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Multi-mode RWM Analysis of NSTX High Beta Plasmas

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J.M. Bialek[1], S.A. Sabbagh[1], A. Boozer[1],
L. Delgado-Aparicio[2], S.P. Gerhardt[2]

[1]*Department of Applied Physics, Columbia University, NY, NY*
[2]*Plasma Physics Laboratory, Princeton University, Princeton, NJ*

2010 NSTX Results & Theory Review

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V1.0

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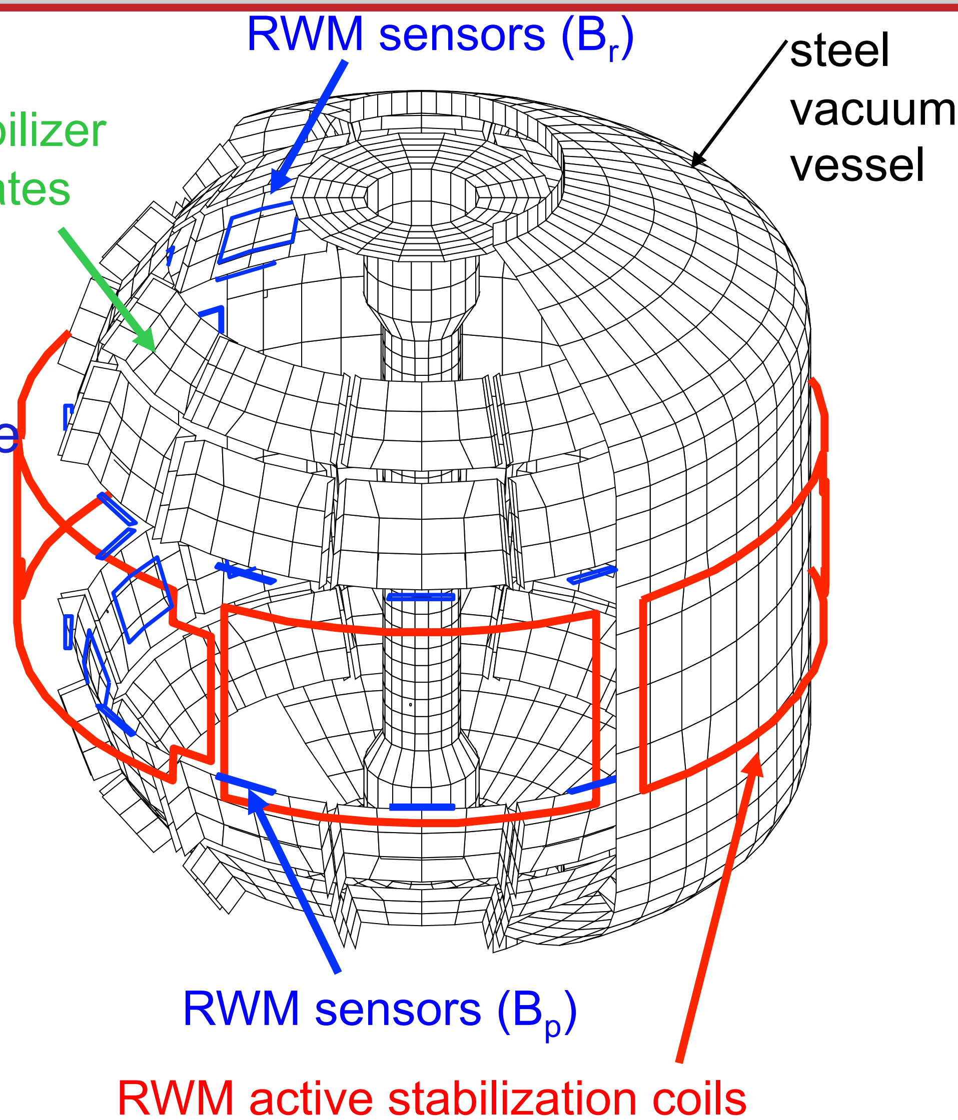
VALEN model of NSTX includes conducting structure, coils, and sensors

- High beta, low aspect ratio
  - R = 0.86 m, A > 1.27
  - $I_p < 1.5$ MA, $B_t = 5.5$ kG
  - $\beta_t < 40\%$, $\beta_N < 7.4$
- Copper stabilizer plates for kink mode stabilization
- Midplane control coils
  - n = 1 – 3 field correction, magnetic braking of ω_ϕ
  - n = 1 resistive wall mode (RWM) control
- Varied sensor combinations used for RWM feedback
  - 48 upper/lower B_p, B_r



The multi mode VALEN formulation includes many ideal plasma eigenfunctions in addition to the 3-D wall and coils

•VALEN formulation of the Resistive Wall Mode uses R-L circuit equations. The plasma response to an external perturbation is

$$\boxed{\Phi_{plasma}^{total}(\phi,\theta) = \left[P\right] \Phi_{plasma}^{external}(\phi,\theta)}$$

•[P] is the 'plasma permeability' [dimensionless], and Φ is the magnetic flux [v*s] on the unperturbed surface of the plasma

•In single mode VALEN [P] depends on only the unstable plasma mode

•In multi-mode VALEN [P] depends on multiple modes (unstable & stable)

•The conducting walls, control coils, and magnetic sensors are represented by L-R circuit equations.

i.e., $\left[L\right]\left\{\dot{I}(t)\right\} + \left[R\right]\left\{I(t)\right\} = \left\{V(t)\right\}$

and $\left\{\Phi^{sensors}(t)\right\} = \left[M\right]\left\{I(t)\right\}$

VALEN equations, multi-mode & original formulation

FLUX on wall, coils, and unperturbed plasma surface from

$\{I_w(t)\}$ = wall currents, $\{I_c(t)\}$ = coil currents,

$\{I_d(t)\}$ = dissipative plasma circuit currents, $\{I_p(t)\}$ = plasma mode currents:

$$\left[L_{ww}\right]\left\{I_w(t)\right\} + \left[L_{wc}\right]\left\{I_c(t)\right\} + \left[L_{wp}\right]\left\{I_d(t)\right\} + \left[L_{wp}\right]\left\{I_p(t)\right\} = \Phi_w(t) \quad = \Phi_{WALL}(t)$$

