NSTX

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RWM stability calculations using VALEN with multiple modes

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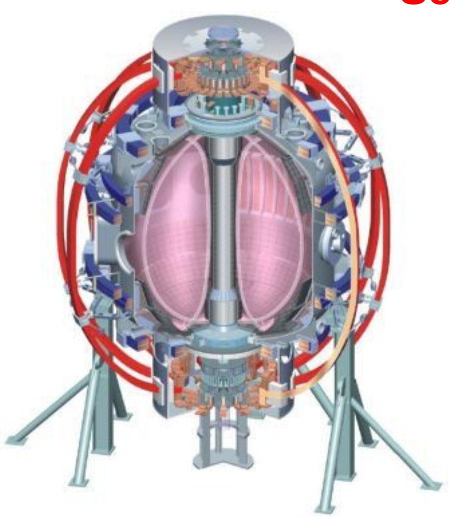
For the NSTX Research Team

Results / Theory Review
B318, LSB
September 16. 2009



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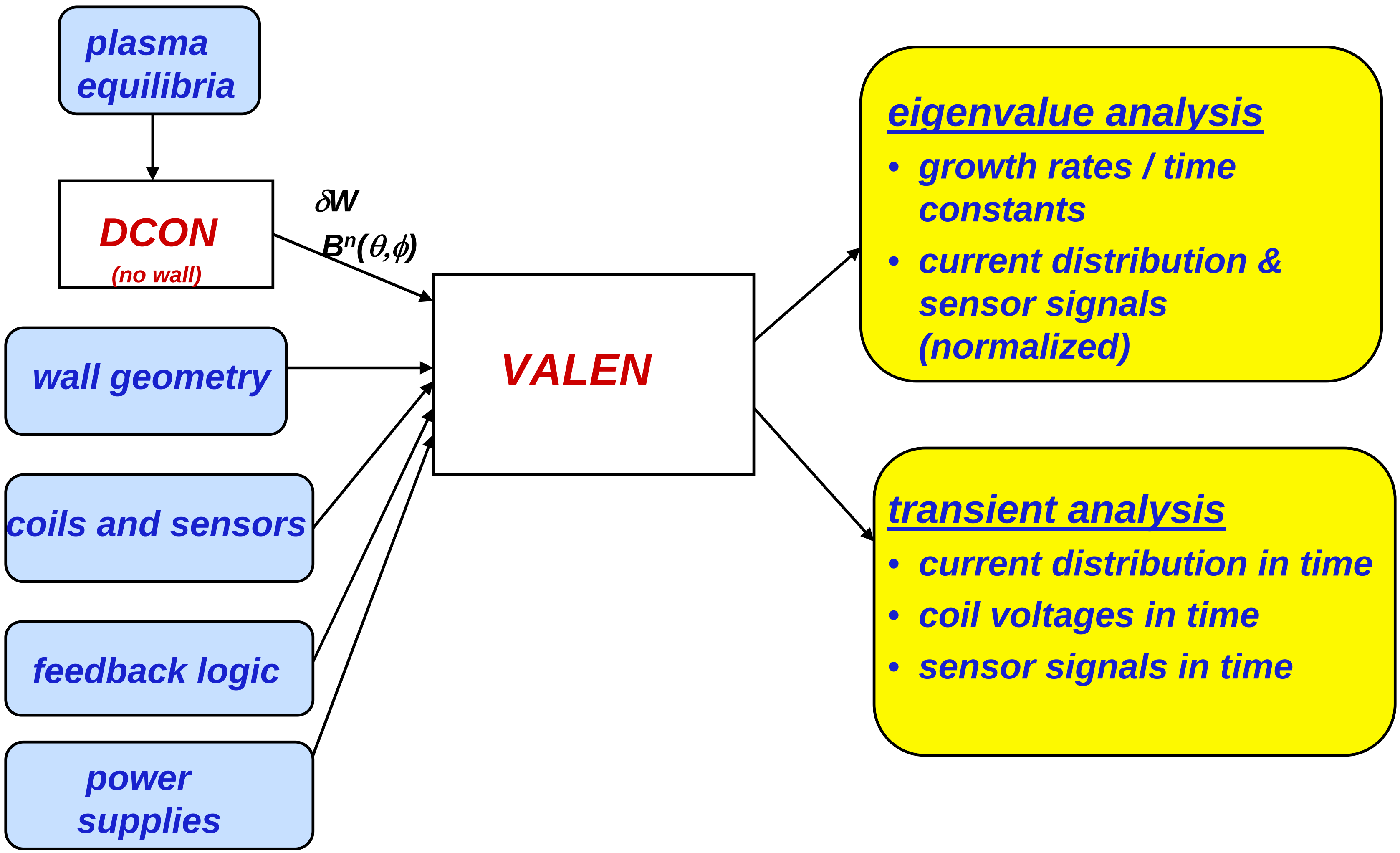
VALEN code models general RWM problem as L / R circuit equations in thin wall formulation



VALEN L / R circuit formulation

$$\frac{d}{dt}\begin{Bmatrix}\{\Phi^{wall}\}\\ \{\Phi^{coil}\}\\ \{\Phi^{plasma}\}\end{Bmatrix}+\begin{bmatrix}[R_{ww}] & [0] & [0]\\ [0] & [R_{cc}] & [0]\\ [0] & [0] & [R_{pd}]\end{bmatrix}\begin{Bmatrix}\{I_w\}\\ \{I_c\}\\ \{I_d\}\end{Bmatrix}=\begin{Bmatrix}\{0\}\\ \{V_c\}\\ \{0\}\end{Bmatrix}\begin{matrix}\text{1000's of equations}\\ \text{10's of equations}\\ \text{\# of plasma modes}\end{matrix}$$

where :

