Model Predictive Control with Integral Action for the Rotational Transform Profile Tracking in NSTX-U

Zeki O. Ilhan[1], William P. Wehner[1], Eugenio Schuster[1], Mark D. Boyer[2], David A. Gates[2], Stefan P. Gerhardt[2], and Jonathan E. Menard[2]

[1]Lehigh University (Plasma Control Group), Bethlehem, PA, USA
[2]Princeton Plasma Physics Laboratory (PPPL), Princeton, NJ, USA

E-mail: *zeki_ilhan@lehigh.edu*

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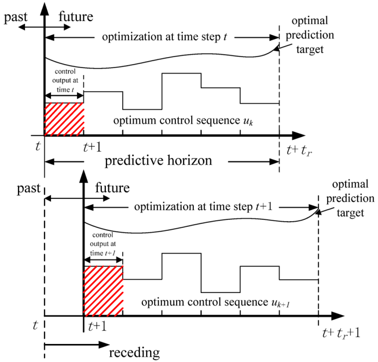
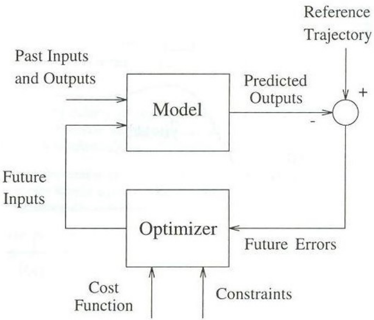
Introduction to Model Predictive Control (MPC)

1. A **dynamic model** of the system is used to predict the system output for a future time horizon.
2. Control sequence is calculated to **optimize an objective function.**
3. ***Receding strategy:*** Only first element of the control sequence is applied at each step!



Hu, C. et al., Energies (2015)



Camacho and Bordons, Springer-Verlag (1999)

Magnetic Diffusion Equation

- The evolution of the **poloidal magnetic flux,** ψ is given by the **Magnetic Diffusion Equation:**

$$\boxed{\frac{\partial\psi}{\partial t} = \frac{\eta(T_e)}{\mu_0\rho_b^2\hat{F}^2}\frac{1}{\hat{\rho}}\frac{\partial}{\partial\hat{\rho}}\left(\hat{\rho}D_\psi\frac{\partial\psi}{\partial\hat{\rho}}\right) + R_0\hat{H}\eta(T_e)\frac{<\bar{j}_{NI}\cdot\bar{B}>}{B_{\phi,0}},} \qquad (1)$$

Simplified Models for

$n_e, T_e, \eta, \bar{j}_{NI}$

$$\frac{\partial\varphi}{\partial t} = \frac{\eta(T_e)}{\mu_0\rho_b^2\hat{F}^2}\frac{1}{\hat{\rho}}\frac{\partial}{\partial\hat{\rho}}\left(\hat{\rho}D_\varphi\frac{\partial\varphi}{\partial\hat{\rho}}\right) + R_0\hat{H}\eta(T_e)\frac{\langle\bar{j}_{NI}\cdot\bar{B}\rangle}{B_{\phi,0}}$$

Magnetic Diffusion Equation

$Control - Oriented\ Current\ Profile\ Evolution\ Model$

Reduction of Control-Oriented FPD Model

"FPD Model"

$$\frac{\partial\psi}{\partial t} = f\left(\psi, \frac{\partial\psi}{\partial\hat{\rho}}, \frac{\partial^2\psi}{\partial\hat{\rho}^2}, \bar{u}, t\right)$$

$$\downarrow$$

$$\dot{\theta}(t) = g(\theta(t), \bar{u}(t))$$

$$\downarrow$$

$$\dot{\tilde{\theta}}(t) = A(t)\tilde{\theta}(t) + B(t)\tilde{u}(t)$$

$$\downarrow$$

$$\dot{\tilde{\theta}}(t) = A\tilde{\theta}(t) + B\tilde{u}(t)$$

$$\downarrow$$

$$\dot{\tilde{\iota}}(t) = \bar{A}\tilde{\iota}(t) + \bar{B}\tilde{u}(t)$$

$$\downarrow$$

$$\tilde{\iota}(k+1) = \bar{A}_d\,\tilde{\iota}(k) + \bar{B}_d\,\tilde{u}(k)$$
$$y(k) = \bar{C}_d\,\tilde{\iota}(k)$$

"Discrete LTI Model"