$$\left[L_{cw}\right]\left\{I_w(t)\right\} + \left[L_{cc}\right]\left\{I_c(t)\right\} + \left[L_{cp}\right]\left\{I_d(t)\right\} + \left[L_{cp}\right]\left\{I_p(t)\right\} = \Phi_c(t) \quad = \Phi_{COIL}(t)$$

$$\left[L_{pw}\right]\left\{I_w(t)\right\} + \left[L_{pc}\right]\left\{I_c(t)\right\} + \left[L_{pp}\right]\left\{I_d(t)\right\} + \left[L_{pp}(t)\right]\left\{I_p\right\} = \Phi_p^{total}(t) \quad = \Phi_{PLASMA}^{total}(t)$$

$$\left[L_{pw}\right]\left\{I_w(t)\right\} + \left[L_{pc}\right]\left\{I_c(t)\right\} + \left[L_{pp}\right]\left\{I_d(t)\right\} = \Phi_p^{external}(t) = \Phi_{PLASMA}^{external}(t)$$

$$\Phi_p^{external}(t) \quad + \quad \left[L_{pp}\right]\left\{I_p\right\} = \Phi_p^{total}(t) = [P]\Phi_p^{external}(t)$$

where: plasma permeability $[P] = [\Lambda][L_{pp}]^{-1}$ and $[\Lambda]^{-1} = 2[G][\varepsilon][G]^t$

$[G]$ = gram schmidt transformation and $[\varepsilon]$ = ideal δW values (from DCON)

i.e., $\{f_i(\theta,\phi)\} = A\sum_\alpha [G_{i\alpha}]\ \{b_\alpha^{DCON-Bn}(\theta,\phi)\}$, *and*, $\oint \dfrac{f_i(\theta,\phi)f_j(\theta,\phi)}{A}da = \delta_{ij}$, $\oint da = A$

VALEN equations, multi-mode & original formulation

single mode VALEN

$$[P]=\left(\frac{-1}{s}\right) \quad or \quad [\mathrm{P}]=\frac{\begin{bmatrix}-s & \alpha \\ -\alpha & -s\end{bmatrix}}{\left(s^2+\alpha^2\right)} \text{ (with rotation)}$$

this is same as: $[P]=[\Lambda]\left[L_{pp}\right]^{-1}$

$$\mathrm{s}=-\Lambda^{-1}L=-2G_{11}\varepsilon G_{11}L$$

$$G_{11}=\frac{1}{A\left[\oint B^n B^n dA/A\right]^{1/2}}$$

$$s=-\frac{2}{A^2}\frac{1}{\left[\oint B^n B^n dA/A\right]^{1/2}}\varepsilon\frac{1}{\left[\oint B^n B^n dA/A\right]^{1/2}}L$$

$$s=\frac{-2L(\delta W)}{\Phi^2}=\frac{-\delta W}{LI^2/2}$$

multi-mode VALEN

$$[P]=[\Lambda]\left[L_{pp}\right]^{-1} \quad and \quad [\Lambda]^{-1}=2[G][\varepsilon][G]^t$$

$$[\varepsilon]=\begin{bmatrix}
\delta W_1 & 0 & 0 & 0 & 0 & \dots & & & & \\
0 & \delta W_1 & 0 & 0 & 0 & \dots & & & & \\
 & & \delta W_2 & 0 & 0 & \dots & & & & \\
 & & 0 & \delta W_2 & 0 & \dots & & & & \\
 & \cdot & & & & & & & & \\
 & \cdot & & & & & & & & \\
0 & 0 & 0 & 0 & & \dots & 0 & \delta W_n & 0 \\
0 & 0 & 0 & 0 & & \dots & 0 & 0 & \delta W_n
\end{bmatrix}$$

or (with rotation)

$$[\varepsilon]=\begin{bmatrix}
\delta W_1 & \Gamma_1 & 0 & 0 & 0 & \dots & & & \\
-\Gamma_1 & \delta W_1 & 0 & 0 & 0 & \dots & & & \\
 & & \delta W_2 & \Gamma_2 & 0 & \dots & & & \\
 & & -\Gamma_2 & \delta W_2 & 0 & \dots & & & \\
 & \cdot & & & & & & & \\
 & \cdot & & & & & & & \\
0 & 0 & 0 & 0 & & \dots & 0 & \delta W_n & \Gamma_n \\
0 & 0 & 0 & 0 & & \dots & 0 & -\Gamma_n & \delta W_n
\end{bmatrix}$$

VALEN equations (continued), & post processing to obtain Bnormal from DCON modes on the plasma surface

single mode VALEN

solve for surface current potential:

$g(\vec{r}) \quad or \quad g(\theta,\phi)$

$\vec{J} = \nabla \times g\, \hat{n}\, \delta\left(\vec{r} - \vec{r}_{surface}\right) \quad and$

$$\vec{B}^{unstable}_{\substack{DCON \\ normal}}(\vec{r}) = \frac{\mu_0}{4\pi} \nabla \times \oint_{\substack{\text{plasma} \\ \text{surface}}} \frac{\vec{J}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}$$

calculate one plasma L using $g(\vec{r})$

multi-mode VALEN

use gram schmidt vectors $\{f_i\}$ for plasma surface current potential

$\vec{J}_i(\vec{r}) = \nabla \times f_i(\vec{r})\, \hat{n}_{surface}\ \delta\left(\vec{r} - \vec{r}_{surface}\right)$

calculate all plasma L_{ik} using $f_i(\vec{r})$ & $f_j(\vec{r})$

multi-mode post processing

$$\{I_p\} = \left[L^{\substack{plasma \\ modes}}_{pp}\right]^{-1} \left([P] - [1]\right) \left\{ \left[L^{plasma-wall}_{pw}\right]\{I_w\} + \left[L^{plasma-coil}_{pc}\right]\{I_c\} + \left[L^{\substack{plasma \\ dissipative}}_{pp}\right]\{Id\} \right\}$$

