_r^s(z)) + b_{ij} \right] N_j(z)$$



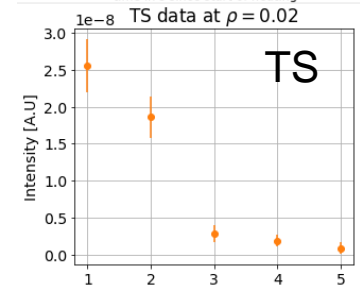
Interferometer



Helium beam diagnostics



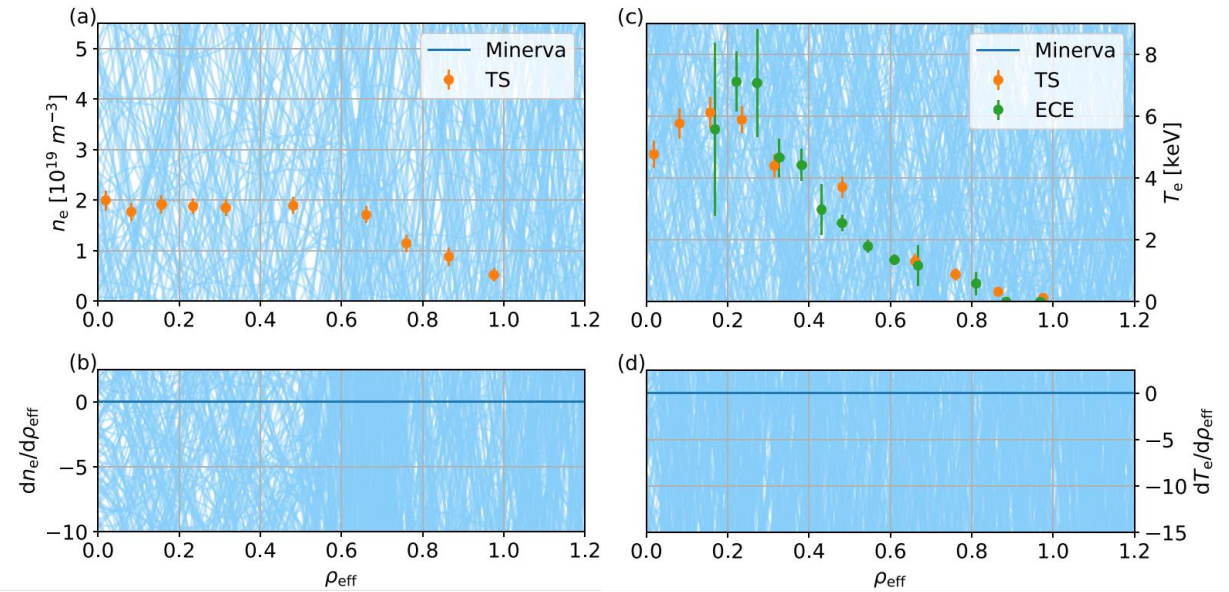
Observations





Application: profile diagnostics for n_e and T_e

$$P(n_e, T_e | D_{int}, D_{TS}, D_{He})$$



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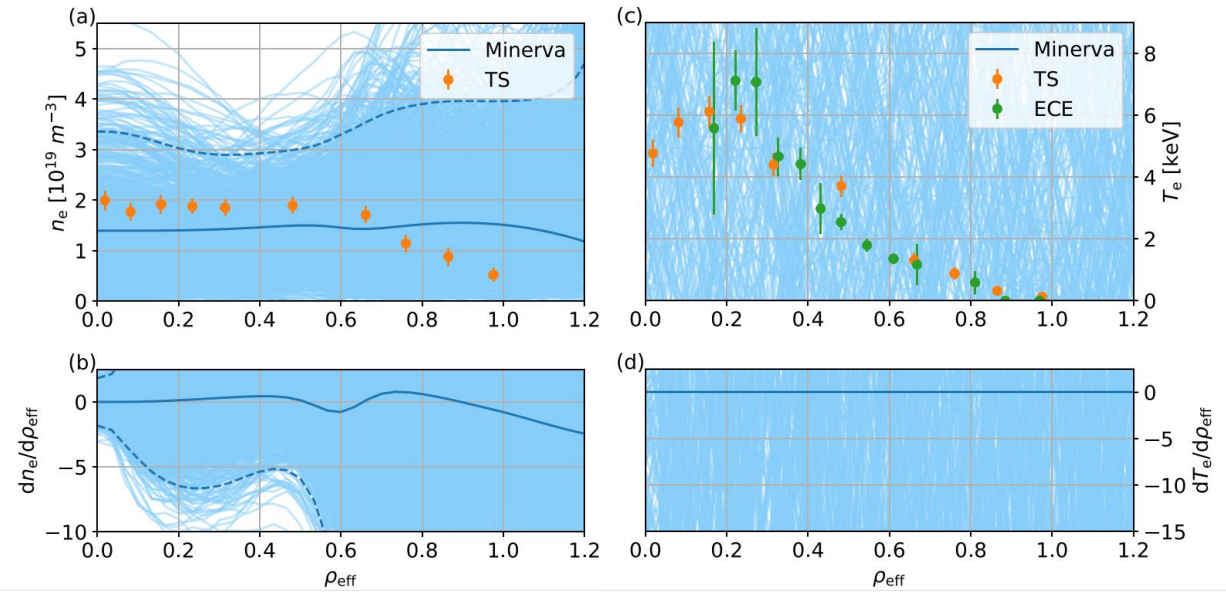
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Profiles are inferred from OP1.1 data



Application: profile diagnostics for n_e and T_e

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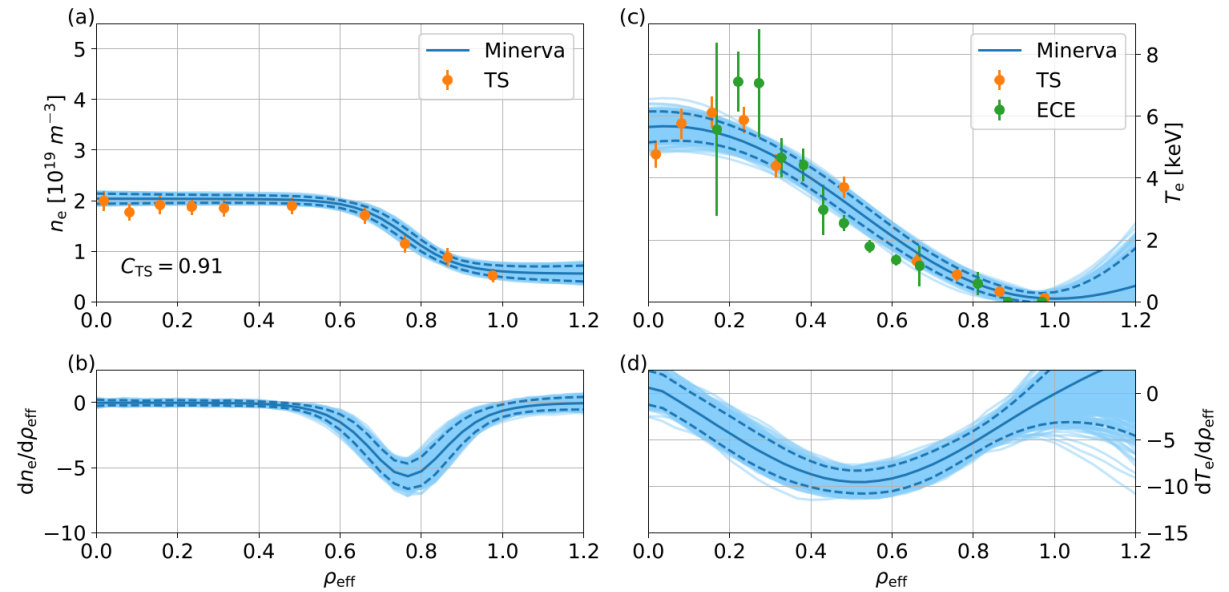
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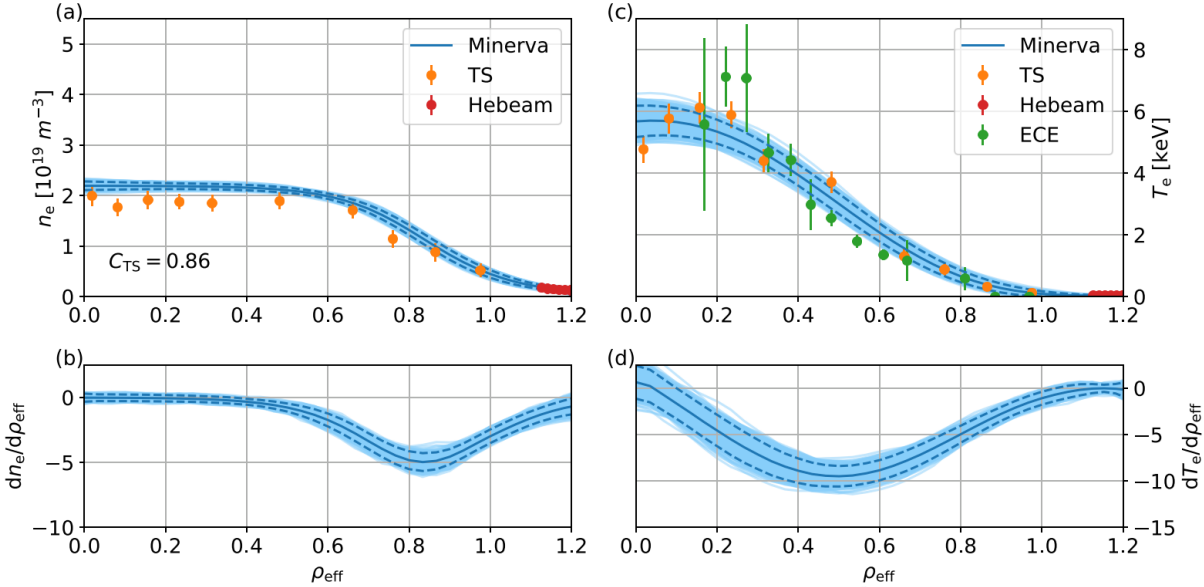
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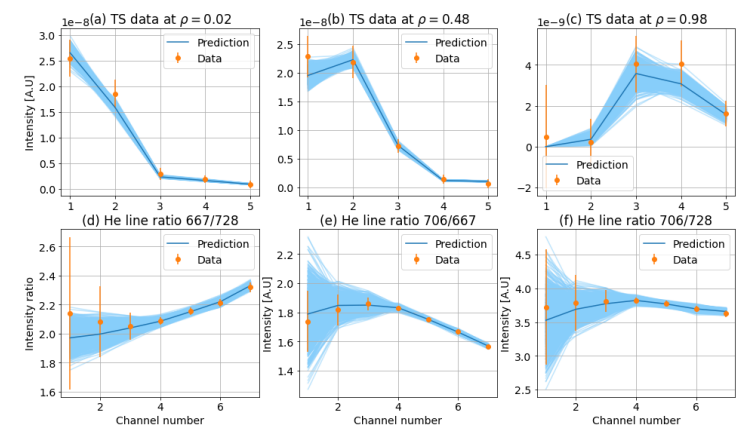
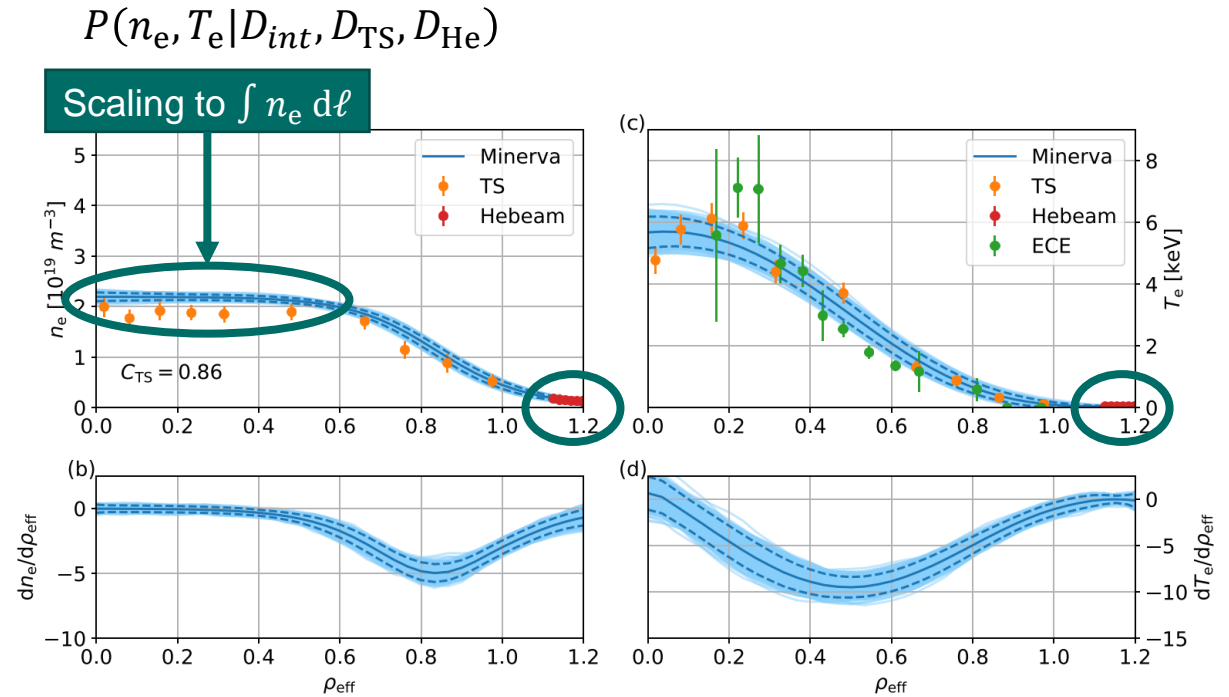


