Vertical stability calculations

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NSTX-U Results Review
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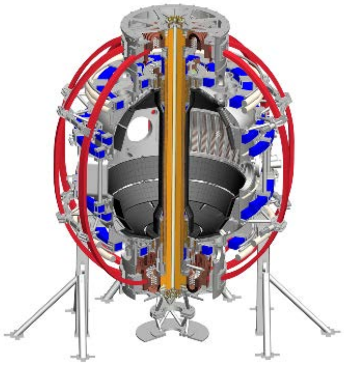
PPPL



Background

- Recently published ST-FNSF / Pilot Plant paper required vertical stability projections
- Used rigid plasma model, LRDFIT / ISOLVER structure model and code infrastructure
- Benchmarked against NSTX natural VDE data with coil voltages frozen → plasma drift
- Assessed FNSF drift recovery vs. vertical offset
  - Determined marginal $\Delta Z/a$ vs l_i and wall position

Rigid plasma vertical growth rate model

For a vertically unstable plasma with velocity far below the Alfven speed, plasma inertia can be ignored and the plasma motion away from the neutral point z_0 can modeled as an equilibrium force balance with the $\vec{J} \times \vec{B}$ force balanced by the opposing force from image currents I_v in nearby vessel or other passive conducting structures:

$$\hat{z} \cdot \int dV \rho_m \frac{d\vec{v}}{dt} = \hat{z} \cdot \int dV \vec{J} \times \vec{B} + \Gamma_v I_v \approx 0 \qquad (1)$$

The vessel or conducting wall current I_v is induced by the plasma motion or velocity $\dot{z} = dz/dt$. If the vessel / passive conductors have self-inductance L_v, mutual inductance to the plasma M_{vp}, and current decay rate $\lambda_v = R_v/L_v$, then:

$$\Gamma_v I_v = Y(t)\Delta F_z \qquad Y(t) \equiv e^{-\lambda_v t}\int^t e^{\lambda_v t'}(\dot{z}/z_0)dt' \qquad (2)$$

$$\Delta F_z \equiv z_0 \Gamma_v \frac{M_{vp}}{L_v}\frac{\partial I_P}{\partial z} \qquad F_z \equiv \hat{z} \cdot \int dV \vec{J}_\phi \times \vec{B}_R \approx \left.\frac{\partial F_z}{\partial z}\right|_{z_0}(z - z_0) \qquad (3)$$

$$\Rightarrow \left.\frac{\partial F_z}{\partial z}\right|_{z_0}(z - z_0) + Y(t)\Delta F_z = 0 \qquad (4)$$

Growth rate dependence on stability index *f*

$$\Rightarrow \frac{\partial F_z}{\partial z}\bigg|_{z_0}(z - z_0) + Y(t)\Delta F_z = 0 \tag{4}$$

$$\dot{Y} = -\lambda_v Y + \dot{z}/z_0 \quad \Rightarrow \quad \dot{z} + \gamma z = \gamma z_0 \tag{5}$$

$$f \equiv -\left(\frac{\partial F_z}{\partial z}\right) / \left(\frac{\Delta F_z}{z_0}\right) \tag{6}$$

$$\gamma \equiv \frac{\lambda_v f}{1 - f} \tag{7}$$

$$f \leq 0 \Rightarrow stable \quad f > 0 \Rightarrow unstable \quad f \geq 1 \Rightarrow ideally - unstable \tag{8}$$

Two eigenmode growth rate model

In general, the force on the plasma from plasma-motion induced image currents can be expressed as:

$$\Gamma_v I_v = \sum_{k=1} Y_k(t)\Delta F_k \qquad Y_k(t) \equiv e^{-\lambda_k t}\int^t e^{\lambda_k t'}(\dot{z}/z_0)dt' \qquad (8)$$

