NSTX

Supported by U.S. DEPARTMENT OF ENERGY Office of Science

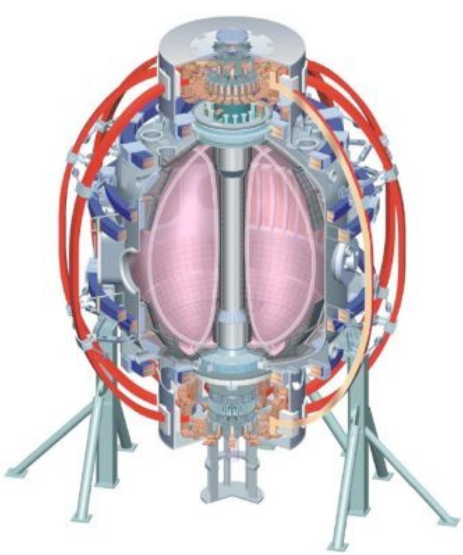
A Model for the Rotational Control of Plasma in NSTX

College W&M
Colorado Sch Mines
Columbia U
CompX
General Atomics
INEL
Johns Hopkins U
LANL
LLNL
Lodestar
MIT
Nova Photonics
New York U
Old Dominion U
ORNL
PPPL
PSI
Princeton U
Purdue U
SNL
Think Tank, Inc.
UC Davis
UC Irvine
UCLA
UCSD
U Colorado
U Illinois
U Maryland
U Rochester
U Washington
U Wisconsin

Kunihiko Taira[1], Egemen Kolemen[1], David Gates[2], Clancy Rowley[1], Jeremy Kasdin[1]

[1]Mechanical and Aerospace Engineering, Princeton University,
[2]Princeton Plasma Physics Laboratory

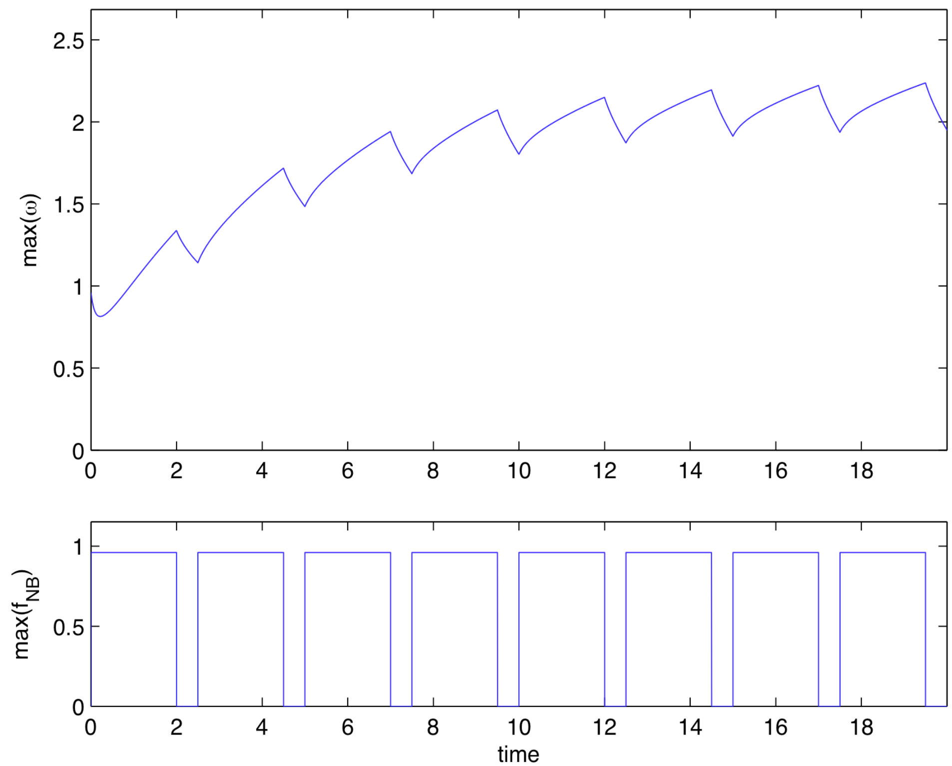
Results Review
B318, PPPL
September 16, 2009

