An Analytic Fit to the TRANSP Energetic Particle Distribution Function for NSTX

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Jack Berkery

Department of Applied Physics, Columbia University, New York, NY, USA

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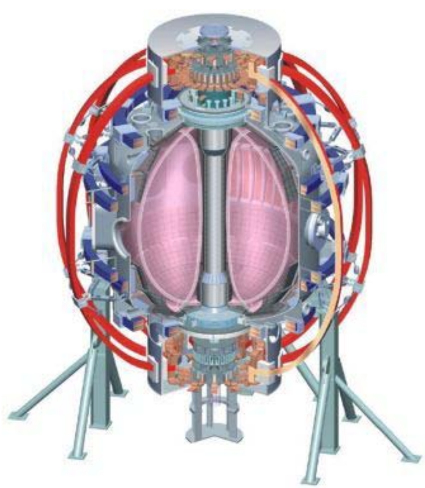

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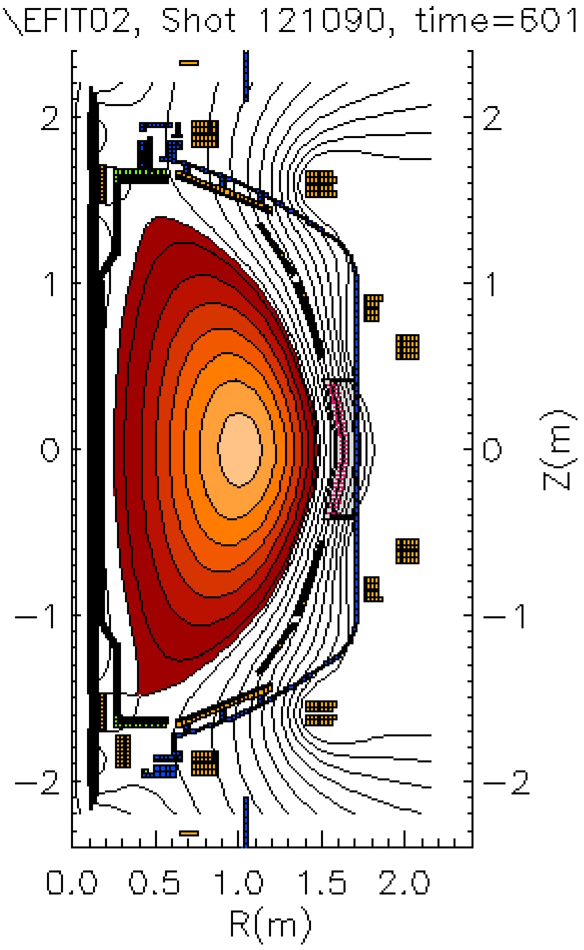
A general expression for δW_K depends on d*f*/dε and d*f*/dΨ

Kinetic stabilization of the resistive wall mode is calculated with the MISK code, using:

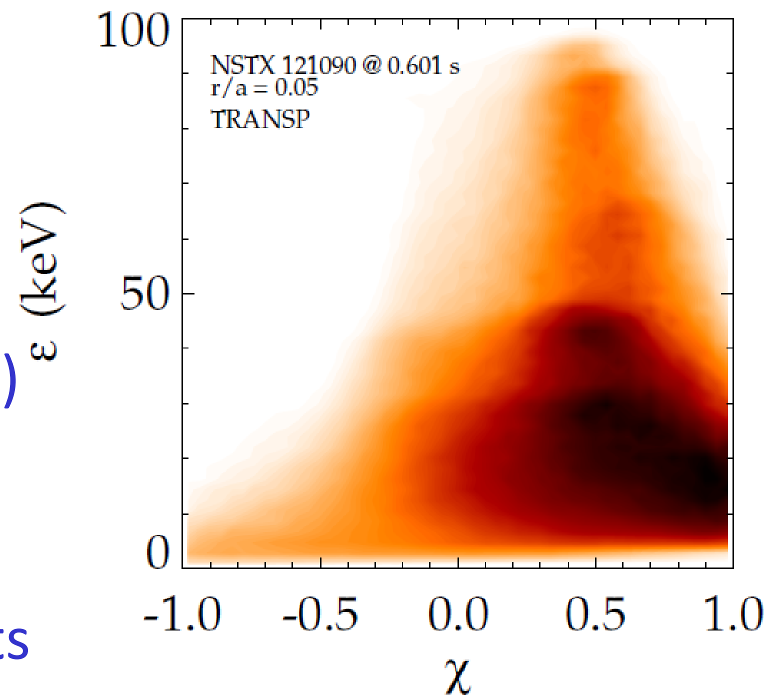
$$\delta W_K = \frac{2\sqrt{2}\pi^2}{m^{\frac{3}{2}}} \sum_{l=-\infty}^{\infty} \int d\varepsilon \int d\chi \int d\Psi \chi \frac{\hat{\tau}}{B} \frac{(\omega_r + i\gamma)\frac{\partial f}{\partial \varepsilon} + \frac{\partial f}{\partial \Psi}}{\langle \omega_D \rangle + l\omega_b - i\nu_{\text{eff}} + \omega_E - \omega_r - i\gamma} \varepsilon^{\frac{5}{2}} \left|\langle H/\hat{\varepsilon} \rangle\right|^2$$

Maxwellian	Isotropic Slowing-down	Actual NSTX E.P.
Thermal ions and electrons	Alpha particles	Beam ions
$f(\varepsilon,\Psi) = n\left(\frac{m}{2\pi T}\right)^{\frac{3}{2}} e^{-\hat{\varepsilon}}$	$f(\varepsilon,\Psi) = \frac{3n_a}{8\sqrt{2}\pi}\left(\ln\left(1+\hat{\varepsilon}_c^{-\frac{3}{2}}\right)\right)^{-1}\left(\frac{m_a}{\varepsilon_a}\right)^{\frac{3}{2}} \frac{1}{\hat{\varepsilon}^{\frac{3}{2}} + \hat{\varepsilon}_c^{\frac{3}{2}}}$	$f(\varepsilon,\Psi,\chi) = \ ?$
No problem	Should be fine for alphas. Presently being used for beam ions, despite its limitations.	

