ECCD models with parallel momentum conservation, finite collisionality, and high T_e

by N.B. Marushchenko

with

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Contents

- Introduction
  - ECCD physics
  - Adjoint approach
- Models for calculation
  - collisionless limit for high T_e
    - momentum conservation & high-speed-limit
    - examples of application
  - finite collisionality
    - simplified analytical and heuristic models + example
    - preliminary results from NEO-2 + example
- Summary and outlook

Introduction: ECCD physics

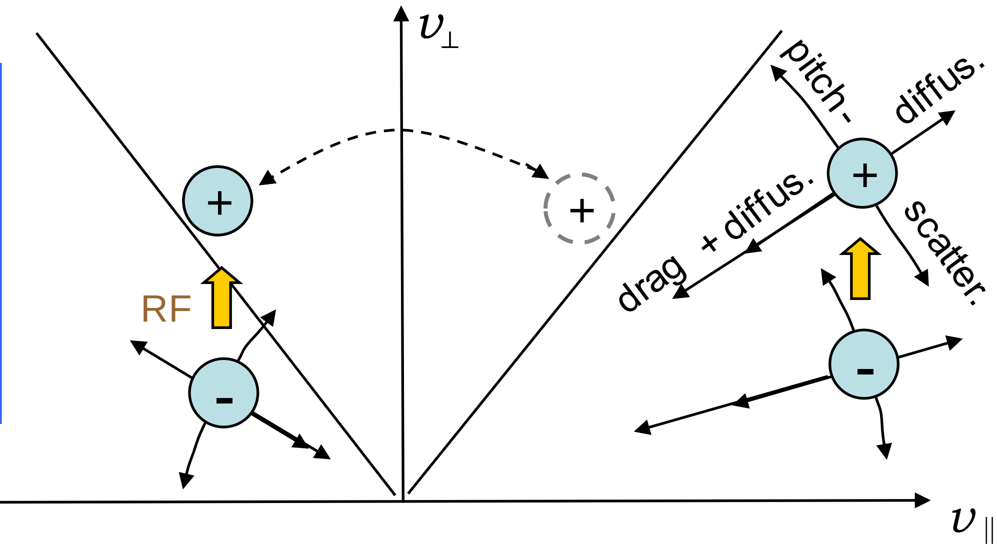
- Electron Cyclotron Current Drive

Ohkawa effect:
induced imbalance of passing particles
$n_e(v_\parallel < 0) < n_e(v_\parallel > 0)$

Fisch-Boozer effect:
induced asymmetry of collisionality

RF diffusion is balanced by

- e/i+e/e pitch-angle scattering
- e/e drag (along v)
- e/e diffusion (along v)



- different mechanisms are dominating for different plasma parameters: careful choice of the model is necessary
- conservation of parallel momentum in collisions of test-electrons with Maxwellian is mandatory: $\int d\mathbf{u}\, u_\parallel [C^{e/e}(\delta f_e, F_{eM}) + C^{e/e}(F_{eM}, \delta f_e)] = 0$

Introduction: adjoint approach

- current driven by RF source,

$$j_{\parallel} = -e\int d\mathbf{u}\ v_{\parallel}\ \delta f_e \quad \text{with} \quad \delta f_e = f_e - F_{eM} \quad \text{and} \quad u = \gamma v$$

can be calculated by solving DKE,

$$\frac{d\delta f_e}{dt} - C^{lin}(\delta f_e) = Q_{RF}(F_{eM}) \equiv -\frac{\partial}{\partial \mathbf{u}}\cdot \mathbf{\Gamma}_{RF} \longleftarrow$$

must *be in relativistic formulation* (both resonance and polarization)

- idea: exploiting the self-adjoint properties of $C^{lin}(\delta f_e)$ to express CD through the response function formally identical to the solution of (generalized) Spitzer-Härm problem (Hirshman, 1980; Antonsen & Chu, 1982; Taguchi, 1983)

  - If solution of the adjoint kinetic eq-n is known,

$$\frac{dg}{dt} + C^{lin}(g) = \nu_{e0}\frac{v_{\parallel}}{v_{th}} b\, F_{eM} \quad \text{with} \quad b = B/B_{\max},$$

then with $g(s;\ u,\xi) = \chi(s;\ u,\xi)F_{eM}(u)$ and $\xi = v_{\parallel}/v$

$$\boxed{\langle j_{\parallel}\rangle = \frac{ev_{th}}{\nu_{e0}}\cdot\frac{\langle b\rangle}{\langle b^2\rangle}\cdot\langle\int d\mathbf{u}\ \frac{\partial\chi}{\partial\mathbf{u}}\cdot\mathbf{\Gamma}_{RF}\rangle}$$

- Presently, adjoint approach is most common for ray- and beam-tracing

Adjoint approach: different models

- Drift kinetic equation (adjoint): $$\boxed{\frac{dg}{dt} + C^{lin}(g) = \nu_{e0}\frac{v_\parallel}{v_{th}} b\, F_{eM}}$$ (here, $\frac{d}{dt} = v_\parallel \mathbf{h}\cdot\nabla$)

$\sim 1/\tau_b$

$\sim 1/\tau_e$

in the analytic theory, two limits are possible,

☺ classical limit, $\tau_b >> \tau_e$, i.e. $\nu^* = \nu_e R/\iota v \rightarrow \infty$ (no trapped particles)

$$g(s;\, u,\xi) \rightarrow \xi g_1(u) \quad \Rightarrow \quad \boxed{C_1^{lin}(g_1) = (v\,/\,v_{th})\, F_{eM}} \quad \text{with} \quad g_1 = \tfrac{3}{2}\int_{-1}^{1} g\, d\xi$$

☺ collisionless limit, $\tau_b << \tau_e$, i.e. $\nu^* \rightarrow 0$ (trapped particles exist)

$$g(s;u,\xi) \rightarrow g(u,\lambda) \text{ with } \lambda=(1-\xi^2)/b \;\Rightarrow\; \boxed{\left\langle \frac{b}{|\xi|} C^{lin}(g)\right\rangle = \mathrm{sgn}(\xi)\,\nu_{e0}\frac{v}{v_{th}}\langle b^2\rangle F_{eM}}$$

ECCD w/o momentum conserv.: Fisch (1984), Cohen (1987); Lin-Liu (2003) ECCD
with momentum conserv.: Taguchi (1989); Karney (1989); Rome (1997), Marushchenko (2008)

😐 finite collisionality, $\tau_b \sim \tau_e$, i.e. $\nu^* \sim 1$ (barely trapped particles contribute)

- ✓ for tokamaks, 3D Fokker-Planck eqn. (Lin-Liu, 1999; Sauter, 1999)
- ✓ for stellarators, 4D Spitzer problem has to be solved:
  to date, no widely accepted approach developed (NEO-2 ?)