$\{\Phi^{\rm wall}\},\{\Phi^{\rm coil}\},\{\Phi^{\rm plasma}\}$ are vectors of magnetic flux in the wall elements, coils, and plasma modes

$\{I_w\},\{I_c\},\{I_p\},\{I_d\}$ are solution variables, i.e. vectors of mesh currents, coil currents, and plasma currents

The wall and coil equations are derived from standard circuit theory, the plasma circuit model is described in terms of plasma permeability i.e., P(δW_i $B^n{}_i(\theta,\phi)$) (no wall δW and B_normal dist):

ref:

A. Boozer, Physics of Plasmas, V5, No. 9, Sept. 1998, pg3350

A. Boozer, Physics of Plasmas, V10, No. 5, May 2003, pg1458

VALEN formulation (continued)

$$\{\Phi_w^{wall}\} = [L_{ww}]\{I_w\} + [L_{wc}]\{I_c\} + [L_{wp}]\{I_d\} + [L_{wp}]\{I_p\}$$ flux on wall elements

$$\{\Phi_c^{coil}\} = [L_{cw}]\{I_w\} + [L_{cc}]\{I_c\} + [L_{cp}]\{I_d\} + [L_{cp}]\{I_p\}$$ flux on coils

$$\left\{\Phi_p^{\substack{plasma\\total}}\right\} = [L_{pw}]\{I_w\} + [L_{pc}]\{I_c\} + [L_{pp}]\{I_d\} + [L_{pp}]\{I_p\} = [P]\left\{\Phi_p^{\substack{plasma\\ext}}\right\}$$

$$\left\{\Phi_p^{\substack{plasma\\ext}}\right\} = [L_{pw}]\{I_w\} + [L_{pc}]\{I_c\} + [L_{pp}]\{I_d\}$$

$$\{\Phi_s^{sensors}\} = [L_{sw}]\{I_w\} + [L_{sc}]\{I_c\} + [L_{sp}]\{I_d\} + [L_{sp}]\{I_p\}$$ flux in magnetic sensors

this gives:

$$\{I_p\} = [L_{pp}]^{-1}[P-1]\{[L_{pw}]\{I_w\} + [L_{pc}]\{I_c\} + [L_{pp}]\{I_d\}\}$$

$[P(B_i^n(\theta,\phi),\delta W)]$ ***is 'plasma permeability', the plasma response***

plasma permeability [P] in single mode VALEN

in single mode VALEN we solve for the current potential $\kappa(\theta,\phi)$ of a single unstable mode B_normal distribution $B^n_u(\theta,\phi)$

$$\vec{j}(\theta,\phi) = \delta(r-a)\nabla\kappa(\theta,\phi)\times\nabla r$$ current density defined by current potential which produces $B^n_u(\theta,\phi)$

$$P = \frac{1}{s} \quad \text{where:} \quad s = \frac{-\delta W}{L_B I_B^2/2}, \quad s > 0 \text{ is unstable}, \quad s < 0 \text{ stable}$$

$$\Phi_B^2 = \oint_{\text{surface}} (B^n_u(\theta,\phi))^2 dA \oint_{\text{surface}} dA, \quad I_B = \frac{\oint_{\text{surface}} B^n_u(\theta,\phi)\kappa(\theta,\phi)dA}{\Phi_B}, \quad L_B = \frac{\Phi_B}{I_B}$$

notice that plasma response has shape of $B^n_u(\theta,\phi)$
notice that plasma response is greatest for small δW (RFA !)

plasma permeability [P] in multi mode VALEN

in multi mode VALEN we use collection of dcon $B^n{}_i(\theta,\phi)$ to define a basis set $\{f_i(\theta,\phi)\}$, these orthonormal functions define $\{\kappa_i(\theta,\phi)\}$, $[L_{pp}]$ & $[P_{pp}]$

$$\text{define inner product}: \ \langle a,b\rangle = \oint_{\text{surface}} \frac{a(\theta,\phi)b(\theta,\phi)}{S} dA, \quad \text{where } S = \oint_{\text{surface}} dA$$

$$\{f_i(\theta,\phi)\} = \sum_{\#\ \text{modes}} G_{i\alpha}\{B^n_\alpha(\theta,\phi)\} \ \text{where}: \ \oint_{\text{surface}} \frac{f_i(\theta,\phi) f_j(\theta,\phi)}{S} = \delta_{ij}, \quad \oint_{\text{surface}} f_i(\theta,\phi) dA = 0$$

$$\boxed{[P_{pp}] = [\Lambda_{pp}][L_{pp}]^{-1}} \quad \text{where}: \ [\Lambda_{pp}]^{-1} = \frac{2[G][\varepsilon][G]^t}{S^2} \ \text{where } [G] \text{ is gram schmidt xform}$$