1. MDE combined with the simplified models of n_e, T_e, η, and $\bar{j}_{ni}$ can be written as an infinite-dimensional PDE, where $\psi(\hat{\rho}, t)$ is the poloidal magnetic flux, and $\bar{u}$ is the nonlinear iputs, i.e., $\bar{u} = p(u)$.
2. FPD model is discretized in space to generate a set of nonlinear ODEs, where $\theta(t) = [\theta_1(t), \ldots, \theta_n(t)]^T$, with $\theta(\hat{\rho}, t) = \partial\psi/\partial\hat{\rho}$ is the poloidal flux gradient.
3. The model is linearized around a set of reference physical inputs u_r, and states θ_r, yielding an LTV model, where $\tilde{\theta}(t) = \theta(t) - \theta_r(t)$, and $\tilde{u}(t) = u(t) - u_r(t)$.
4. Further simplification leading to an LTI model is possible by setting $A = A(t_s)$ and $B = B(t_s)$, where t_s is some time during the flat-top phase of the discharge.
5. Since $\iota(\hat{\rho}, t) = -\theta(\hat{\rho}, t)/B_{\phi,0}\rho_b^2\hat{\rho}$, the LTI model for $\tilde{\theta}$ can be converted into an LTI model for $\tilde{\iota}$, where $\bar{A} = T^{-1}AT$, $\bar{B} = T^{-1}B$, and $T = -\text{diag}(B_0\rho_b^2\hat{\rho}_i)$.
6. Finally, the $\tilde{\iota}$ model is converted to discrete-time, and an output equation is added to select the reference-tracking states.

MPC Formulation with Integral Action

- Rewrite the discrete, LTI model of the ι-profile in terms of the **state increment, $\Delta\tilde{\iota}(k+1)$** and **output increment, $\Delta y(k+1)$** so that input is the **control increment, $\Delta\tilde{u}(k)$.**

$$\overbrace{\Delta\tilde{\iota}(k+1)}^{\tilde{\iota}(k+1)-\tilde{\iota}(k)} = \bar{A}_d\Delta\tilde{\iota}(k) + \bar{B}_d\Delta\tilde{u}(k) \tag{2}$$

$$\underbrace{\Delta y(k+1)}_{y(k+1)-y(k)} = \bar{C}_d\bar{A}_d\Delta\tilde{\iota}(k) + \bar{C}_d\bar{B}_d\underbrace{\Delta\tilde{u}(k)}_{\tilde{u}(k)-\tilde{u}(k-1)} \tag{3}$$

- Defining an enlarged state variable as $x(k) = [\Delta\tilde{\iota}(k) \;\; y(k)]^T$, equations (2) and (3) are combined together to form

$$\underbrace{\begin{bmatrix}\Delta\tilde{\iota}(k+1)\\ y(k+1)\end{bmatrix}}_{x(k+1)} = \underbrace{\begin{bmatrix}\bar{A}_d & 0_{n\times m}\\ \bar{C}_d\bar{A}_d & I_{m\times m}\end{bmatrix}}_{\widetilde{A}}\underbrace{\begin{bmatrix}\Delta\tilde{\iota}(k)\\ y(k)\end{bmatrix}}_{x(k)} + \underbrace{\begin{bmatrix}\bar{B}_d\\ \bar{C}_d\bar{B}_d\end{bmatrix}}_{\widetilde{B}}\Delta\tilde{u}(k) \tag{4}$$

- The enlarged plant can then be written as

$$x(k+1) = \widetilde{A}x(k) + \widetilde{B}\Delta\tilde{u}(k), \tag{5}$$

$$y(k) = \widetilde{C}x(k), \tag{6}$$

where, $\widetilde{C} = \begin{bmatrix}0_{m\times n} & I_{m\times m}\end{bmatrix}$.

Integral MPC Solution

- **Future feedback control increments** $(\Delta\tilde{u}^*_{k|N})$ are obtained by minimizing the **quadratic performance index** while satisfying the **input constraints**, i.e.,

$$\Delta\tilde{u}^*_{k|N} = \text{arg} \min_{\Delta\tilde{u}_{k|N}} \left\{ \Delta\tilde{u}^T_{k|N} H \Delta\tilde{u}_{k|N} + 2x^T(k) f^T \Delta\tilde{u}_{k|N} \right\} \tag{7}$$

$$\text{subject to} \quad \mathcal{A}\Delta\tilde{u}_{k|N} \leq b_k \tag{8}$$

- (7)-(8) define a standard **Q**uadratic **P**rogramming (**QP**) problem.
- A ***receding horizon strategy*** is used and only the first control increment $\Delta\tilde{u}^*(k)$ in the calculated $\Delta\tilde{u}^*_{k|N}$ is used for control.
- Optimal feedback control action becomes

$$\tilde{u}(k) = \Delta\tilde{u}^*(k) + \tilde{u}(k-1). \tag{9}$$

Closed-Loop Integral MPC Simulation Study in MATLAB

- The target state trajectory $\iota_r(\rho, t)$ is generated through an open-loop TRANSP simulation with the following constant reference inputs.

