Recent Progress on NTV (Neoclassical Toroidal Viscoscity) Theory and Computation

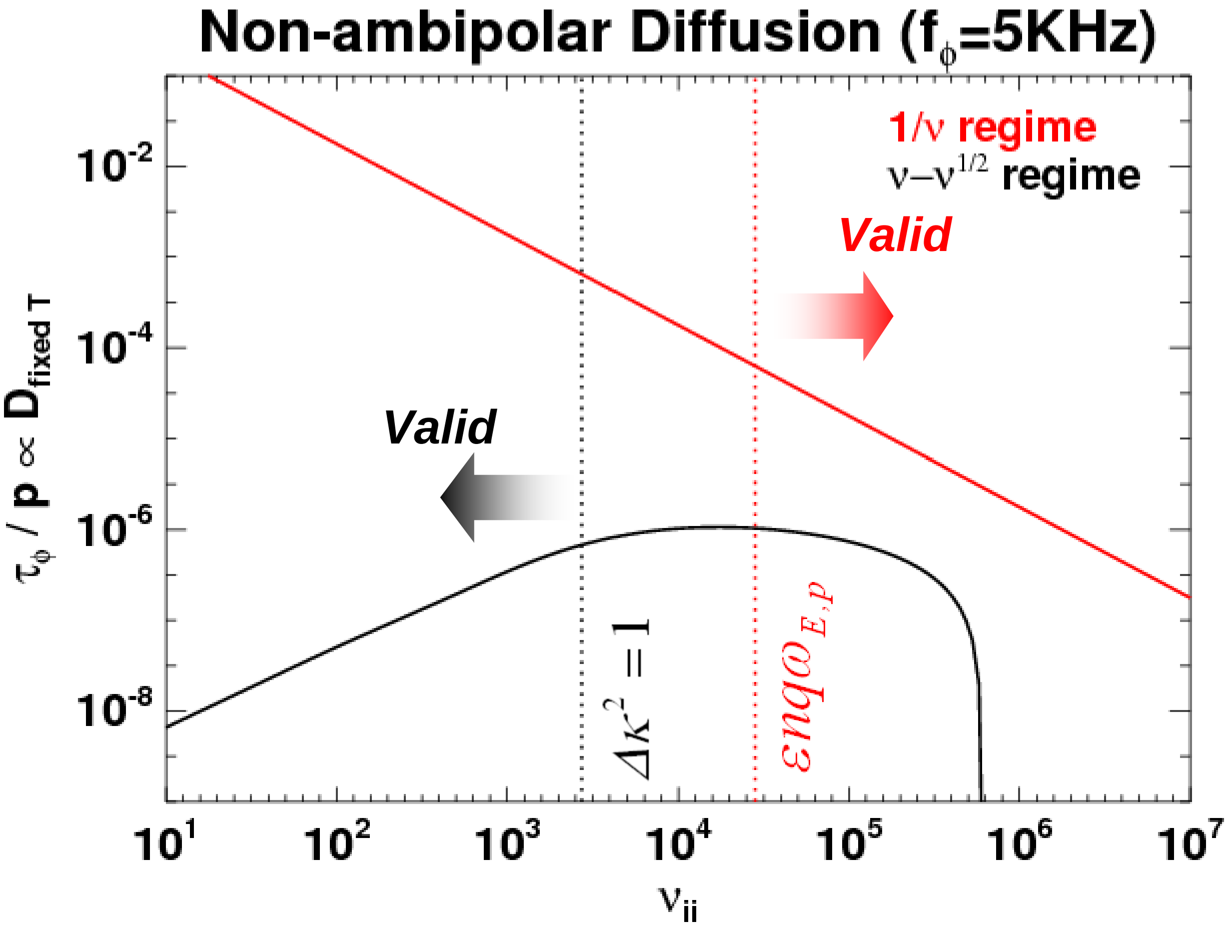
Jong-kyu Park,
Allen Boozer, Jonathan Menard,
Harry Mynick, Roscoe White, Andrea Garofalo,
Stefan Gerhardt, Steve Sabbagh

