

# Simulations of NBI-driven Global Alfvén Eigenmodes in NSTX

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## Motivation:

- Multiple sub-cyclotron frequency modes are observed in NSTX during NBI injection.
- CAE and GAE modes are predicted to be driven unstable by super Alfvénic NBI ions with  $V_b \sim 3V_A$  (80 keV) through the Doppler shifted cyclotron resonance.
- Strong anisotropy in the fast-ion pitch-angle distribution provides the energy source for these instabilities.
- New observations of sub-cyclotron frequency modes during NBI injection show the spectrum line intersections, corresponding to GAE dispersion (with q-profile relaxation), and nonlinear evolution of GAE spectrum.
- Both CAE and GAE modes are observed.
- Numerical simulations are needed to include: self-consistent anisotropic equilibrium, FLR effects, thermal ion and nonlinear effects.

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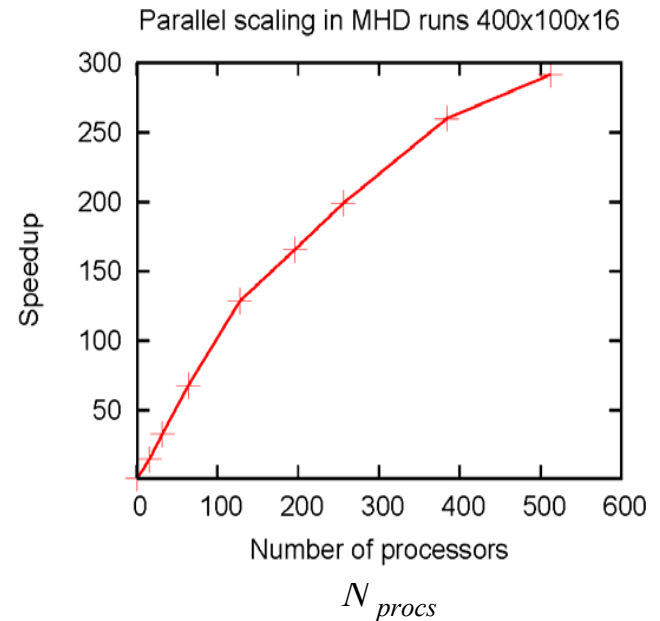
# HYM – Parallel Hybrid/MHD Code

HYM code developed at PPPL and used to investigate kinetic effects on MHD modes in toroidal geometry (FRCs and NSTX)

- 3-D nonlinear.
- Three different physical models:
  - Resistive MHD & Hall-MHD.
  - Hybrid (fluid electrons, particle ions).
  - MHD/particle (one fluid thermal plasma, + energetic particle ions)<sup>1</sup>
- Full-orbit kinetic ions.
- For particles: delta-f / full-f numerical scheme.
- Parallel (3D domain decomposition, MPI)<sup>2</sup>.

<sup>1</sup>Beam ions:  $V_0 > V_A$ , large Larmor radius  $\rho_i / L \sim 0.1-0.3$ .  
High-frequency modes:  $\omega \sim \omega_{ci}$

<sup>2</sup>Simulations are performed at NERSC.



New MPI version of HYM shows good parallel scaling up to 500 processors for production-size jobs, and allows high-resolution nonlinear simulations.

# Self-consistent anisotropic equilibrium the energetic NBI ions in NSTX

Grad-Shafranov equation for two-component plasma: MHD plasma (bulk) and fast ions

$$\frac{\partial^2 \psi}{\partial z^2} + R \frac{\partial}{\partial R} \left( \frac{1}{R} \frac{\partial \psi}{\partial R} \right) = -R^2 p' - HH' - GH' + RJ_{i\phi}$$

$$\mathbf{B} = \nabla \phi \times \nabla \psi + h \nabla \phi$$

$$h(R, z) = H(\psi) + G(R, z)$$

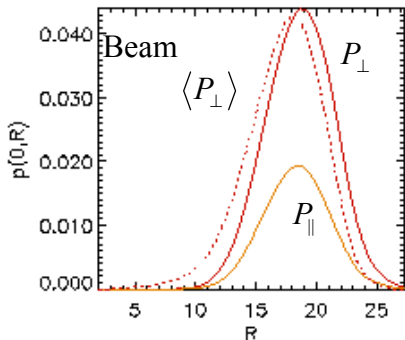
$$\mathbf{J}_{ip} = \nabla G \times \nabla \phi \quad , \quad G - \text{poloidal stream function}$$