Collisionless (lmfp) model

Representing the linearized collision operator as

$$C^{lin}(g) \equiv C^{e/e}(g;F_{eM}) + \cancel{C^{e/e}(F_{eM},g)} + C^{e/i}(g;F_{iM}) \approx$$

$$\left[\cancel{\frac{\xi}{u^2}\frac{\partial}{\partial u}u^2 D_{uu}^{e/e}\frac{\partial \chi_1}{\partial u}} \underbrace{- \xi F_u^{e/e}\frac{\partial \chi_1}{\partial u} + (\nu_{ei}+\nu_{ee})L(\chi)}_{\text{high-speed limit (hsl)}} + \cancel{\xi I^{e/e}(\chi_1)} \right] F_{eM},$$

response (Green's) function, $\chi(u,\lambda)$, can be written as (Taguchi, 1989; Lin-Liu, 2003)

$$\chi(u,\lambda) = -\,\mathrm{sgn}(u_{\|})H(\lambda)K(u),$$

$$H(\lambda) = \frac{\langle b^2\rangle}{2f_c}\Theta(1-\lambda)\int_{\lambda}^{1}\frac{d\lambda}{\langle\sqrt{1-\lambda b}\rangle} \quad \text{with} \quad f_c = 1 - f_{tr} = \frac{3}{4}\langle b^2\rangle\int_0^1 \frac{\lambda\, d\lambda}{\langle\sqrt{1-\lambda b}\rangle},$$

$$K(u) = \frac{3}{2}\int_0^1 d\lambda\ \chi(u,\lambda),$$

simplest model of Cohen (1987):
hsl + magnetic square well (CURBA)

Spitzer function, $K(u)$, is solution of integro-differential eq-n (non-relat.: Taguchi, 1989)

$$\boxed{\frac{1}{F_{eM}}C_1^e(F_{eM}K) - \frac{f_{tr}}{f_c}\left[\nu_{ee}(u) + \nu_{ei}(u)\right]K = \nu_{e0}\frac{u}{\gamma v_{th}}}$$

weak.relat: Marushchenko, 2008
(TRAVIS code)

parallel momentum conservation (pmc)

1st Legendre harmonic

High-speed limit: validity range

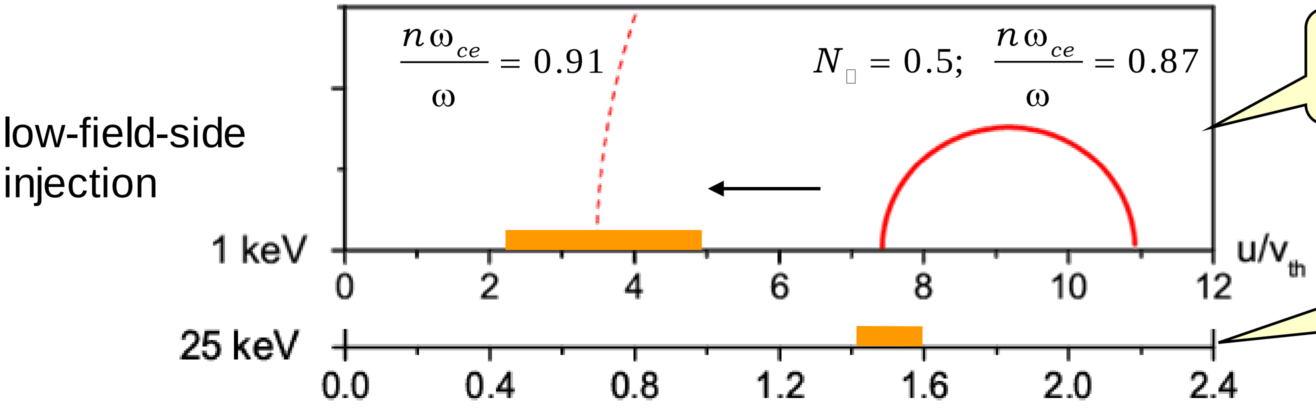
- In *hsl* approach, where $v >> v_{\mathrm{th}}$, Spitzer problem can be solved analytically (Fisch, 1984; Cohen, 1987; Lin-Liu, 2003):

$$\nu_e(u) \approx \nu_{e0}\frac{\gamma v_{th}^3}{u^3}(1+Z_{eff}) \quad \text{and} \quad F_u^{e/e} \approx \nu_{e0}\frac{\gamma^2 v_{th}^2}{u^2}$$

Fisch-Taguchi ($c \to \infty$)

$$K^{hsl}(u) = \frac{1}{v_{th}^4}\left(\frac{\gamma+1}{\gamma-1}\right)^{\rho/2}\int_0^u du'\left(\frac{u'}{\gamma'}\right)^3\left(\frac{\gamma'-1}{\gamma'+1}\right)^{\rho/2} \to \frac{f_c\cdot(u/v_{th})^4}{Z_{eff}+1+4f_c} \quad \text{with} \quad \rho = \frac{Z_{eff}+1}{f_c}$$