Background for NTV (Neoclassical Toroidal Viscoscity)

- The symmetry-braking by the field causes non-ambipolar transport in addition to ambipolar transport
- The non-ambipolar transport produces a loss of the toroidal momentum
- **Ambipolar transport tends to be proportional to ν, because collisional detrapping is the main process of the transport**
- **Non-ambipolar transport tends to be proportional to $1/\nu$, because the variations of banana motion cause radial drifts before collisional detrapping, and also depends on precessions.**
- NTV (Neoclassical Toroidal Viscoscity) has been used as a “special name” of this non-ambipolar transport in tokamaks

Connection between different regimes required in NTV theory

- ❑ NTV torque has been derived in 1/ν (*Shaing POP03*) and ν-$\nu^{1/2}$ (*Shaing POP08*) regime, but still connection is required



Note Log Scale

Parameters

n_e=10^{19}cm^{-3}

f = 5kHz

B=2T

ε=0.3

q=2.2

Field

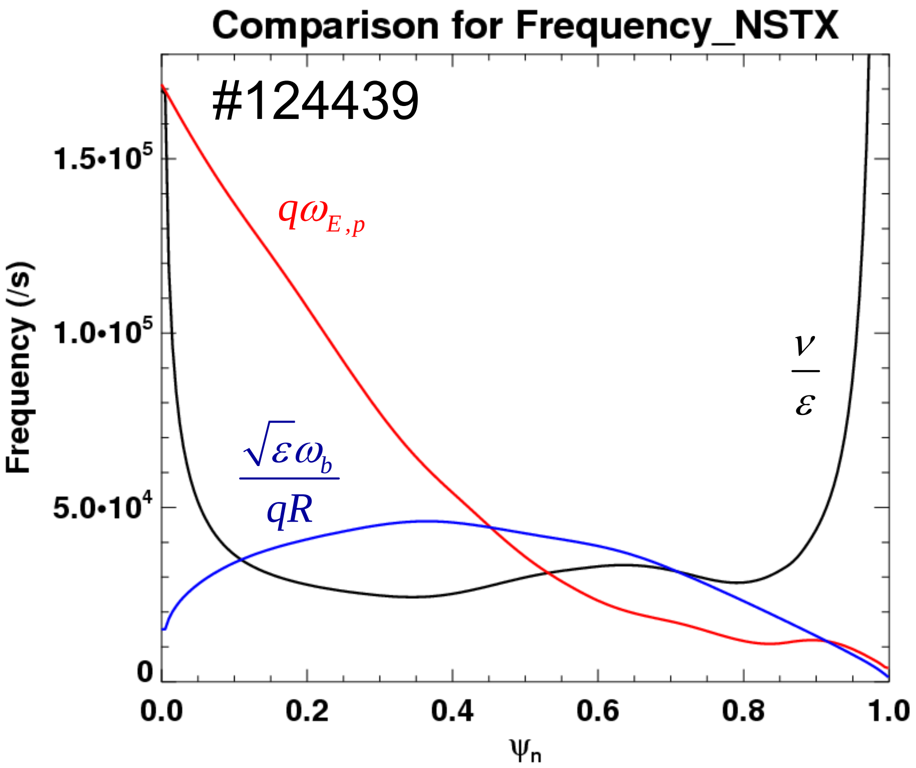
n=3

-10<m<30

Significant overlapping between NTV regimes

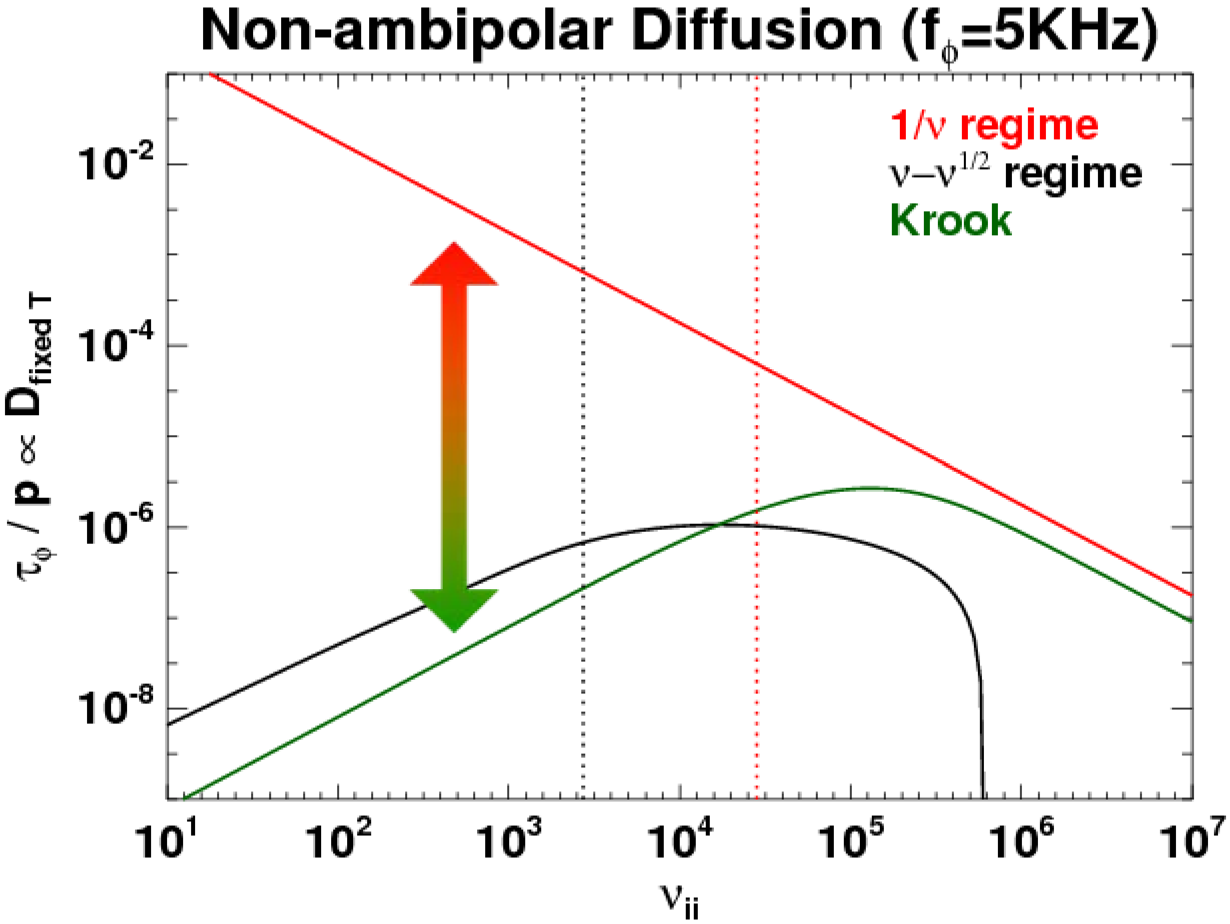
- Significant overlapping exists for most of tokamaks

$$\underbrace{\frac{\nu}{\varepsilon}}_{\nu\text{-}\nu^{1/2}} < q\omega_{E,p} < \underbrace{\frac{\nu}{\varepsilon}}_{1/\nu} < \frac{\sqrt{\varepsilon}\,\omega_b}{qR} < \underbrace{\frac{\nu}{\varepsilon}}_{\text{Passing}}$$



Connection obtained between ν and 1/ν

- ☐ A Krook collisional operator gives an analytic expression for both regimes and a reasonable connection, but the expectation of torque is too small compared to experimental observations



Non-ambipolar Diffusion (f_ϕ=5KHz)

Note Log Scale

Parameters

$n_e=10^{19}cm^{-3}$

f = 5kHz

B=2T

ε=0.3

q=2.2

Field

n=3

-10<m<30

Missing components in NTV theory

- ☐ The small expectation of torque in theory is due to very large E×B toroidal precession in practical tokamaks