Bn (gram schmidt functions) $\longrightarrow$

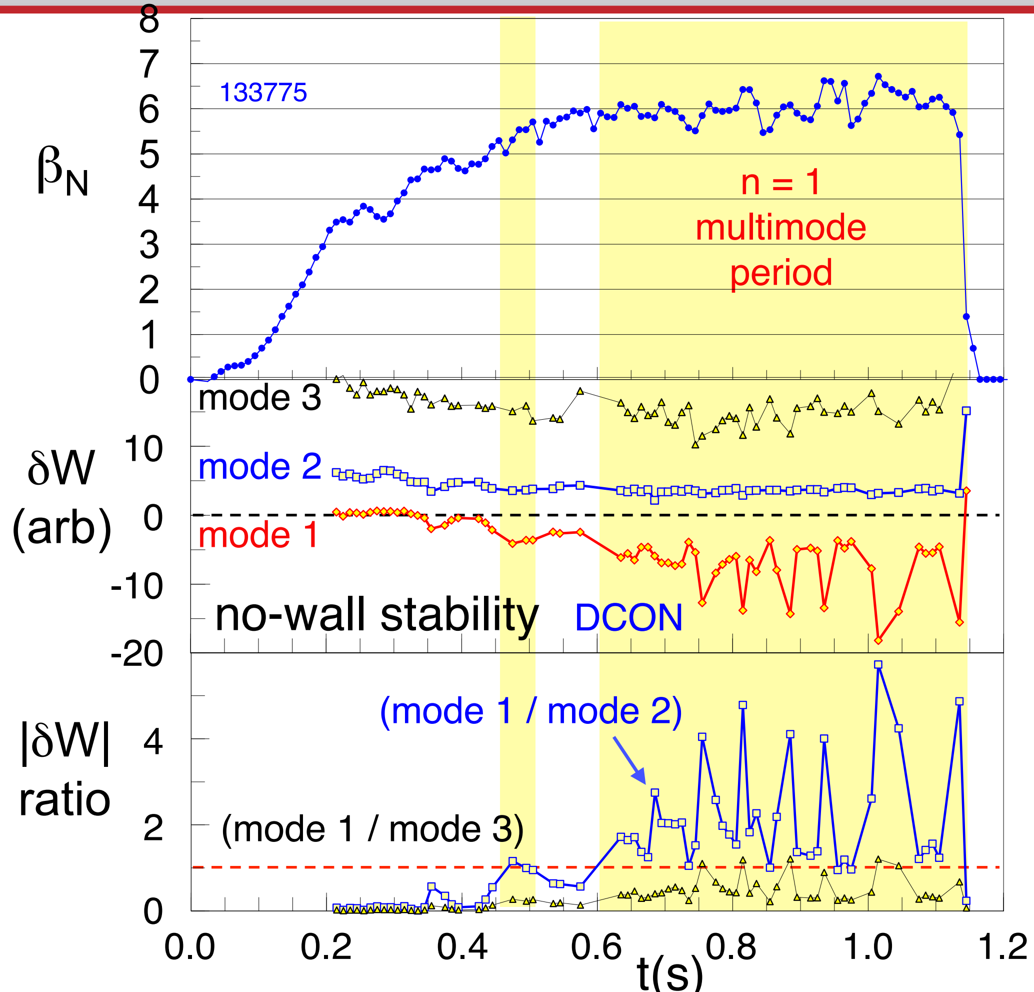
$$\left\{\vec{B}_{plasma}(t) \cdot \hat{n}_{surface}(\theta,\phi)\right\}_i = \left\{ \left[L_{pp}\right]_{ij} \left(\left(f_j(\theta,\phi)\right) I^{plasma}_j(t) + \left(f_j(\theta,\phi)\right) I^{\substack{plasma \\ dissipative}}_j(t) \right) \right\} / A$$

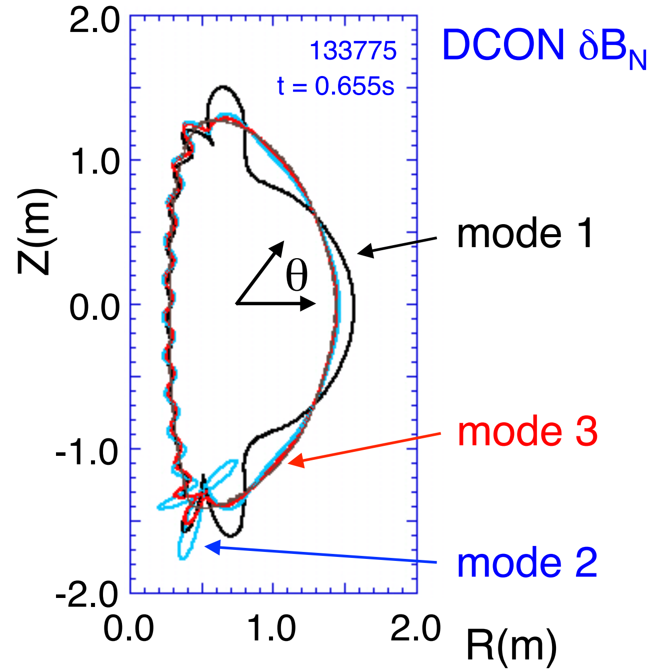
recall: $\quad f_j(\theta,\phi) = [G]_{jk} B^n_{\text{DCON\_mode}}(\theta,\phi)$

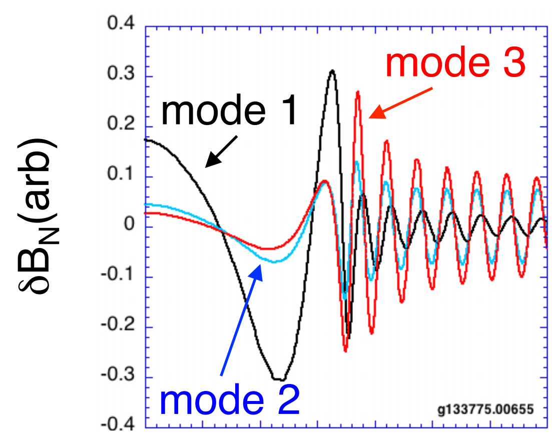
Bn (DCON eigenmodes) $\longrightarrow$

$$\left\{B^{normal}(\theta,\phi,t)\right\} = \left\{ \left[L_{pp}\right]_{ij} \left(\left([G]_{jk} B^n_{\text{DCON\_mode}}(\theta,\phi)\right) I^{plasma}_j(t) + \left([G]_{jk} B^n_{\text{DCON\_mode}}(\theta,\phi)\right) I^{\substack{plasma \\ dissipative}}_j(t) \right) \right\} / A$$

