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Theory of Kinetic Stabilization of RWMs

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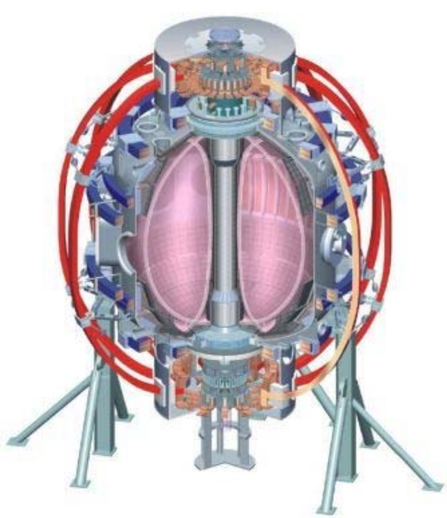

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The Kinetic Approach to δW

$$mn\frac{d\mathbf{u}}{dt} = \mathbf{j} \times \mathbf{B} - \boldsymbol{\nabla} \cdot \mathbf{P} \qquad \omega^2 \frac{1}{2} \int mn|\xi|^2 d\mathbf{V} = \frac{1}{2} \int \xi^* \cdot \left(\tilde{\mathbf{j}} \times \mathbf{B} + \mathbf{j} \times \tilde{\mathbf{B}} - \boldsymbol{\nabla} \tilde{p}_F - \boldsymbol{\nabla} \tilde{p}_K \right) d\mathbf{V}$$

$$\omega^2 K = \delta W_F + \delta W_K$$

$$\delta W_K = -\frac{1}{2} \int \xi^* \cdot \boldsymbol{\nabla} \tilde{p}_K d\mathbf{V}$$

- For the trapped ion contribution, the bounce-averaged drift kinetic equation is used to find:

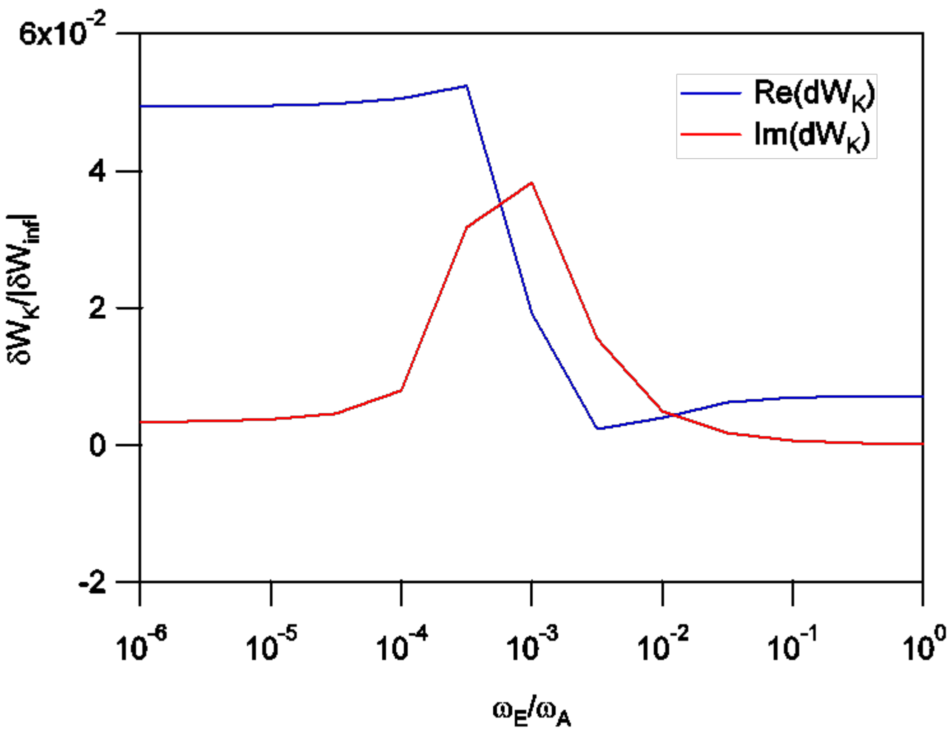
$$\delta W_K^{ti} = \int_0^{\Psi_a} \frac{d\Psi}{B_0} \left(\frac{p}{1 + \frac{T_e}{T_i}} \right) \sqrt{\pi} \sum_{l=-\infty}^{\infty} \int_{B_0/B_{max}}^{B_0/B_{min}} d\Lambda \hat{\tau}_b$$
$$\times \int_0^{\infty} \left[\frac{\omega_{*N} + \left(\hat{\varepsilon} - \frac{3}{2}\right) \omega_{*T} + \omega_E - \omega}{\langle \omega_D \rangle + l\omega_b - i\nu_{\text{eff}} + \omega_E - \omega} \right] \hat{\varepsilon}^{5/2} e^{-\hat{\varepsilon}} d\hat{\varepsilon}$$
$$\times \left\langle \left(2 - 3\frac{\Lambda}{B_0/B} \right) (\boldsymbol{\kappa} \cdot \xi_\perp) - \left(\frac{\Lambda}{B_0/B} \right) (\boldsymbol{\nabla} \cdot \xi_\perp) \right\rangle^2$$