- ☐ Missing components in NTV theory are

1) **The bouncing motion can resonate with E×B precession :**
   L-class of particles having L-times of the full toroidal precession during one bounce have a small E×B precession effectively (Mynick NF83)

2) **The magnetic precession can cancel E×B precession :**
   The curvature and grad-B drift for ions (when counter-injection) or for electrons (when co-injection) can locally cancel E×B precession in Ohmically heated plasma (Linsker,Boozer PF82)

3) **Local retrapping and detrapping in the secondary wells :**
   Tokamak in perturbations seems not plausible in driving this mechanism

A general expression obtained

- A general formula has been derived by including the missing components by solving bounce-averaged drift kinetic equation

$$\langle \hat{\phi} \cdot \vec{\nabla} \cdot \overleftrightarrow{\Pi}_a \rangle = \frac{\epsilon^{-1/2}}{\sqrt{2}\pi^{3/2}} p_a \left\langle \frac{1}{R} \right\rangle \underbrace{\sum_{\ell}}_{\textit{l-class}} \underbrace{\left(\int_0^1 d\kappa^2 S_\delta^\ell \right)}_{\textit{(1)}} \underbrace{\left(\int_0^\infty dx \mathcal{Y}_{a1}^\ell \right)}_{\textit{(2)}} \underbrace{\left(u^\zeta + 2.0\sigma \left| \frac{c}{e} \frac{dT_a}{d\chi} \right| \right)}_{\textit{Rotation with offset}}$$

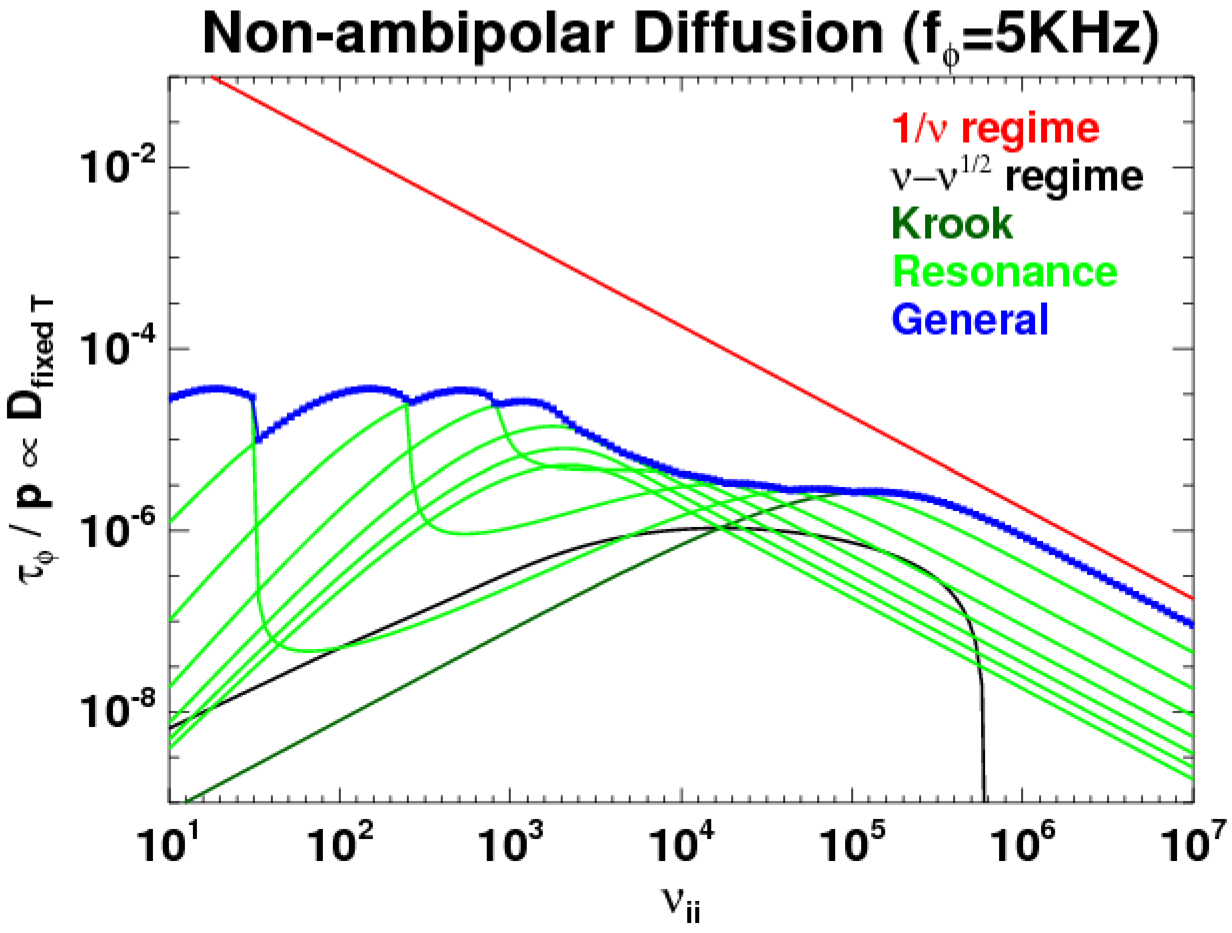
(1) $$S_\delta^\ell = \sum_{nmm'} n^2 \delta_{nmm'\ell}^2 \frac{F_{nmlc}^{-1/2} F_{nm'lc}^{-1/2}}{4K(\kappa)},$$