Multimode response theoretically is expected to be significant at high β_N







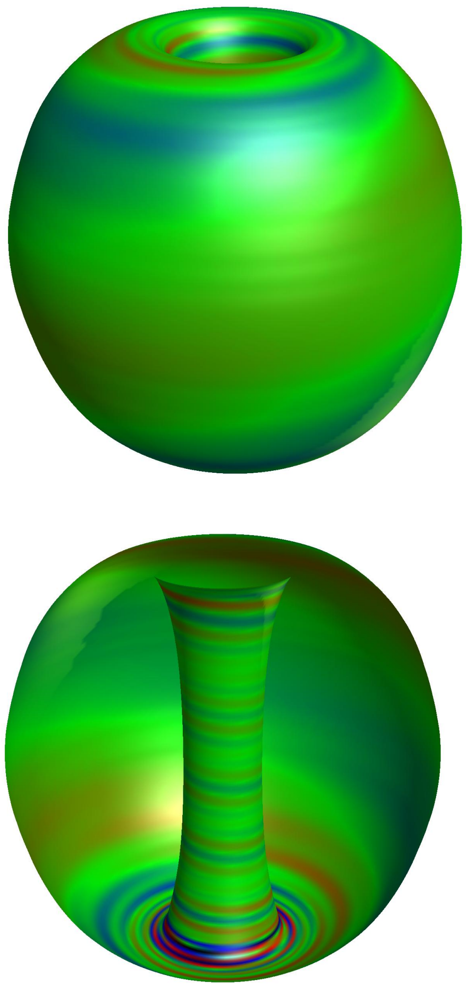
- Boozer multimode criterion for n = 1 met at high β_N
  - |δW| smallest for 2[nd] n = 1 eigenfunction (PoP 10 (2003) 1458.)
  - Ratio of |δW| for 3[rd] vs. 1[st] least stable mode sometimes also > 1

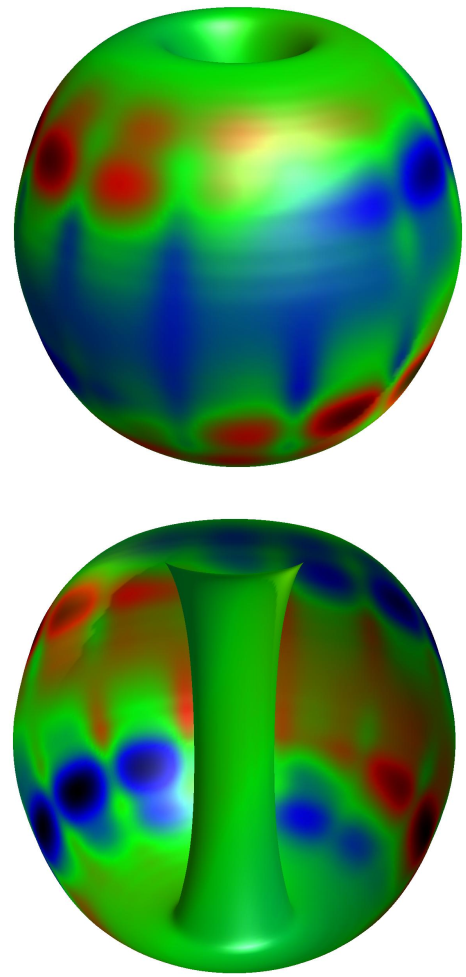
Passive growth @ β_N = 5.54 multi mode VALEN results for NSTX shot 133775.00495

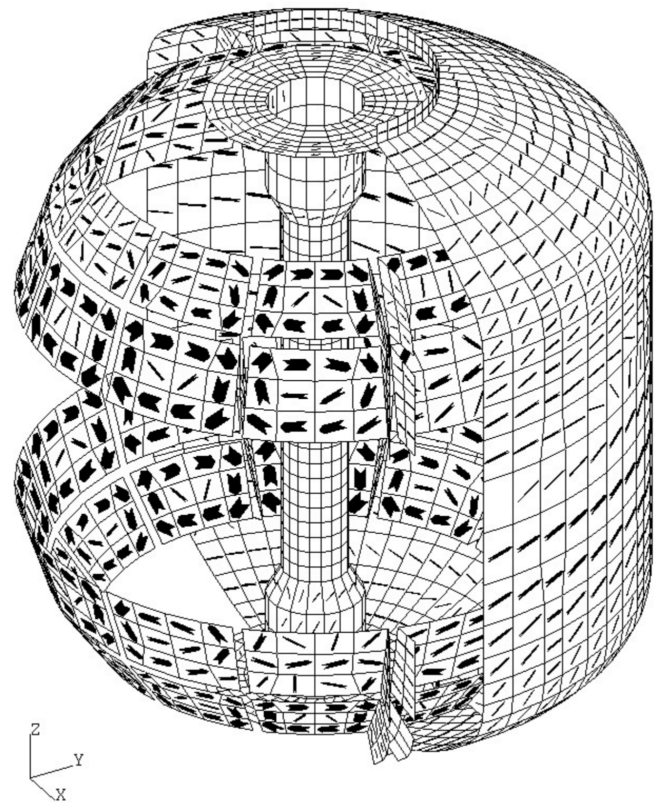
total B_n on plasma surface

B_n on plasma surface from only wall currents

cut away view of
wall currents
β_N = 5.54
growth rates =93, 101 [1/s]

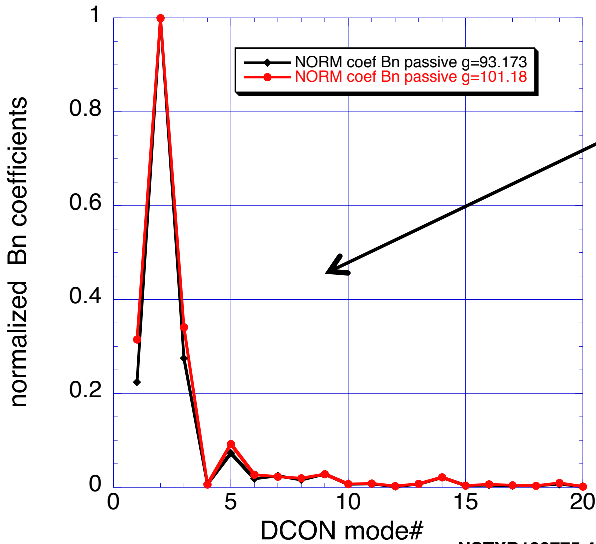






color contours: dark red ~big negative, green ~ zero, dark blue ~ big positive

Multi-mode spectra passive RWM in NSTX @ β_N=5.54



relative weight of DCON modes in passive growing RWM

NSTX shot #133775
t = 0.495 [s]