Application: profile diagnostics for n_e and T_e

- Thomson scattering:
 - Determining overall profile shapes
- Helium beam line ratios:
 - Well-constrained n_e and T_e in the edge region
- Interferometer:
 - Automatically corrected absolute n_e scaling
- Predictions and observations: good agreements

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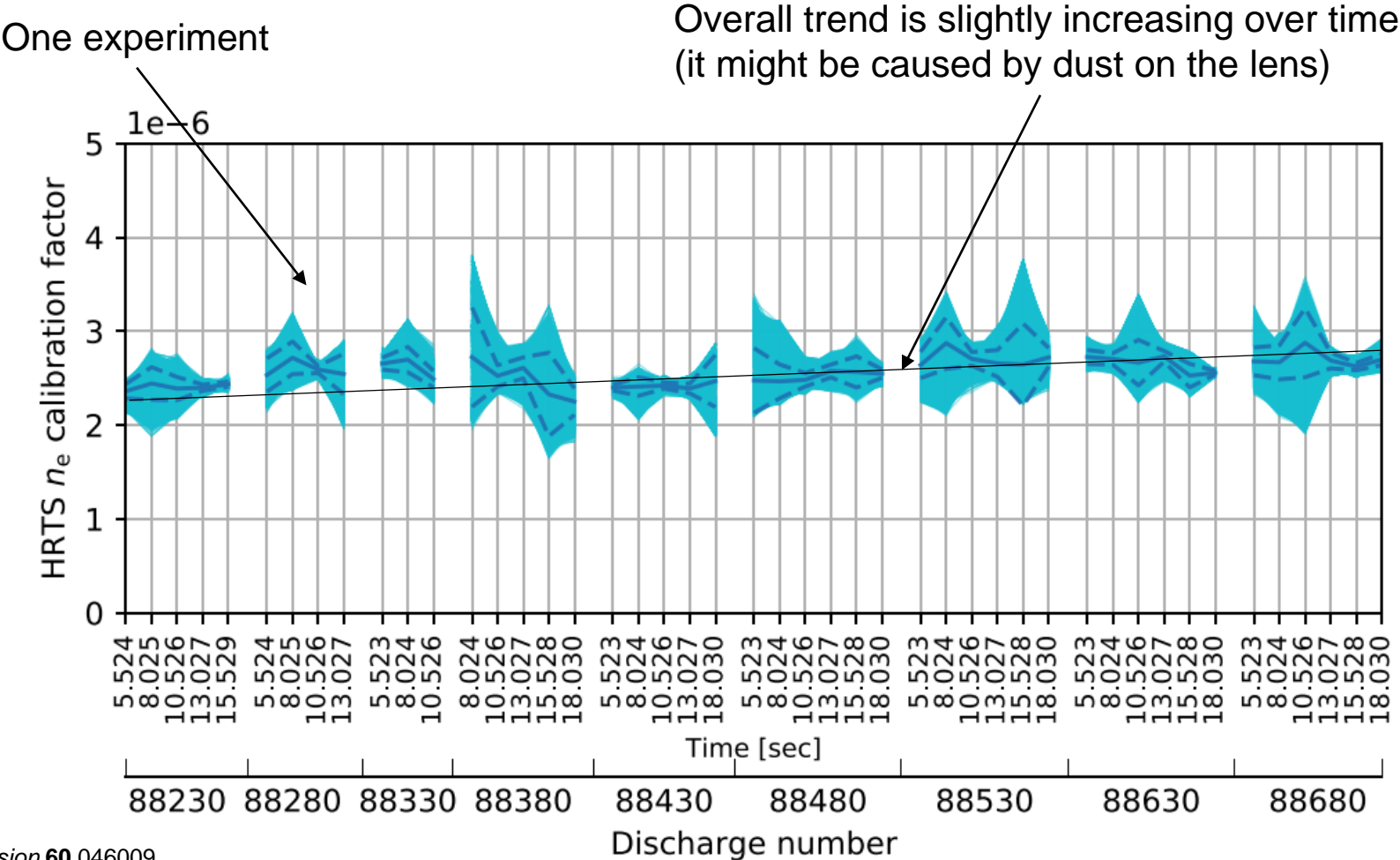


inferred from OP1.1 data



Application: profile diagnostics for n_e and T_e (JET)

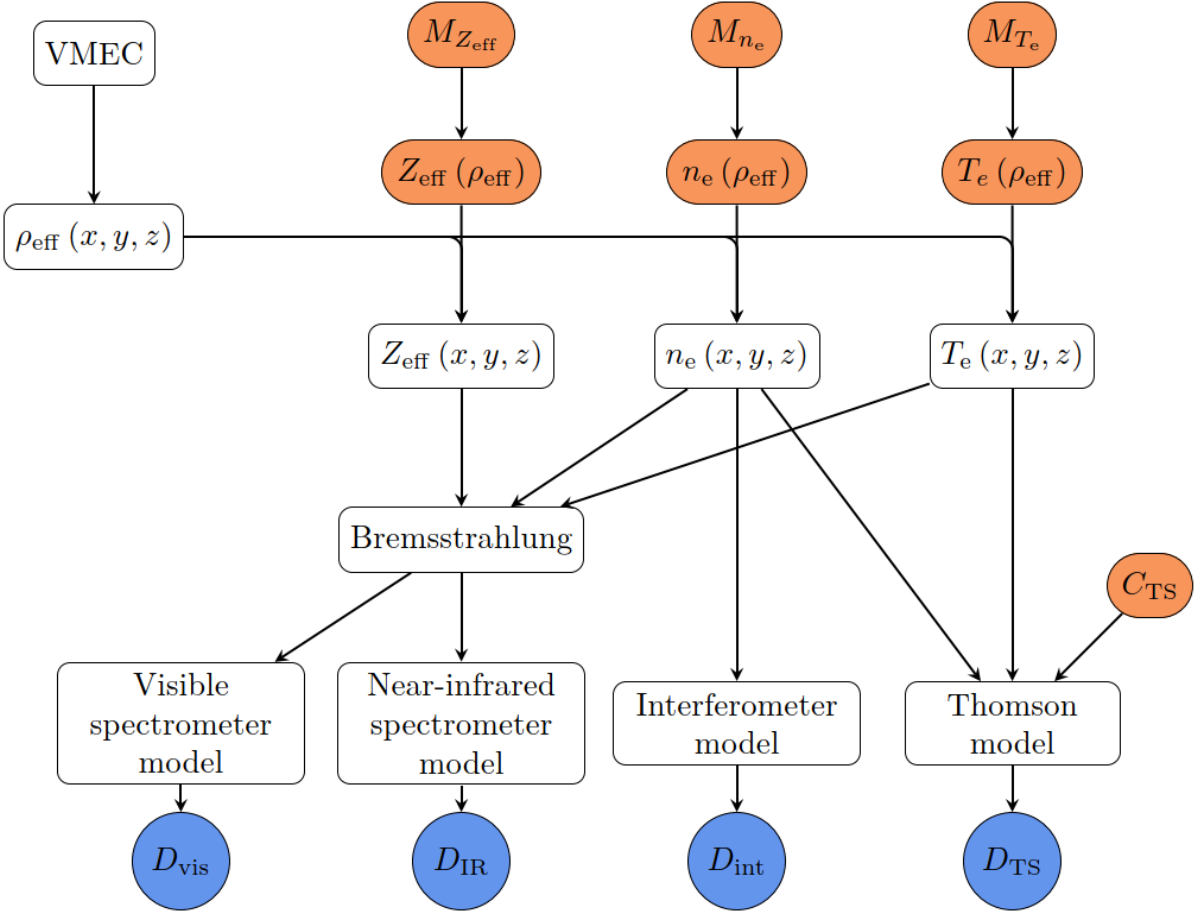
- Automatic calibration of the high-resolution TS system at JET



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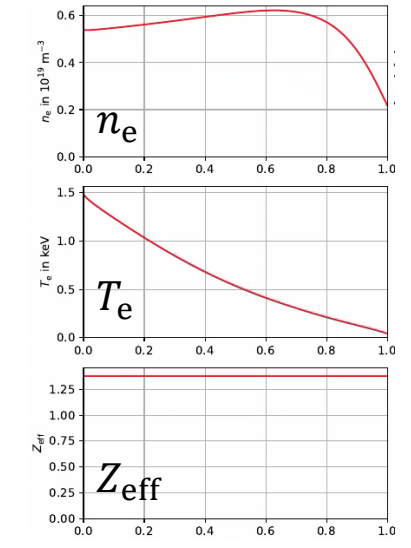
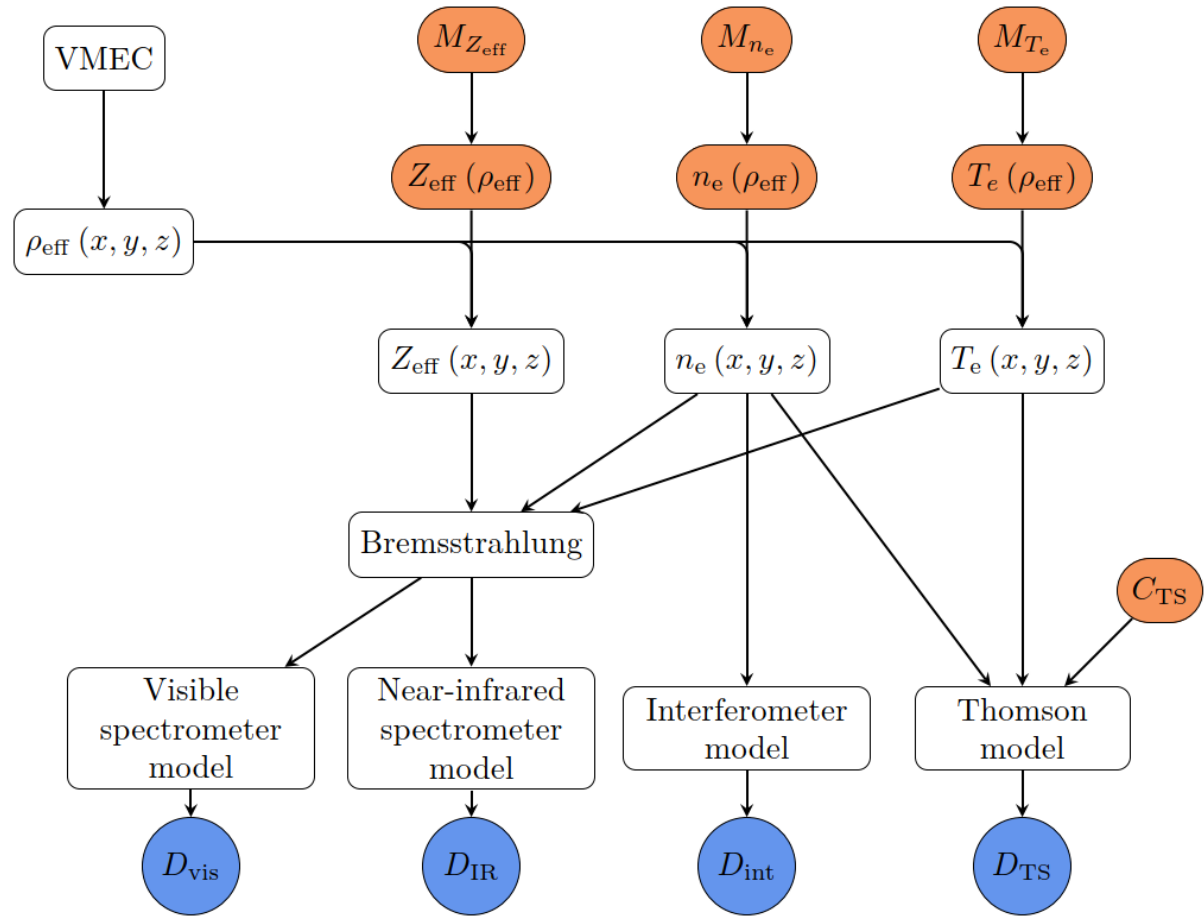