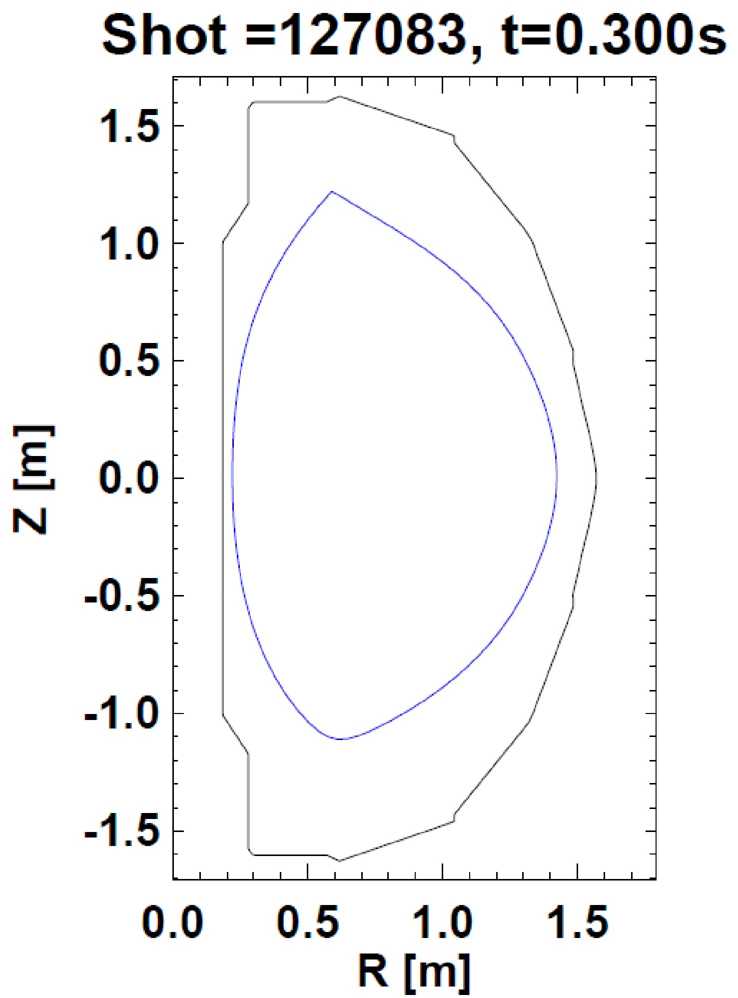
A more accurate yet analytically tractable approximation is to fit the full response force to a reduced model with two image current decay times λ_1 and λ_2 and corresponding force coefficients ΔF_1 and ΔF_2. The corresponding equation for the vertical growth-rate γ then becomes:

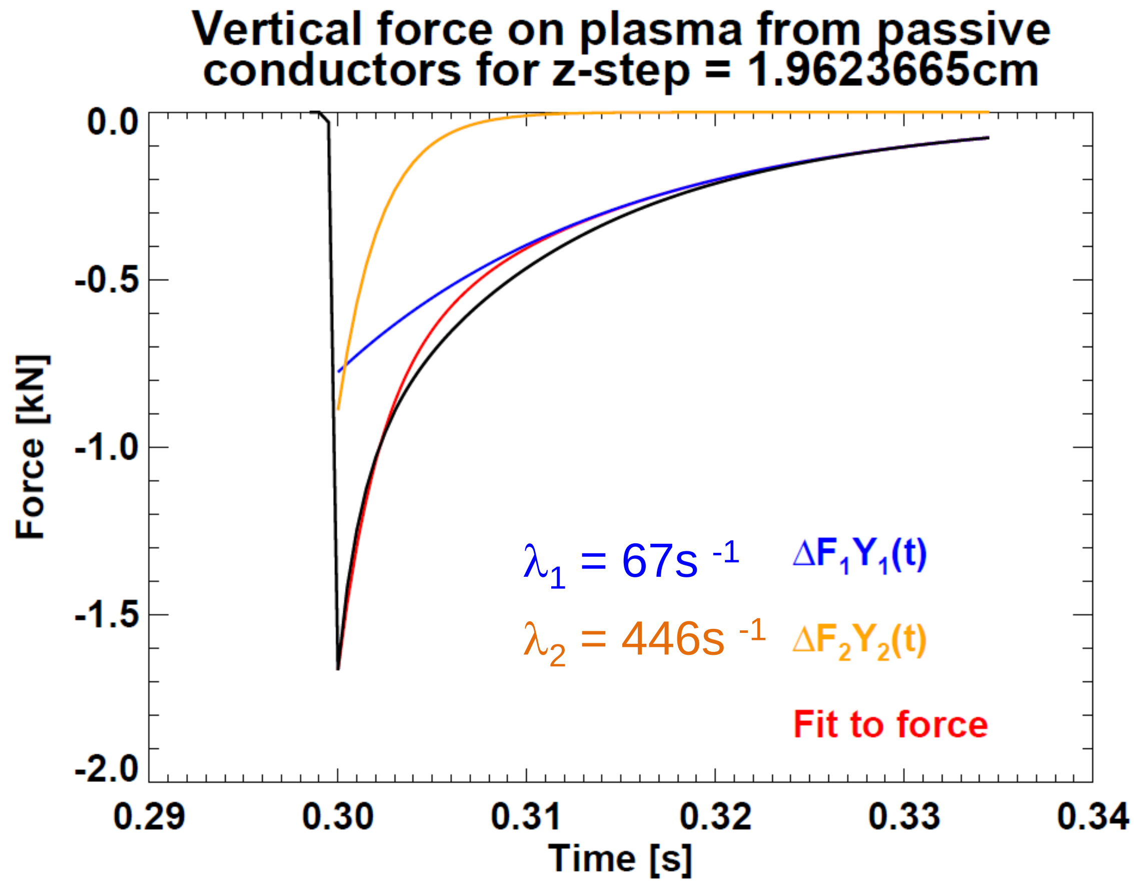
$$d + s_1\frac{\gamma}{\gamma+\lambda_1} + s_2\frac{\gamma}{\gamma+\lambda_2} = 0 \quad d \equiv \left.\frac{\partial F_z}{\partial z}\right|_{z_0} \quad s_1 \equiv \frac{\Delta F_1}{z_0} \quad s_2 \equiv \frac{\Delta F_2}{z_0} \qquad (9)$$

$$a \equiv s_1 + s_2 + d \qquad b \equiv d(\lambda_1+\lambda_2) + s_1\lambda_2 + s_2\lambda_1 \qquad c \equiv d\lambda_1\lambda_2 \qquad (10)$$

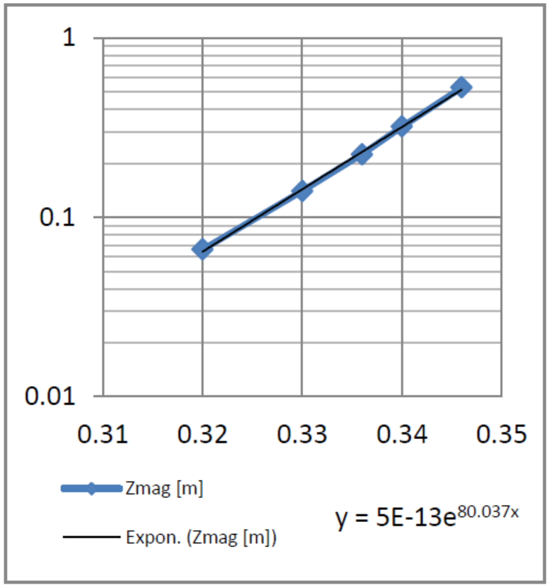
$$\Rightarrow \gamma = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Passive current force in reduced model



