High-frequency shear Alfvén instability driven by circulating energetic ions in NSTX

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It is shown that a number of features of an instability with the frequency comparable to the ion gyrofrequency observed in the National Spherical Torus Experiment [E. D. Fredrickson *et al.*, "Observation of hole-clump pair generation by global or compressional Alfvén eigenmodes," Contributed Papers, 33rd European Physical Society Conference on Plasma Physics, Rome, 2006, Europhysics Conference Abstracts (European Physical Society, Petit-Lancy, 2006), Report P5.058 (unpublished)] is consistent with the features of the Alfvén instability with large, about the inverse, Larmor radius of the energetic ions (ρ_b^{-1}) longitudinal wavenumbers. The conclusions drawn are based on an analysis of the resonant interaction of the energetic circulating ions and the waves, as well as on the calculation of the instability growth rate taking into account effects of the finite Larmor radius, ρ_b. *© 2006 American Institute of Physics*. [DOI: 10.1063/1.2402129]

I. INTRODUCTION

In a pioneer work on Alfvén instabilities driven by the energetic ions, an instability with the wavenumbers along and across the magnetic field ($k_\parallel$ and $k_\perp$) comparable to the inverse fast ion gyroradius, ρ_b^{-1}, was predicted.[1,2] Later, energetic-ion driven Alfvén instabilities were observed in many experiments on tokamaks, spherical tori, and stellarators; see, e.g., overviews in Refs. 3 and 4. However, in contrast to the instability considered in Ref. 1, all the reported shear Alfvén instabilities observed experimentally had very small longitudinal wavenumbers, $k_\parallel \ll \rho_b^{-1}$, and the frequencies which were much, by several orders, less than the ion gyrofrequency, ω_B. An exception is the instabilities of Alfvénic type with higher frequencies, which were observed in the National Spherical Torus Experiment (NSTX).[5,6] It is natural to assume that the mechanism of destabilization of these waves differs from that of the conventional Alfvén instabilities by involving the cyclotron wave-particle interaction. This possibility is investigated below for the instabilities reported in Ref. 6. An Alfvén instability with the frequency about the energetic ion gyrofrequency is considered taking into account both effects of the finite Larmor radius of the energetic ions and the particle drift motion induced by the toroidicity. A comparison is made of the features of this instability and the features of the experimentally observed waves.

The structure of the work is as follows. In Sec. II we carry out a qualitative analysis of a possible resonant wave-particle interaction in an NSTX experiment. In Sec. III an expression for the instability growth rate is derived; it is used to evaluate the magnitude of the instability growth rate in the experiment and analyze the role of the anisotropy and the spatial distribution of the energetic ions.

II. QUALITATIVE ANALYSIS OF THE DESTABILIZATION OF HIGH-FREQUENCY ALFVÉN WAVES

We consider the case when the energetic ions are produced by the tangential neutral beam injection in the direction of the magnetic field, so that the energetic ion population consists of circulating particles with

$$0 < v_\parallel < v_b, \tag{1}$$

where $v_\parallel$ is the longitudinal velocity of the energetic ions and v_b is their birth velocity. These particles can lead to Alfvén instabilities through the resonant interaction with the waves. Typically, the destabilized waves have small longitudinal wavenumbers, $k_\parallel \sim 1/qR$ (R is the major radius of the torus, q is the safety factor), and wave frequencies, ω, negligible in comparison with the ion cyclotron frequency, ω_B. However, we are interested in waves with ω comparable to ω_B. In this case $k_\parallel qR \gg 1$ and, therefore, the conventional sideband resonance, $\omega=[k_\parallel \pm 1/(qR)]v_\parallel$, responsible for instabilities with $k_\parallel qR \sim 1$, leads to the resonant velocity $v_\parallel^{\rm res} \approx \sigma_k v_A$, with $\sigma_k = k_\parallel/|k_\parallel|$ and v_A the Alfvén velocity (we assume without loss of generality that $\omega > 0$). However, the efficiency of this resonance is rather low in systems with weak magnetic field because of the low ratio v_A/v_b (in the NSTX experiment considered in this work $v_A/v_b < 1/3$). Moreover, this resonance does not work for the waves propagating in the counter direction with respect to the beam, which was the case in the NSTX experiment. Therefore, we have to assume that the instability is associated with a resonance involving the gyrofrequency. The corresponding resonance condition can be written as

$$\omega = k_s \langle v_\parallel \rangle + l \langle \omega_B \rangle, \tag{2}$$