## Self-consistent MHD + fast ions coupling scheme

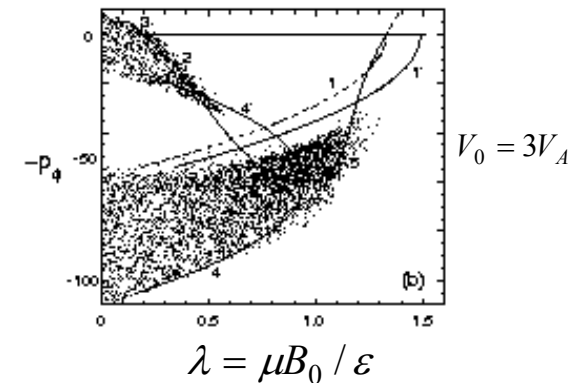
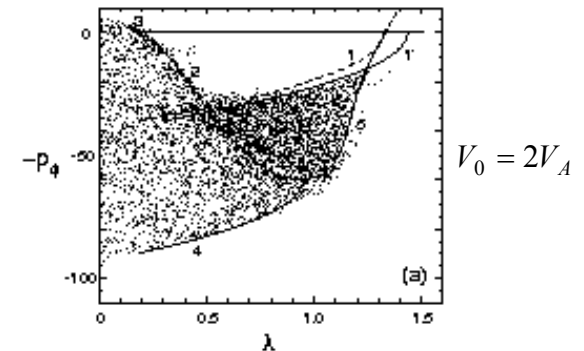
Background plasma - fluid:  $\rho \frac{d\mathbf{V}}{dt} = -\nabla p + (\mathbf{j} - \mathbf{j}_i) \times \mathbf{B} - n_i (\mathbf{E} - \eta \mathbf{j})$

**Fast ions** – delta-f scheme:  $F_0 = F_0(\varepsilon, \mu, p_\phi)$ , where  $\mu$  is calculated up to 1<sup>st</sup> order in  $\rho_i / L$ ; realistic  $F_0$  to match the distribution functions computed from the TRANSP code:

-The prompt-loss condition, anisotropy, the large Larmor radius of the beam ions and the strong pitch-angle scattering at low energies have been included in order to match the distribution functions computed from the TRANSP code.



Strong modifications of equilibrium profiles due to beam ions: more peaked current profile, anisotropic pressure, increase in Shafranov shift and reduction of  $q_{\text{axis}}$  – indirect effect on stability.



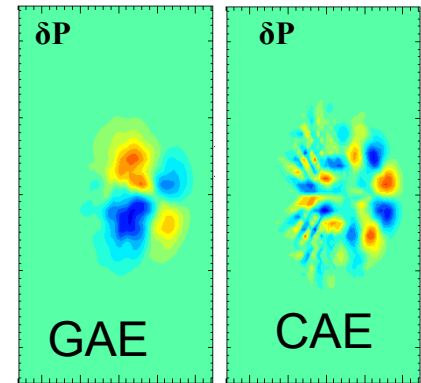
# 3D simulations of energetic ion-driven instabilities in NSTX

## Linearized delta-f simulations

- Low- $n$  simulations ( $n=1-6$ ) show instability of Global Alfvén Eigenmode (GAE) with large  $k_{\parallel}$ , and significant compressional component  $\delta B_{\parallel} \approx 1/3 \delta B_{\perp}$ .

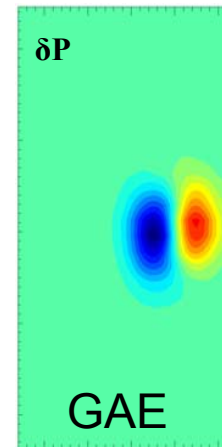
For  $n=4$  and  $m=-2$ ,  $\gamma = 0.005\omega_{ci}$  and  $\omega = 0.3\omega_{ci}$  for  $n_b=3\%$   
 $\gamma = 0.016\omega_{ci}$  and  $\omega = 0.3\omega_{ci}$  for  $n_b=5\%$  ( $E=80\text{keV}$ )