Application: Z_{eff} profiles from line integrated bremsstrahlung



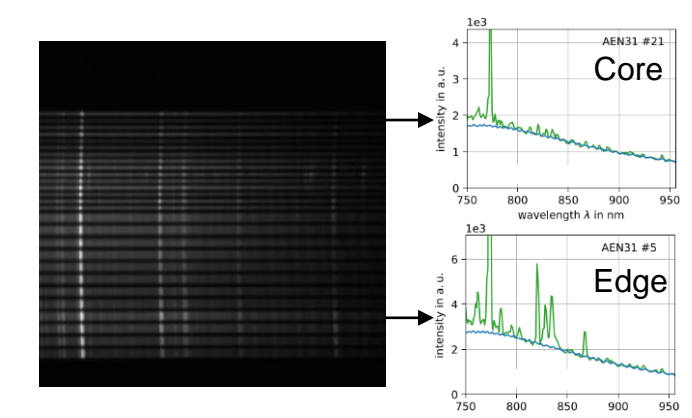
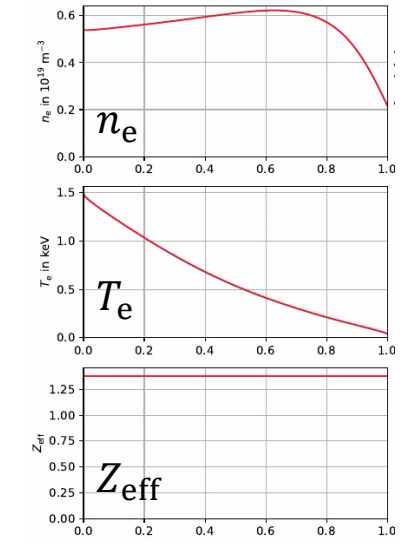
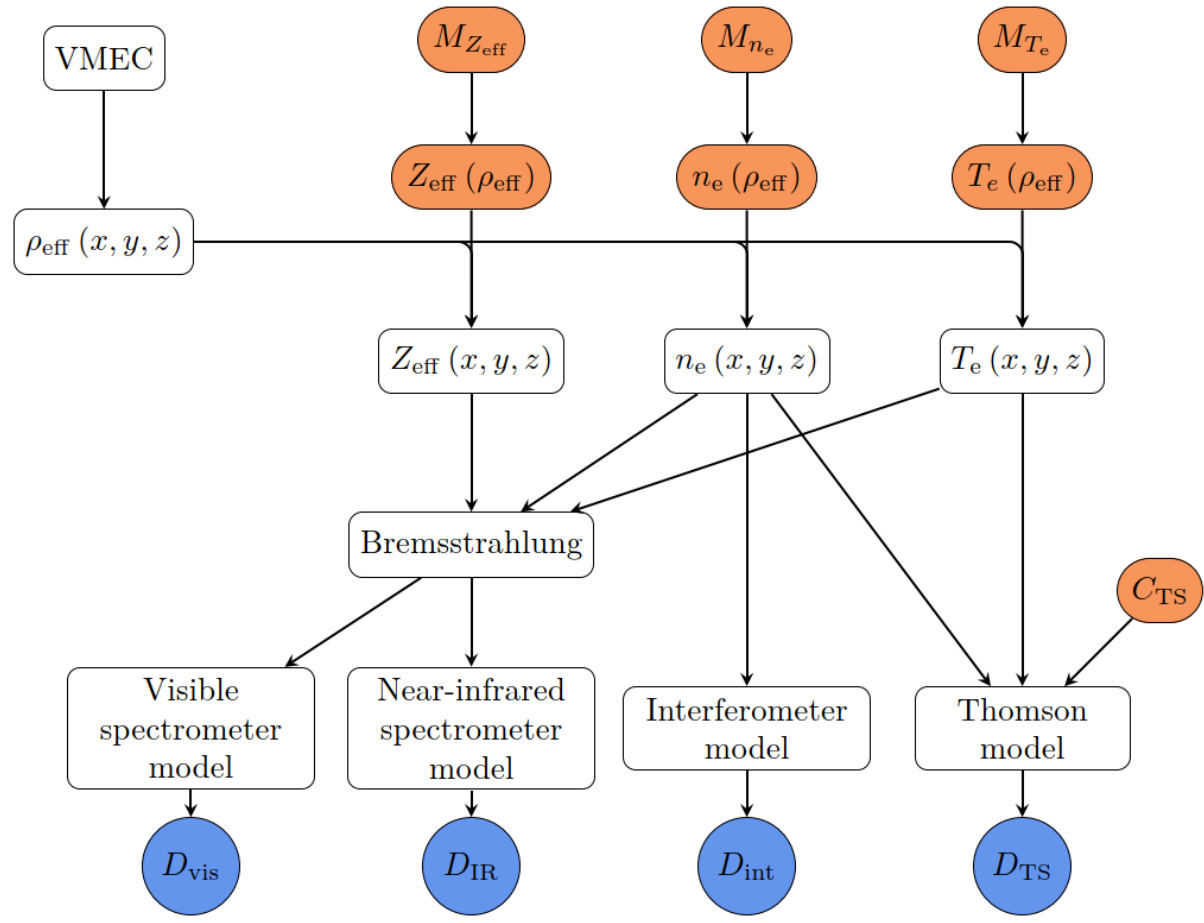