where $k_s = k_\parallel + s/(qR)$, s and l are integers, $\langle \cdots \rangle = \oint dt(\cdots)/\tau_b$, and τ_b is the particle transit time. For Alfvén waves, the longitudinal wavenumber is $k_\parallel \approx \sigma_k \omega / v_A$. Taking this into account and assuming $v_{\parallel}^{\mathrm{res}}>v_A$, we obtain from Eqs. (1) and (2) that the considered injected ions and waves can be in the resonance provided that the wave frequency is not less than a certain magnitude, $\omega_{\min}$, taking place for $l=1$ and given by

$$\omega_{\min}=\frac{\omega_B I_s}{v_b/v_A-\sigma_k}, \tag{3}$$

where $I_s=1-|s|\rho_b/(qR)$, $\rho_b=v_b/\omega_B$. Note that Eq. (3) was obtained with the assumption that it is mainly low-s resonances with $|s|<qR/\rho_b$ that contribute to the destabilization of the waves. For $v_b\gg v_A$ and $I_s\sim 1$, it follows from Eq. (3) that the longitudinal wavenumber of the waves interacting resonantly with the particles is $k_{\parallel}\sim\rho_b^{-1}$.

Equation (3) shows that the minimum frequency of the counterpropagating waves ($k_{\parallel}<0$) is smaller than that of the copropagating waves. In particular, for the NSTX experiment with $v_b/v_A=3.5$, $R=100$ cm, $q=2$, $v_b=3\times10^8$ cm s^{-1}, $\omega_B=1.4\times10^7$ s^{-1}, we obtain $\omega_{\min}\approx 450$ kHz for $k_{\parallel}<0$ and $\omega_{\min}\approx 800$ kHz for $k_{\parallel}>0$. These magnitudes take place when $s=-1$.

When the plasma rotates toroidally in the beam direction and $\mathrm{sgn}(k_{\varphi})=\mathrm{sgn}(k_{\parallel})$ (φ is the toroidal angle), the Doppler effect decreases the frequency of counterpropagating waves and increases the frequency of copropagating waves in the laboratory frame ($\omega^{\mathrm{lab}}=\omega+\mathbf{k}\cdot\mathbf{u}$, with $\mathbf{u}$ the velocity of the plasma rotation). As a result, the difference between $\omega_{\min}$ of copropagating waves and $\omega_{\min}$ of counterpropagating waves increases. To evaluate this effect in the NSTX experiment, we take the toroidal mode number $n=5$ and $u=10^7$ cm$\cdot$s^{-1}. Then $k_{\varphi}=-n/R=-0.05$ cm^{-1} [we take perturbations in the form $\propto\exp(-i\omega t+im\vartheta-in\varphi)$, where ϑ is the poloidal angle, m and n are the poloidal and toroidal mode numbers, respectively], and the Doppler shift is 80 kHz. This leads to $\omega_{\min}^{\mathrm{lab}}=370$ kHz for the counterpropagating waves and $\omega_{\min}^{\mathrm{lab}}=880$ kHz for the copropagating waves.

This simple analysis enables us to conclude the following. First, the resonance (2) is consistent with the frequencies of the destabilized waves in the NSTX experiment (in particular, the frequencies at the beginning of the instability bursts were about 400 kHz in shot #114147 and 500 kHz in shot #114154). Second, only counterpropagating waves can be destabilized in the frequency range relevant to the experiment. Third, the resonance condition imposes restrictions on the instability mode numbers. They are $n>0$ and $m<nq$, which follows from two conditions, $k_{\varphi}=-n/R<0$ found experimentally, and $k_{\parallel}=(m-nq)/(qR)<0$ required for the frequency of ~400–500 kHz to satisfy the resonance condition. The obtained restriction on the poloidal mode numbers is valuable because it may help to select the mode among those which can be found by numerical simulation (the poloidal mode numbers are not available from experimental measurements). The restriction on n is also important in connection with the following.

Note that the resonance condition given by Eq. (2) describes the wave-particle interaction not only in the considered case of the shear Alfvén waves, but also when fast magnetoacoustic waves (FMW) are destabilized. However, probably, the FMW were not destabilized in the considered NSTX experiment. At least, Eq. (2) cannot explain why copropagating waves were not observed and why waves with the frequencies below several hundred kilohertz were not destabilized (whereas these facts are explained for the Alfvén waves). The matter is that in the case of the FMW ($\omega=kv_A$, where $k=\sqrt{k_{\parallel}^2+k_{\perp}^2}$), Eq. (2) can be satisfied for any sign of $k_{\parallel}$ and does not lead to a restriction on the possible frequencies, $\omega\geq\omega_{\min}$, like that given by Eq. (3).