$$\text{and}: \ [\varepsilon] = \begin{bmatrix} \delta W_1 & 0 & \cdots & 0 \\ 0 & \delta W_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \delta W_{\substack{\text{lst}\\\text{mode}}} \end{bmatrix} \ \text{or} \ [\varepsilon] = \begin{bmatrix} \delta W_1 & \Gamma_1 & \cdots & 0 \\ -\Gamma_1 & \delta W_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & -\Gamma_{\text{last}} & \delta W_{\substack{\text{lst}\\\text{mode}}} \end{bmatrix}$$

no rotation with rotation

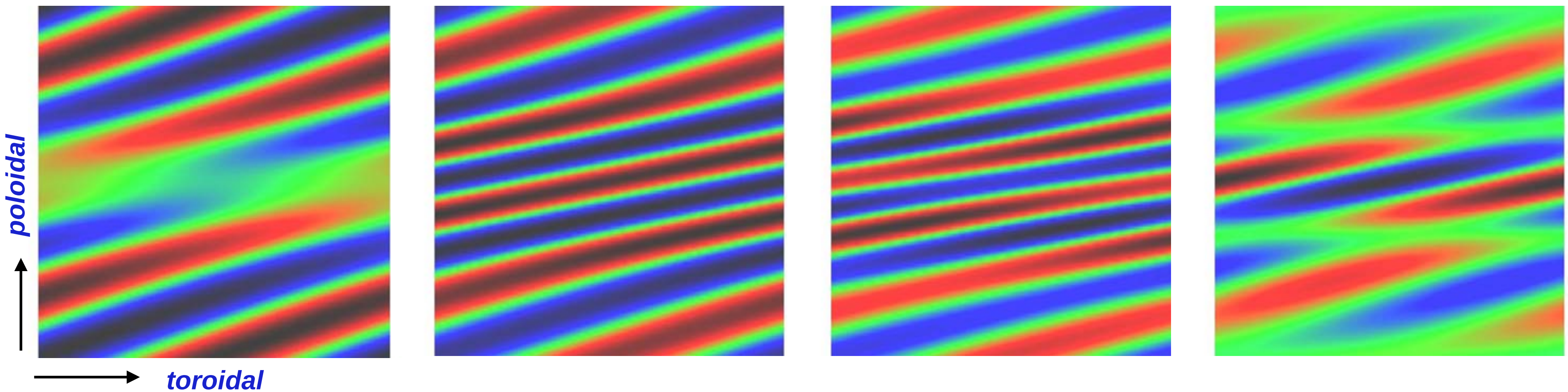
notice that we may combine $\{f_i\}$ from different 'n' values

notice that plasma response is a weighted sum of the $f_i(\theta,\phi)$

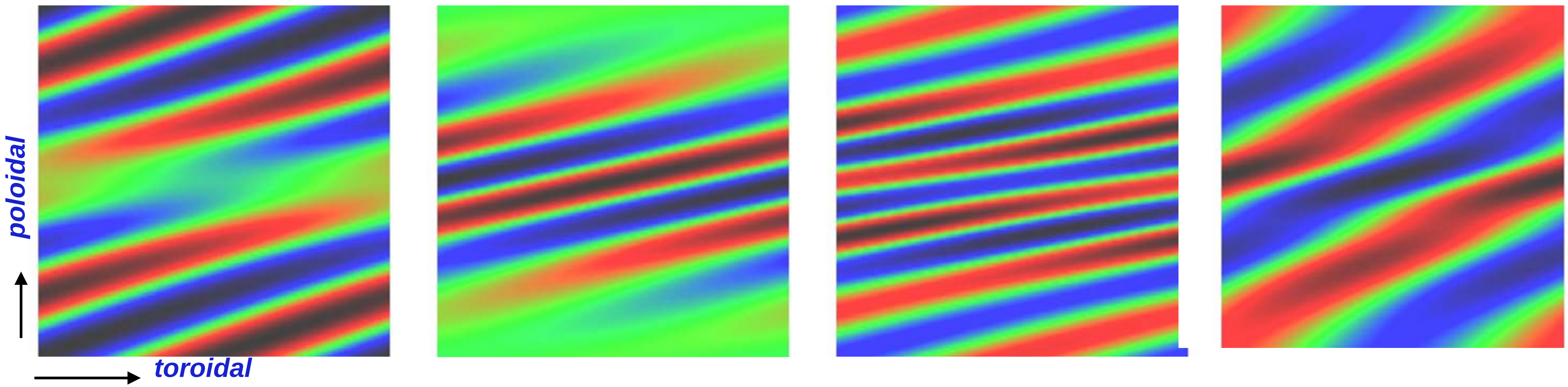
notice that plasma response is greatest for small δW (RFA !)