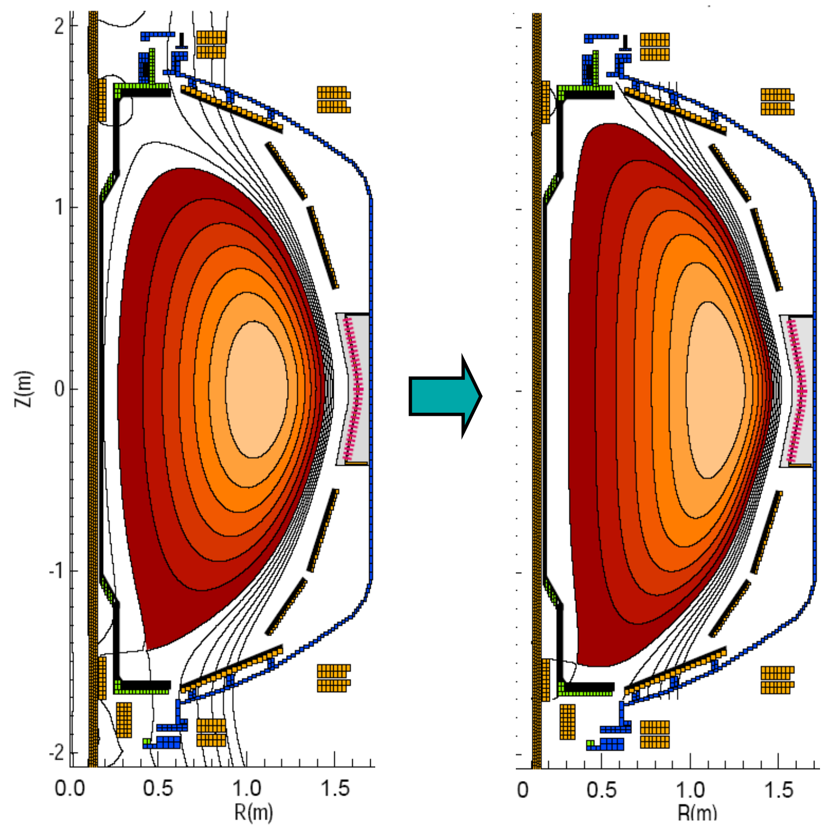


Culham Sci Ctr
U St. Andrews
York U
Chubu U
Fukui U
Hiroshima U
Hyogo U
Kyoto U
Kyushu U
Kyushu Tokai U
NIFS
Niigata U
U Tokyo
JAEA
Hebrew U
Ioffe Inst
RRC Kurchatov Inst
TRINITI
KBSI
KAIST
POSTECH
ASIPP
ENEA, Frascati
CEA, Cadarache
IPP, Jülich
IPP, Garching
ASCR, Czech Rep
U Quebec

Objective

- Develop model and controller for the toroidal momentum of plasma in NSTX
  - to attain desirable <u>temporal</u> & spatial profile





Governing Equation

- Toroidal momentum balance (Goldston, 1986)

$$\sum_i n_i m_i \left\langle R^2 \right\rangle \frac{\partial \omega}{\partial t} + \omega \left\langle R^2 \right\rangle \sum_i m_i \frac{\partial n_i}{\partial t}$$

$$+ \sum_i n_i m_i \omega \frac{\partial \left\langle R^2 \right\rangle}{\partial t} + \sum_i n_i m_i \left\langle R^2 \right\rangle \omega \left(\frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial t} \frac{\partial V}{\partial \rho}$$

•*Temporal change*

$$= \left(\frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial \rho} \left[\frac{\partial V}{\partial \rho} \sum_i n_i m_i \chi_\phi \left\langle R^2 (\nabla \rho)^2 \right\rangle \frac{\partial \omega}{\partial \rho} \right]$$

•*Diffusion*

$$- \left(\frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial \rho} \left[\frac{\partial V}{\partial \rho} \sum_i n_i m_i \omega \left\langle R^2 (\nabla \rho)^2 \right\rangle \frac{v_\rho}{|\nabla \rho|} \right] \quad \textit{0}$$

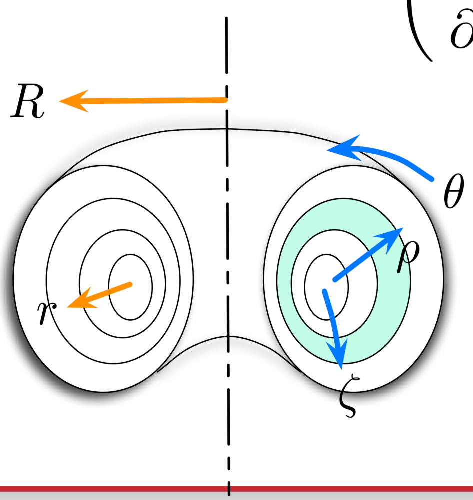
•*Pinch*

$$+ T_{\mathrm{col}} + T_{J \times B} + T_{\mathrm{bth}} + T_{iz}$$

•*Torque inputs*

$$- \sum_i n_i m_i \left\langle R^2 \right\rangle \omega \left(\frac{1}{\tau_{\phi cx}} + \frac{1}{\tau_{c\delta}} \right) \quad \textit{0}$$