Various resonances determine the kinetic stability: low collisionality example

$$\delta W_K^{ti} \propto \left[\frac{\omega_{*N} + \left(\hat{\varepsilon} - \frac{3}{2}\right)\omega_{*T} + \omega_E - \omega}{\langle \omega_D \rangle + l\omega_b - i\nu_{\text{eff}} + \omega_E - \omega} \right]$$

low collisionality

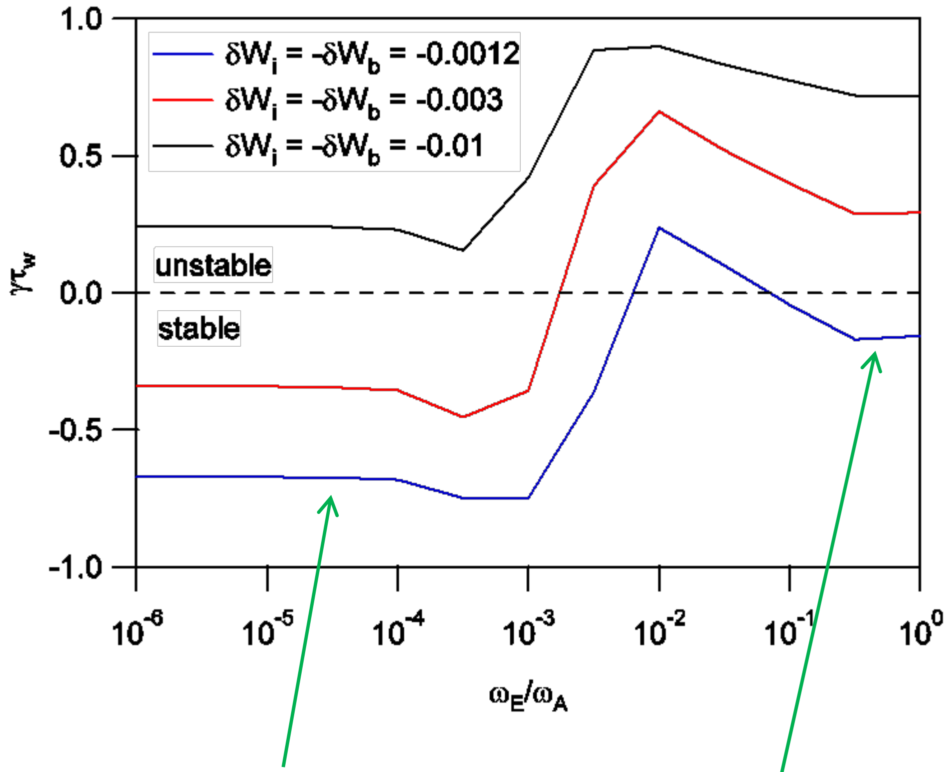
$$\omega \approx 0, \nu_{\text{eff}} = 0: \qquad \delta W_K^{ti} \sim \left[\frac{\omega_{*i} + \omega_E}{\langle \omega_D \rangle + l\omega_b + \omega_E} \right]$$