- Hybrid simulation with  $n=8$  show weakly unstable CAE mode with  $m=8-10$ ,  $\omega = 0.4\omega_{ci}$ ,  $\gamma \approx 0.001\omega_{ci}$ , and  $\delta B_{\parallel} > \delta B_{\perp}$
- GAE modes are more unstable than CAE (agrees with analytical calculations) with  $\gamma/\omega \sim n_b/n_0$ .
- Main damping mechanism for GAE is continuum damping (modeled in HYM with artificial viscosity):  $\gamma_d/\omega \sim (r/r_{res})^{2m+\delta}$
- Simulations with different  $n$  show that most unstable modes have  $(n-m) \sim 6-7$ , ie same  $k_{\parallel}$ .
- Modes with larger- $m$  have smaller radial extent.



$n=4$  ( $m=-2$ )

$n=8$



$n=6$  ( $m=-1$ )

# Low- $n$ most unstable modes have a character of **GAE** modes

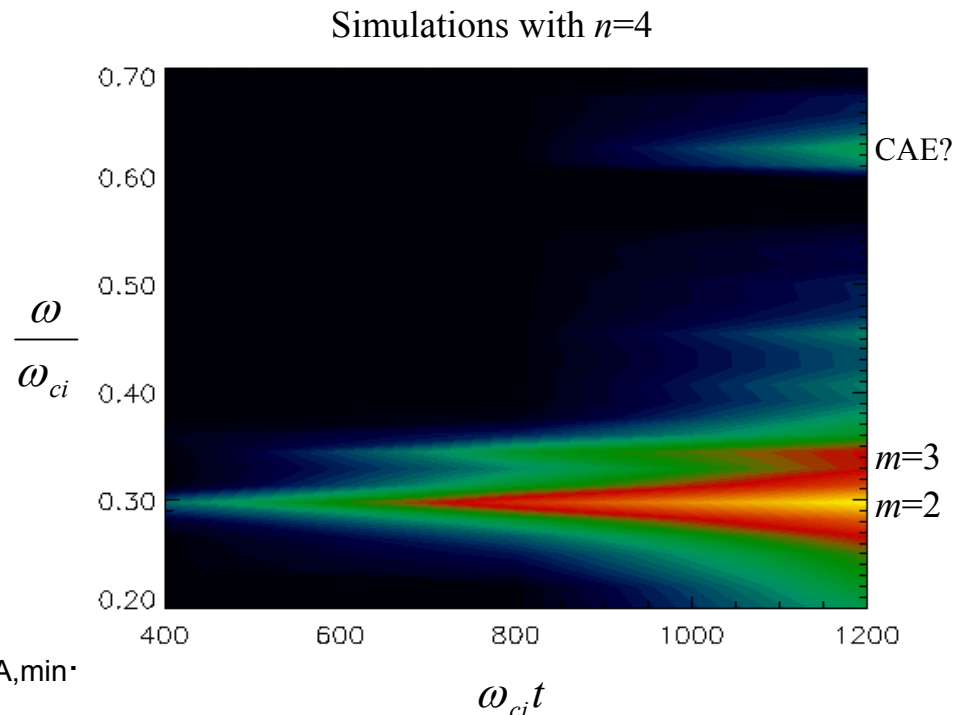
- Modes with  $2 < n < 7$  are unstable.
- For each  $n$ , several  $m$  are unstable with large  $k_{\parallel}$  and  $nm < 0$ .
- Localized near magnetic axis.
- Large  $\delta B_{\perp}$  component.

- observed features agree with that of GAE mode, which exists just below the lower edge of the Alfvén continuum [Appert et al., 1982].

GAE modes are primarily cylindrical and localized radially near  $\min(\omega_A)$  with  $\omega < \omega_{A,\min}$ .

For  $n=4$ ,  $m=-2$  case  $\omega/\omega_{ci}=0.3 < \omega_A/\omega_{ci}=0.33$ .