- *hsl* approach may be sufficient for plasmas with low and moderate T_e

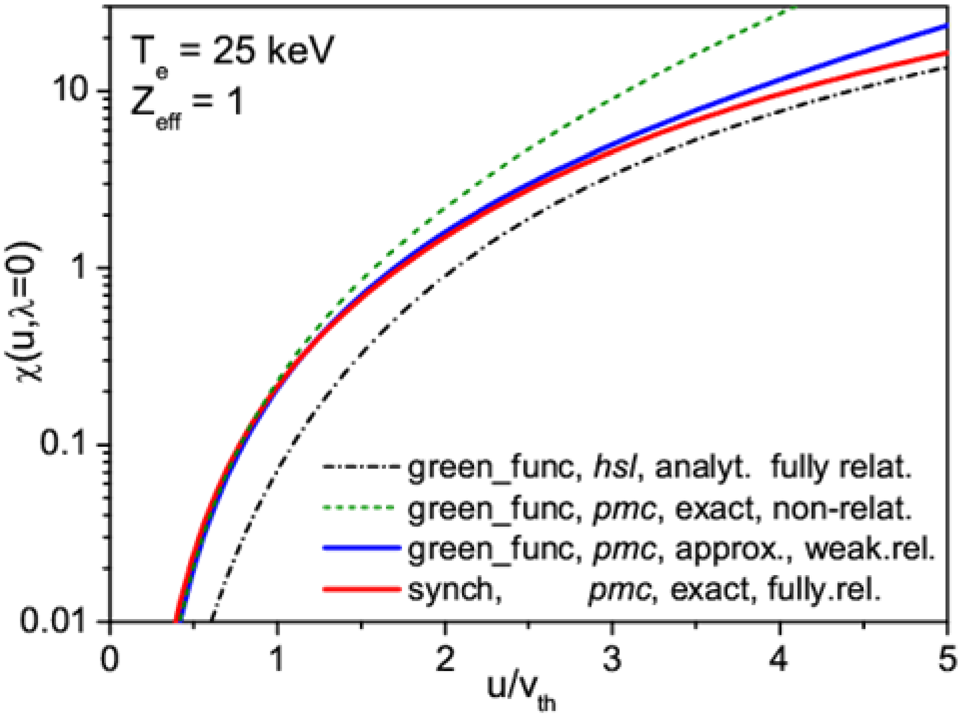


hsl marginally applicable

hsl not applicable

- for the modern devices with hot plasmas – *hsl* surely not sufficient

Spitzer function (lmfp-regime): comparison of models



few models considered:

- non-relativistic *pmc*-solution (Taguchi appr., exact)
- weakly relativ. *pmc*-solution (Taguchi appr.+ polinom. fit): green_func - solver
- fully relativistic *pmc*-solution (midplane Legendre harm. series): SYNCH - solver
- fully relativistic *hsl*-model (Taguchi appr., analyt. solution)

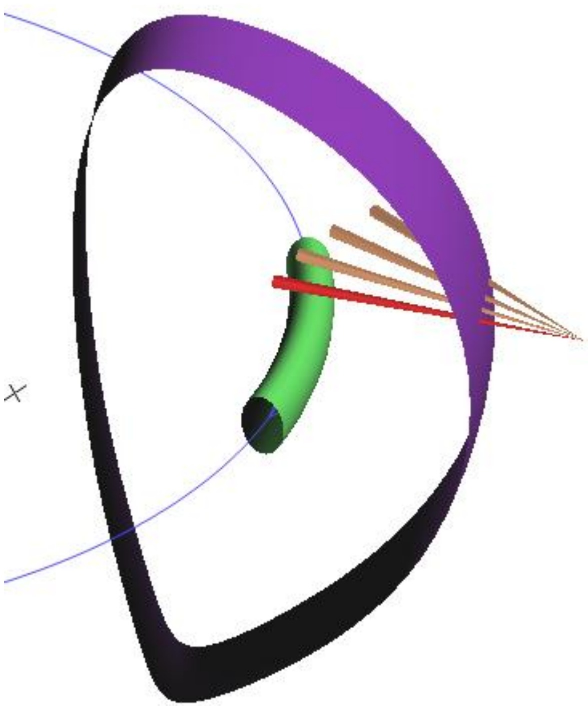
Numerical results for the Spitzer function:

- for $mc^2/T_e \approx 20$ (typical for ITER), the relativistic effects are significant
- *pmc*-solutions significantly larger than *hsl*-solution (but converge for $u >> v_{\mathrm{th}}$)
- both approximate weakly relativistic and exact fully relativistic *pmc*-solutions coincide well apart from very fast electrons

Examples: ITER, equatorial launcher

- Scenario 2, O1-mode: $n_e(0) = 10^{20}$ m^{-3}, $T_e(0) = 25$ keV, $Z_{eff}(0) = 1.7$

  Plasma parameters guarantee
  i) relativistic effects and **ii)** validity of collisionless limit