The fulfillment of the resonance condition is not sufficient for the instability to arise. The factors which can lead to the destabilization of the waves are spatial inhomogeneity of the energetic ions and their velocity anisotropy. The spatial inhomogeneity drives the instability when $n\,dn_b/dr>0$ (n_b is the beam density). Because $n>0$, the beam particles with monotonically decreasing radial profile have a stabilizing (rather than destabilizing) influence on the waves in the case of counter-rotating modes. However, calculations show that in the NSTX case the beam radial distribution has an off-axis maximum, which suggests that the beam spatial inhomogeneity tends to destabilize modes localized in the core region. A more detailed analysis is required to clarify the role of the velocity anisotropy. This will be done in the next section.

III. CALCULATION OF THE INSTABILITY GROWTH RATE

In order to evaluate the instability growth rate (γ), we use the local approximation. Neglecting the wave damping caused by the bulk plasma, we can write

$$\frac{\gamma}{\omega}=-\frac{v_A^2}{2c^2}\epsilon_1, \tag{4}$$

where ϵ_1 is the fast ion dielectric permeability tensor component calculated in Ref. 7. We take into account that $F=F(r,\mathcal{E},\lambda,\sigma_v)$, where F is the unperturbed distribution function of the energetic ions, $\int d^3vF=n_b$, $\mathcal{E}$ is the particle energy, $\lambda=\mu B_0/\mathcal{E}$, μ is the particle magnetic moment, B_0 is the magnetic field at the magnetic axis, $\sigma_v=\mathrm{sgn}(v_{\parallel})$. Using the resonance condition given by Eq. (2), we calculate the integral over the energy in the expression for ϵ_1. Then we obtain

$$\frac{\gamma}{\omega}=\frac{\pi^2\omega_B^4}{k_{\perp}^2 n_i\omega^2}\sum_{l,s,\sigma_v}\frac{l^2}{|k_s|}\int\frac{d\lambda}{1-\lambda}\mathcal{E}J_l^2(\xi)J_s^2(\zeta)\hat{\Pi}F\Bigg|_{v^{\mathrm{res}}}, \tag{5}$$

where n_i is the bulk ion density, $v^{\mathrm{res}}=(\omega-l\omega_B)/(k_s\sqrt{1-\lambda})$ is the resonant velocity, $J_l(\xi)$ and $J_s(\zeta)$ are the Bessel functions, $\xi=k_{\perp}\rho\sqrt{\lambda}$, $\zeta=(q/\sqrt{1-\lambda})\sqrt{k_{\perp}^2\rho^2(1-0.5\lambda)^2/\kappa^2+l^2r^2/\rho^2}$, $\rho=v/\omega_B$, $k_{\perp}$ is the transverse wavenumber, κ is the plasma elongation,

$$\hat{\Pi}=\omega\frac{\partial}{\partial\mathcal{E}}+(l\omega_B-\lambda\omega)\frac{1}{\mathcal{E}}\frac{\partial}{\partial\lambda}+\frac{nq}{\omega_B M}\frac{1}{r}\frac{\partial}{\partial r}. \tag{6}$$

Equation (5) takes into account effects of finite transverse Larmor radius of the energetic ions ($\rho_{\perp}=\rho\sqrt{\lambda}$). Moreover, the growth rate is considerable only when $k_{\perp}\rho_{\perp}$ is not small. Note that the growth rates of shear Alfvén instabilities with the wavenumbers comparable to the reciprocal of the Larmor radius of the energetic ions were obtained in a number of works, in particular, in Refs. 1 and 5. However, the expressions for the growth rate in these works neglected the drive due to spatial inhomogeneity of the energetic ions and the effects of the particle drift motion.