Application: Z_{eff} profiles from line integrated bremsstrahlung



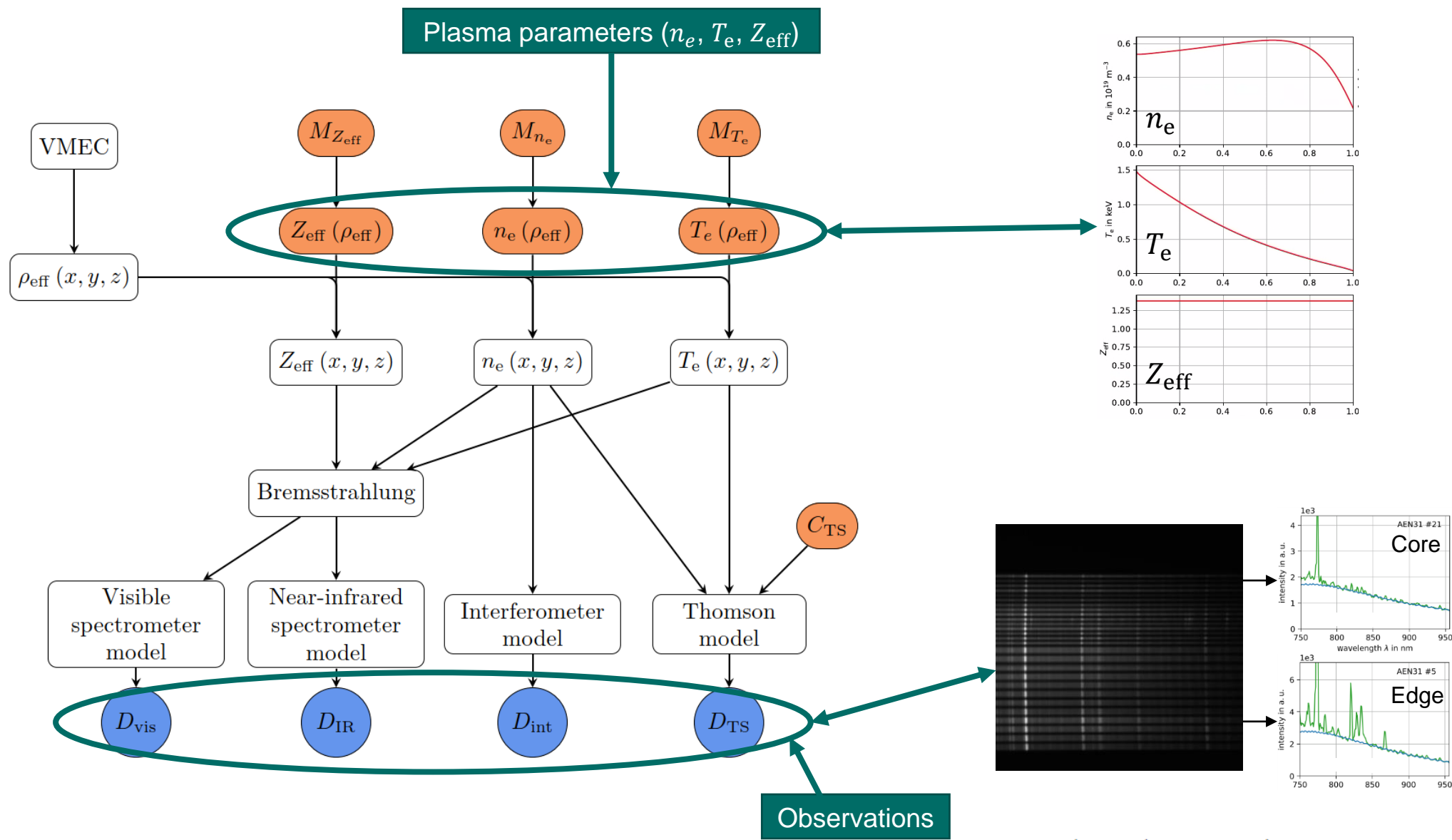


Application: Z_{eff} profiles from line integrated bremsstrahlung



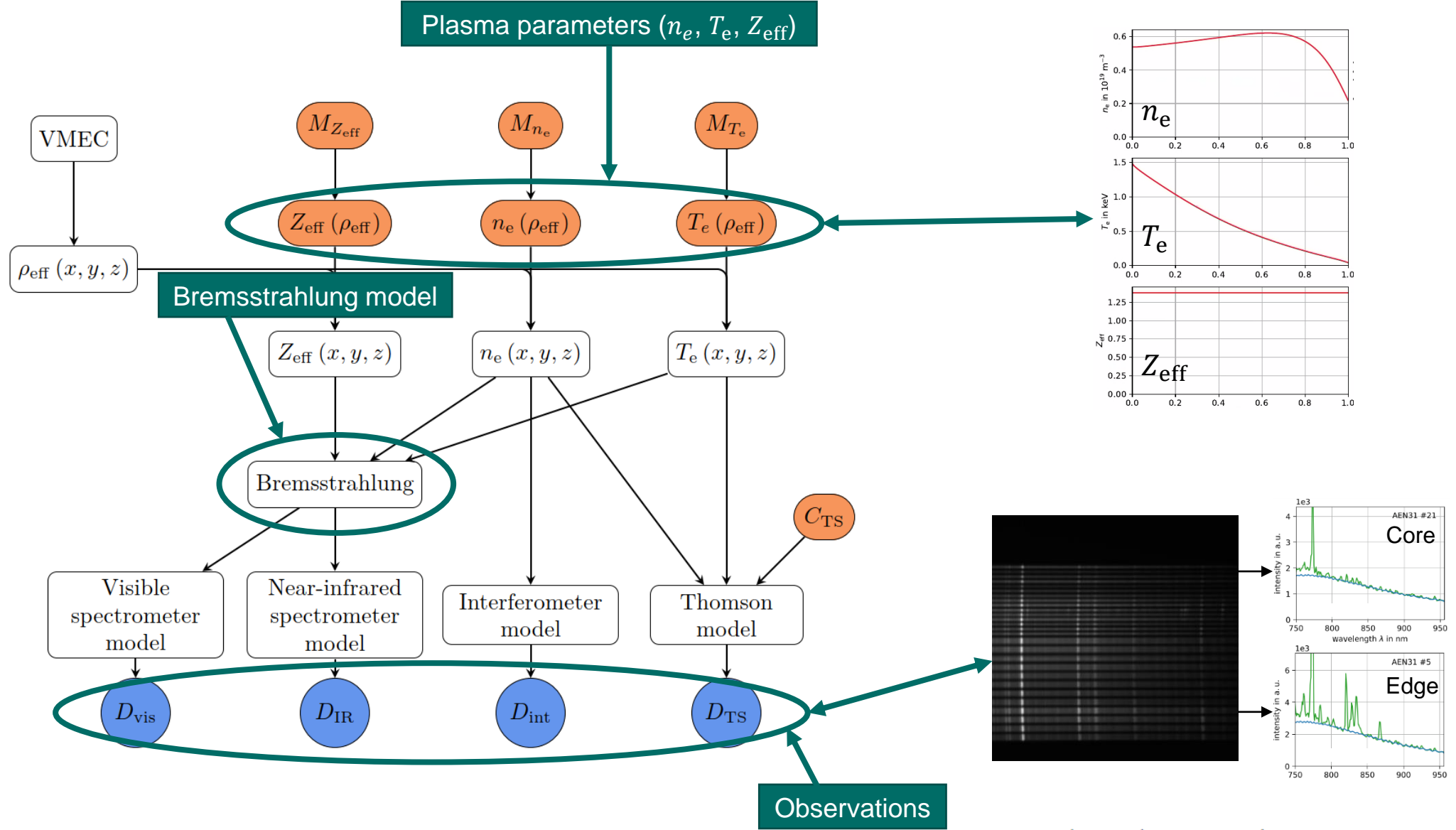


Application: Z_{eff} profiles from line integrated bremsstrahlung



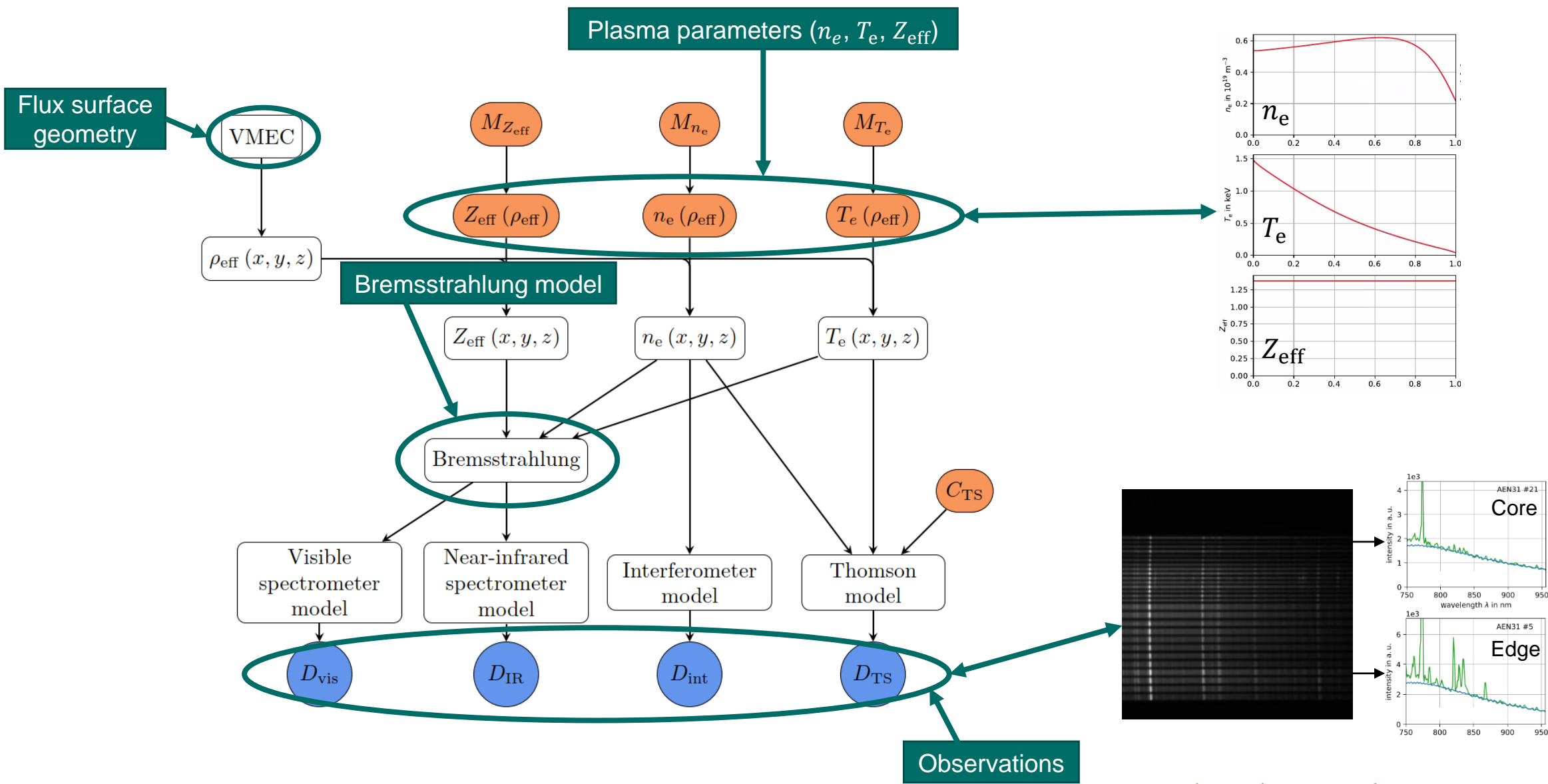


Application: Z_{eff} profiles from line integrated bremsstrahlung



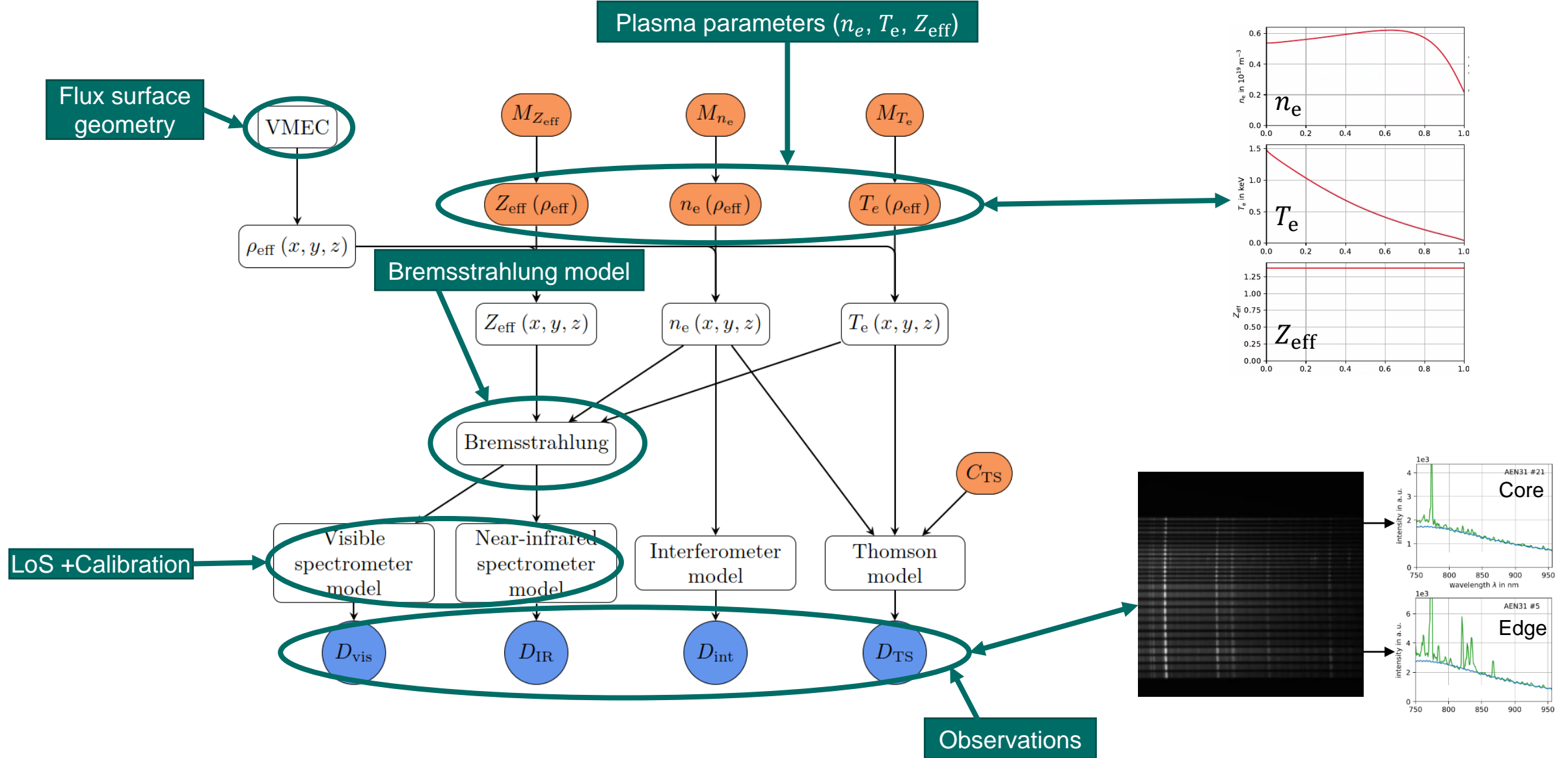


Application: Z_{eff} profiles from line integrated bremsstrahlung





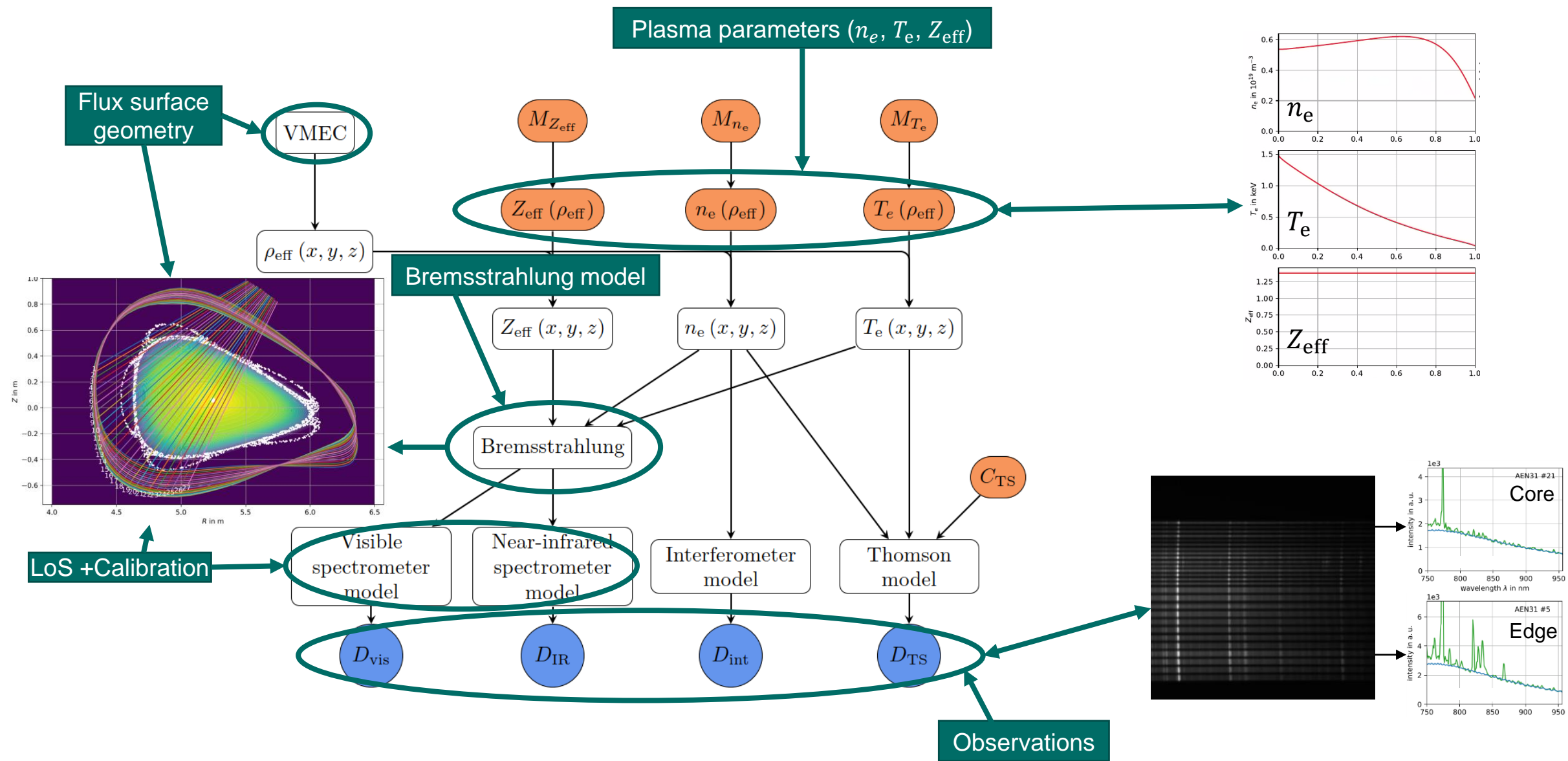
Application: Z_{eff} profiles from line integrated bremsstrahlung



Kwak S. et al. 2021 Rev. Sci. Instrum. 92 043505



Application: Z_{eff} profiles from line integrated bremsstrahlung





Application: Z_{eff} profiles from line integrated bremsstrahlung

Plasma parameters (n_e , T_e , Z_{eff})