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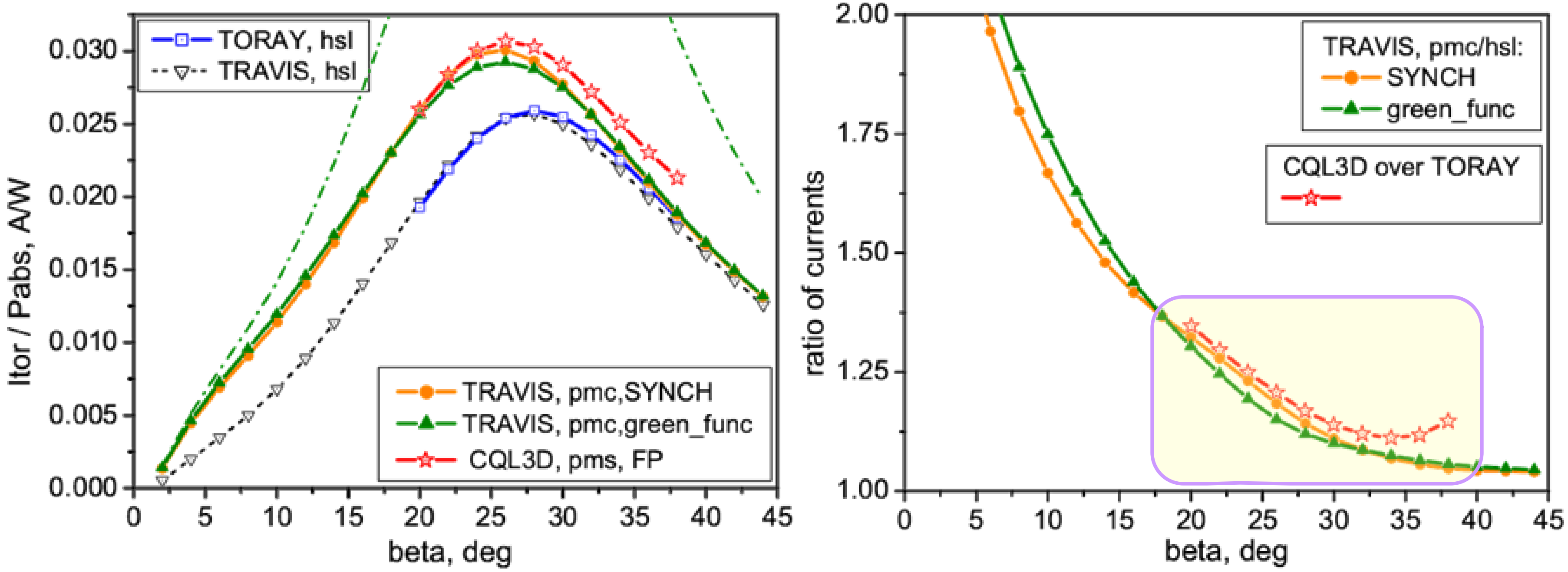
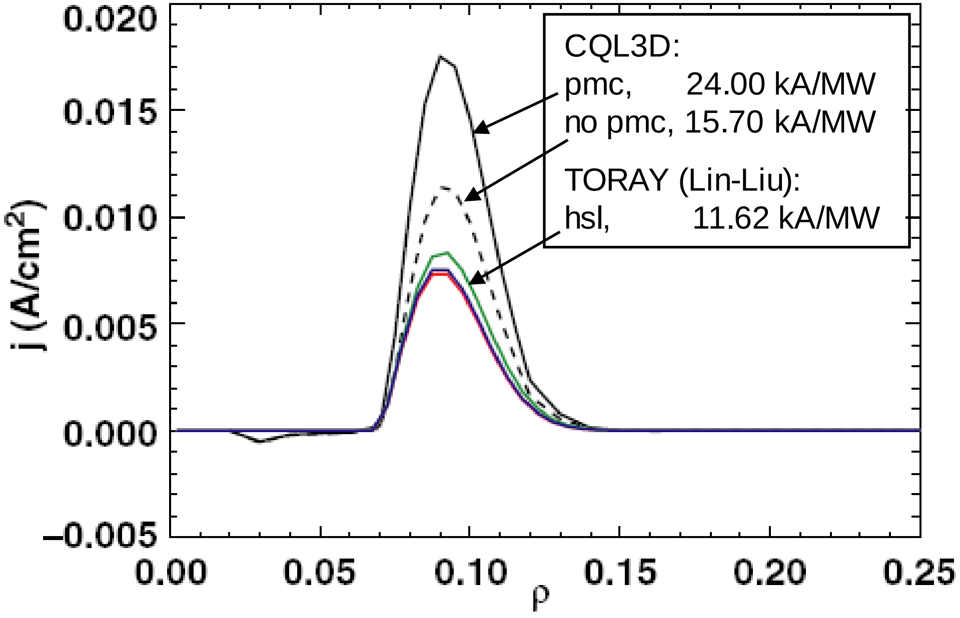
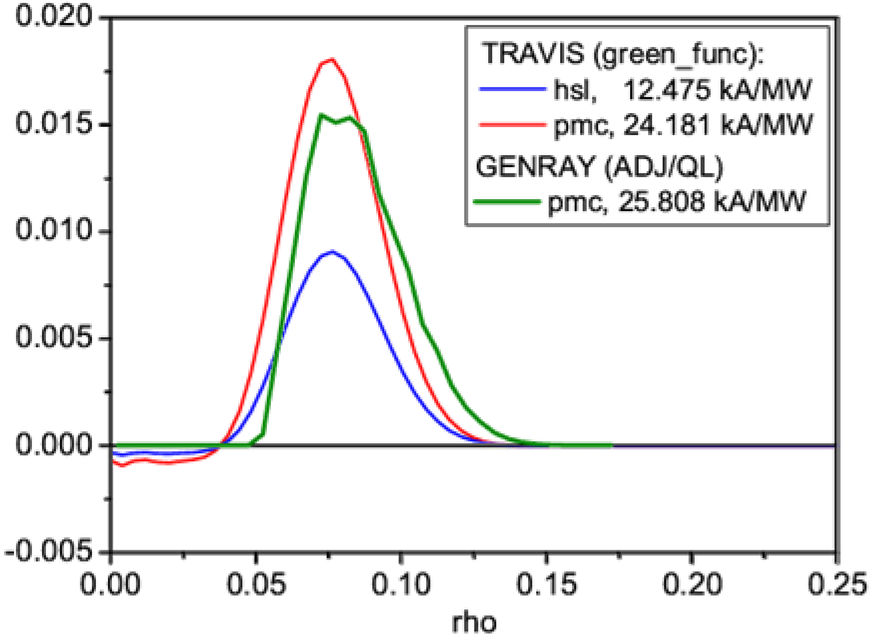


- ECCD calculated by **TORAY** and **TRAVIS** in *hsl* approach coincide well
- both **TRAVIS**-*pmc* and **CQL3D** (FP-*pmc*) calculations are in a good agreement
- in the main range of interest, *hsl* significantly underestimates ECCD (10% – 30%)

Examples: ITER, Prater's Gedankenexperiment (NF 2008)

- increased *B* (5.63 T) → absorption near axis, small fraction of trapped particles
- small angle (12.3°) → bulk electrons responsible for ECCD, *hsl* not valid