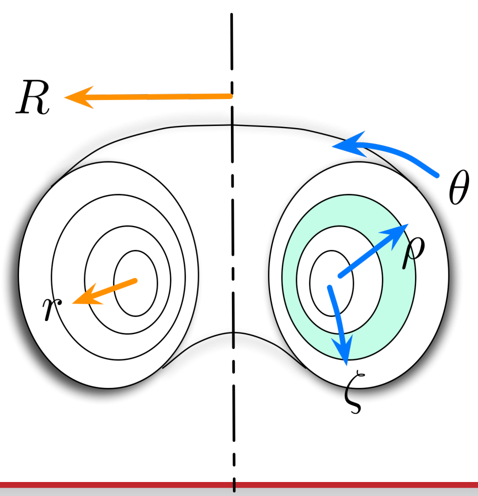
•*Loss (charge ex, ripple)*



Governing Equation

- Toroidal momentum balance (Goldston, 1986)

$$\sum_i n_i m_i \langle R^2 \rangle \frac{\partial \omega}{\partial t} + \omega \langle R^2 \rangle \sum_i m_i \frac{\partial n_i}{\partial t}$$
$$+ \sum_i n_i m_i \omega \frac{\partial \langle R^2 \rangle}{\partial t} + \sum_i n_i m_i \langle R^2 \rangle \omega \left(\frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial t} \frac{\partial V}{\partial \rho}$$
$$= \left(\frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial \rho} \left[\frac{\partial V}{\partial \rho} \sum_i n_i m_i \chi_\phi \langle R^2 (\nabla \rho)^2 \rangle \frac{\partial \omega}{\partial \rho} \right]$$
$$+ \sum_j T_j$$



•*4 shape (geometric) variables*

$$\langle R^2 \rangle \qquad \langle R^2 (\nabla \rho)^2 \rangle \qquad \frac{\partial V}{\partial \rho} \qquad \frac{T(\rho, t)}{\max_\rho T(\rho, t)}$$