Flux surface geometry

VMEC

$\rho_{\text{eff}}(x, y, z)$

Bremsstrahlung model

$M_{Z_{\text{eff}}}$

M_{n_e}

M_{T_e}

$Z_{\text{eff}}(\rho_{\text{eff}})$

$n_e(\rho_{\text{eff}})$

$T_e(\rho_{\text{eff}})$

$Z_{\text{eff}}(x, y, z)$

$n_e(x, y, z)$

$T_e(x, y, z)$

Bremsstrahlung

Visible spectrometer model

Near-infrared spectrometer model

Interferometer model

Thomson model

C_{TS}

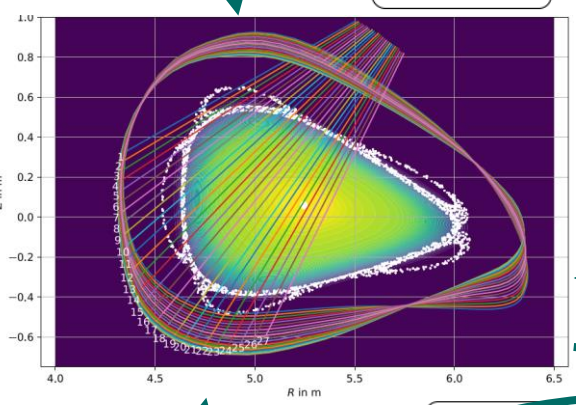
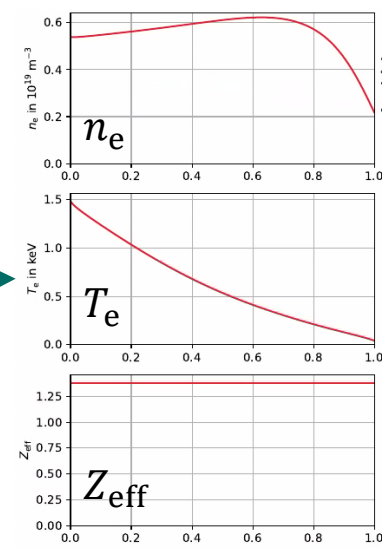
D_{vis}

D_{IR}

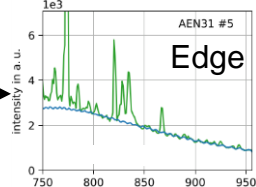
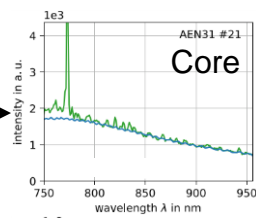
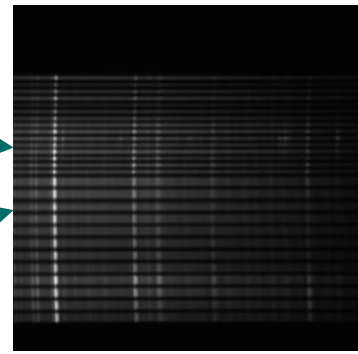
D_{int}

D_{TS}

Observations



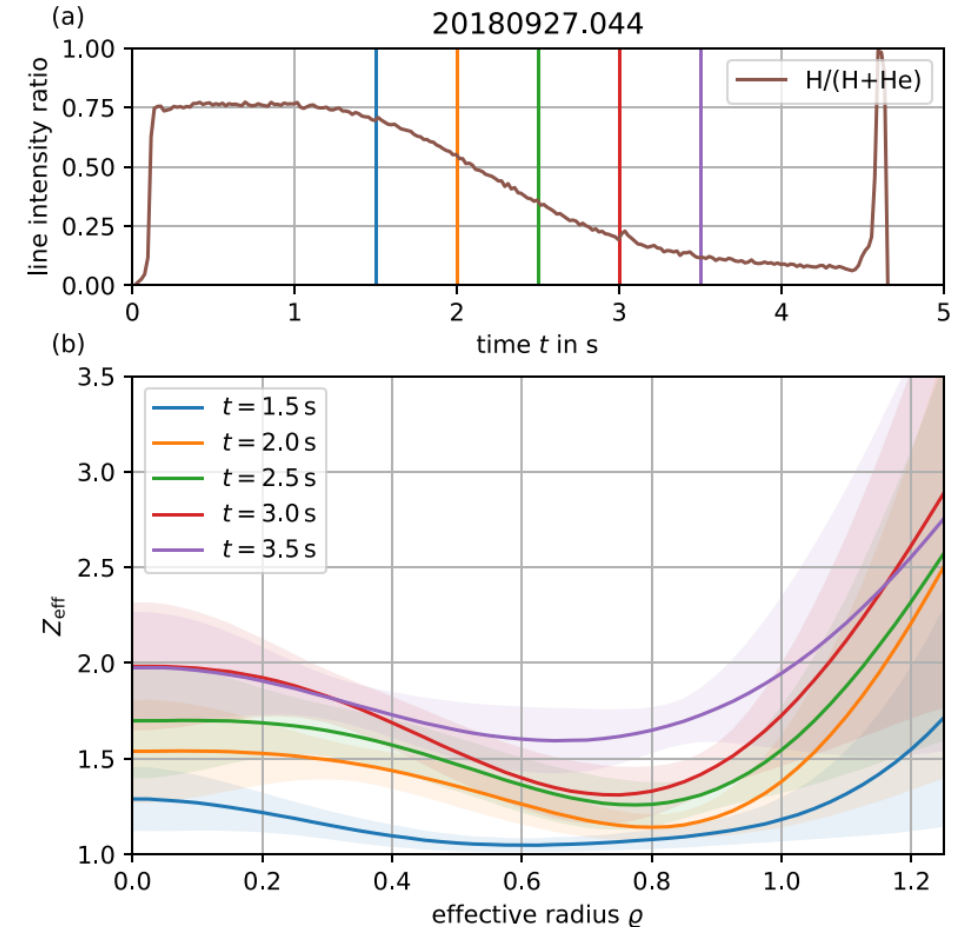
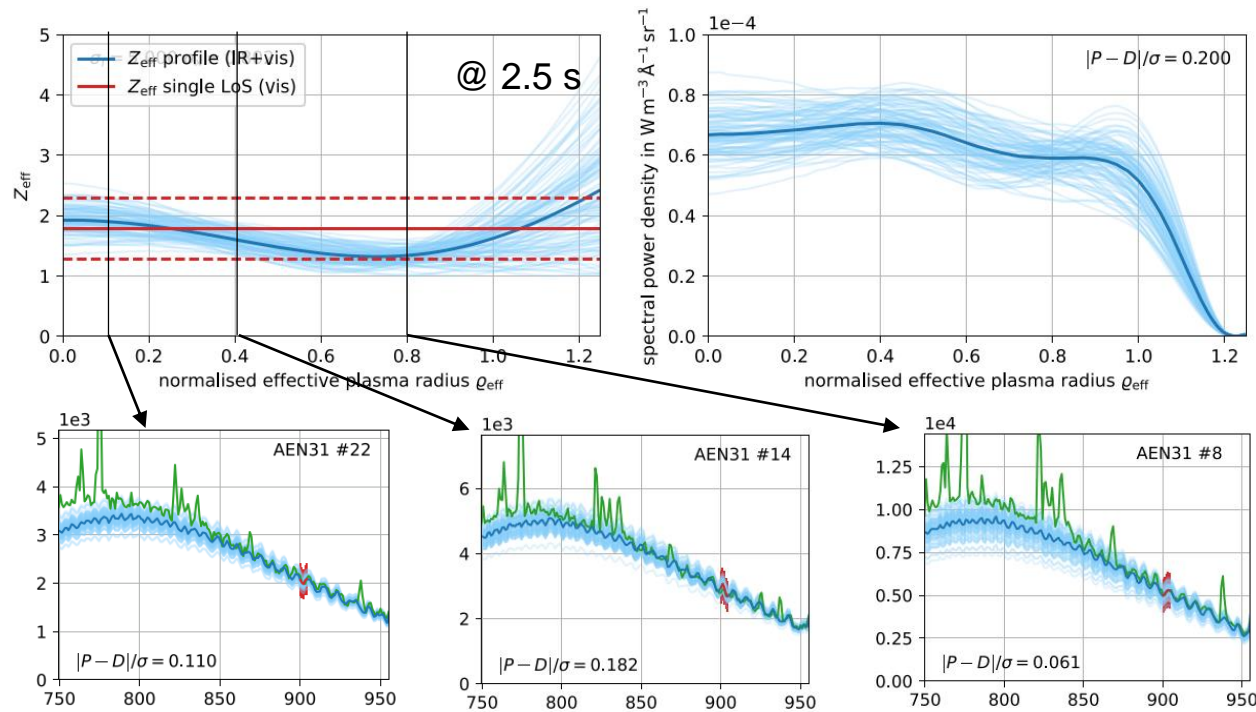
LoS + Calibration





Application: Z_{eff} profiles from line integrated bremsstrahlung

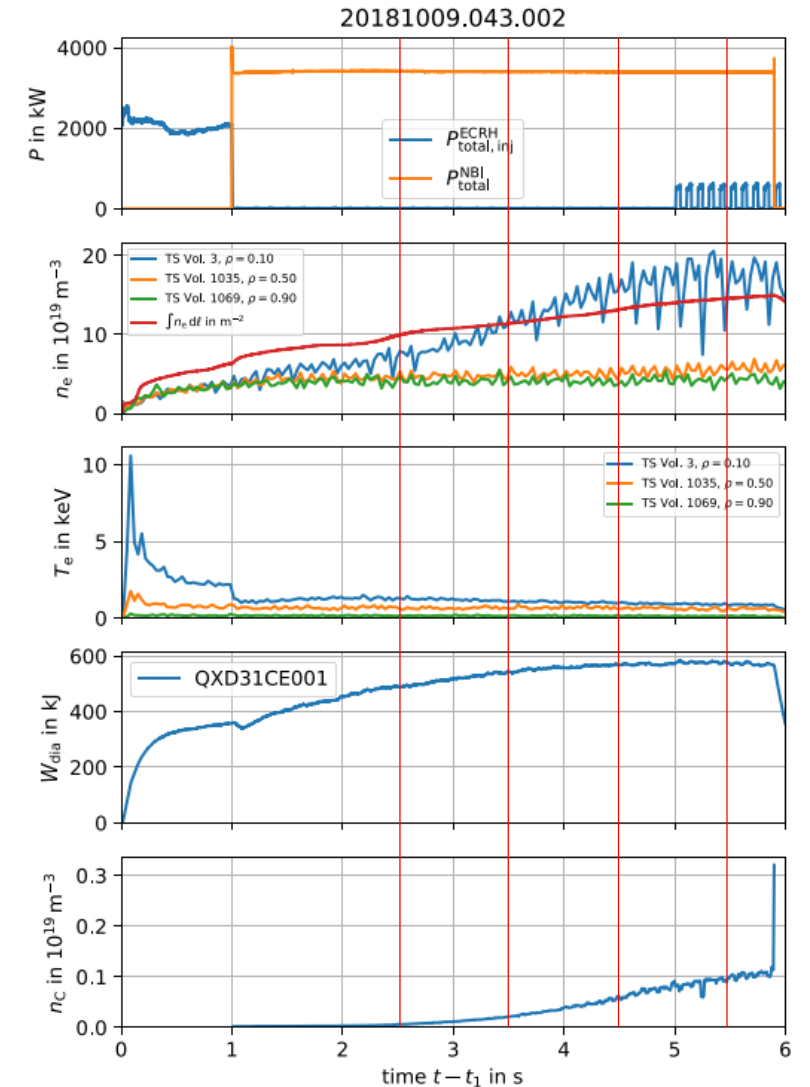
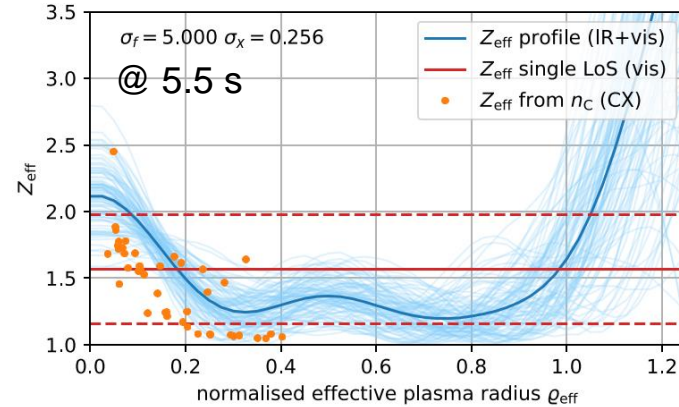
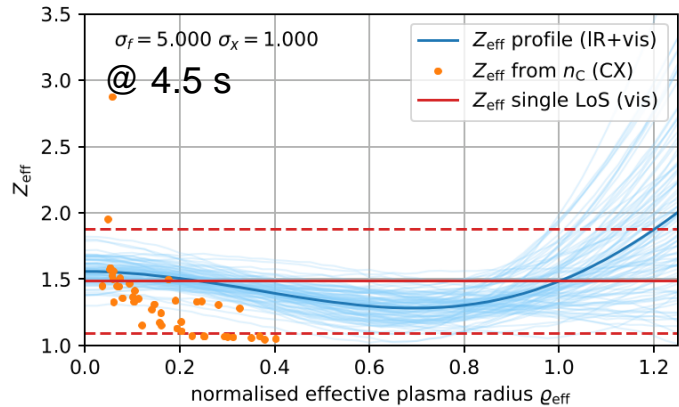
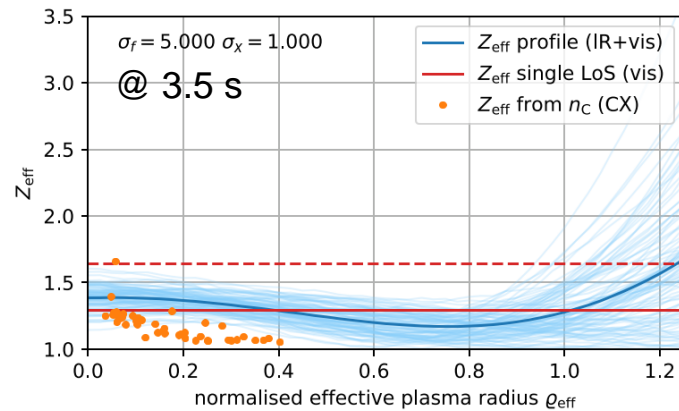
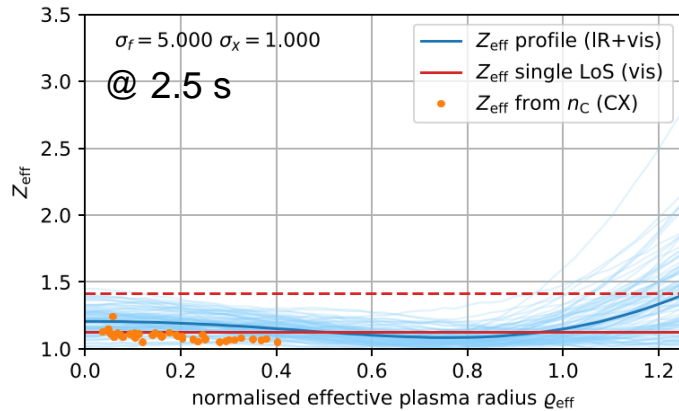
- Example discharge: H \rightarrow He plasma
- Increasing overall Z_{eff} over time
- Well predicted line integrated bremsstrahlung spectra