DCON mode weighted sums shown below

Illustration of B_normal signature for RWM mode

normal distance from plasma surface corresponds to B_normal

γ = 93.17 [1/s]
β_N = 5.54

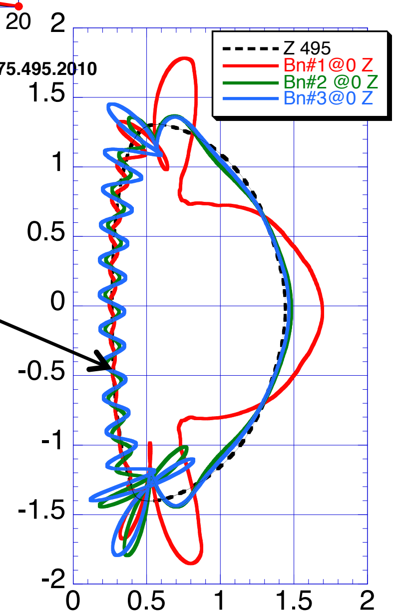
γ = 101.18 [1/s]
β_N = 5.54

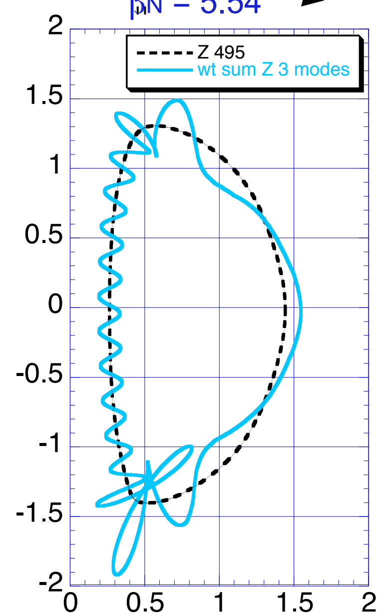
Illustration of DCON modes equal weights

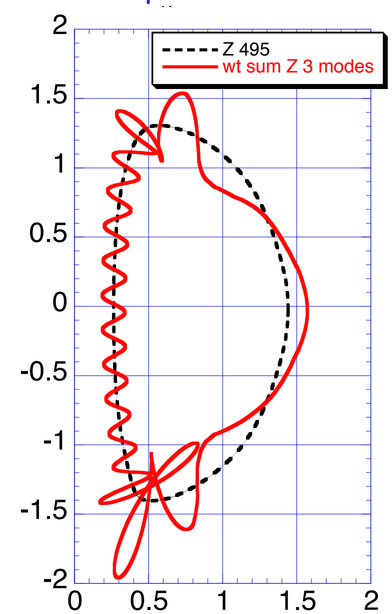
#1 thru #3,

each DCON mode shown with max B_n = 0.5[m]

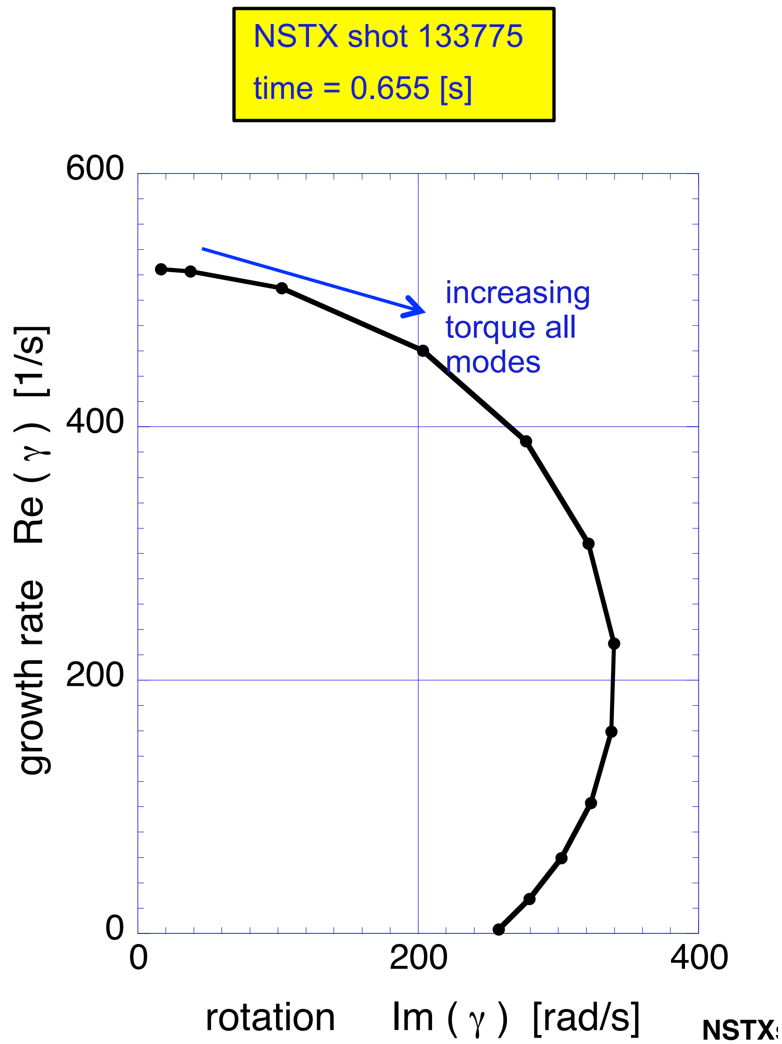
normal distance from plasma surface (black dashed line) corresponds to B_normal