•*Torque inputs*

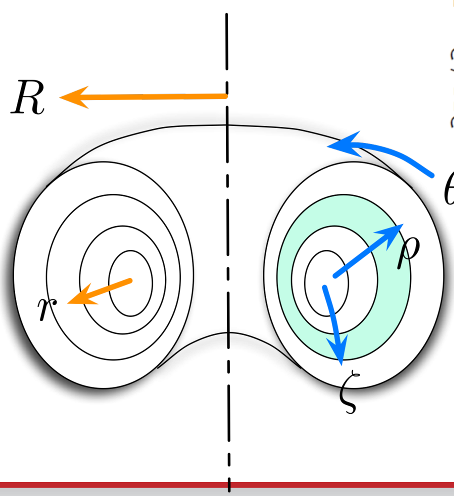
$$T(\rho, t) = g_1(t) g_2(\rho)$$

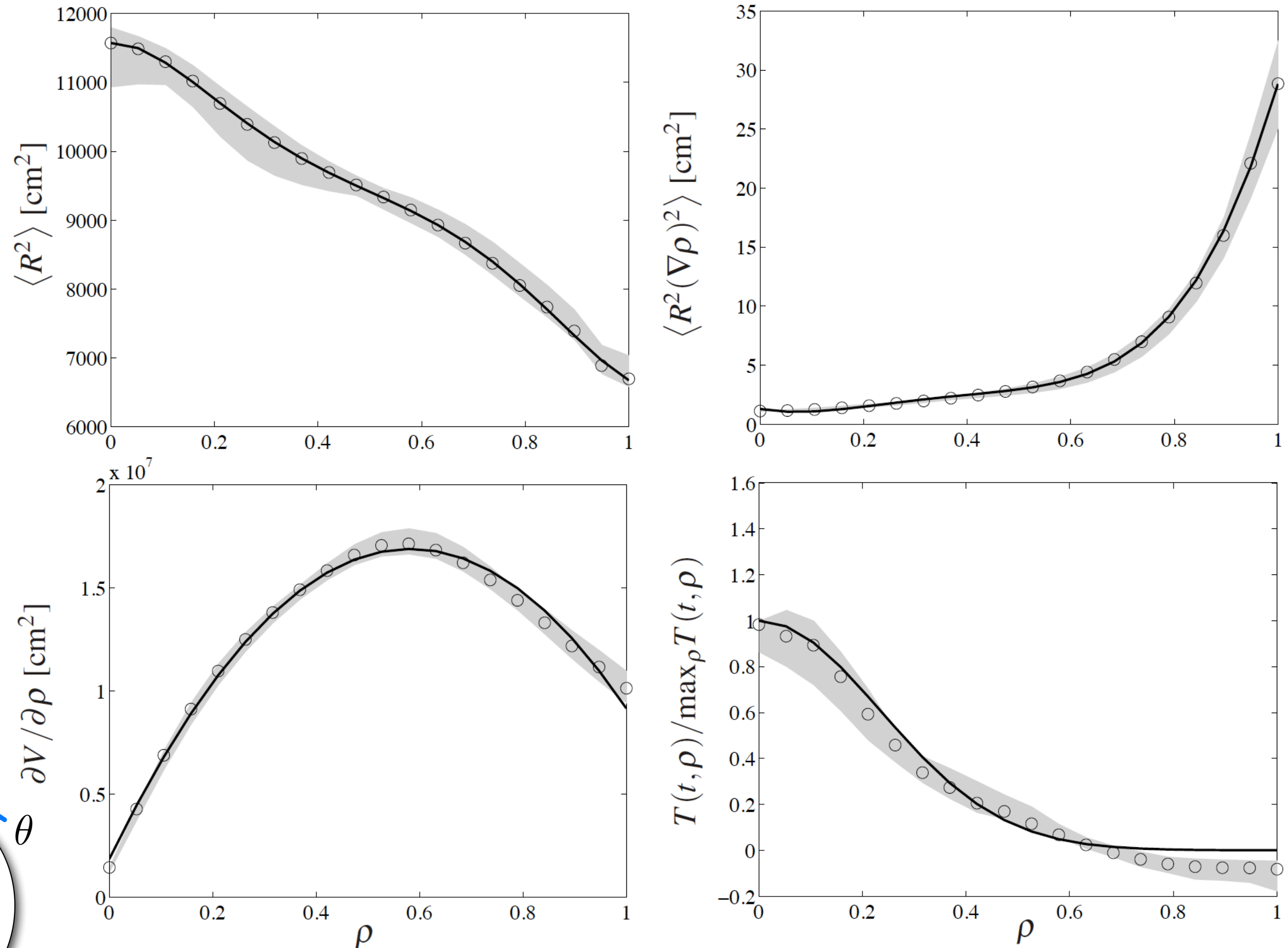
Shape Functions

- *Circle: time avg*
- *Shade: variation*
- *Line: curve fit*

- *Shot 120001*

$$\frac{\partial}{\partial t} \equiv 0$$





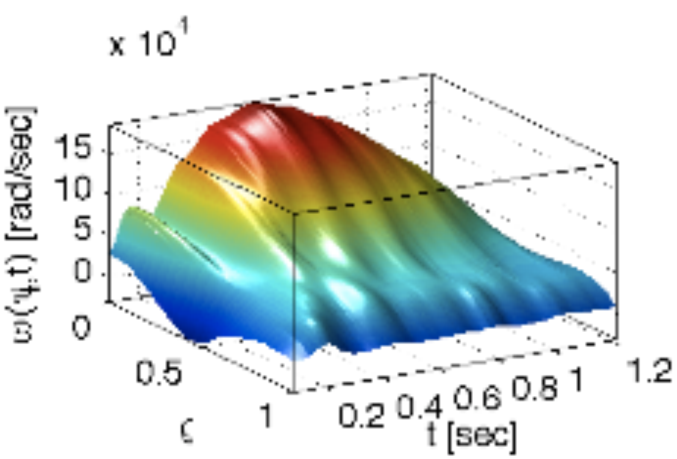
- *Model shape functions with curve fit of time averages*

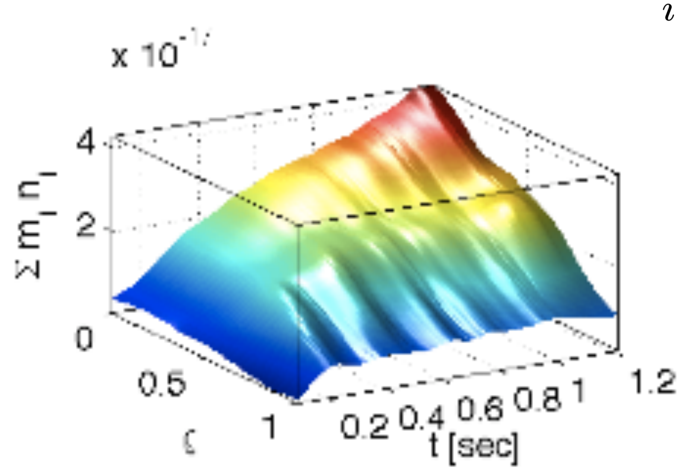
Model Equation

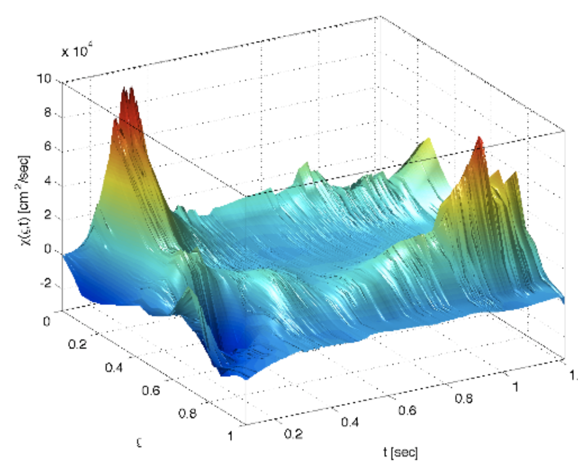
- Toroidal momentum balance

$$\sum_i n_i m_i \langle R^2 \rangle \frac{\partial \omega}{\partial t} = \left(\frac{\partial V}{\partial \rho}\right)^{-1} \frac{\partial}{\partial \rho}\left[\frac{\partial V}{\partial \rho}\sum_i n_i m_i \chi_\phi \langle R^2 (\nabla \rho)^2 \rangle \frac{\partial \omega}{\partial \rho}\right] + \sum_j T_j$$

  - 1D Linear PDE (parabolic) – diffusion equation with forcing
  - Neumann (ρ=0) and Dirichlet (ρ=1) BCs
  - Curve fit coefficients (4 geometric variables)
  - Coefficients to be supplied from data: χ_ϕ and $\sum_i n_i m_i$