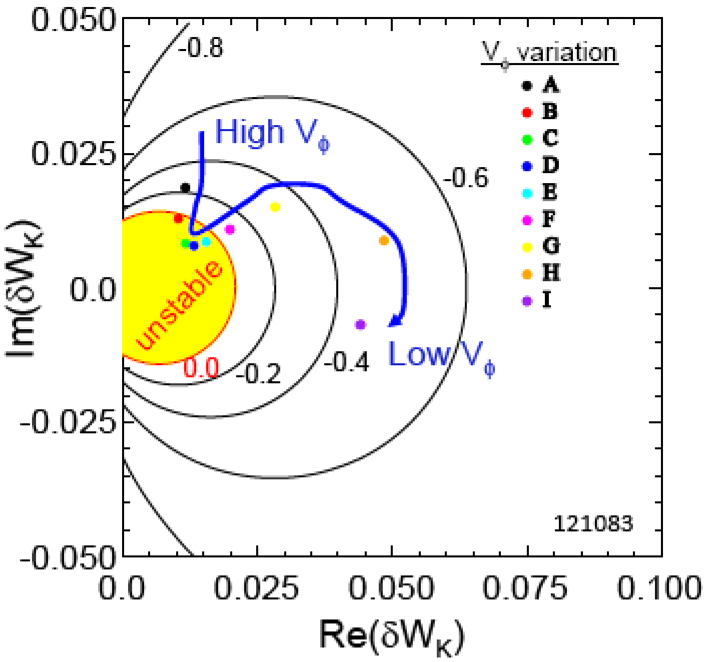
- At large ω_E the frequency resonance term [.] goes to 1 for the real part and 0 for the imaginary part.
- At small ω_E the real part goes like the diamagnetic frequency divided by precession drift plus bounce frequencies.
- What does this all mean for stability?

The growth rate depends on dWK resonances *and* its relative magnitude

$$\gamma_K \tau_w = -\frac{\delta W_\infty + \delta W_K}{\delta W_b + \delta W_K} \qquad Re(\gamma_K \tau_w) = -\frac{\delta W_\infty \delta W_b + (Im(\delta W_K))^2 + Re(\delta W_K)(\delta W_\infty + \delta W_b + Re(\delta W_K))}{(\delta W_b + Re(\delta W_K))^2 + (Im(\delta W_K))^2}$$



RWM stability vs. V_ϕ (contours of $\gamma\tau_w$)



low rotation stability!! *high rotation stability*

Various resonances determine the kinetic stability: high collisionality example

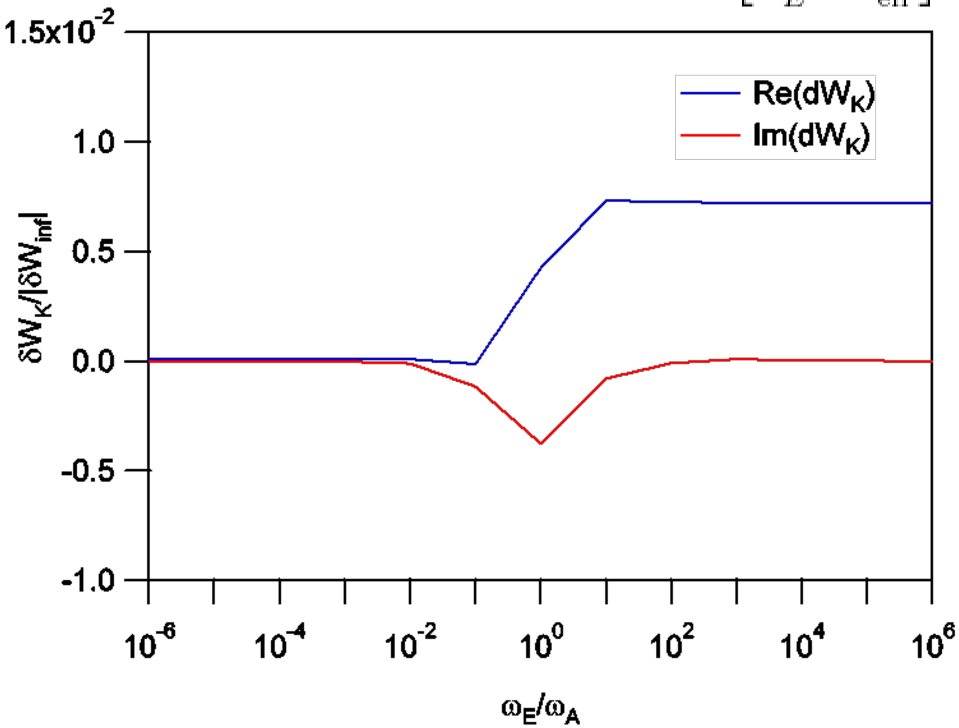
$$\delta W_K^{ti} \propto \left[\frac{\omega_{*N} + \left(\hat{\varepsilon} - \frac{3}{2}\right)\omega_{*T} + \omega_E - \omega}{\langle\omega_D\rangle + l\omega_b - i\nu_{\mathrm{eff}} + \omega_E - \omega}\right]$$