Benchmark mmVALEN HBT_EP test case

original $B^n(\theta,\phi)$ modes from dcon (first 4 modes) for HBT analysis



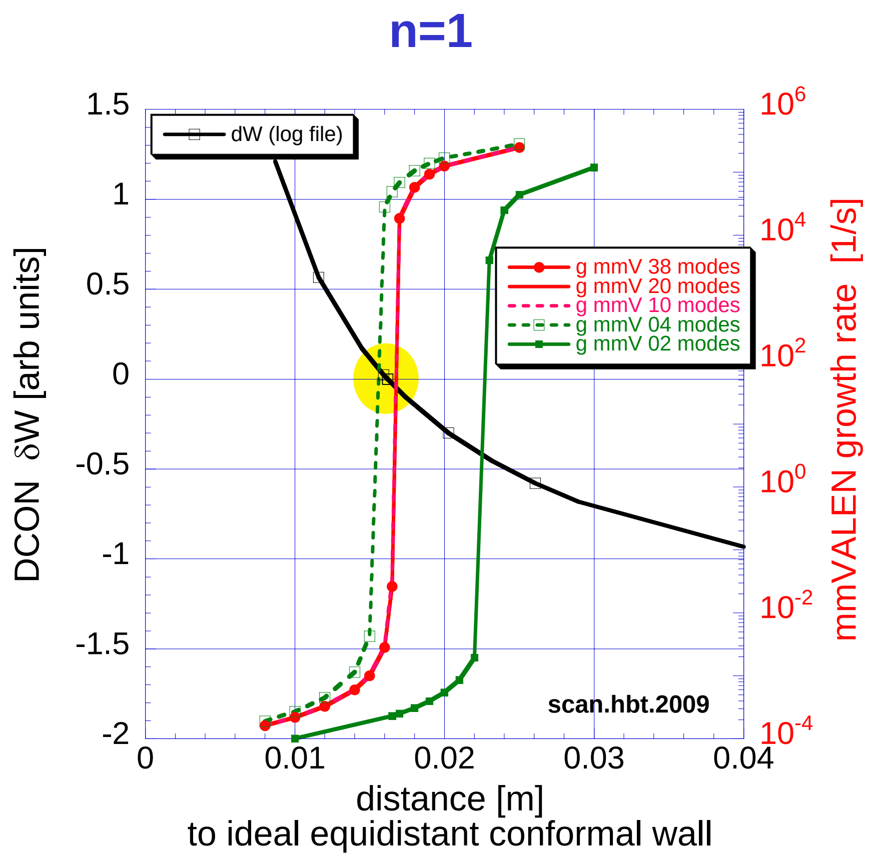
orthonormal basis set derived from above $B^n(\theta,\phi)$ modes



Successful benchmark of mmVALEN HBT_EP test case

We compare estimates of the n=1 critical wall position, from DCON & mmVALEN using a complete ideal equidistant conformal wall.

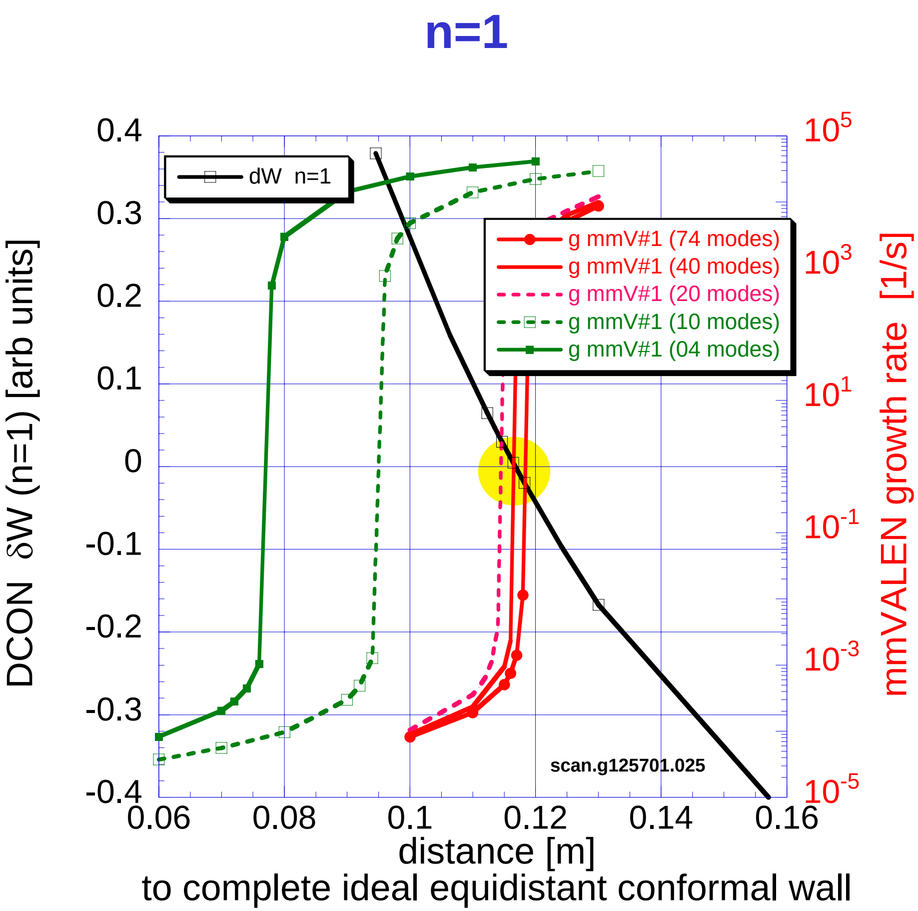
- the DCON zero crossing in δW (black curve use left axis) compares well with vertical part of the mmVALEN dispersion curve (red & green curves, use right axis)
- good agreement and good convergence with >=4 modes
- Note: 4 mmVALEN modes = 2 dcon modes, (modes in dcon have real & imaginary components)



Successful benchmark of mmVALEN DIII-D test case

We compare estimates of the n=1 critical wall position, DCON & mmVALEN using a complete ideal equidistant conformal wall