Equation (5) can be used to calculate the growth rate numerically. We, however, restrict ourselves to an analytical evaluation of γ. With this purpose we approximate the distribution function of the energetic ions as follows:

$$F = n_b f_{\mathcal{E}} f_\lambda \ \eta(v_b - v)\eta(\sigma_v), \tag{7}$$

with $f_{\mathcal{E}}=C/\mathcal{E}^{3/2}$, $C^{-1}=M^{-3/2}\pi\sqrt{2}\Lambda\int d\lambda f_\lambda/\sqrt{1-\lambda}$ is a constant found from the equation $\int d^3 v f_{\mathcal{E}}=1$, $\Lambda=\ln v_b/v_c$ with $v_c\sim(M_i/M_e)^{1/3}$, $f_\lambda=f_\lambda(\lambda)$, $\int f_\lambda d\lambda=1$, $\eta(x)=\int_{-\infty}^{x}\delta(x)dx$. For the considered waves the first term in Eq. (6) can be neglected. The term associated with the velocity anisotropy [proportional to the λ derivative in Eq. (6)] can be transformed by integrating by parts. Assuming that particles with small λ dominate in the energetic-ion population, we neglect for simplicity the terms proportional to $\partial J_s(\zeta)/\partial\lambda$ and use the approximation $d\xi(v^{\mathrm{res}})/d\lambda=\xi^{\mathrm{res}}/(2\lambda)$. Then we obtain

$$\frac{\gamma}{\omega}=\frac{\pi}{\Lambda}\frac{n_b}{n_i}\frac{\omega_B^2}{\omega^2}\left(\int\frac{d\lambda f_\lambda}{\sqrt{1-\lambda}}\right)^{-1} \times\sum_{l,s}\frac{l^2}{|l-\omega/\omega_B|}\int\frac{d\lambda f_\lambda}{\sqrt{1-\lambda}}J_s^2(\zeta) \times\left[-\frac{l}{\xi}\frac{\partial J_l^2(\xi)}{\partial\xi}+J_l^2(\xi)\frac{nq}{k_\perp^2 r}\frac{\partial\ln n_b}{\partial r}\right]_{v^{\mathrm{res}}}. \tag{8}$$

In the case relevant to the NSTX experiment, only the $l=+1$ harmonic contribute (there are no resonant particles for other l). Therefore, the velocity anisotropy of the energetic ion distribution is a destabilizing factor only when the particle Larmor radius is sufficiently large to lead to a negative derivative of $J_l^2(\xi)$. One can see that this condition is satisfied even for a low-m instability when the mode is well localized. For $f_\lambda\propto\delta(\lambda-\lambda_0)$ the term associated with the anisotropy dominates, unless ξ is about a point where $dJ_l^2/d\xi=0$.

Let us evaluate the growth rate in the NSTX shot #114147. We take $n_b/n_i\sim 1/20$, $\omega_B/\omega=4.7$, $\zeta\sim 10$, $\xi\sim 2$, and $J_n^2(x)\sim(\pi x)^{-1}$. Then we obtain $\gamma/\omega\sim(\pi\zeta\xi^2)^{-1}$, which leads to $\gamma/\omega\sim 10^{-2}$ for $\xi\sim 2$ and $\zeta\sim 10$. Numerical calculations using Eq. (5) give a smaller growth rate because in reality the beam distribution function is not peaked at very small λ.

Thus, the driving part of the instability growth rate is rather small. Therefore, the perturbative approach is justified in spite of the fact that the beam pressure was comparable to the plasma pressure in the core region in the considered NSTX shot.

IV. SUMMARY AND CONCLUSIONS

Our analysis shows that the destabilized waves with the frequencies $\omega\sim\omega_B/5$ observed in NSTX in shots #114147 and #114154 during neutral beam injection can be identified as the high-frequency Alfvén instability with the $k_\parallel\rho_b\sim 1$. This instability arises due to the cyclotron resonant interaction of Alfvén waves and the beam particles. The following facts support this conclusion. First, the wave frequencies determined by the resonance condition are close to the observed frequencies. Second, our analysis predicts that only counterpropagating (with respect to the beam direction) waves can be destabilized, in agreement with the experiment. Third, the resonance condition leads to the toroidal mode number $n>0$, again in agreement with the experiment. In addition, it imposes a restriction on the poloidal mode number, $m<nq$. The considered instability arises mainly because of the velocity anisotropy of the injected ions, although the spatial inhomogeneity—the hollow radial distribution of the energetic ions—contributes to the destabilization of the waves with $n>0$. The instability growth rate (the driving part) is relatively small, which justifies the use of the perturbative approach in the analysis.

Note that the most striking feature of the modes described in Ref. 6 and considered in this work is the fact that they simultaneously chirp up and down in frequency. Such chirping must be due to a nonlinear process, such as that described in Ref. 8. The first stage in understanding this phenomenon consists in identifying the instability and calculating its linear growth rate. This is what is done in the present work. At the next stage, a nonlinear theory of Alfvén modes is to be developed, which should take into account the realistic structure of the mode and all three degrees of freedom of the energetic ions.

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