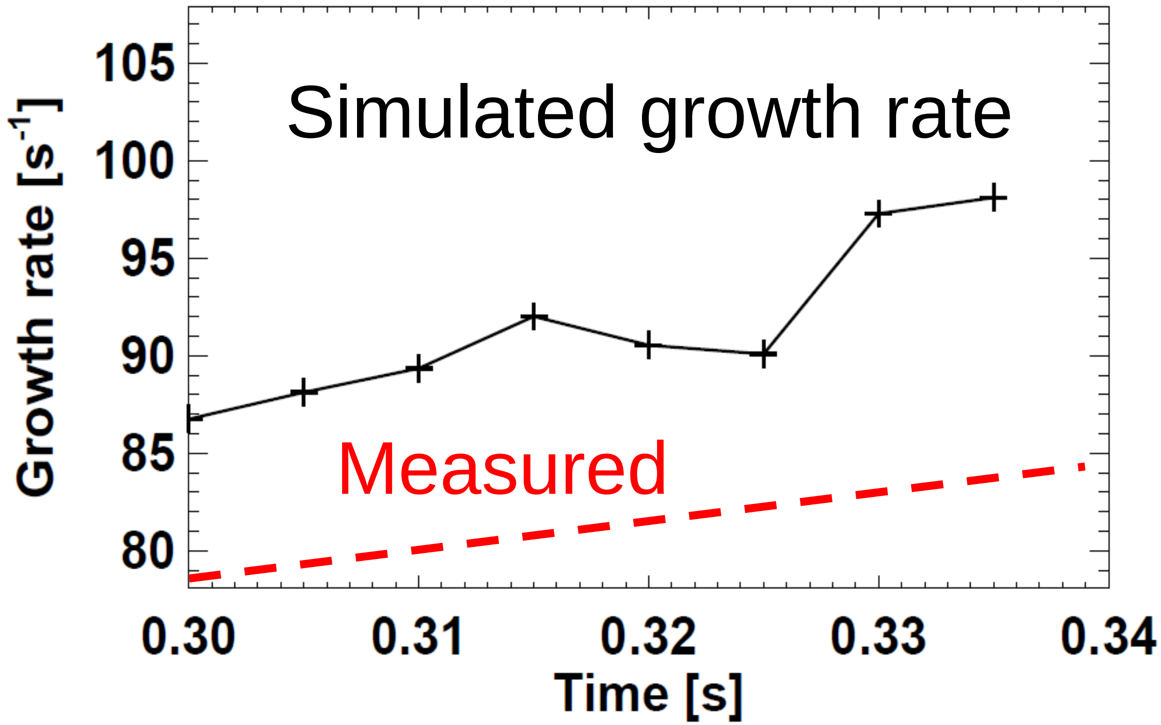


Simulated γ 10-15% higher than expt



127083
LRDFIT01

time [s]	Zmag [m]
0.306	0.022
0.31	0.03
0.32	0.066
0.33	0.14
0.336	0.224
0.34	0.32
0.346	0.529



Future: Need to assess variation in predicted growth rate vs. # of conducting wall eigenvalues retained

Model for vertical position dynamics

$$z_j = z(t = t_j) \qquad F_z(z_j) + \frac{\partial F_z}{\partial z}\Big|_{z_j} \Delta z_{j+1} + \sum_{k=1} Y_k(t_{j+1}) \Delta F_k + \chi_{j+1} = 0$$