TRANSP gives out an energetic particle distribution function

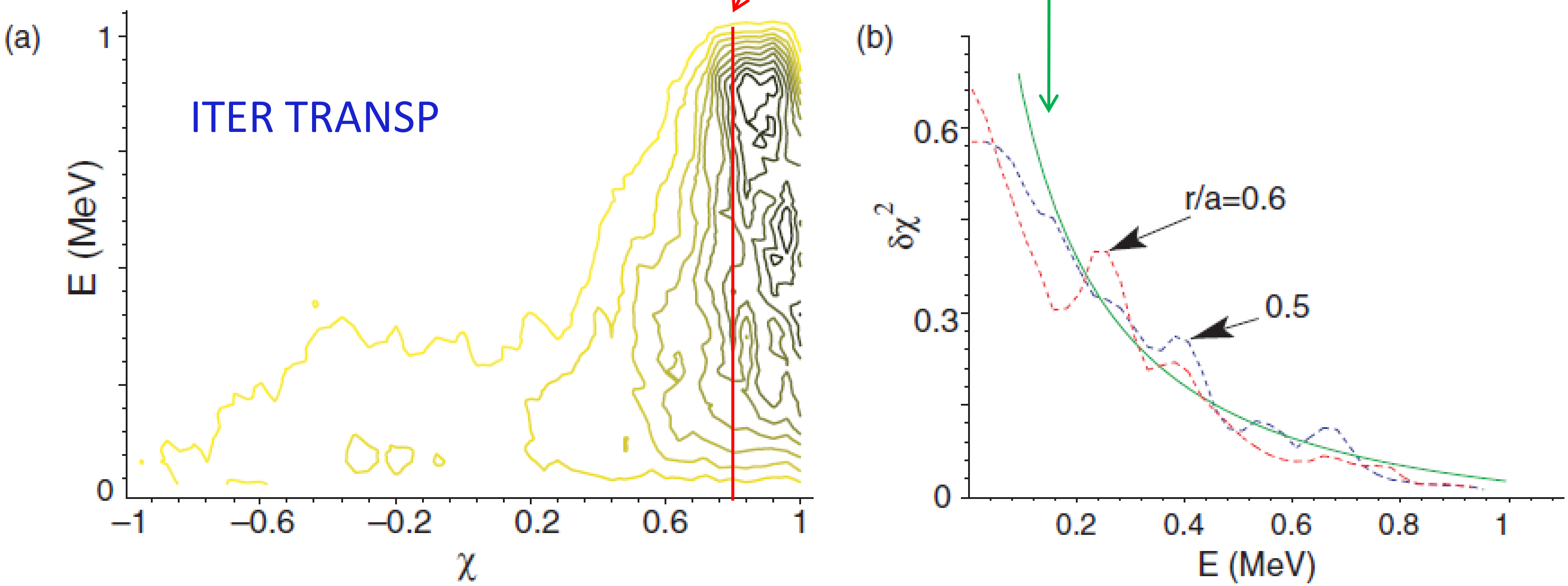


- To catch particles on their "home" surfaces we take a vertical slice, rather than a horizontal slice
  - Gorelenkov suggestion
- What we actually get out of TRANSP:
  - 75 ε pts (keV)
  - 50 χ (=$v_{||}/v$) pts
  - 10 x (~r/a) pts



Analytic expressions for E.P. *f* have been derived before

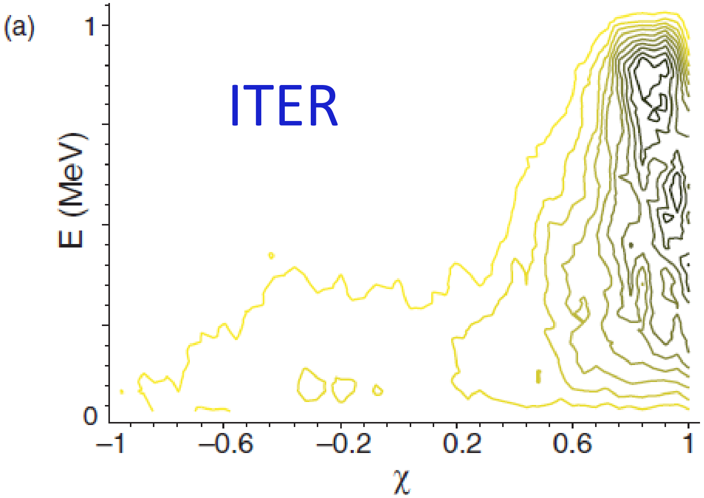
$$f(\varepsilon,\Psi,\chi) = M(\varepsilon,\Psi)\frac{e^{-(\chi-\chi_0(\Psi))^2/\delta\chi^2(\varepsilon,\Psi)}}{\sqrt{\delta\chi^2(\varepsilon,\Psi)}} \qquad \chi_0(\Psi) = C_2(\Psi) \qquad \delta\chi^2(\varepsilon,\Psi) = C_3(\Psi) - \frac{1}{3}\ln\left[\frac{\varepsilon^{\frac{3}{2}}}{\varepsilon^{\frac{3}{2}} + C_4(\Psi)}\right]$$