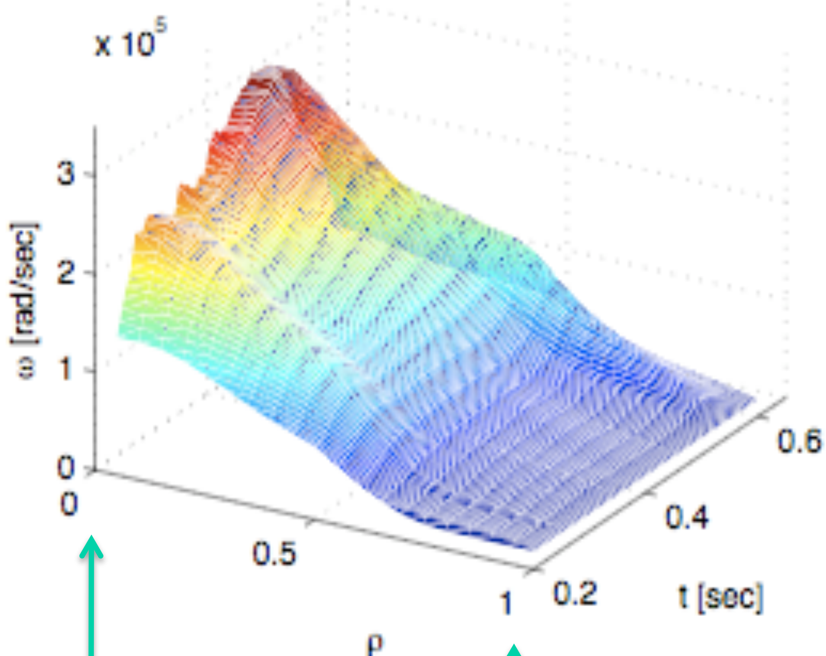




Results

- Numerically solved with MATLAB
  - adaptive time steps (parabolic PDE solver)
  - Ref: Skeel & Berzins, SIAM 1990