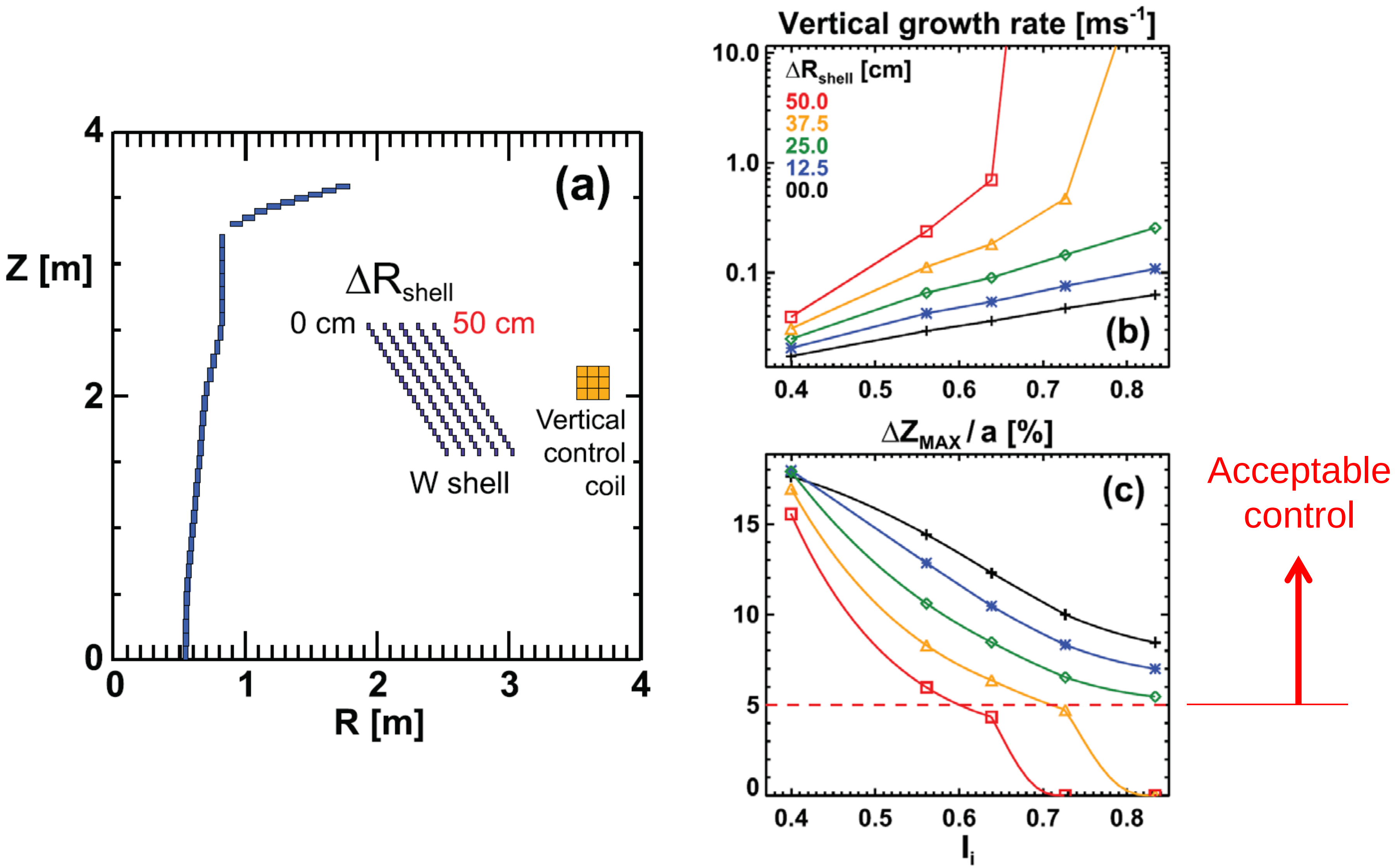
Depends linearly on Δz_{j+1}

➔ Solve for $\Delta z_{j+1}(\Delta z_j, \ldots)$

Force from control coils

- Assess ability to recover plasma from vertical offset:
  - Let plasma drift exponentially to vertical offset ΔZ
  - Apply step voltage to control coils
  - Determine if coil force can return plasma to mid-plane
  - Assess power vs. max offset ΔZ_{max} as metric for controllability

Example controllability calculation for FNSF



Possible future work / ideas

- Benchmark against NSTX-U (when we get some uncontrolled drift rate data)
- Correlate NSTX / NSTX-U controllability vs. open loop growth rate and/or stability index
- Complete / extend dynamical model for modular closed-loop control simulations
  - Optimize sensor positions, assess EFC/RWM coils for n=0
  - Implement n=0 stability calculator in PCS and/or TRANSP
  - Adjust elongation based on calculated marginal point?