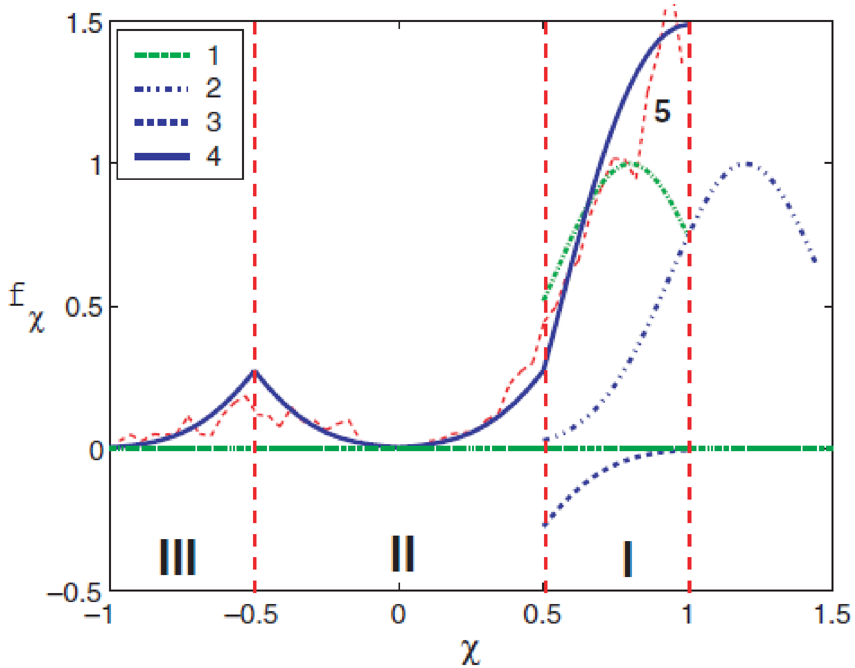


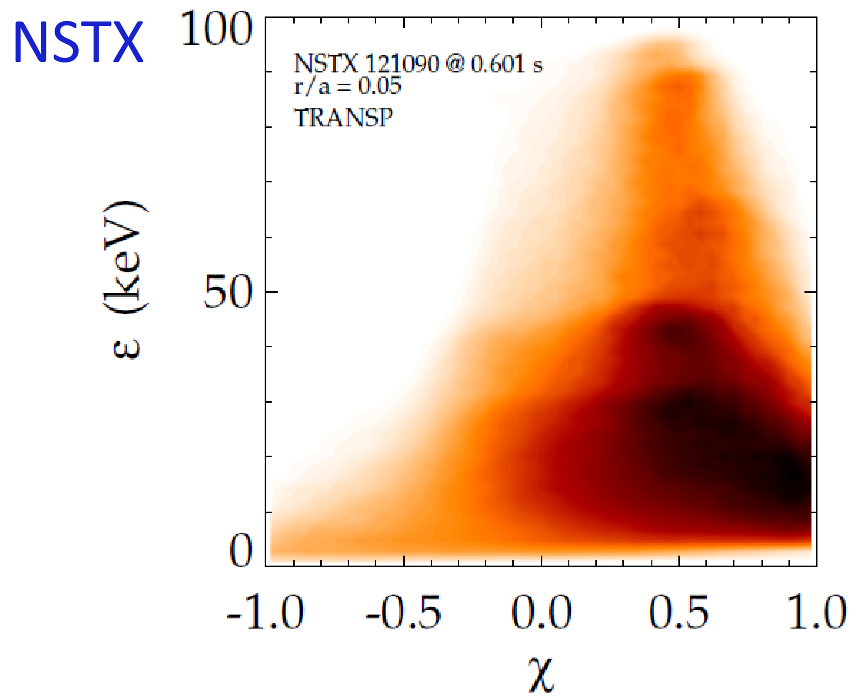
[N.N. Gorelenkov, H.L. Berk, and R.V. Budny, Nucl. Fusion **45**, 226 (2005)]

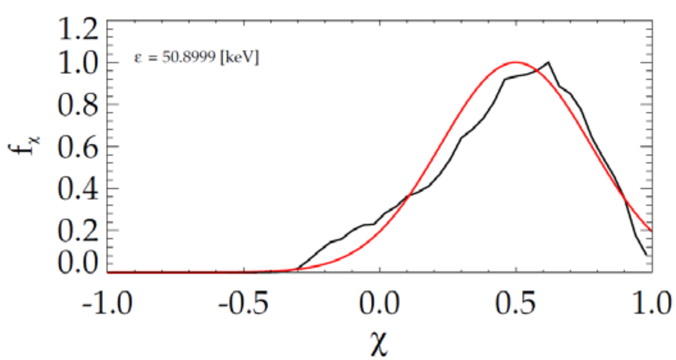
- Physics based
  - Actually more complex than shown above (trapped vs. circulating)
- But, doesn't work for NSTX (I tried it)