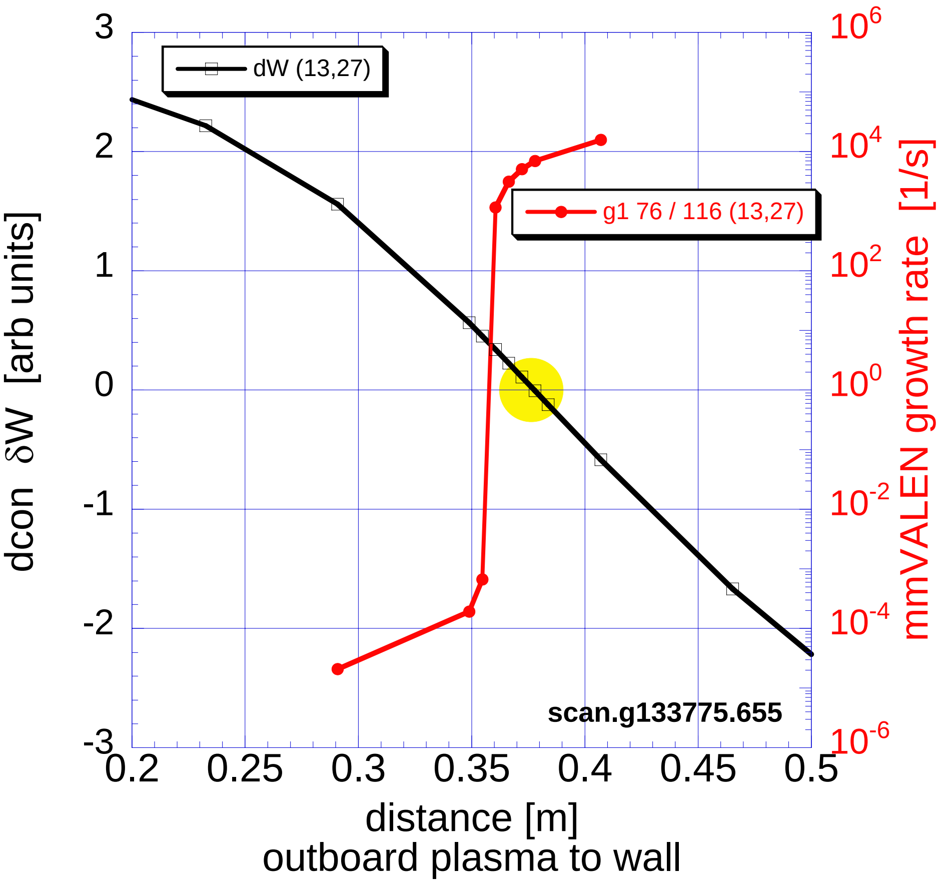
- the DCON zero crossing in δW (black curve use left axis) compares well with vertical part of the mmVALEN dispersion curve (red & green curves, use right axis)
- good agreement and good convergence with >=20 modes
- Note: 20 mmVALEN modes = 10 dcon modes



Successful benchmark of mmVALEN NSTX test case

We compare estimates of the n=1 critical wall position, from DCON & mmVALEN using a complete ideal D-shaped wall. This is required because of the NSTX aspect ratio. (option ishape=5 in VACUUM).

- the DCON zero crossing in δW (black curve use left axis) compares well with vertical part of the mmVALEN dispersion curve (red & green curves, use right axis)
- fair convergence with =76 modes, calculations continue
- testing continues, reasonable estimates of growth rate in NSTX have been obtained (ref sabbagh)



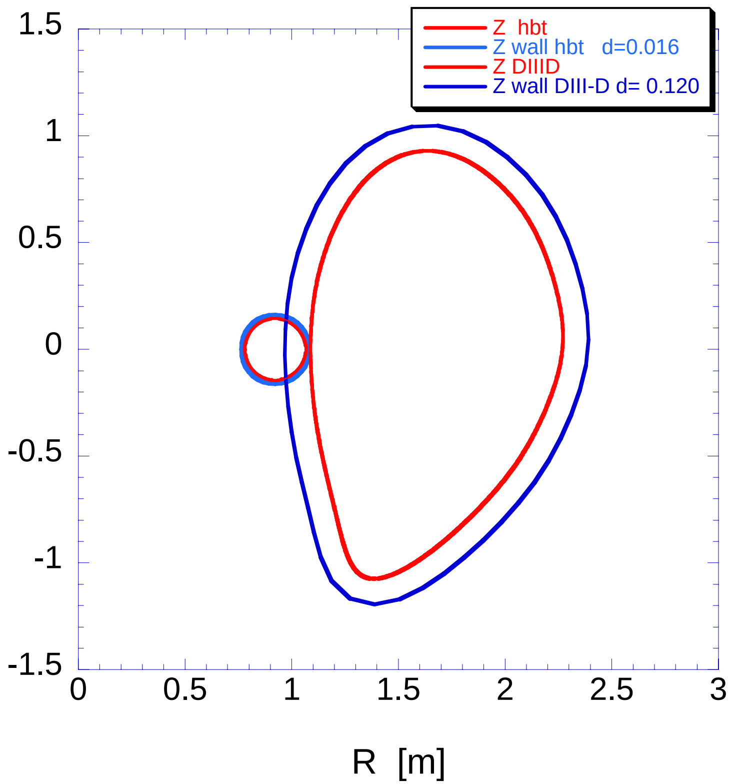
RWM stability calculations underway with mmVALEN