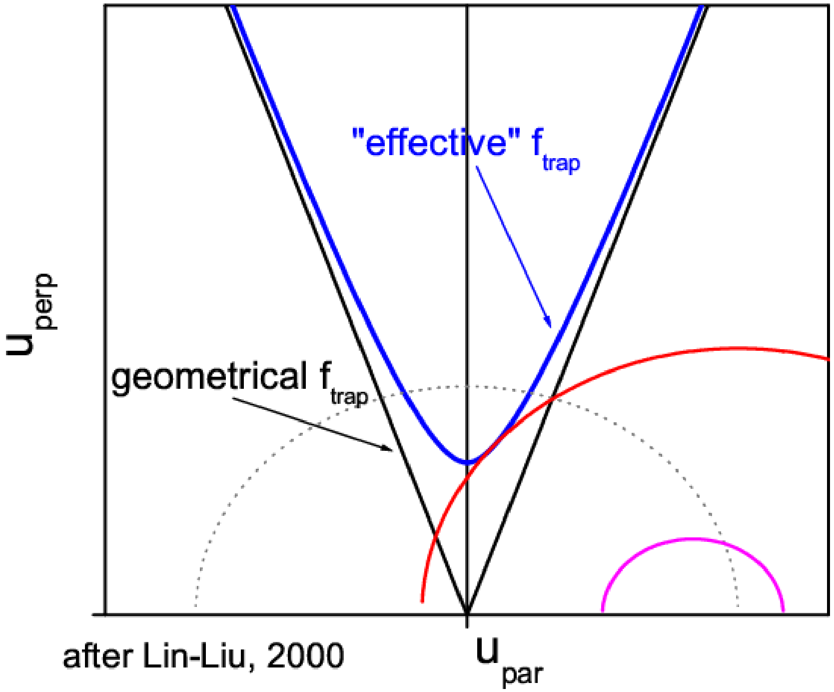




- ECCD calculated in *hsl* approach by TRAVIS and other codes coincide well
- both TRAVIS and GENRAY calculations with *pmc*-models reproduce CQL3D Fokker-Planck results with high accuracy (few %)

Finite collisionality: qualitative picture

- intermediate regime: barely trapped electrons affect on the current drive



after Lin-Liu, 2000

For ECCD, two factors must be considered

- ✓ contribution of barely trapped electrons:

  for tokamaks, $\Delta j_{||} \approx (\varepsilon\nu^*)^{1/2} j_c$

  (Chan & Chiu, 1981; Lin-Liu, 1999)

- ✓ reduction of drag over trapped fraction

  $$f_{tr}^{eff}(u) = f_{tr}^{geom} - f_{tr}^{b}(u)$$

  (Maaßberg, 2008)

- if Ohkawa effect appears, simplified analytical model of Lin-Liu can be applied
- if no Ohkawa effect, the concept of “effective” trapped particle fraction can be applied: momentum conservation is important

- as general (but “expensive”) alternative, NEO-2 code can be used

Finite collisionality: simplified analytical model (Lin-Liu, 1999)

- tokamak, $\varepsilon = r/R \ll 1$: only pitch scattering is accounted, $C^{lin}(g) \approx \nu_e \hat{L} g$

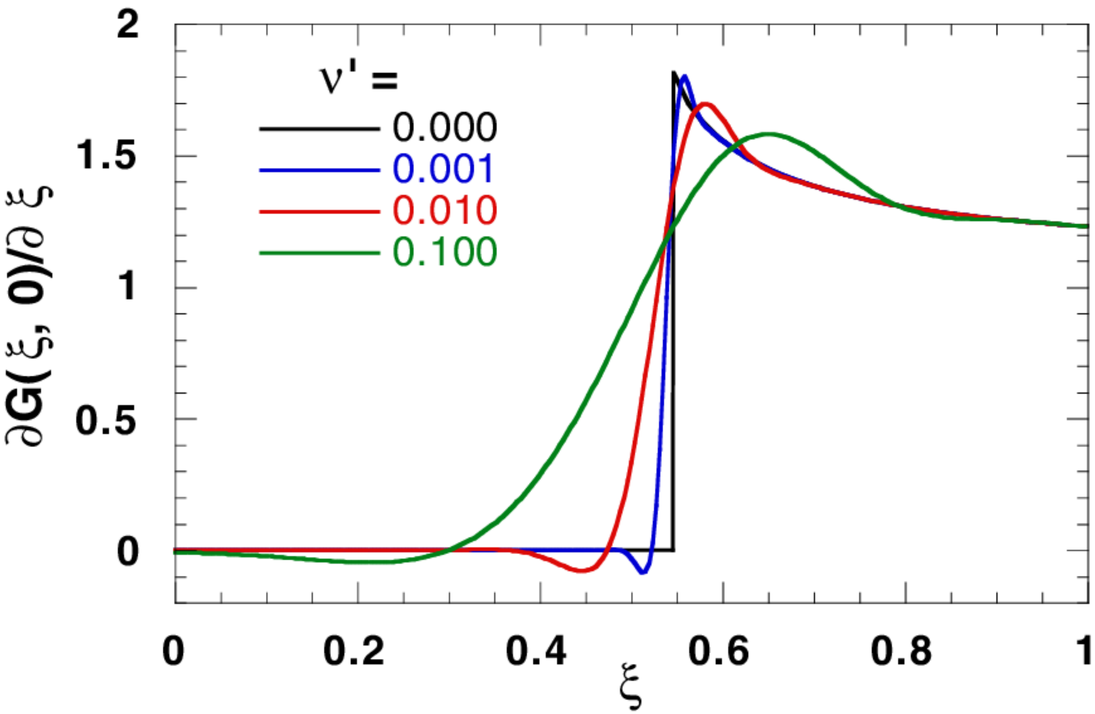
Spitzer function: classical + banana solutions

$$g(\theta,u,\xi) = -\mathrm{sgn}(\xi)\left[H_c(\theta,\xi) + H_{tr}(\theta,\xi)\right] K(u)$$

banana (barely trapped) solution satisfies to

$$\boxed{\mathrm{sgn}(\xi)\frac{\partial H_{tr}}{\partial \theta} + \frac{\nu_e}{2}\frac{\partial}{\partial \lambda}(\lambda\,|\xi|)\frac{\partial H_{tr}}{\partial \lambda} = \alpha(\xi,\theta) \equiv -\frac{\partial H_c}{\partial \theta}}$$

with $H_c = |\xi|\, b(\theta)$



- the model gives qualitative picture
- valid only for tokamaks with large aspect ratio and thin boundary layer (i.e. if $\Delta j_{||} \ll j_c$)

