

# M3D-C<sup>1</sup>-K

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NSTX-U Results Review

September 22, 2016

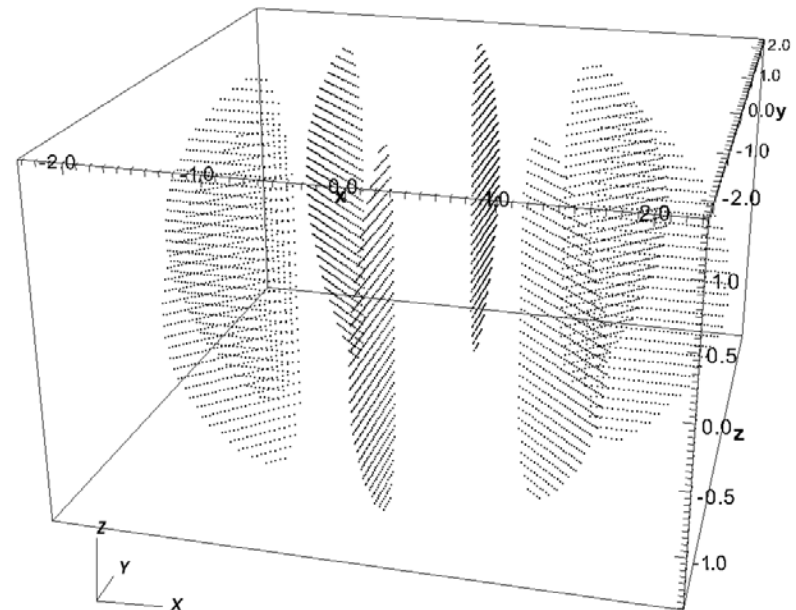
# Background & Motivation

- M3D-C<sup>1</sup> is a mature nonlinear extended MHD code that has superseded the old M3D code for most tokamak applications because of its
  - Efficient, high-accuracy 5<sup>th</sup>-order-polynomial finite-element field representation with C<sup>1</sup> continuity
  - Fully implicit time advance scheme
  - More accurate model of gyroviscous stress and Hall terms, etc.
  - Fast 2D-complex mode of operation for linear problems
  - Finite-thickness resistive wall capability and in-mesh coils
  - Clean, modern Fortran programming paradigm
- A key remaining step to achieving feature parity with M3D-K is the development of a hybrid kinetic capability for energetic ions.

# Particle loading

- Physical space initialization: uniform over  $(R, \varphi, z)$  cube with Jacobian to ensure uniformity over  $d^3x$ . Particles outside mesh rejected.
- Velocity space initialization: use Jacobian to initialize distribution uniformly on  $d^3v$ , with  $0 < |\mathbf{v}| < \sqrt{2E_{\max}/m}$ .
- Uniform particle weights.

*Sample spatial distribution  
over four-partition KSTAR  
mesh:  
 $5840 / 8192 = 32 \times 8 \times 32$   
particles deposited.*

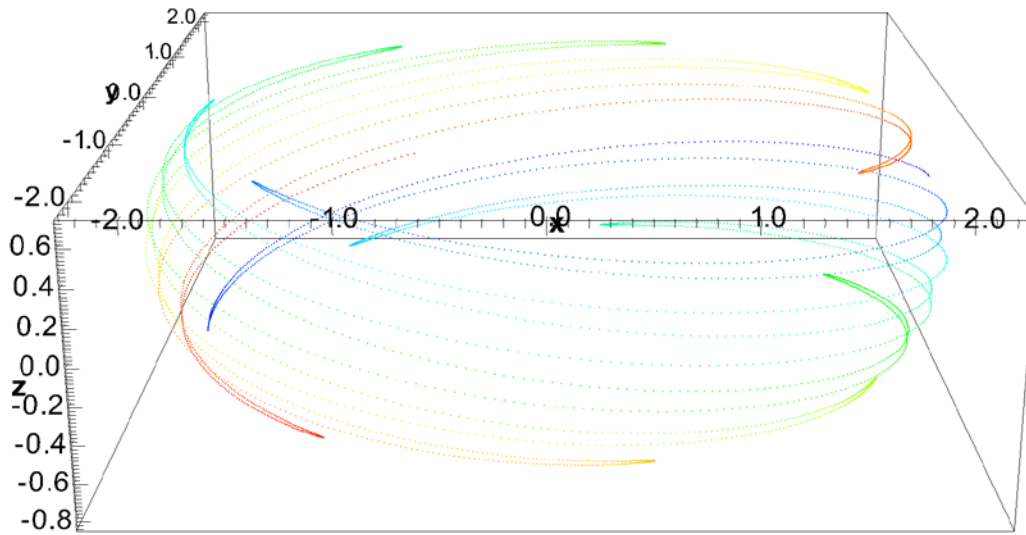
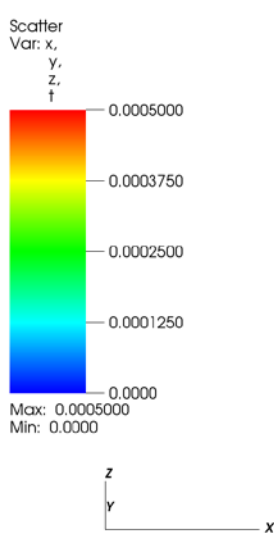