•Model

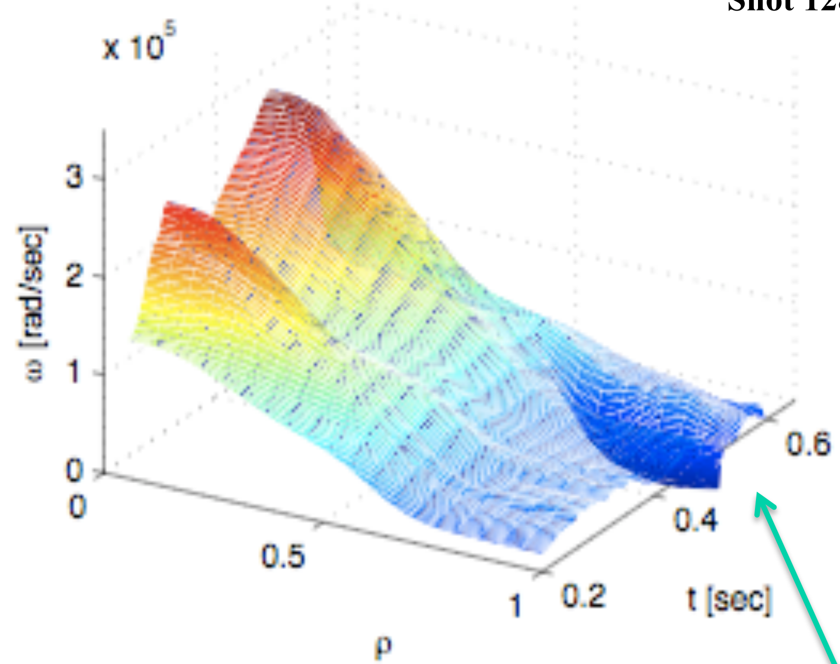


center

edge

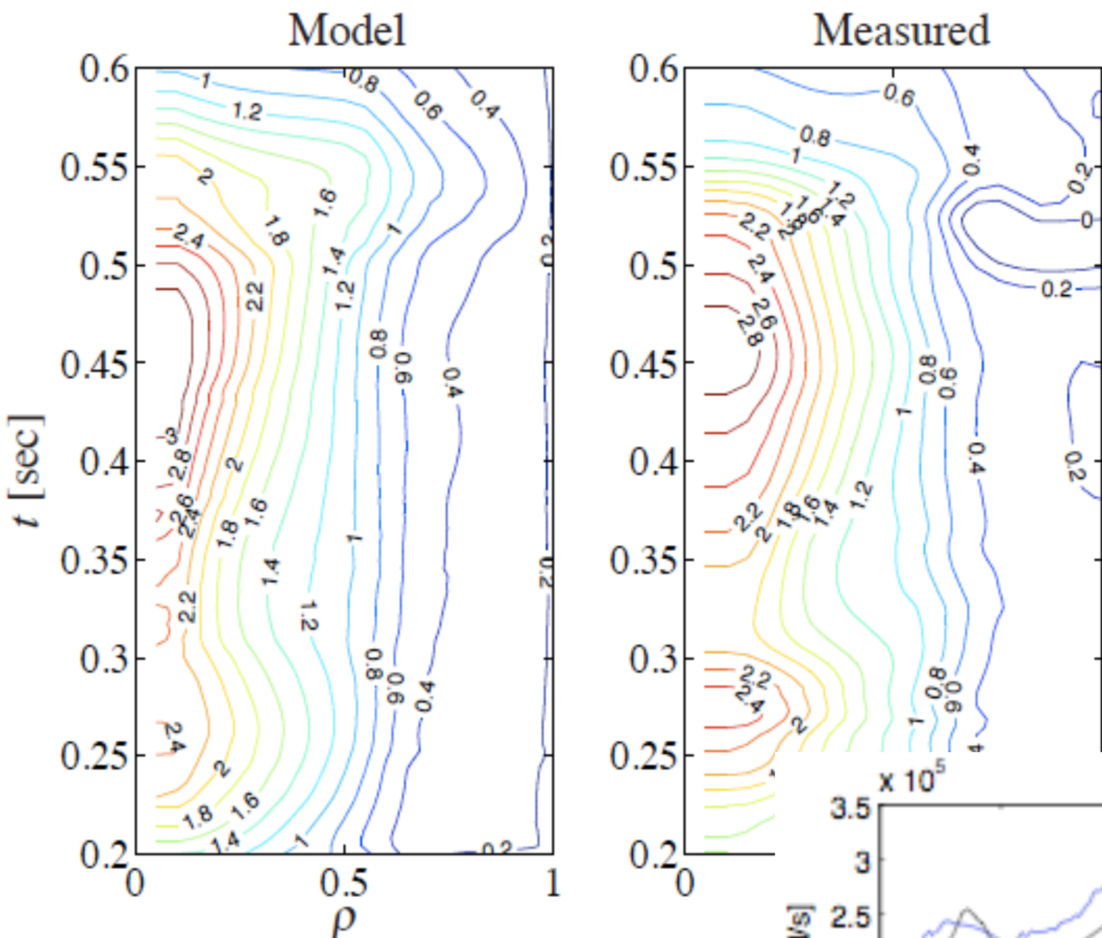
•Experiment

Shot 128020



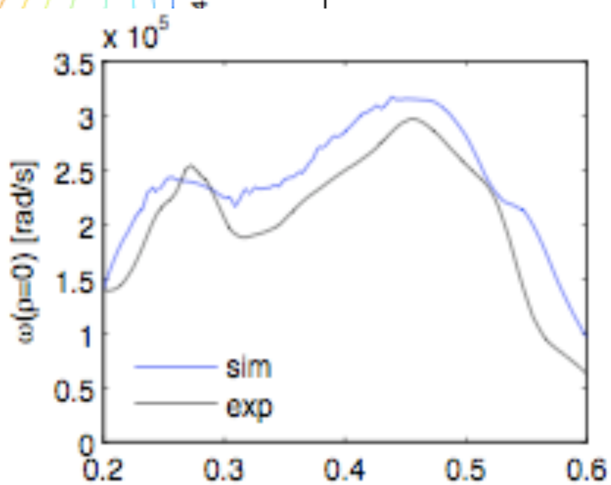
H- to L-mode transition

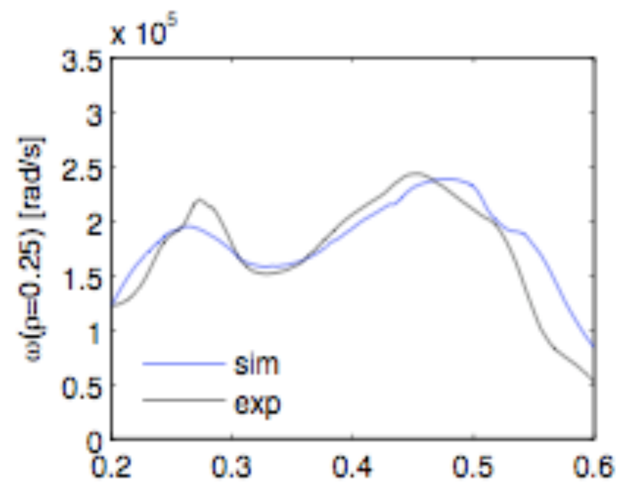
Results



•H- to L-mode transition

Shot 128020



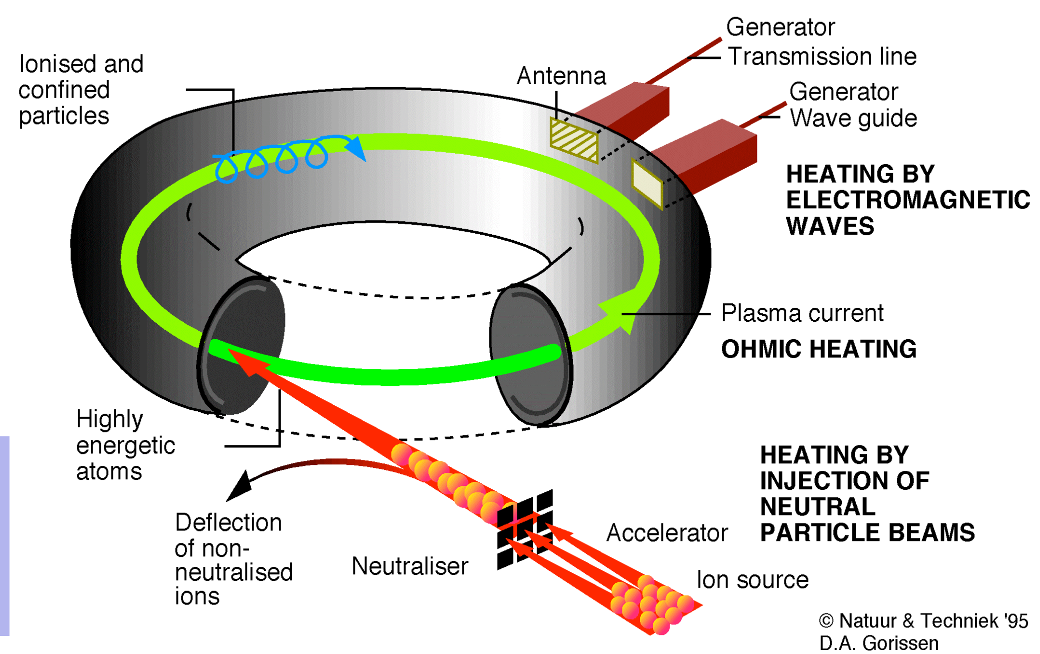


Rotational Control

$$\sum_i n_i m_i \langle R^2 \rangle \frac{\partial \omega}{\partial t} = \left(\frac{\partial V}{\partial \rho}\right)^{-1} \frac{\partial}{\partial \rho}\left[\frac{\partial V}{\partial \rho}\sum_i n_i m_i \chi_\phi \langle R^2 (\nabla\rho)^2\rangle \frac{\partial \omega}{\partial \rho}\right]$$