Finite collisionality: heuristic model (Maaßberg, 2008)

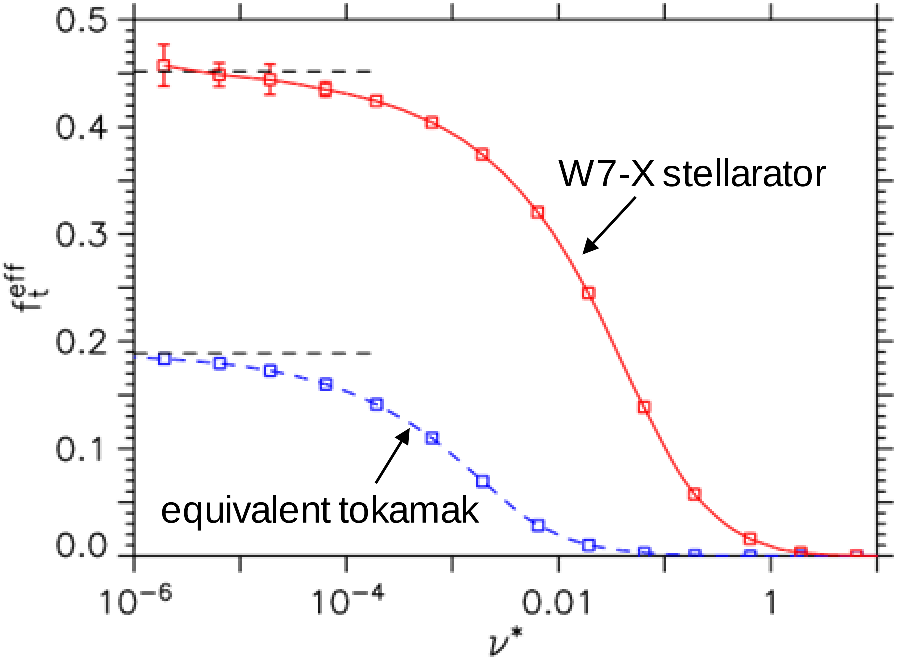
- for arbitrary ν*, “effective trapped fraction” can be estimated (heuristically) from the mono-energetic conductivity $D_{33}(\nu^*)$ (DKES solver)

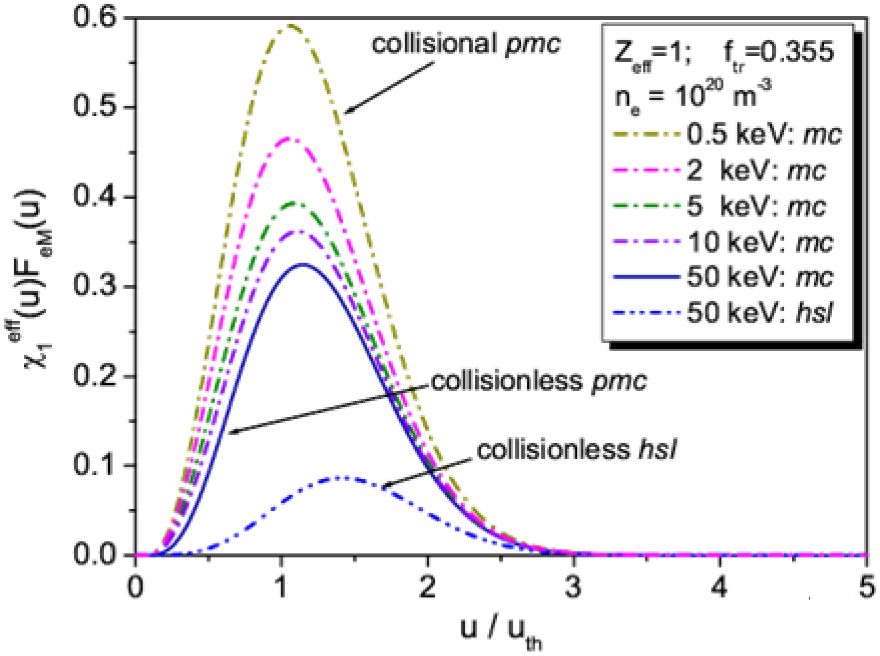
$$\boxed{f_{tr}^{eff}(\nu^*) \rightarrow f_{tr}^{eff}(u) = 1 - f_c^{eff}(u) = 1 - 3\frac{B_0^2}{\langle B^2 \rangle}\cdot\frac{\nu_e^*(u)}{(u/\upsilon_{th})^2}D_{33}(\nu_e^*(u))}$$

“generalized” Spitzer problem for the *global* collisional response:

$$\boxed{\frac{1}{F_{eM}}C_1^e(F_{eM}K^{eff}) - \frac{f_{tr}^{eff}(u)}{f_c^{eff}(u)}\left[\nu_{ee}(u) + \nu_{ei}(u)\right]K^{eff} = \nu_{e0}\frac{u}{\gamma\upsilon_{th}}}$$