- Conclusions
  - Multi mode VALEN (mmVALEN) operational (initial testing complete)
  - Benchmarking mmVALEN successful so far (eigenvalue problems examined)
- Next steps
  - Investigate effects of multiple toroidal mode numbers, mode rotation effects
  - Examine effects of feedback and error fields on multi-mode spectrum (code behavior in time domain)
  - Compare to experimental results

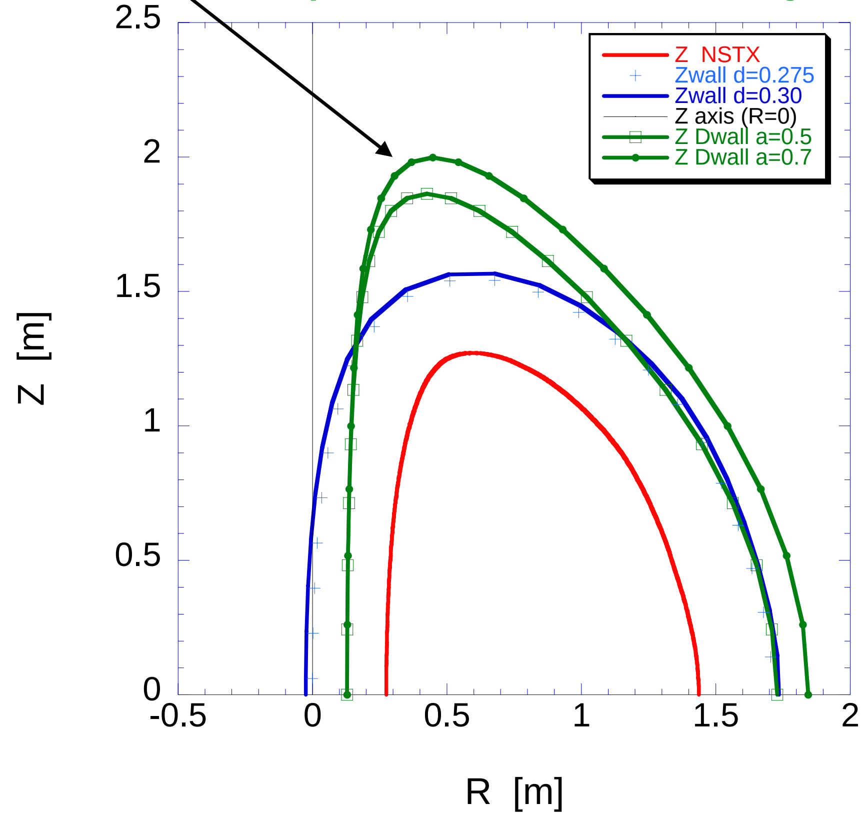
testing mmVALEN with an equidistant conformal wall works for HBT & DIII-D, not valid for NSTX

in NSTX an equidistant conformal wall becomes unphysical at d~0.275[m] when δW(n=1) still >0, we use D-shaped walls for benchmarking VALEN(NSTX)

equidistant conformal walls
near critical location
HBT-EP & DIII-D

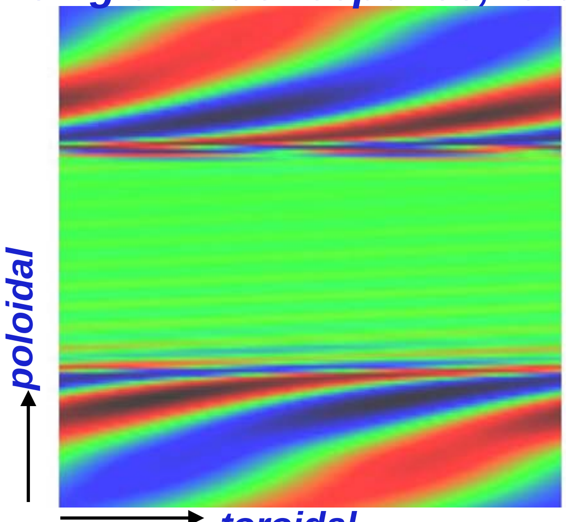


walls used for mmVALEN benchmarking
NSTX plasma surface
equidistant conformal walls
D-shaped walls for benchmarking

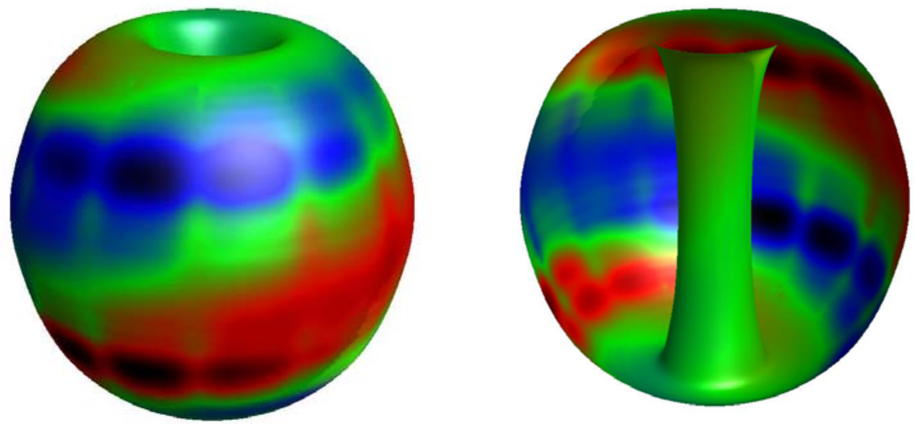


typical results from mmVALEN for NSTX for shot g133775.00655 ($B^n(\theta,\phi)$ on plasma surface)