(2) $$\mathcal{Y}_{a1}^\ell = \frac{1}{2} \frac{((\ell^2+1)\nu_a/\epsilon) x e^{-x}}{\left(\underbrace{\ell \frac{\pi\sqrt{\epsilon}}{4\sqrt{2}} \omega_{ta} \sqrt{x}}_{\textit{Bounce}} - \underbrace{n\omega_E}_{\textit{ExB}} - \underbrace{n\sigma \frac{q^3}{8\pi^2\epsilon} (\omega_{ta}^2/\omega_{ca}) x}_{\textit{Magnetic precession}} \right)^2 + ((\ell^2+1)\nu_a/\epsilon)^2 x^{-3}},$$

Resonance dominates transport (NSTX, DIII-D)

- ☐ A small fraction of particles having resonance with precession can effectively gives 1/ν transport

Note Log Scale



Parameters

$n_e=10^{19}cm^{-3}$

f = 5kHz

B=2T

ε=0.3

q=2.2

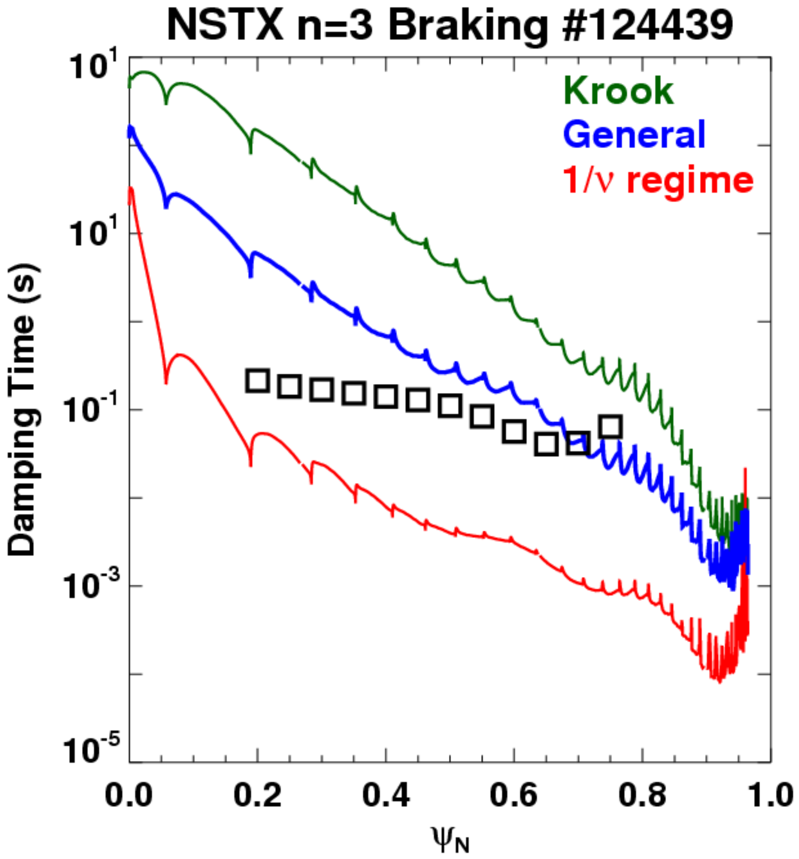
Field

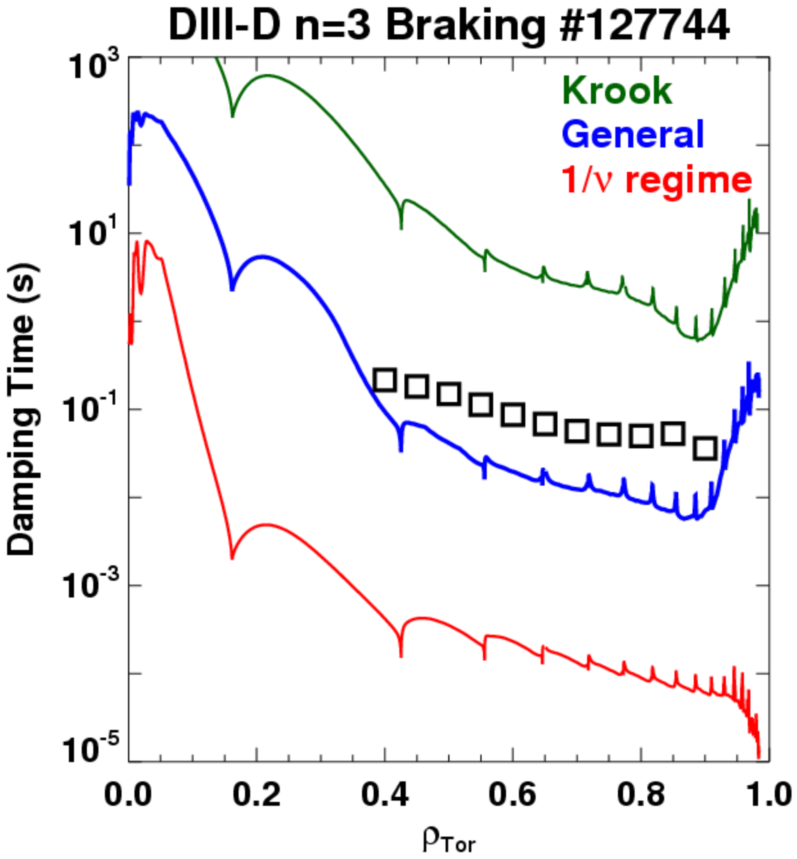
n=3

-10<m<30

Generalized formula gives a way to evaluate magnetic braking in experiments

- ❑ Evaluation of braking can give a reasonable evaluation by order of magnitudes for the profile, and give a good estimation for the total torque


NSTX n=3 Braking #124439


DIII-D n=3 Braking #127744

Summary and Future Work

- NTV theory has been extended to include resonance with precession

- The generalized NTV and IPEC (s > α, or n>1) give the maximum estimation of the torque in existing theory and are close to the experimental observation

- Monte-Carlo (ORBIT) study has been started

- The NTV damping is closely related to the kinetic contribution in δW
  - The imaginary part of the perturbed f_1 gives torque : α
  - The real part of the perturbed f_1 gives kinetic stabilization : s

$$f_1 \propto \frac{\frac{p'}{p} + \frac{e\phi'}{T} + A\frac{T'}{T}}{\left(\ell\omega_b - n(\omega_E + \omega_B)\right) + i\nu}\left(dB^2(\vec{\xi}\,)\right)$$

- Tensor pressure perturbed equilibria can determine both consistently