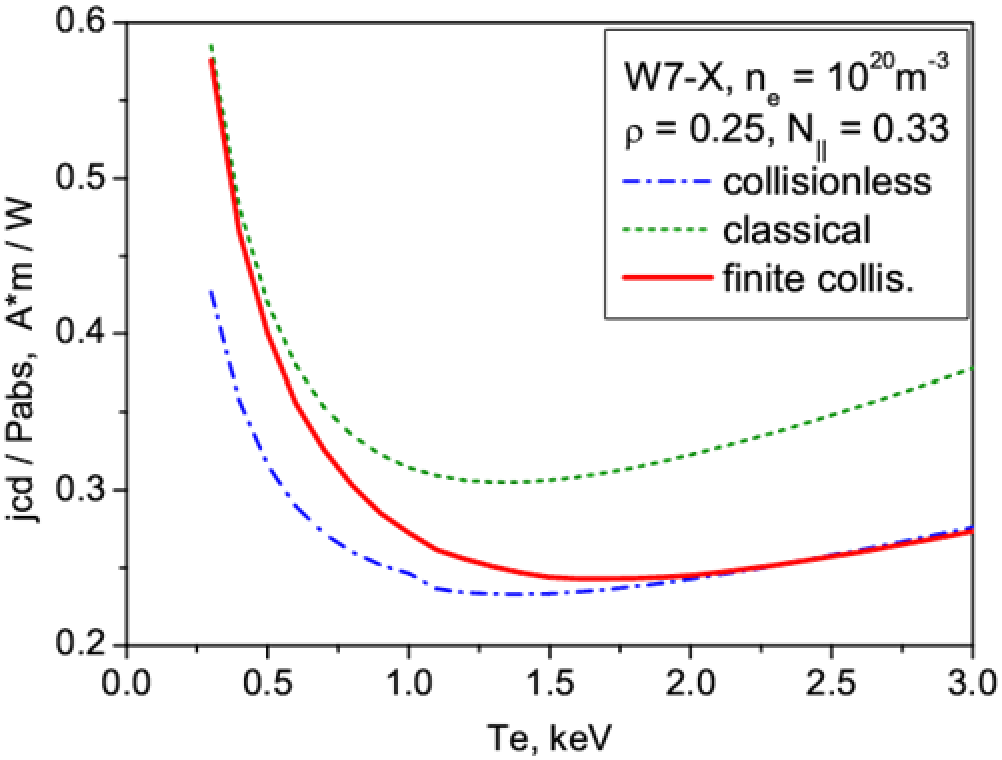
(with *pmc*)





Finite collisionality: heuristic model (Maaßberg, 2008)

- application for ECCD (with TRAVIS)
  - ✓ W7X "standard" config.: T_e - scan = collisionality scan
  - ✓ X2-mode: fixed direction, fixed spatial point ($B = B_{max}$) → no Ohkawa effect
  - ✓ concept of "effective" trapped particles fraction applicable



- ☺ covers well transition from classical to collisionless regime
- ☺ applicable for both tokamaks and stellarators
- 😐 critical only for scenarios with well pronounced Ohkawa effect

- both simplified analytical and advanced numerical models are necessary

Finite collisionality: NEO-2 code

- ❖ NEO-2 solves generalized Spitzer problem by field-line-tracing technique

$$v_{\parallel}\mathbf{h}\cdot\nabla(\chi F_{eM}) - C^{lin}(\chi F_{eM}) = \frac{1}{l_c} v_{\parallel} F_{eM}$$

In kinetic transport theory, series in associated Laguerre polynomials of order 3/2 series is applied

$$\chi(\mathbf{r},u,\lambda) = \sum_{m=0}^{M} \chi_m(\mathbf{r},\lambda) L^{3/2}(u^2/v_{th}^2)$$

full version of NEO-2 only for tokamaks is tested (4D → system of 2D)

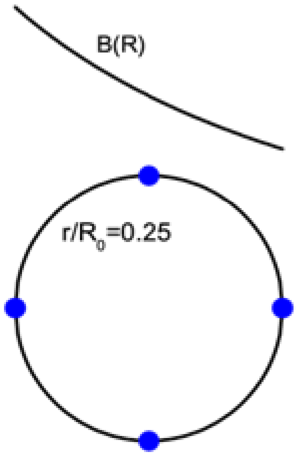
$$\chi_m(\mathbf{r},\lambda) \equiv \chi_m(\theta,\varphi,\lambda) \rightarrow \chi_m(\theta,\lambda)$$

(for stellarators, the mono-energetic version was successfully benchmarked)

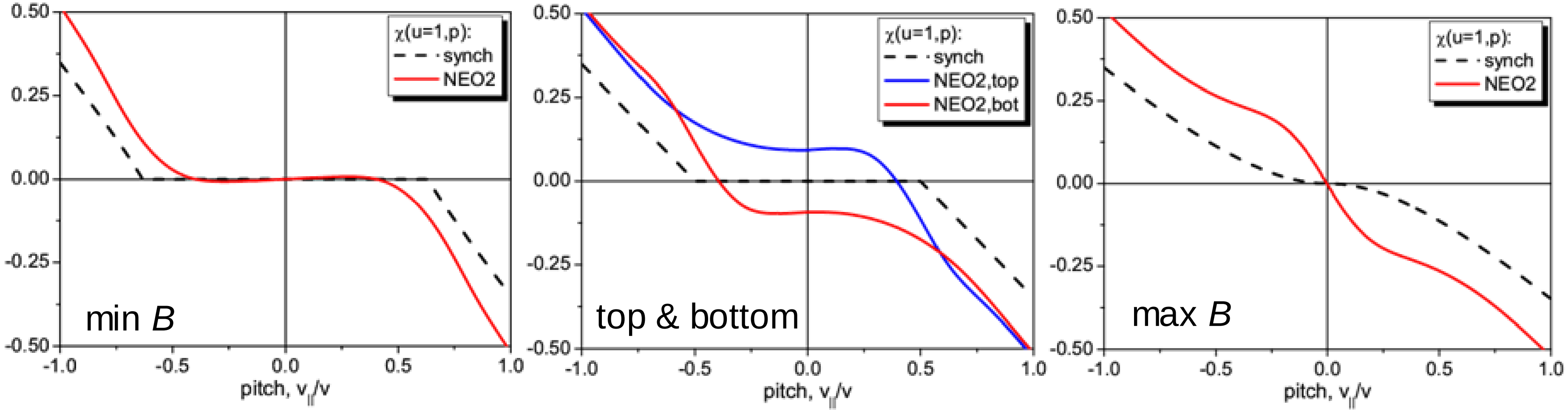
- ❖ Corresponding regimes:
  Pfirsch-Schlüter; plateau; banana regime; deep banana regime
- ❖ Good convergence to asymptotic limits
- ❖ Interesting results due to combination of
  - ✓ magnetic mirroring force
  - ✓ collisional detrapping