high collisionality

$\omega \approx 0, \nu_{\mathrm{eff}} = \mathrm{large}:$

$$Re\left(\delta W_K^{ti}\right) \sim \left[\frac{\omega_E^2}{\omega_E^2 + \nu_{\mathrm{eff}}^2}\right]$$

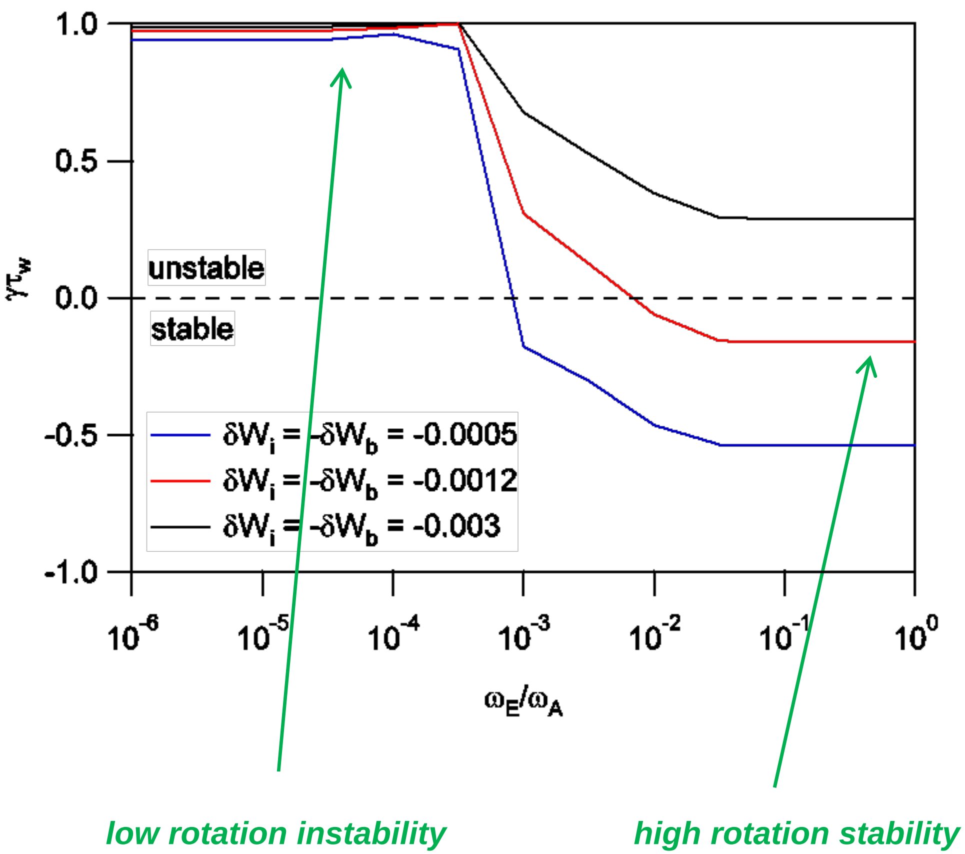
$$Im\left(\delta W_K^{ti}\right) \sim \left[\frac{\omega_E \nu_{\mathrm{eff}}}{\omega_E^2 + \nu_{\mathrm{eff}}^2}\right]$$



- At large ω_E the frequency resonance term [.] goes to 1 for the real part and 0 for the imaginary part (same as before).
- But, now at small ω_E the real and imaginary parts both go to 0.
- What happens in this case?