$$\omega \approx \pm \frac{V_A}{R_0} \left( \frac{m}{q_0} - n \right)$$



# Resonant condition for GAE instability

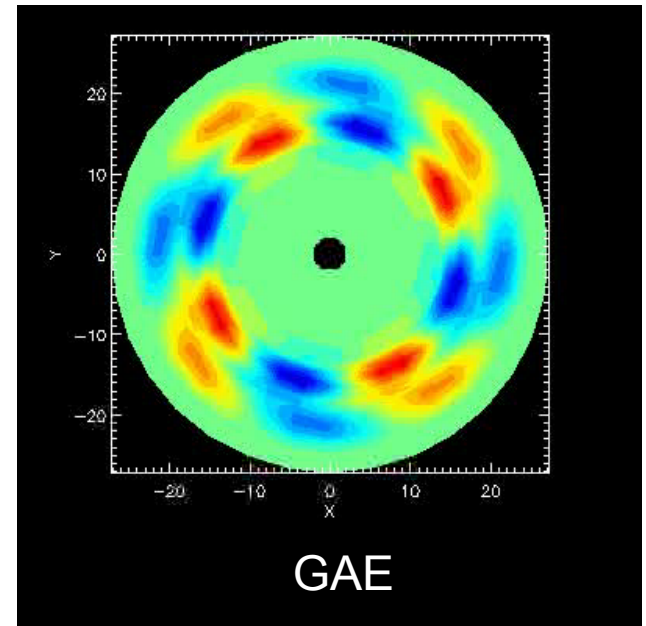
Resonant beam ions satisfy condition:

$$\omega - k_{\parallel} V_{\parallel} - \omega_{ci} \approx 0$$

$$\omega \approx k_{\parallel} V_{\parallel}$$

In the simulations:  $V_{\parallel} > 0$  and  $\omega/k_{\parallel} < 0$ .

For  $\omega = 0.3-0.4\omega_{ci}$  and  $k_{\parallel} V_A \sim 0.3$ , the resonant particles will have  $V_{\parallel} \sim 2V_A \rightarrow$  significant fraction of beam ions can be in resonance.



Contour plot of perturbed fluid pressure at equatorial plane.

# Nonlinear simulations (GAE instability)

## 3D nonlinear hybrid simulations for given toroidal mode number

- Nonlinear results ( $n_b=5\%$ ) show saturation of instability at relatively low amplitudes:

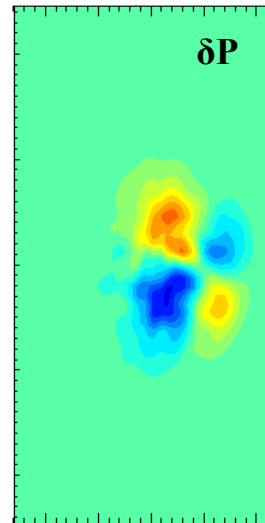
$$\delta P \sim 0.8 \cdot 10^{-3}$$

$$\delta V_R \sim 6 \cdot 10^{-3}$$

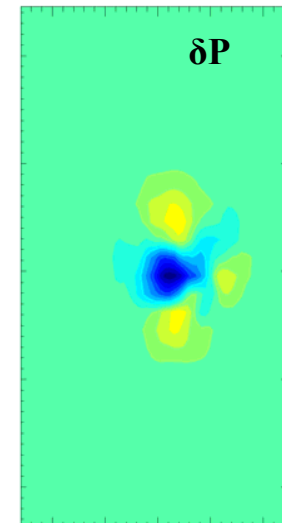
$$\delta V_Z \sim 6 \cdot 10^{-3}$$

$$\delta V_\phi \sim 10^{-3}$$

- Most unstable mode changes nonlinearly from  $n=4$   $m=-2$  mode to  $(4,-3)$  mode.
- Poloidal mode structure is complicated: several modes with different poloidal mode number have comparable amplitudes.
- Initial fully nonlinear simulations (many- $n$ ) show dominant  $n=3$  instability.



Linear mode structure,  $n=4$ ,  $m=-2$



Nonlinear mode structure  $n=4, m=-3$  (at saturation)

# Conclusions

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- Nonlinear 3D HYM code has been modified for ST geometry and parallelized.
- Grad-Shafranov solver for general case of the two component plasma: MHD plasma with  $p=p(\psi)$ , plus energetic ions with realistic (anisotropic) distribution.
- Simulations show that for large injection velocities, and strong anisotropy in the pitch-angle distribution, many Alfvén modes can be excited: GAE modes for  $2 < n < 7$ , and CAE modes for larger  $n$ .
- Instabilities are excited via Doppler-shifted resonance with NBI ions.
- GAE instability saturates at low amplitude, for given- $n$  several poloidal modes are excited with comparable amplitudes in nonlinear regime.