$\mathbf{n_e(m^{-3})}$	5.0×10^{19}		$\mathbf{P_4(W)}$	0.8×10^6
$\mathbf{P_1(W)}$	0.2×10^6		$\mathbf{P_5(W)}$	1.0×10^6
$\mathbf{P_2(W)}$	0.4×10^6		$\mathbf{P_6(W)}$	1.2×10^6
$\mathbf{P_3(W)}$	0.6×10^6		$\mathbf{I_p(A)}$	0.7×10^6

- The prediction horizon is set to $N = 5$ to guarantee closed-loop stability.
- The initial condition perturbation rejection capability is tested by setting

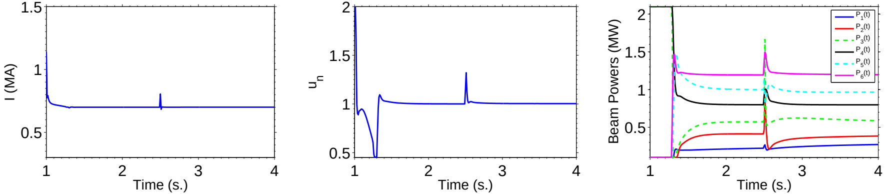
$$\iota(t_0) = \iota_r(t_0) + \delta\iota \tag{10}$$

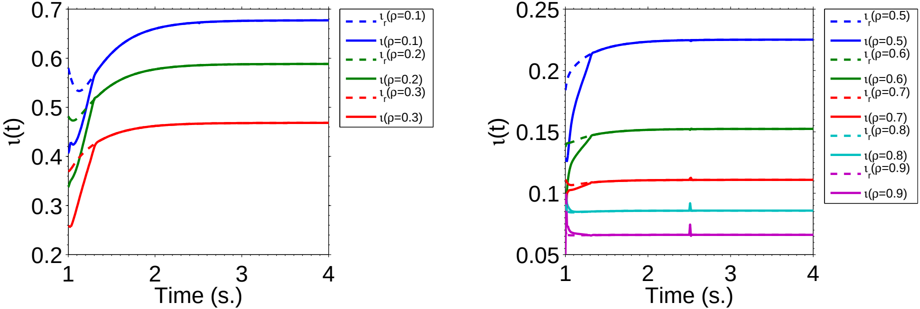
- The controller is also tested against constant input disturbances starting from $t = 2.5$ s. i.e.,

$$\tilde{u}(k) = \begin{cases} \Delta\tilde{u}^*(k) + \tilde{u}(k-1), & t < 2.5\,\text{s.} \\ \Delta\tilde{u}^*(k) + \tilde{u}(k-1) + u_d, & t \geq 2.5\,\text{s.} \end{cases} \tag{11}$$

where $u_d = 0.15u_r$ stands for the constant disturbance inputs.

Results of the Closed-Loop Integral MPC Simulation Study





Upper Figures: (left)Time evolution of the optimal plasma current, (center) time evolution of the optimal n_e regulation, and (right) time evolution of the optimal neutral beam injection powers.

Lower Figures: Time evolution of the optimal outputs (solid) with their respective targets (dashed).

Conclusion and Future Work

- An **NSTX-U-tailored plasma response model** is obtained by combining the MDE with simplified models for various plasma variables.
- A constrained **MPC algorithm** is formulated based on the reduced-order, LTI model to regulate the rotational transform (ι-profile).
- **An integrator** is added to the MPC formulation to achieve offset-free tracking against modeling uncertainties and external disturbances.
- The proposed MPC control scheme is tested via closed-loop numerical simulations based on the control-oriented MDE solver.
- ***First MPC design for NSTX-U for current density profile control.***
  - explicitly handles input and state constraints
  - predicts plasma future state in real time based on current plasma state
  - may be crutial in achieving current profile control + MHD instability avoidance
- **Future work** includes:
  - **Refinement** of the FPD control-oriented model using actual experimental data once NSTX-U achieves relevant plasma scenarios.
  - **Implementation** of MPC algorithm in **TRANSP's Expert routine** and **PCS.**
  - **TRANSP closed-loop simulations** $\Rightarrow$ **Experimental testing in NSTX-U.**