Finite collisionality: NEO-2 code, examples

- circular tokamak with $\varepsilon = r / R = 0.25$
- plasma parameters:
  $n_e = 6.65\times10^{19}$ m^{-3}, $T_e = 1$ keV, $Z_{\mathrm{eff}} = 1$
- four points: max & min of *B*, top & bottom



Local pitch-dependence of the generalized Spitzer function, calculated by NEO-2, $\chi(u = v_{\mathrm{th}}, \xi)$:



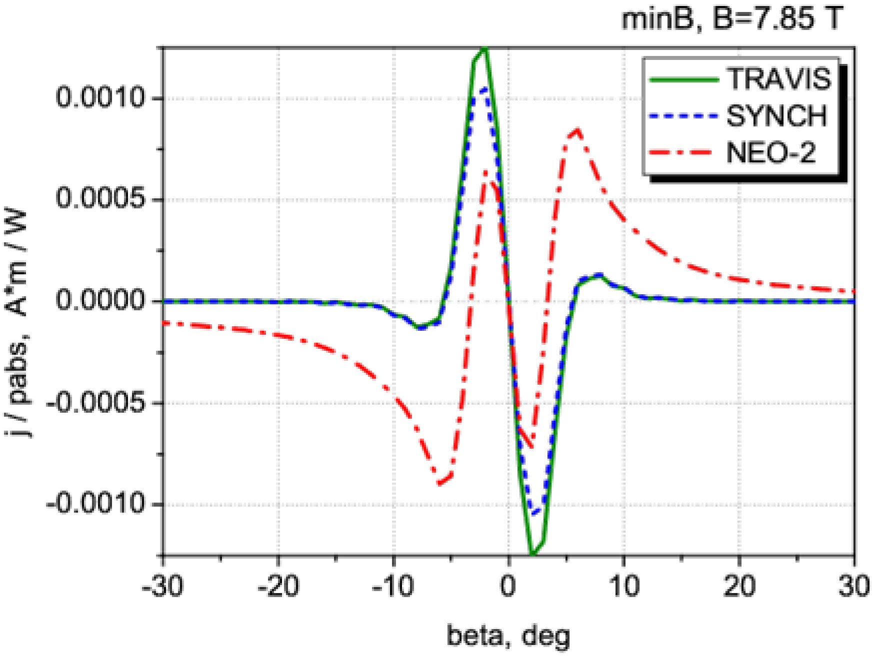
❖ top / bottom asymmetry: disappears for both classical and collisionless limits

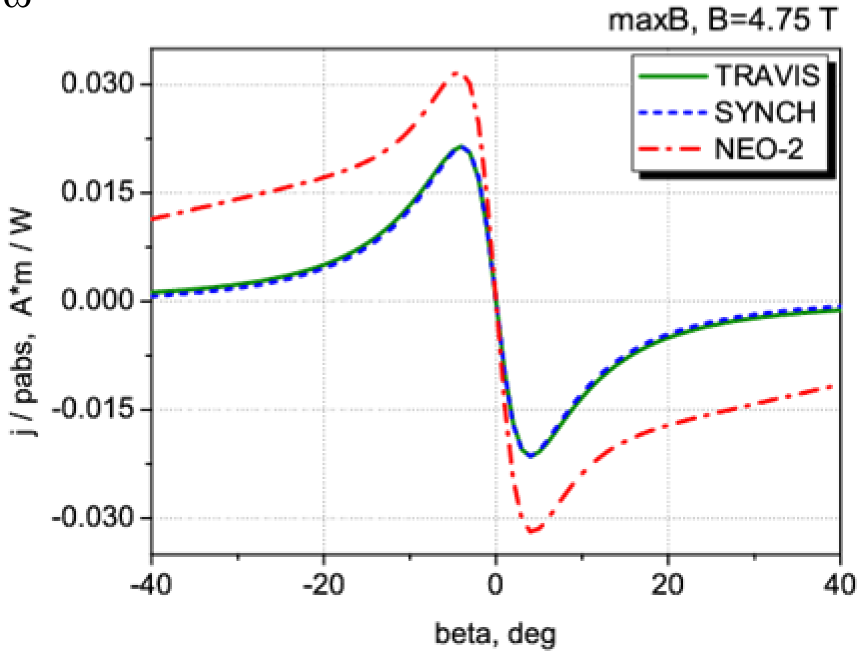
  ✓ physical reason: collisional detrapping of barely trapped electrons

  ✓ predicted by Helander & Catto (2001) through the simplified analyt. model

ECCD with finite collisionality: TRAVIS + NEO-2

- circular tokamak with $\varepsilon = r / R = 0.25$
- plasma parameters: $n_e = 6.65\times10^{19}\ \mathrm{m}^{-3}$, $T_e = 1$ keV, $Z_{\mathrm{eff}} = 1$
- X2-mode: toroidal angle scan for *minB* & *maxB* points
- *B* chosen to have Ohkawa effect: $\dfrac{n\,\omega_{ce}}{\omega} > 1$





- *minB*: pronounced interplay of collisional effects with the Ohkawa effect
- *maxB*: increased integral contribution of the barely trapped particles (due to parallel momentum conservation)

Conclusions and outlook

- Physics of ECCD and the relevant approaches for calculations of efficiency are analyzed
- The dominating role of parallel momentum conservation in the like-particle collisions is well illuminated (*hsl*-model is not sufficiently accurate)
- Numerical models with parallel momentum conservation developed for high temperature (relativistic) plasmas are already implemented in TRAVIS and GENRAY codes
- Ray-tracing calculations for ITER plasma with new solvers benchmarked against the Fokker-Planck code CQL3D: agreement is satisfactory
- Recently developed fast and accurate solvers are well benchmarked and can be recommended for implementation in all beam- & ray-tracing codes

Conclusions and outlook

- The role of collisional effects is demonstrated.
  It's of special interest for high density and moderate temperature plasmas in stellarators
- For some scenarios, simplified analytical and heuristic models can be well applicable. Work in this direction is very desirable
- Further development of the NEO-2 code promises the powerful tool for accurate study of new physics and benchmarking the simplified models

Thanks for your attention