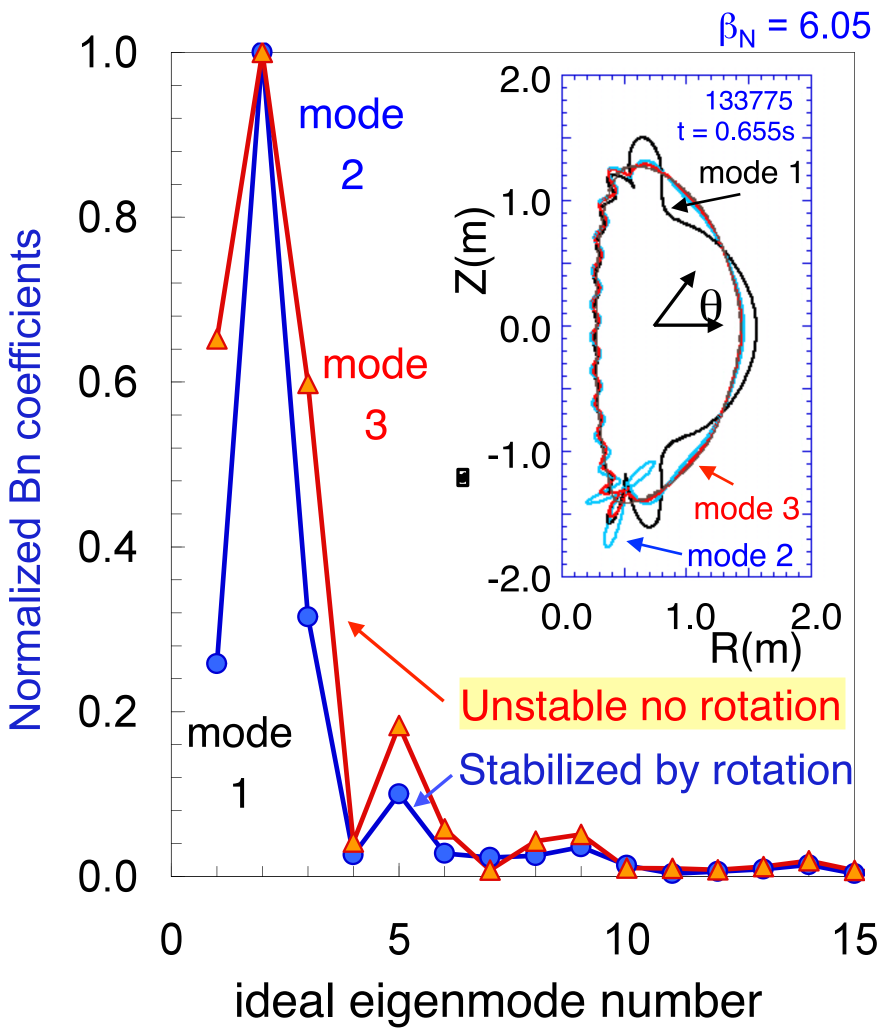
Mode may be stabilized by rotation alone, VALEN predicts RWM growth rate as same torque is applied to all modes



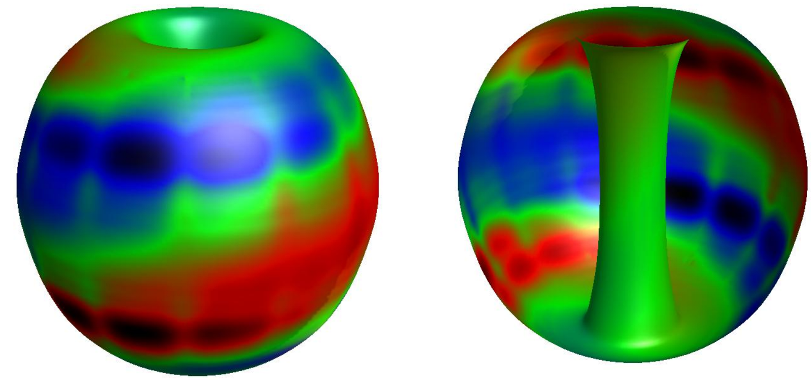
- NSTX RWM stabilized by ω_ϕ
- VALEN calculation of rotation needed for stabilization ~ 41 [Hz] is close to experimental value of ~ 30 [Hz]
- β_N = 6.05

VALEN multi-mode RWM computation shows 2nd eigenmode component usually has dominant amplitude in NSTX at high β_N

δBⁿ RWM multi-mode composition



δBⁿ from wall, multi-mode response



- NSTX unstable RWM
  - Computed growth time consistent with experiment
  - 2nd eigenmode ("divertor") has larger amplitude than ballooning eigenmode
- NSTX RWM stabilized by ω_ϕ
  - Ballooning eigenmode amplitude decreases relative to "divertor" mode

RWM feedback control: Mathematics of VALEN feedback for NSTX

- Analyze upper Bp sensor signals in NSTX for selected 'n' to identify RWM
- Compensate for control coil flux in sensors
- Scan phase angle between the RWM signal and the pattern of voltage applied to the control coils
- Choose gain for feedback [v/flux]
- Run multi-mode VALEN eigenvalue calculation and predict growth rate
- Feedback may cause the plasma modes to rotate

use classic least squares fitting to find $S\ \&\ C$:

$$S*\sin(n*\phi_1^{sensor}) + C*\cos(n*\phi_1^{sensor}) = \Phi_1^{sensor}$$
$$S*\sin(n*\phi_2^{sensor}) + C*\cos(n*\phi_2^{sensor}) = \Phi_2^{sensor}$$
$$\vdots$$
$$S*\sin(n*\phi_N^{sensor}) + C*\cos(n*\phi_N^{sensor}) = \Phi_N^{sensor}$$

or $$[A]_{Nx2}\begin{Bmatrix} S \\ C \end{Bmatrix}_{2x1} = \left\{\Phi^{sensor}\right\}_{Nx1}$$

where : sensors located at angles ϕ_i

and sensors read magnetic flux Φ_i

solution is :

$$\begin{Bmatrix} S \\ C \end{Bmatrix} = \left([A]^t[A]\right)^{-1}_{2x2}[A]_{2xN}\left\{\Phi^{sensor}\right\}_{Nx1}$$

voltages applied to coil are :

$$\begin{Bmatrix} V_1^{coil} \\ \vdots \\ V_C^{coil} \end{Bmatrix} = G_p \begin{bmatrix} \sin(\phi_1^{coil}+\delta) & \cos(\phi_1^{coil}+\delta) \\ \vdots & \vdots \\ \sin(\phi_C^{coil}+\delta) & \cos(\phi_C^{coil}+\delta) \end{bmatrix} \left([A]^t[A]\right)^{-1}_{2x2}[A]_{2xN}\left\{\Phi^{sensor}\right\}_{Nx1}$$