# Particle push

- Particles advance by a specified time increment between fluid steps, using given 2D (real or complex) or 3D fields, subcycling as necessary.
- Hierarchical organization of particles by element, element ensemble, OMP thread, and MPI/mesh partition allows good optimization.
- 4<sup>th</sup>- and 5<sup>th</sup>-order Runge-Kutta ODE integration have been implemented; both show good energy,  $P_\phi$  conservation over many time steps.

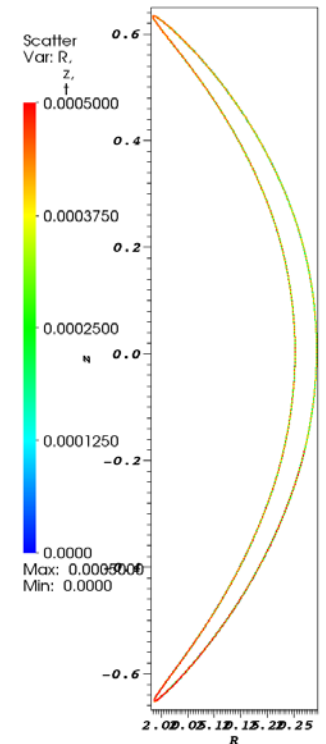
# Sample trapped orbit ( $\lambda_0 = 3\pi/5$ )

- 5000 steps, dt (drift-kinetic)= $10^{-7}$  s  $\approx$  5 gyroperiods.
- Initial KE=9.9995e+03 eV; final=9.9990e+03.
- Initial  $P_\varphi$ =-0.476633; final=-0.476630.



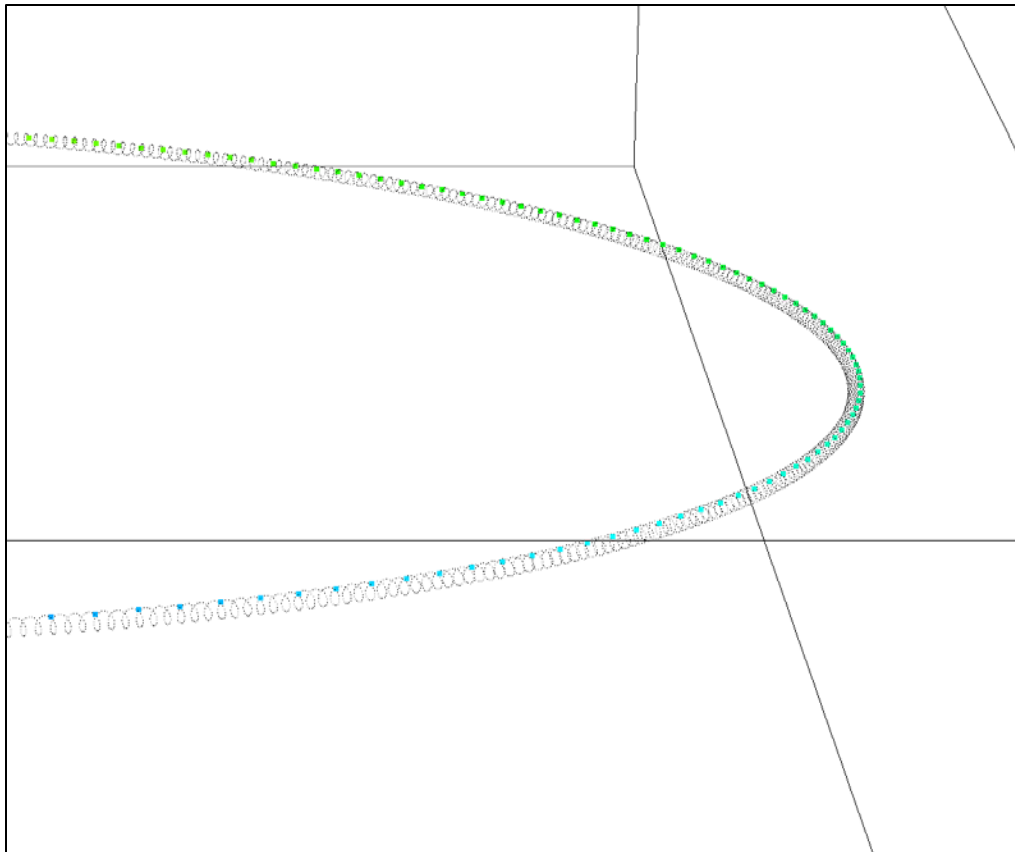
Colors indicate advancing time.