$$+\sum_j T_j + T_{\mathrm{NBI}} + \mu\left(\frac{B_0}{B_{\mathrm{eff}}}\right)^2 (\omega - \omega^*)$$

- Torque Input
- Neutral Beam Injection (NBI)
- Neoclassical Toroidal Visc (NTV)
  - Callen et al (IAEA, 2008)



- ***Response time < 2 - 3 conv time (100 – 150 ms)***
- ***Follow desired temporal trajectory***

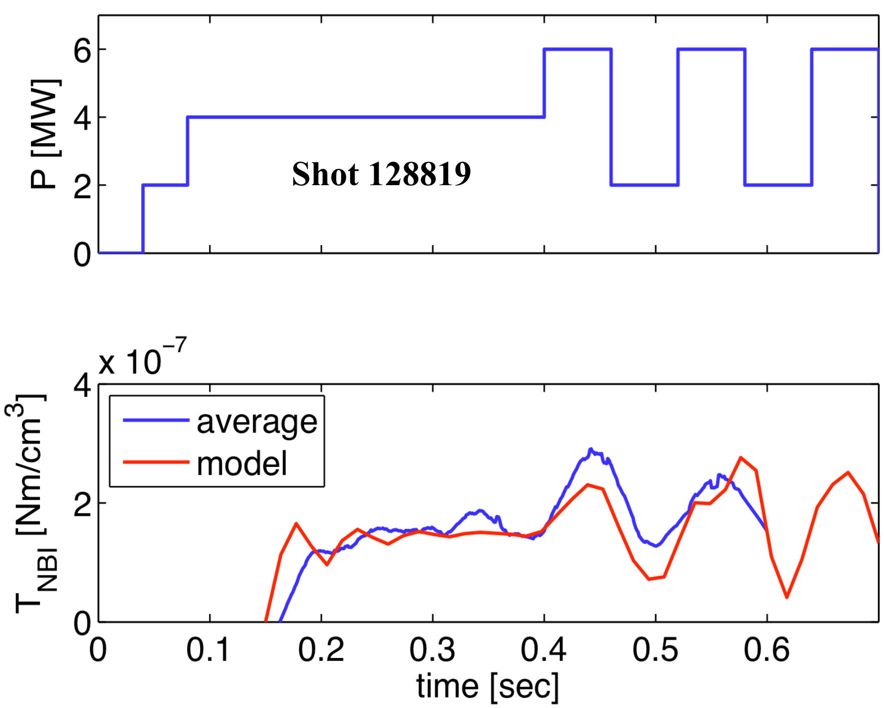
Actuators – NBI & NTV

- Neutral Beam Injection

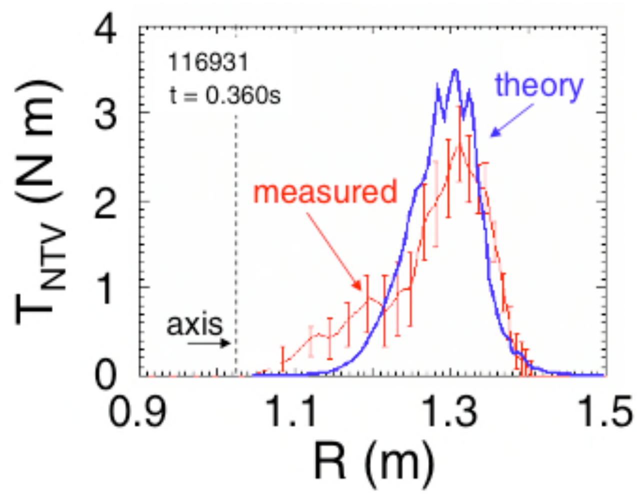
$$\frac{\partial \overline{T}_{\rm NBI}}{\partial t} + \frac{1}{\tau}\overline{T}_{\rm NBI} = \kappa P,$$