where : coils located at angles ϕ_i^{coil}

and we use phase δ

to provide compensation for coil to sensor coupling

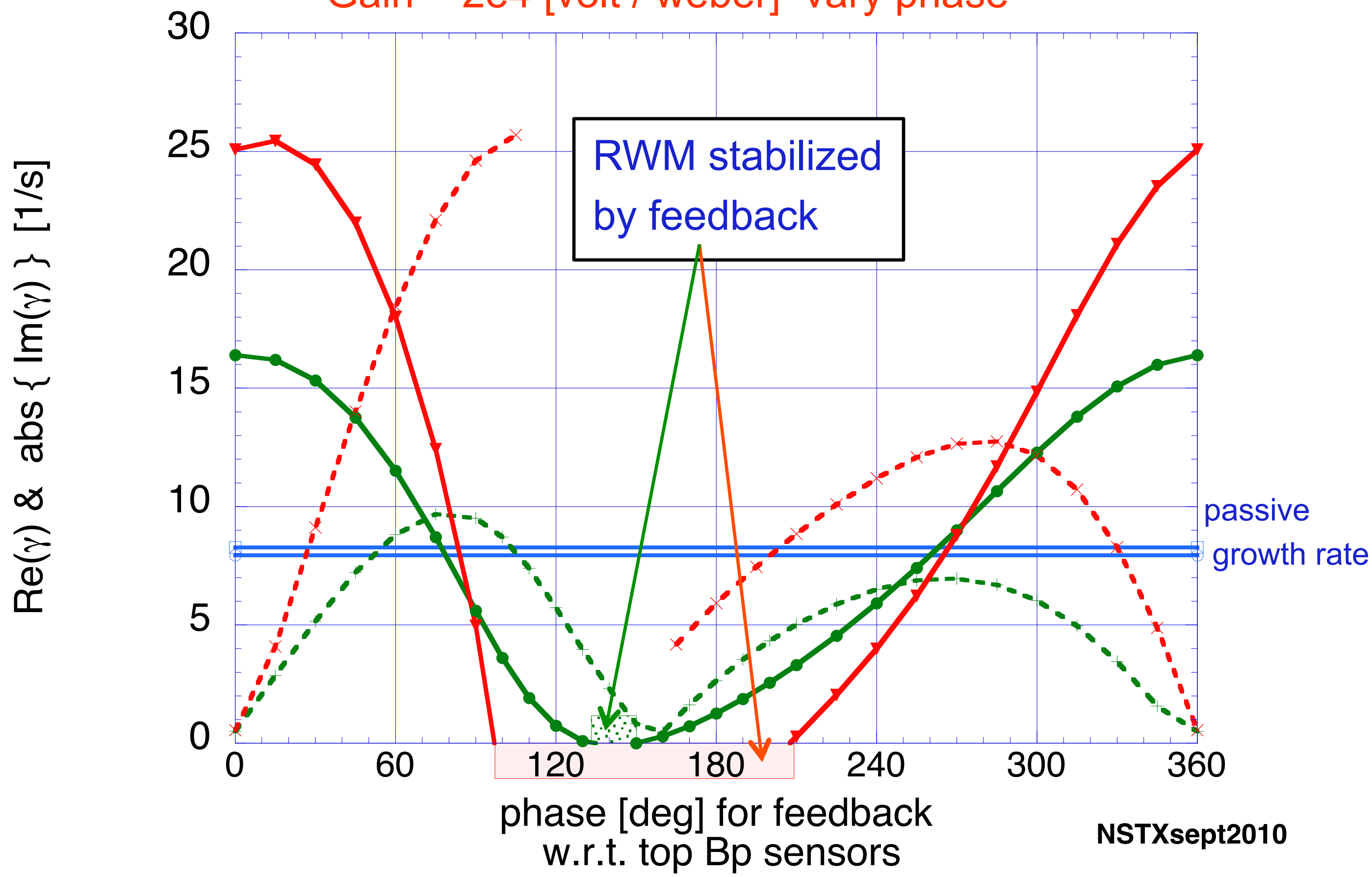
replace $\left\{\Phi^{sensor}\right\}_{Nx1}$ by $\left\{\Phi^{sensor}\right\}_{Nx1} - \left[M_{sensor,coil}\right]_{NxC}\left\{I^{coil}\right\}_{Cx1}$

RWM feedback control:
VALEN results, scan in feedback phase for β_N = 4.65

NSTX shot #133775
t = 0.345 [s]

VALEN eigenvalue calcuations with feedback
Gain = 1e4 [volt / weber] vary phase
Gain = 2e4 [volt / weber] vary phase

- Notes:
- solid lines show growth rate
- dashed lines show rotation
- shaded regions indicate RWM is stabilized by feedback
- horizontal lines show passive growth rate without feedback