R-z plane projection



# Drift-kinetic/full-orbit comparison

- Full-orbit: 20,480 steps,  $dt=10^{-10}$  s  $\approx$  0.005 gyroperiods.



Detail

- KE conservation for full-orbit is good, but angular momentum conservation is relatively poor.
- A drift-kinetic step is about twice as fast as a full-orbit step, and can be around 1600x larger for comparable accuracy.

# Pressure deposition

- RHS vectors for  $p_{||}$ ,  $p_{\perp}$  computed by integrating over particle delta functions within each element.
- LHS vectors computed by mass matrix inversion for each component.
  - Very fast (time is independent of particle count).

# Fluid coupling

- In progress!
- Plan: pressure coupling, i.e.,

$$n \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi_{visc} - \nabla \cdot \Pi_{hot} \quad (1)$$

where

$$\Pi_{hot} \equiv (p_{\parallel} - p_{\perp}) \hat{\mathbf{b}} \hat{\mathbf{b}} + p_{\perp} \mathbf{I} \quad (2)$$

so that if  $\delta p \equiv p_{\parallel} - p_{\perp}$ , then

$$\nabla \cdot \Pi_{hot} = \left[ \nabla (\delta p) \cdot \hat{\mathbf{b}} \right] \hat{\mathbf{b}} + \delta p \nabla \cdot (\hat{\mathbf{b}} \hat{\mathbf{b}}) + \nabla p_{\perp} \quad (3)$$



# Fluid coupling, continued

- All terms in (3) are projected to the M3D-C<sup>1</sup> velocity representation with appropriate operators integrated by parts:

$$U : \iint d^2R R^2 \nabla_{\perp} v_i \times \nabla \varphi_{\bullet},$$

$$\omega : \iint d^2R v_i R^2 \nabla \varphi_{\bullet},$$

$$\chi : \iint d^2R R^{-2} \nabla_{\perp} v_i_{\bullet},$$

e.g.

$$R^2 \nabla \varphi_{\bullet} \left[ \nabla_{\bullet} (\delta p \hat{\mathbf{b}} \hat{\mathbf{b}}) \right] = \frac{F}{B} (\hat{\mathbf{b}}_{\bullet} \nabla \delta p) + R \delta p \hat{\varphi}_{\bullet} \nabla_{\bullet} (\hat{\mathbf{b}} \hat{\mathbf{b}})$$

# I/O & Diagnostics

- The `particle_test()` subroutine writes out the entire trajectory of a predetermined subset of particles, tracking KE and  $P_\phi$ .
- Subroutine `hdf5_write_particles()` uses parallel HDF5 to dump the entire particle distribution at a given time, including positions, velocities, and weights.
  - Utilities exist to extract position data from these to a text file, enabling comparisons and plotting with VisIt.
  - Utilities to visualize velocity distributions, pressure tensor components are now being developed.
- Checkpointing of particle distribution will be based on HDF5.

# Summary

- Particle initialization, full- $f$  push, pressure deposition, and I/O now working, tested and optimized for 2D complex version.
- Weight evolution, fluid coupling in progress.
- First linear validation tests on fishbone mode to be conducted in October.