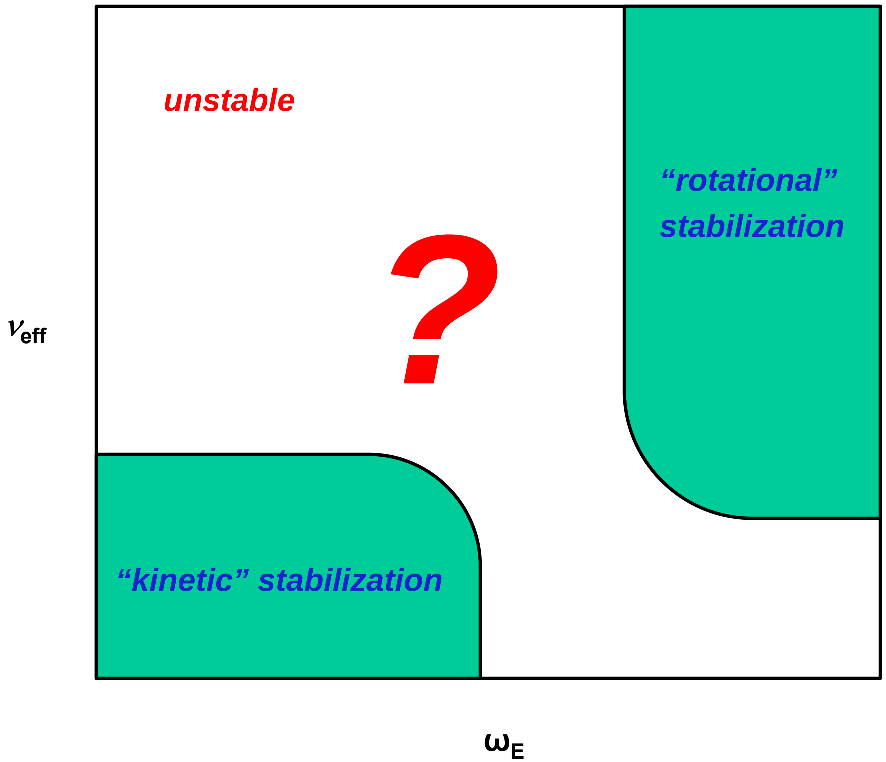
The high collisionality case is more like the "traditional" view of RWM stability

$$\gamma_K \tau_w = -\frac{\delta W_\infty + \delta W_K}{\delta W_b + \delta W_K} \qquad Re(\gamma_K \tau_w) = -\frac{\delta W_\infty \delta W_b + (Im(\delta W_K))^2 + Re(\delta W_K)(\delta W_\infty + \delta W_b + Re(\delta W_K))}{(\delta W_b + Re(\delta W_K))^2 + (Im(\delta W_K))^2}$$



- High collisionality swamps out all other kinetic effects. Only high rotation can stabilize.

A picture of the kinetic RWM stabilization physics is starting to emerge



- Could help determine:
  - What is the best operating space for RWM stability?
  - Why is DIII-D robustly stable while NSTX is not?
  - What happens when we go to lower collisionality in NSTX upgrade?
- Next Steps (theory)
  - Make contour plots like this one, for various C_β.
  - Improve the treatment of collisionality through the drift kinetic equation.

Solov'ev equilibrium benchmarking between Hu/Betti code and MARS is underway

$$P(\psi) = \frac{1+\kappa^2}{\kappa R_0^3 q_0}\psi, F(\psi) = 1$$

$$\psi(R,Z) = \frac{\kappa}{2R_0^3 q_0}\left[\frac{R^2 Z^2}{\kappa^2} + \frac{1}{4}\left(R^2 - R_0^2\right)^2 - a^2 R_0^2\right]$$

- constant density profile (n_e = 1.52e-21 m^{-3}).
- constant rotation profiles (ω_E/ω_A: 1e-6 – 1).