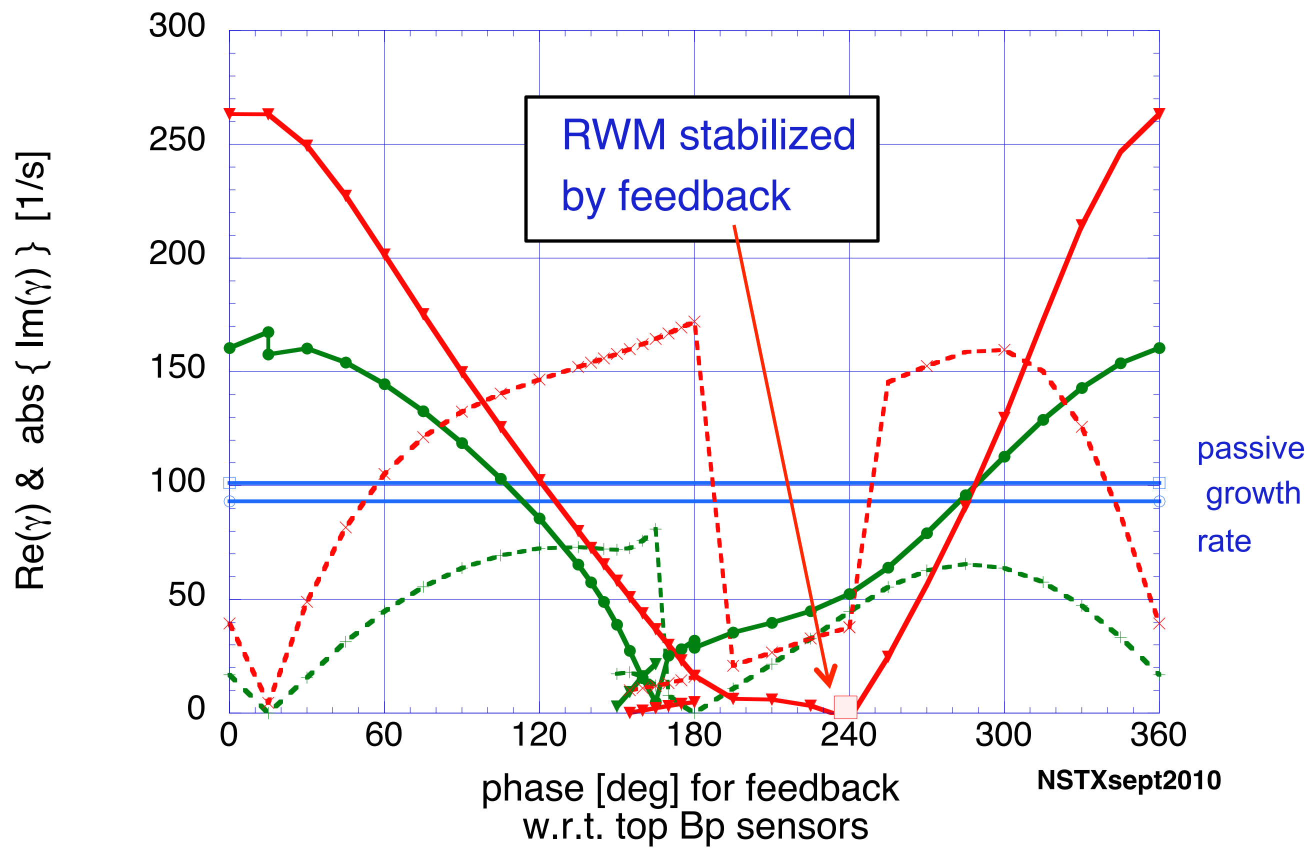


RWM feedback control: VALEN results, scan in feedback phase for β_N = 5.54

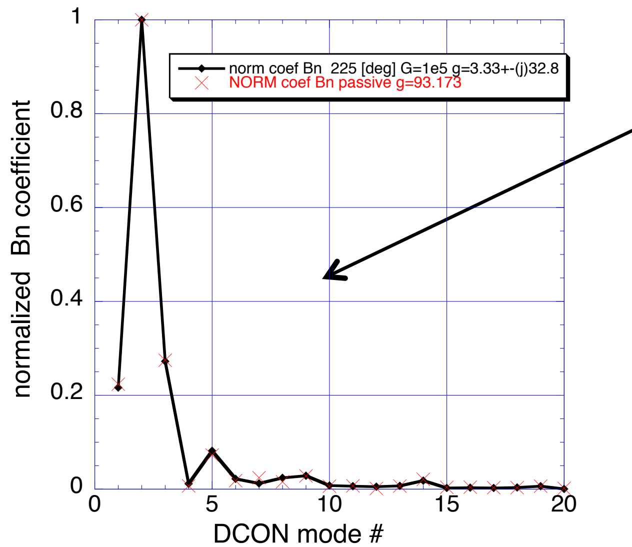
- •Notes:
- •solid lines show growth rate
- •dashed lines show rotation
- •horizontal lines show passive growth rate without feedback
- •only a small region at G=1e5 near phase = 240 is stabilized when βn=6.079

NSTX shot #133775
t = 0.495 [s]

VALEN eigenvalue calcuations with feedback
Gain = 4e4 [volt / weber] vary phase
Gain = 1e5 [volt / weber] vary phase



Multi-mode spectra with RWM feedback in NSTX @ β_N = 5.54



- relative weight of DCON modes in RWM with feedback G=1e5, phase=225 [deg]

x relative weight of DCON modes for passive RWM growth

DCON mode weighted sums shown below

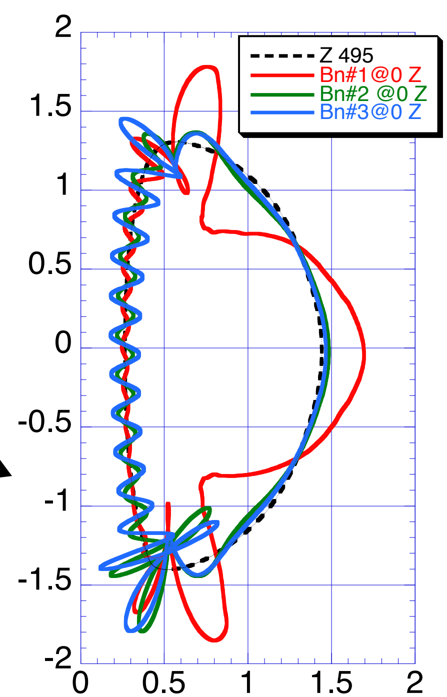
Illustration of B_normal signature for RWM mode with feedback

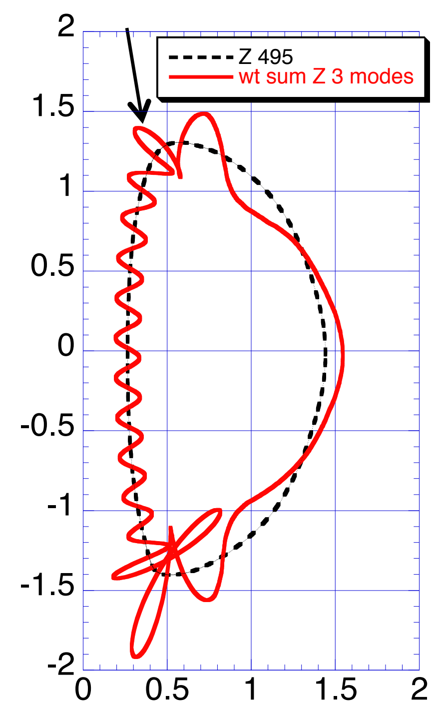
normal distance from plasma surface corresponds to B_normal

Illustration of DCON modes #1 thru #3,

each DCON mode shown with max B_n = 0.5 [m] (equal weights)

normal distance from plasma surface (dashed black line) corresponds to B_normal





Summary - Conclusions

- Control of RWM is important, allows sustained access to high beta regimes

- Multi mode VALEN predictions, both passive growth and performance with applied feedback consistent with experimental observations in NSTX

- Multi mode VALEN calculations indicate lowest order modes dominate the RWM response, both passive and with active feedback
  - The second (stable) DCON mode has the greatest contribution for simple passive growth, passive growth with substantial mode rotation, and when feedback is applied

- Future work planned includes further application to NSTX experimental results, studies of error field amplification, and application of RWM feedback logic both in the time domain and frequency domain.