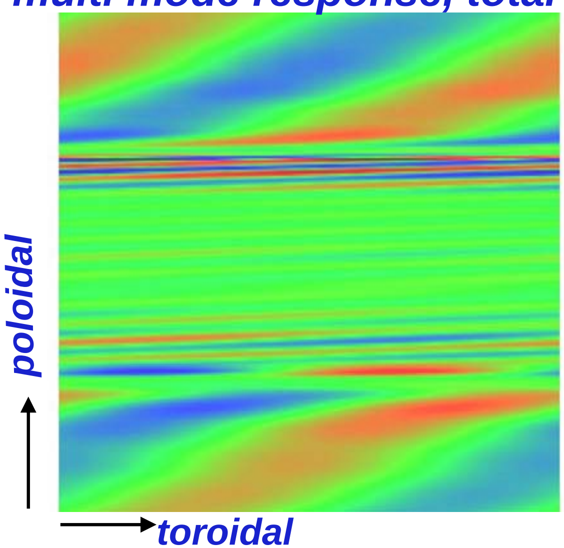
single mode response, total B^n



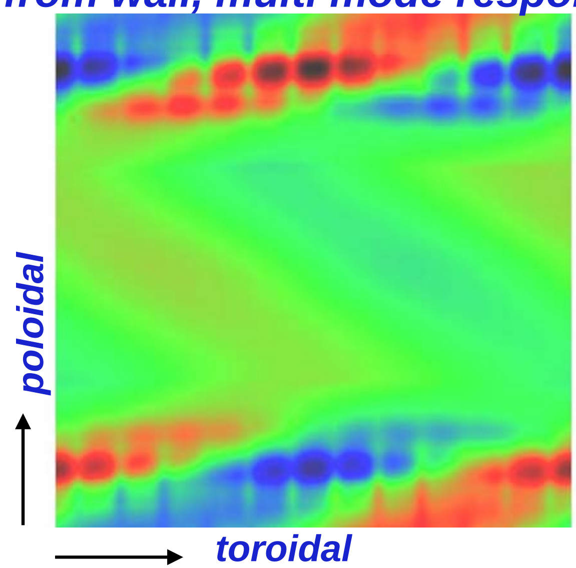
B^n from wall, multi mode response



multi mode response, total B^n

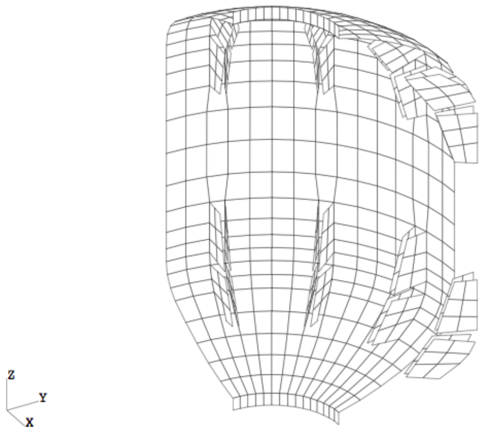


B^n from wall, multi mode response

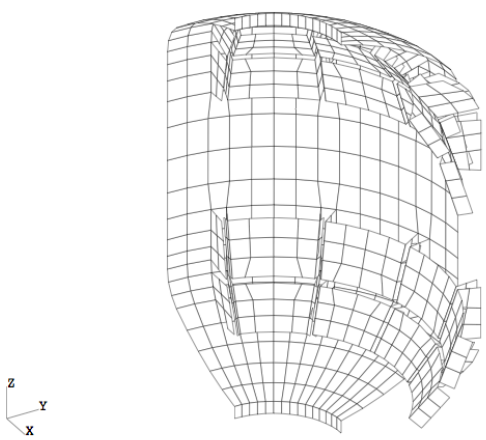


VALEN NSTX model (plots of wall)