Application: Z_{eff} profiles from line integrated bremsstrahlung

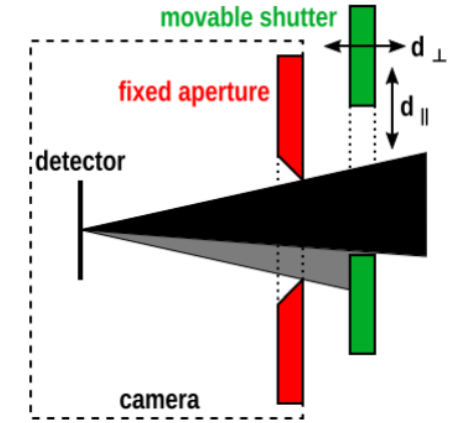
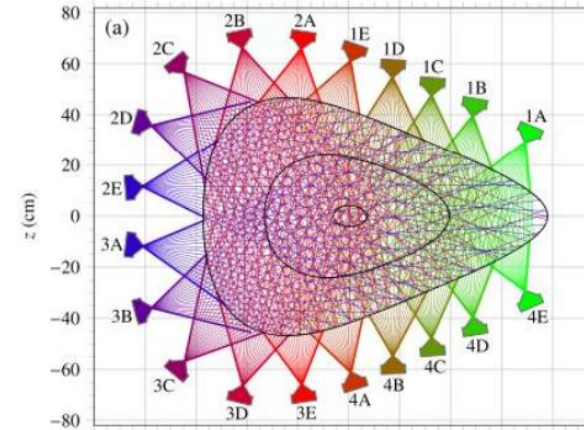
- Example discharge: Carbon accumulation
- Consistent with estimated Z_{eff} values from CX spectrometers



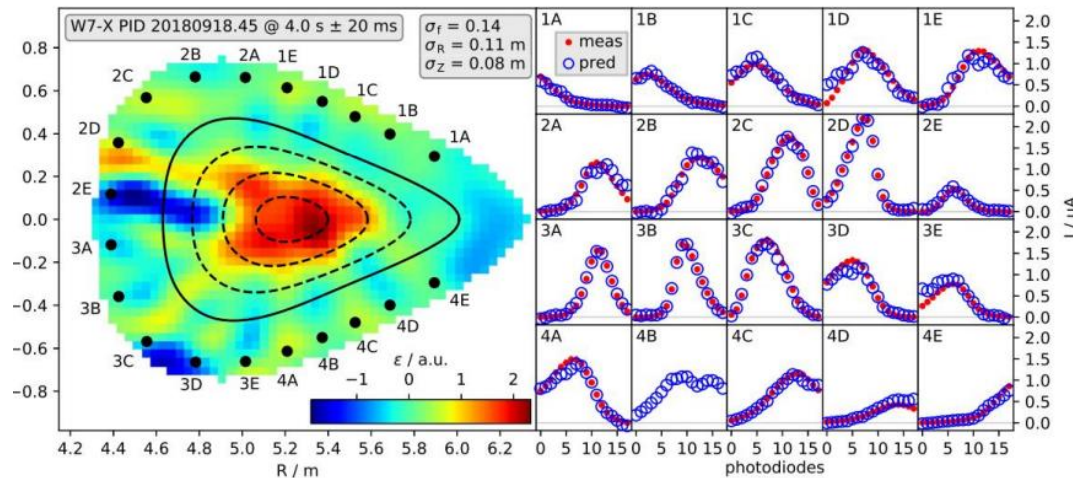


Application: Soft X-ray tomography

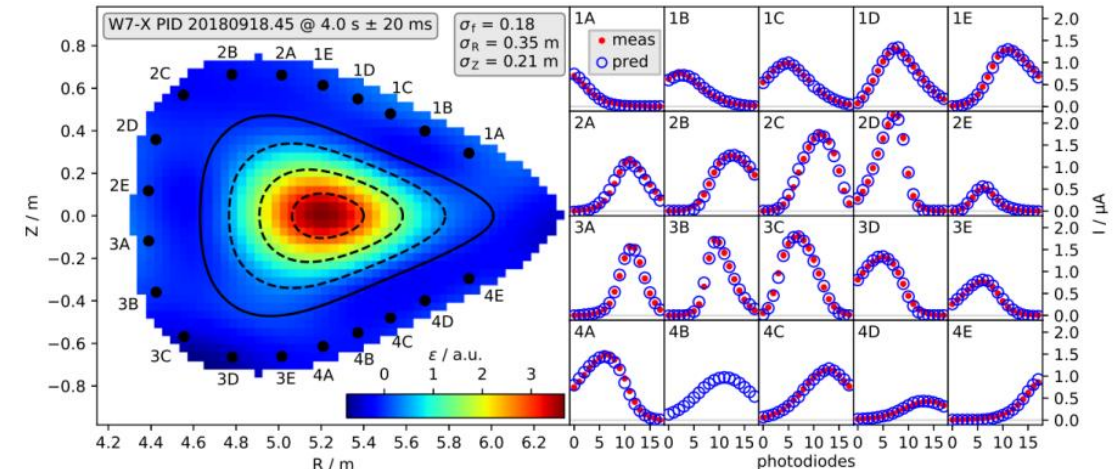
- XMCTS: 20 soft X-ray cameras
 - Shutters open to unknown positions → Tomograms inconsistent with flux surfaces
- Solution: infer shutter positions and calibration factors by exploring the marginal posterior $P(d_{\parallel}, d_{\perp}, c | D)$.



Shutter position not inferred

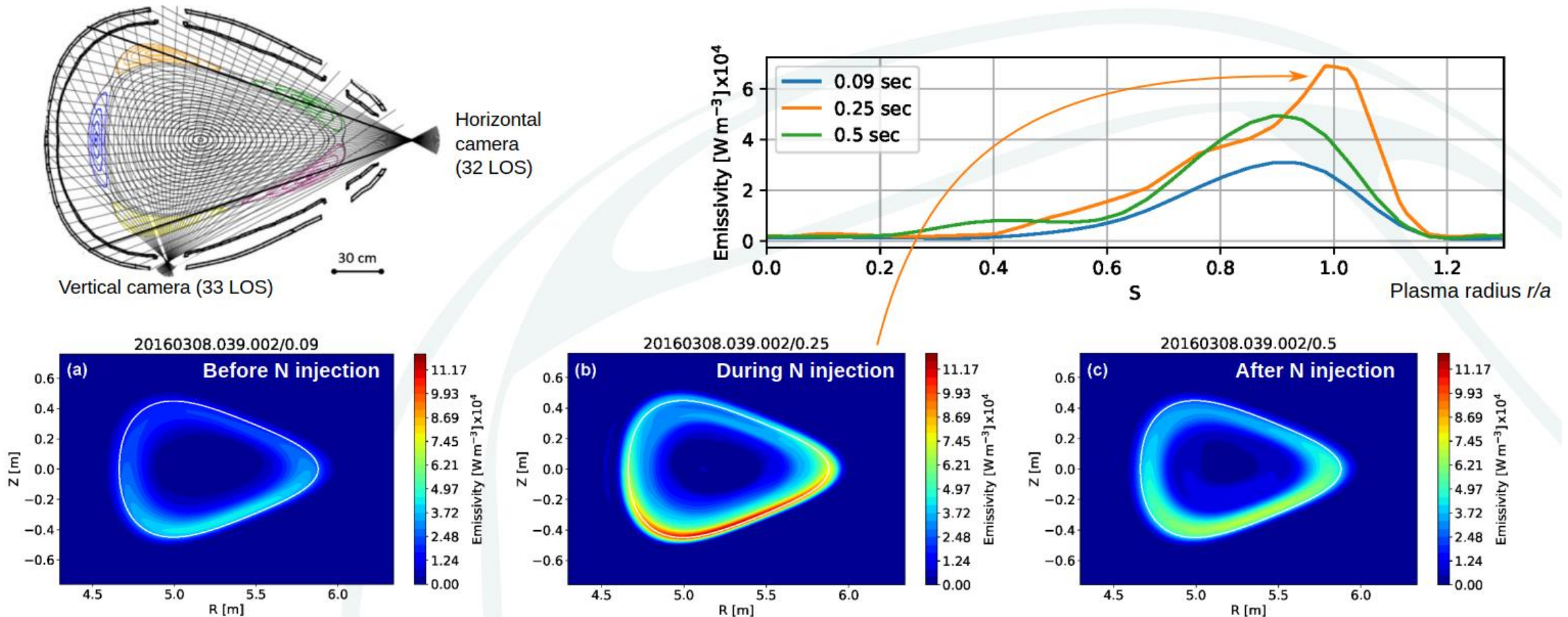


Shutter position inferred



Application: Bolometer tomography

- Advanced Gaussian process tomography resolving asymmetric radiation patterns

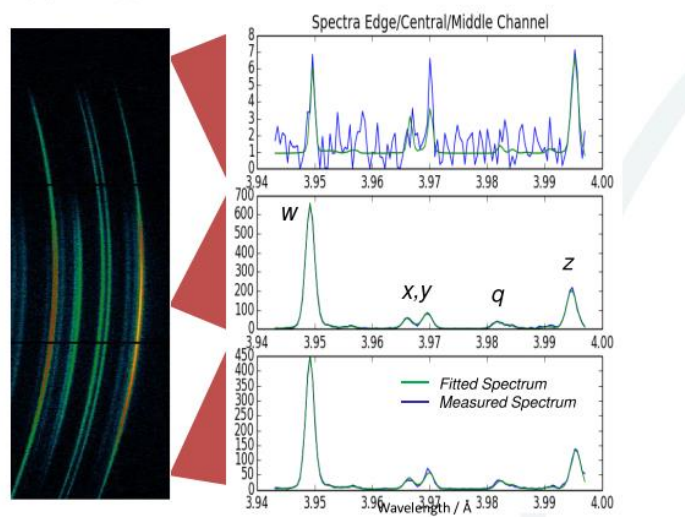
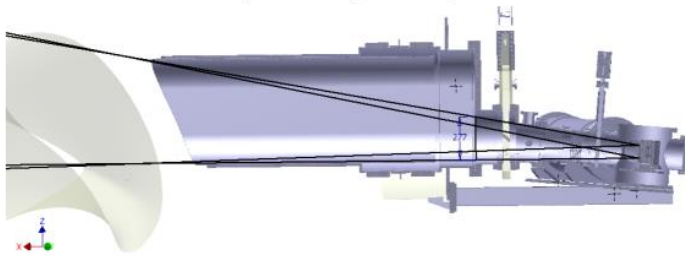