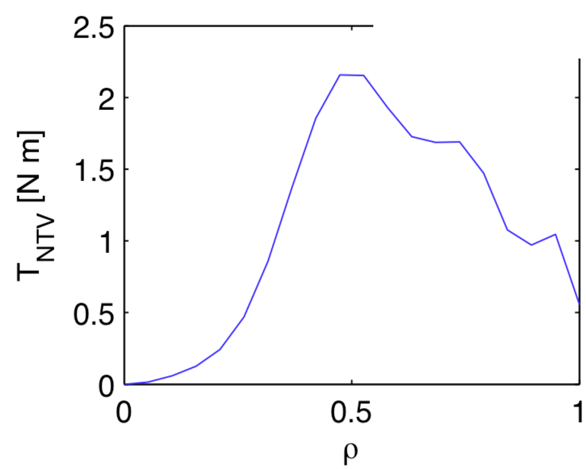
$$\text{time constant: } \tau \approx 0.02 sec$$

$$\kappa \approx 1.9 \times 10^{-6} Nm/cm^3/MW$$



- Neoclassical Toroidal Viscosity





Equivalent 1-D model

$$T_{\rm NTV} \approx m_i n_i \mu_i \frac{B_{\rm eff}^2}{B_0^2} R^2 \omega$$

for $n = 3$

Refs: Zhu et al, PRL (2006), Callen et al, NF (2009)

Preliminary Controller Design

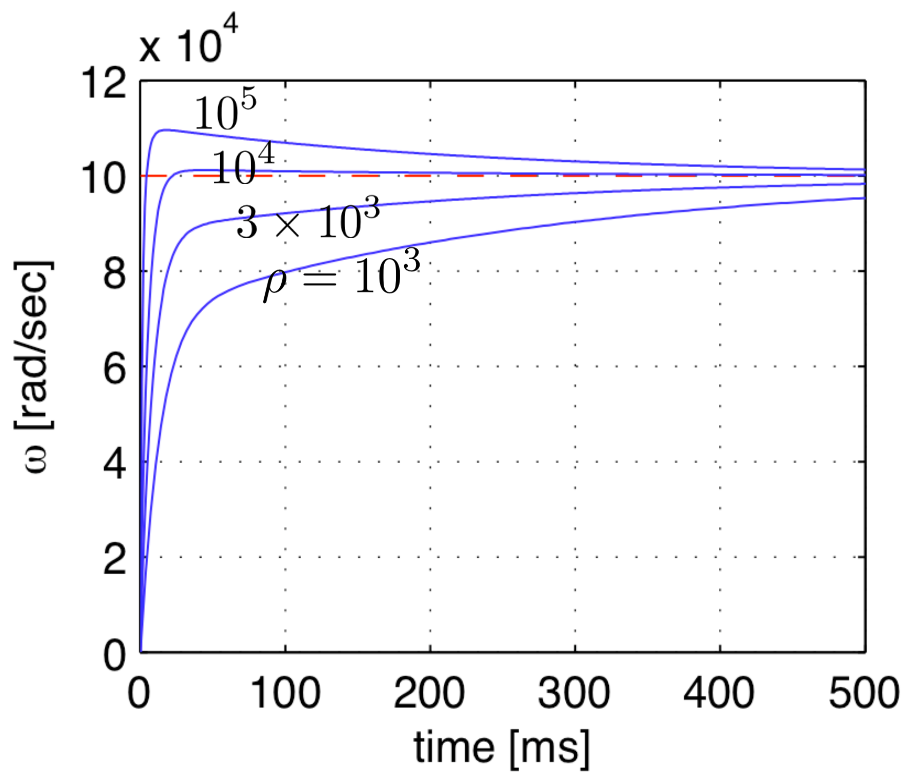
- Integral action
  - introduce an extra state x_i (integral of output error)
  - achieves zero SS error
  - robust to perturbations in the system matrix (A, B, C, D)
  - automatically adjusts the integrator state to hold output at ref: $r = Cx_d$

- Desired steady state within 100 ms

- Cost function for LQR

$$J = \int_{t_0}^{\infty} \left(\tilde{x}^T Q \tilde{x} + u^T R u \right) dt$$

$$\tilde{x} \equiv \begin{bmatrix} x \\ x_i \end{bmatrix} \quad Q = \rho I \quad R = I$$



Upcoming work

- Further validation of models.
- Implement NBI and NTV models (with time constants) into the control studies.
- Linear time varying control problem.
- Trajectory tracking.
- Control for spatial profile.