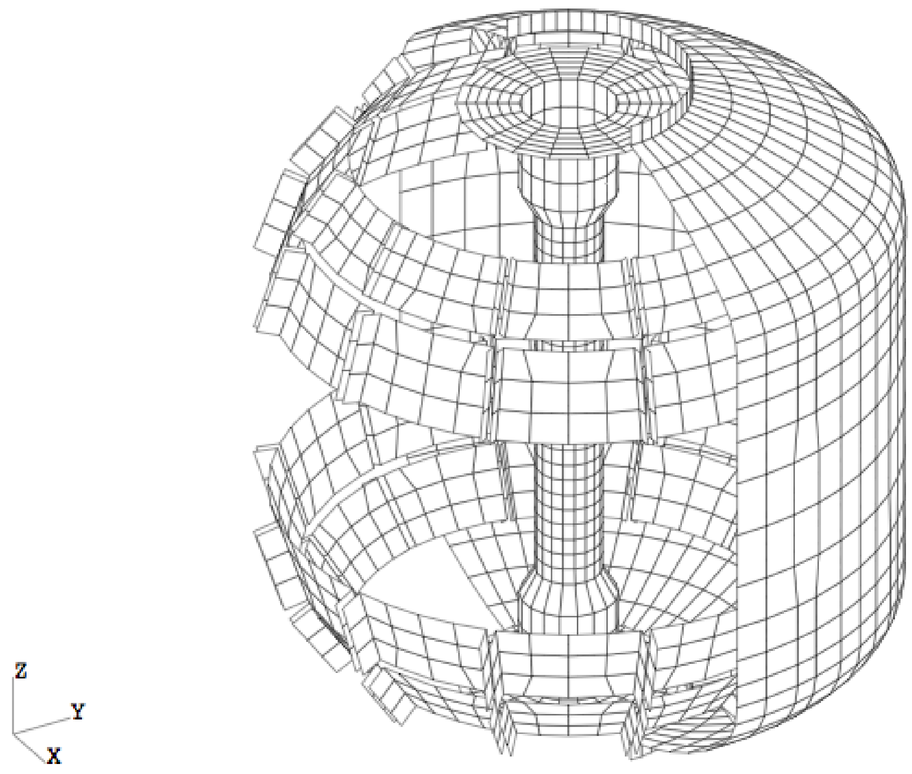
View of wall and mounting brackets



View of wall, mounting brackets
And copper passive plates

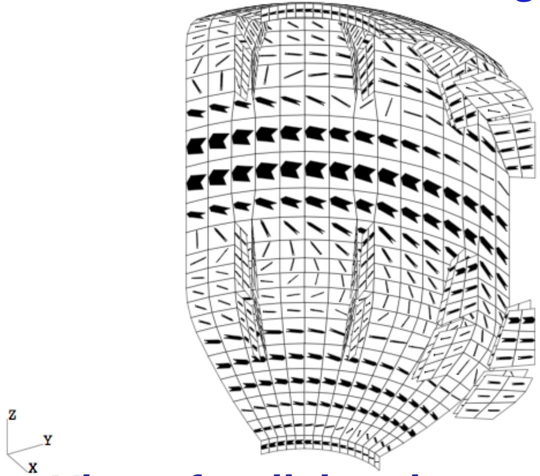


View with half VV cut away

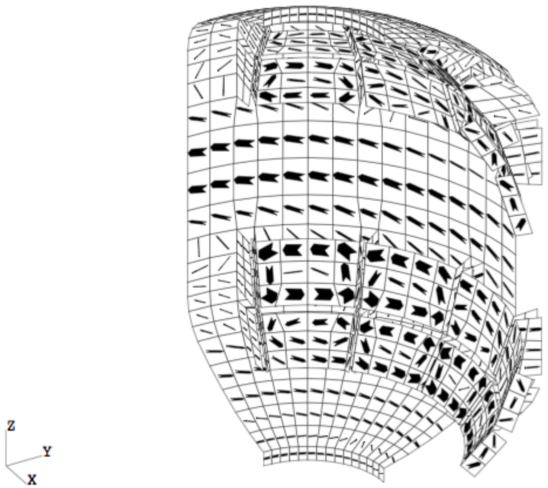


typical results from mmVALEN for NSTX, shot g133775.00655 (plots of wall currents)

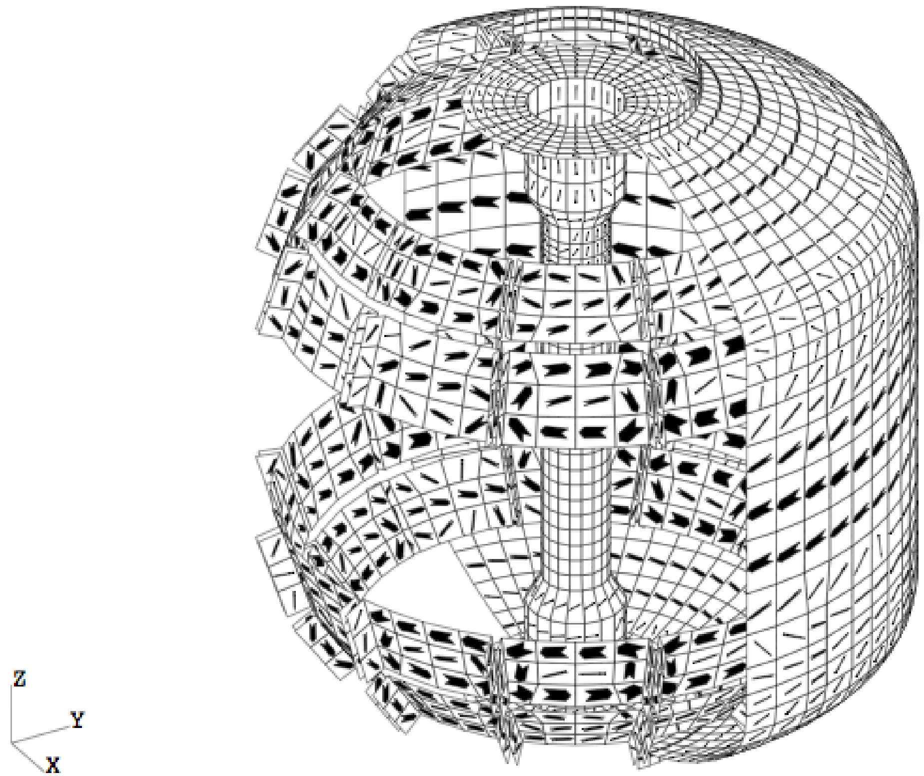
View of wall & mounting brackets



View of wall, brackets, and copper passive plates



View with half VV cut away



Growth rate = 407. [1/s]