J. Svensson, S. Kwak *et al*, to be published

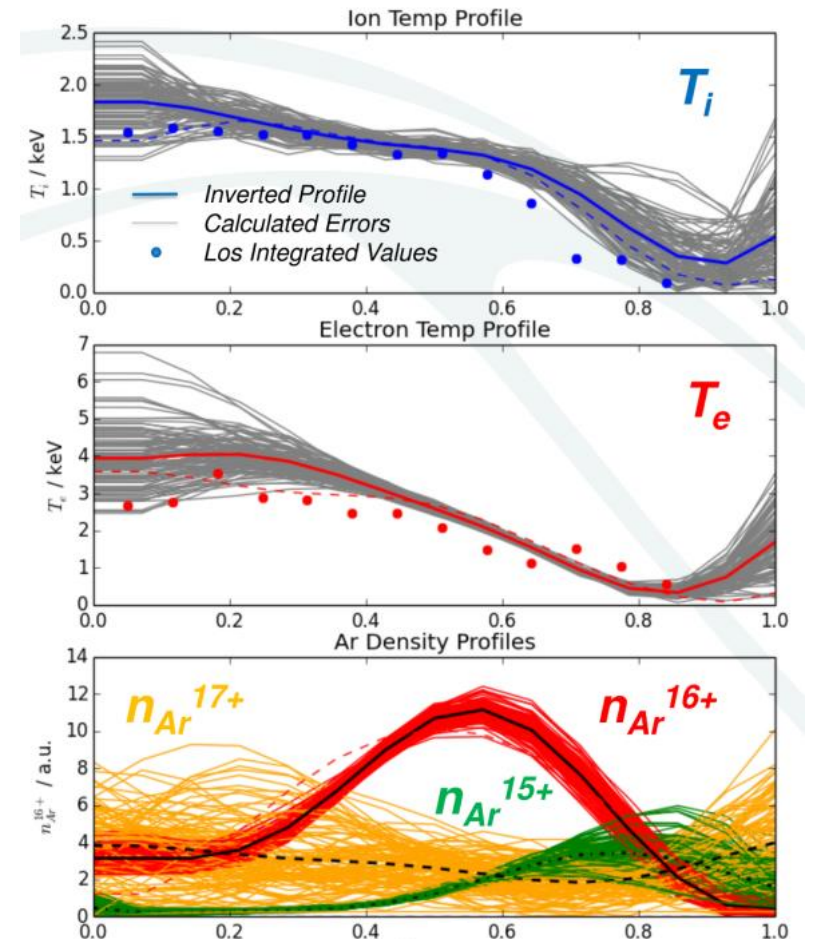
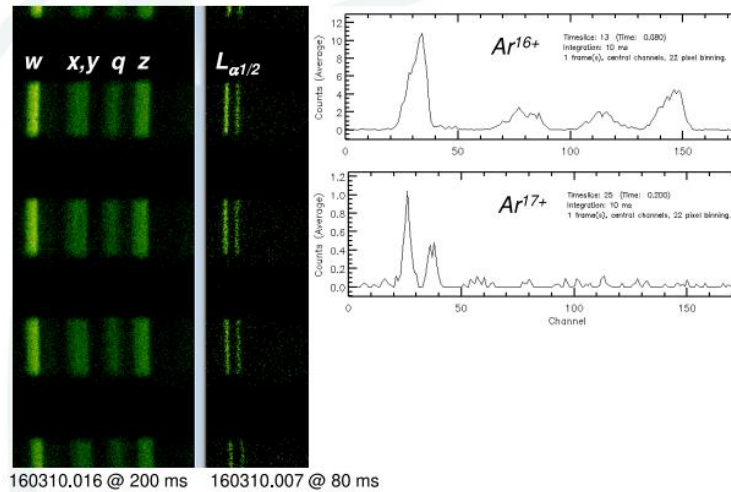
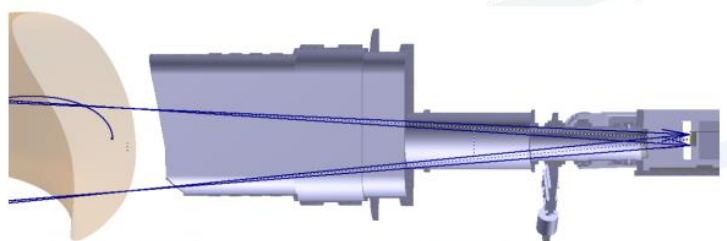
Application: X-ray imaging spectrometers

- Tomographically inferred profiles given line-integrated X-ray spectra
 - Delivering T_e, T_i and n_z profiles for transport studies, compared well with fast analysis by Novimir Pablant

XICS He-like Ar Spectra (PPPL)



HR-XIS He- and H-like Ar Spectra (FZ Jülich)

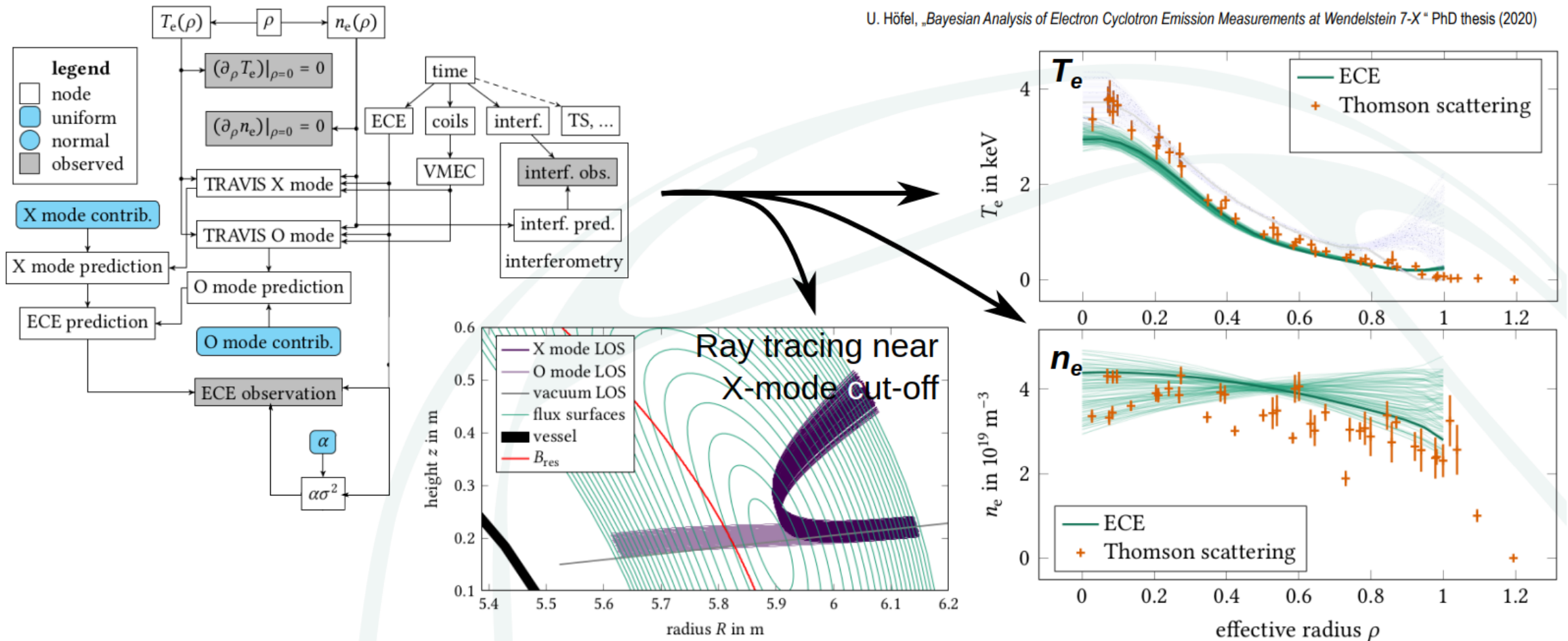




Application: ECE

- T_e profiles from ECE measurements by including the TRAVIS code
- Extract n_e profiles by combining ECE and interferometer data sets

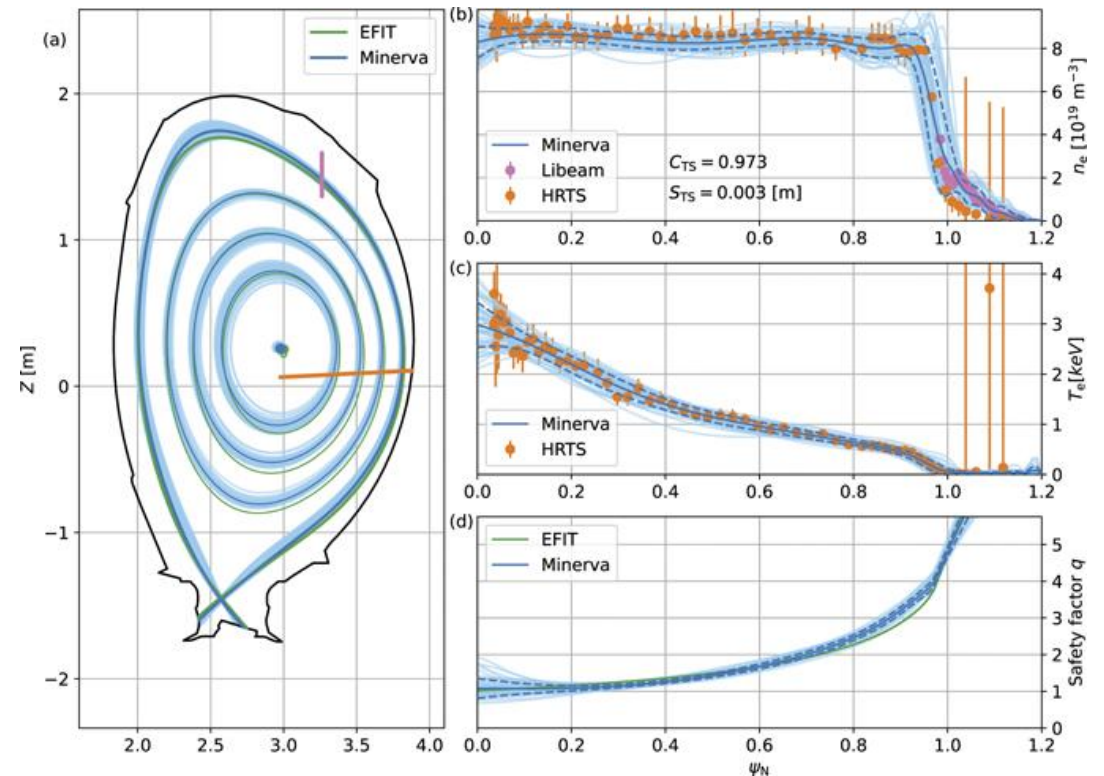
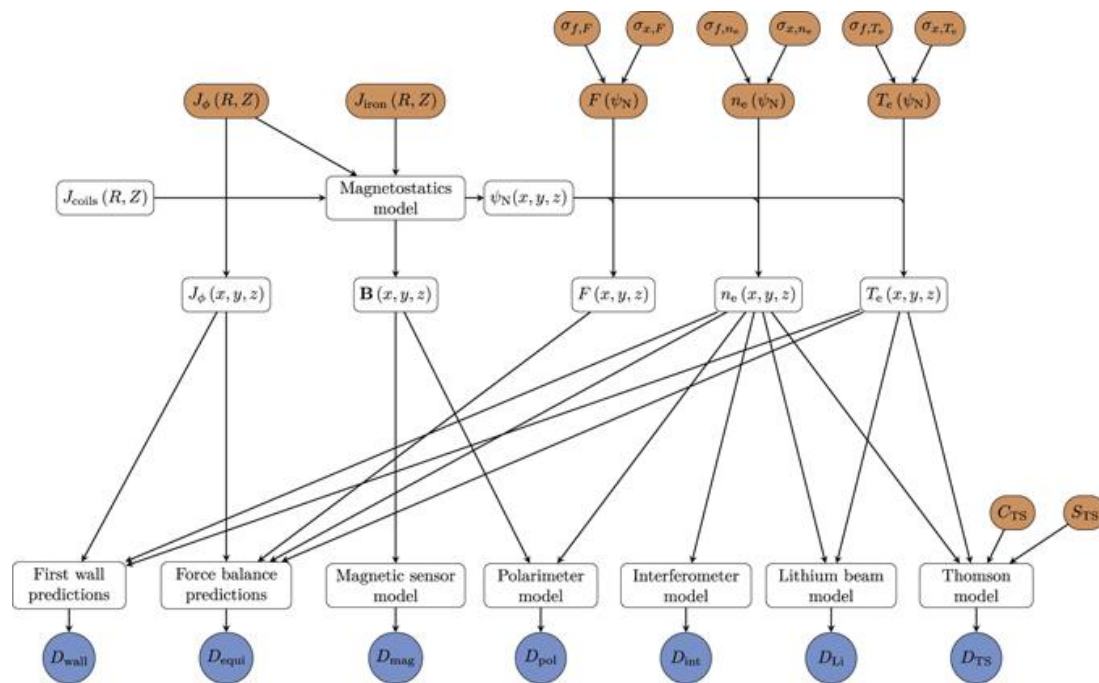
U. Höfel, „Bayesian Analysis of Electron Cyclotron Emission Measurements at Wendelstein 7-X“ PhD thesis (2020)





Application: Inference of equilibria (JET)

- Bayesian modelling of plasma equilibria at JET
- Find equilibria fulfilling the force balance and multiple diagnostics (magnetics – pickups, saddles, flux loops, interferometers/polarimeters and profile diagnostics)
- MHD force balance implemented as a Bayesian prior through virtual observations

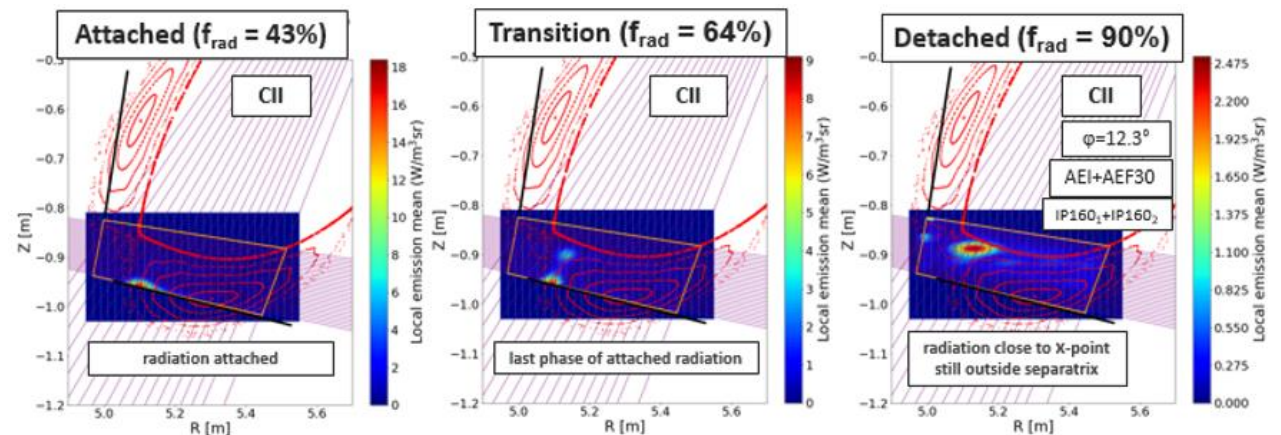
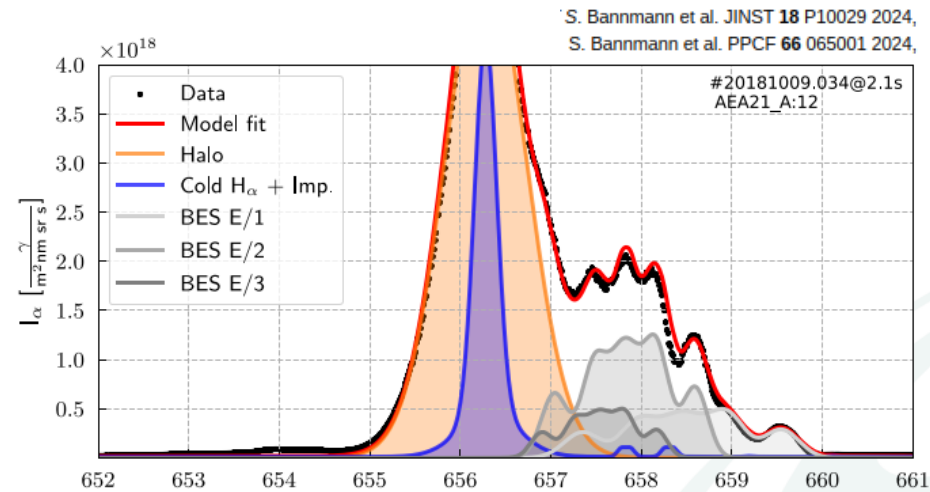
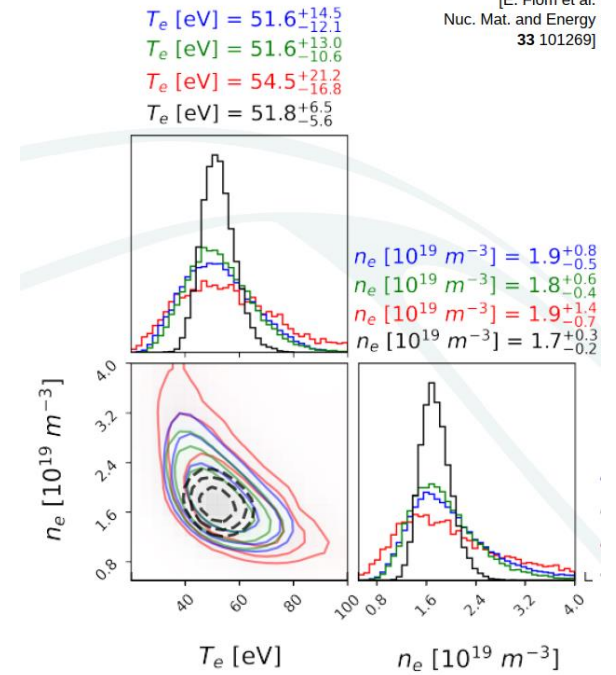




Other applications

- Beam emission spectrometers with beam model and halo emission
- Divertor impurity emissivity tomography
- Helium beam diagnostics, atomic physics studies
- ECE calibrations and profiles, Michelson interferometers
- Langmuir probes, magnetics, single LoS Z_{eff} , power deposition, etc.
- Easily transferable to other devices: also employed in ITER, JET, etc.

[E. Flom et al.
Nuc. Mat. and Energy
33 101269]

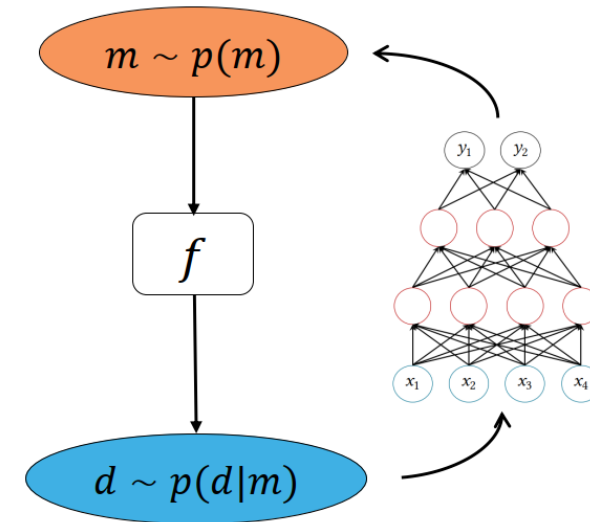


M. Krychowiak et al, EPS 2021

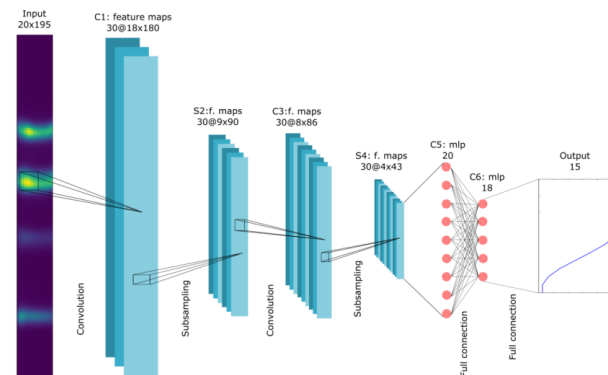


Deep learning surrogates within the Minerva framework

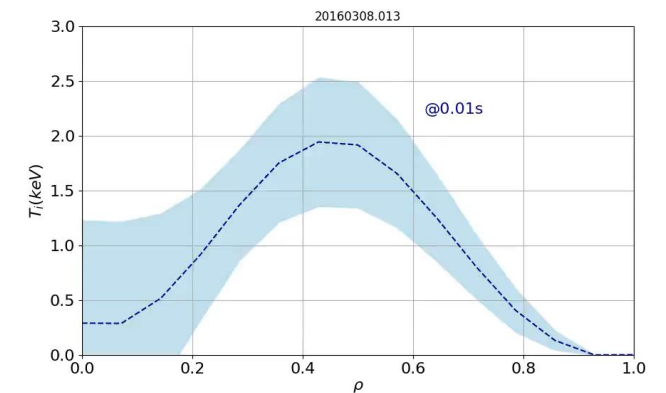
- Main obstacle of Bayesian inference: inversion time
- Can be overcome by accelerating Minerva models through the use of deep learning surrogates:
 - Training a deep learning model with synthetic data generated by the Minerva models
- Applicable to any Minerva model for fast inference ($\approx 100 \mu\text{s}$)
- Accelerated applications:
 - X-ray imaging spectrometers (T_i profiles)
 - Single LoS Z_{eff}
 - VMEC equilibria
 - Lithium beam diagnostics at JET (n_e profiles)



A. Pavone, Machine learning approximation of Bayesian inference in nuclear fusion PhD thesis



A. PAVONE et al. »Neural network approximation of Bayesian models for the inference of ion and electron temperature profiles at W7-X«. In: *Plasma Physics and Controlled Fusion*, Vol. 61.7 (May 2019), page 075012. doi: 10.1088/1361-6587/ab1d26.





Summary

Bayesian modelling of plasma diagnostics within the Minerva framework:

- Consistent inferences obtained from various observation combinations
- Uncertainty quantification without losing any information
- Well-established method for developing and maintaining the applications
- Broad application spectrum: from profile inferences using multiple diagnostics to advanced tomographic techniques for bremsstrahlung and soft X-ray data
- Easily transferable to other devices: also employed in ITER, JET, etc.
- Fast inferences ($\approx 100 \mu\text{s}$) based on deep learning: applicable to any Minerva model



References

- List of diagnostics implemented within Minerva
 - Visible Bremsstrahlung [S Kwak RSI 92, 043505 2021] + Neural network fast surrogate [A. Pavone et. al. PPCF 62 045019]
 - Soft X-ray cameras [J. Schilling et al. PPCF 63 055010]
 - X-Ray spectroscopy [A. Langenberg Nucl. Fus. 61 116018] + Neural network fast surrogate [A. Pavone et al. 2019 Plasma Phys. Control. Fusion 61 075012]
 - Bolometry [Contact Seed eScience Ltd] [J Svensson, S Kwak et al, to be published]
 - Beam emission spectroscopy (not fluctuations) [S. Bannmann et al. JINST 18 P10029 2024] + Neural network fast surrogate in development
 - ECE [U. Höfel, PhD Thesis <https://depositonce.tu-berlin.de/items/1000194b-7825-4e4e-acec-7415665d7708>]
 - Thomson Scattering / Interferometry [S Kwak et al, to be published]
 - Thermal helium beam [E. Flom et al. Nuc. Mat. and Energy 33 101269]
 - Divertor visible spectroscopy [M. Krychowiak et al. EPS 2022]
 - Langmuir probes [L. Rudischhauser RSI. 91, 063505]
 - 3D Equilibrium magnetics [J. Schilling et. al. MSc Thesis Kiel University 2018] + Neural network fast surrogate [A. Merlot Nucl. Fus. 61 096039]
 - Heavy-ion beam probe [H. Trimino Mora et al. HTPD 2024]
 - Ellipsometry (Stand-alone) [M. Krychowiak et al, HTPD 2024]
- ITER diagnostics: interferometers, polarimeters, magnetics (pickups, flux loops, Rogowskis), XRCS, Hard X-ray, Soft X-ray, ECE, visible reference spectrometers (bremsstrahlung, H-alpha, synchrotron, real-time Z_{eff} , n_e) [internal ITER reports]



References

- JET diagnostics
 - Interferometers (Stand alone GPT application for profiles) [Svensson J. 2011 *EFDA-JET-PR(11)24* JET-EFDA]
 - High-resolution TS system [Kwak S et al. 2020 *Nucl. Fusion* **60** 046009]
 - Lithium beam diagnostics [Kwak S. et al. 2017 *Nucl. Fusion* **57** 036017] + **Neural network fast surrogate** [A Pavone et al 2020 *PPCF* 62 045019]
 - ECE [S. Schmuck et al 2020 *Nucl. Fusion* **60** 066009]
 - Polarimeters [Ford O. et al. 2008 *Rev. Sci. Instrum.* **79** 10F324]
 - Magnetics (pickups, saddles, flux loops) [Svensson J. and Werner A. 2008 *Plasma Phys. Control. Fusion* **50** 085002]
 - Current tomography [Svensson J. and Werner A. 2008 *Plasma Phys. Control. Fusion* **50** 085002]
 - Equilibrium [Sehyun Kwak et al 2022 *Nucl. Fusion* **62** 126069]
 - Z_{eff} profiles [Svensson J., JET Internal report]
 - Divertor camera [Svensson J., JET Internal report]
 - Soft X-ray [Li D. et al. 2013 *Rev. Sci. Instrum.* **84** 083506]
- For Minerva license, contact Seed eScience Research Ltd.



Thank you very much for your attention!