ITER and NSTX cases are quite different

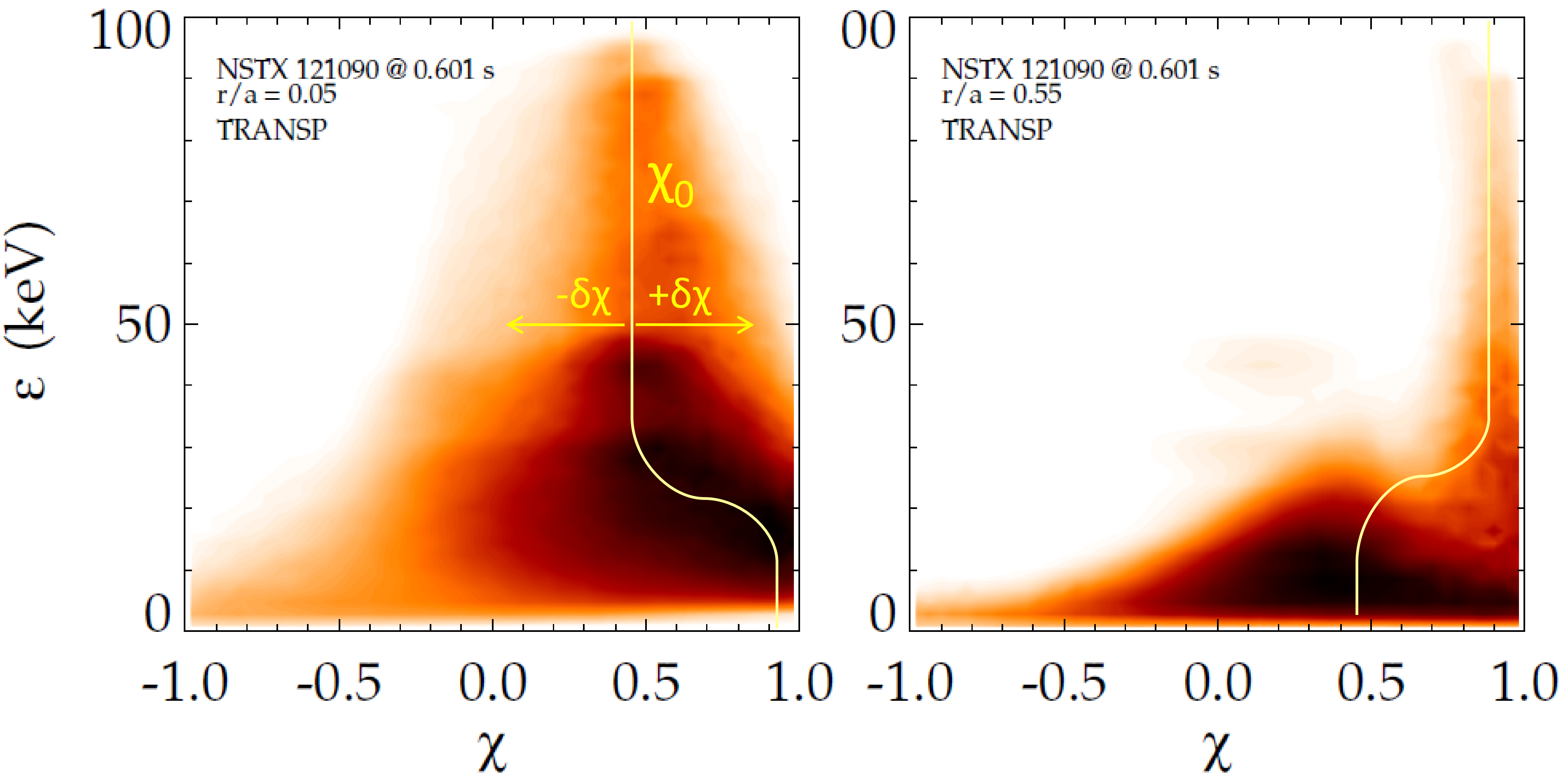






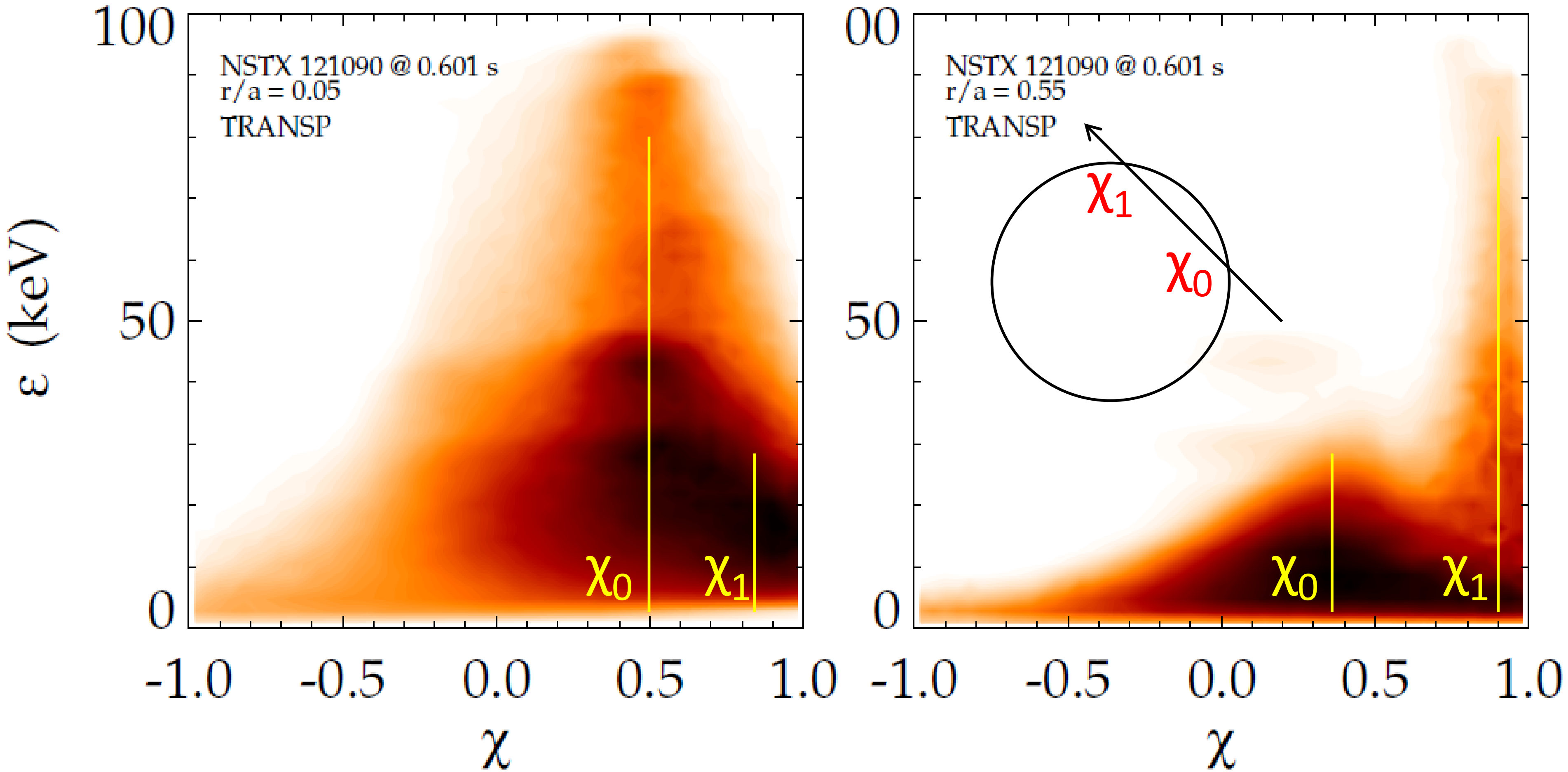


First idea: χ_0 is ε dependent



$$f(\varepsilon,\Psi,\chi) = M(\varepsilon,\Psi)\frac{e^{-(\chi-\chi_0(\varepsilon,\Psi))^2/\delta\chi^2(\varepsilon,\Psi)}}{\sqrt{\delta\chi^2(\varepsilon,\Psi)}}$$?

Second idea: Double Gaussian?



$$f(\varepsilon, \Psi, \chi) = C_0(\Psi) \left[M_0(\varepsilon, \Psi) \frac{e^{-(\chi-\chi_0(\Psi))^2/\delta\chi_0^2(\varepsilon,\Psi)}}{\sqrt{\delta\chi_0^2(\varepsilon, \Psi)}} + M_1(\varepsilon, \Psi) \frac{e^{-(\chi-\chi_1(\Psi))^2/\delta\chi_1^2(\varepsilon,\Psi)}}{\sqrt{\delta\chi_1^2(\varepsilon, \Psi)}} \right]$$?

Finding an analytic fit is not an easy process

- My best analytic expression so far is:

$$f(\varepsilon,\Psi,\chi) = \frac{C_0(\Psi)\varepsilon^{C_1(\Psi)}}{\varepsilon^{C_2(\Psi)} + C_3(\Psi)} \frac{e^{-(\chi-\chi_0(\varepsilon,\Psi))^2/\delta\chi^2(\varepsilon,\Psi)}}{\sqrt{\delta\chi^2(\varepsilon,\Psi)}}$$

$$\chi_0(\varepsilon,\Psi) = C_4(\Psi) + \frac{C_5(\Psi)}{1 + e^{-C_6(\Psi)(\varepsilon - C_7(\Psi))}}$$

$$\delta\chi^2(\varepsilon,\Psi) = C_8(\Psi) - \frac{1}{3}\ln\left[\frac{\varepsilon^{\frac{3}{2}}}{\varepsilon^{\frac{3}{2}} + C_9(\Psi)}\right] + C_{10}(\Psi)\left|\chi_0(\varepsilon,\Psi) - \chi_0(\varepsilon_{\max},\Psi)\right|$$

Two major changes:

- "magnitude" term is more generalized
- χ_0 is dependent on ε (which causes correction to δχ as well)

- Gorelenkov:

$$f(\varepsilon,\Psi,\chi) = \frac{C_0(\Psi)}{\varepsilon^{\frac{3}{2}} + C_1(\Psi)} \frac{e^{-(\chi-\chi_0(\Psi))^2/\delta\chi^2(\varepsilon,\Psi)}}{\sqrt{\delta\chi^2(\varepsilon,\Psi)}} \quad \chi_0(\Psi) = C_2(\Psi) \quad \delta\chi^2(\varepsilon,\Psi) = C_3(\Psi) - \frac{1}{3}\ln\left[\frac{\varepsilon^{\frac{3}{2}}}{\varepsilon^{\frac{3}{2}} + C_4(\Psi)}\right]$$

Finding an analytic fit is not an easy process

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$$f(\varepsilon,\Psi,\chi) = \frac{C_0(\Psi)\varepsilon^{C_1(\Psi)}}{\varepsilon^{C_2(\Psi)} + C_3(\Psi)} \frac{e^{-(\chi-\chi_0(\varepsilon,\Psi))^2/\delta\chi^2(\varepsilon,\Psi)}}{\sqrt{\delta\chi^2(\varepsilon,\Psi)}}$$

$$\chi_0(\varepsilon,\Psi) = C_4(\Psi) + \frac{C_5(\Psi)}{1 + e^{-C_6(\Psi)(\varepsilon - C_7(\Psi))}}$$

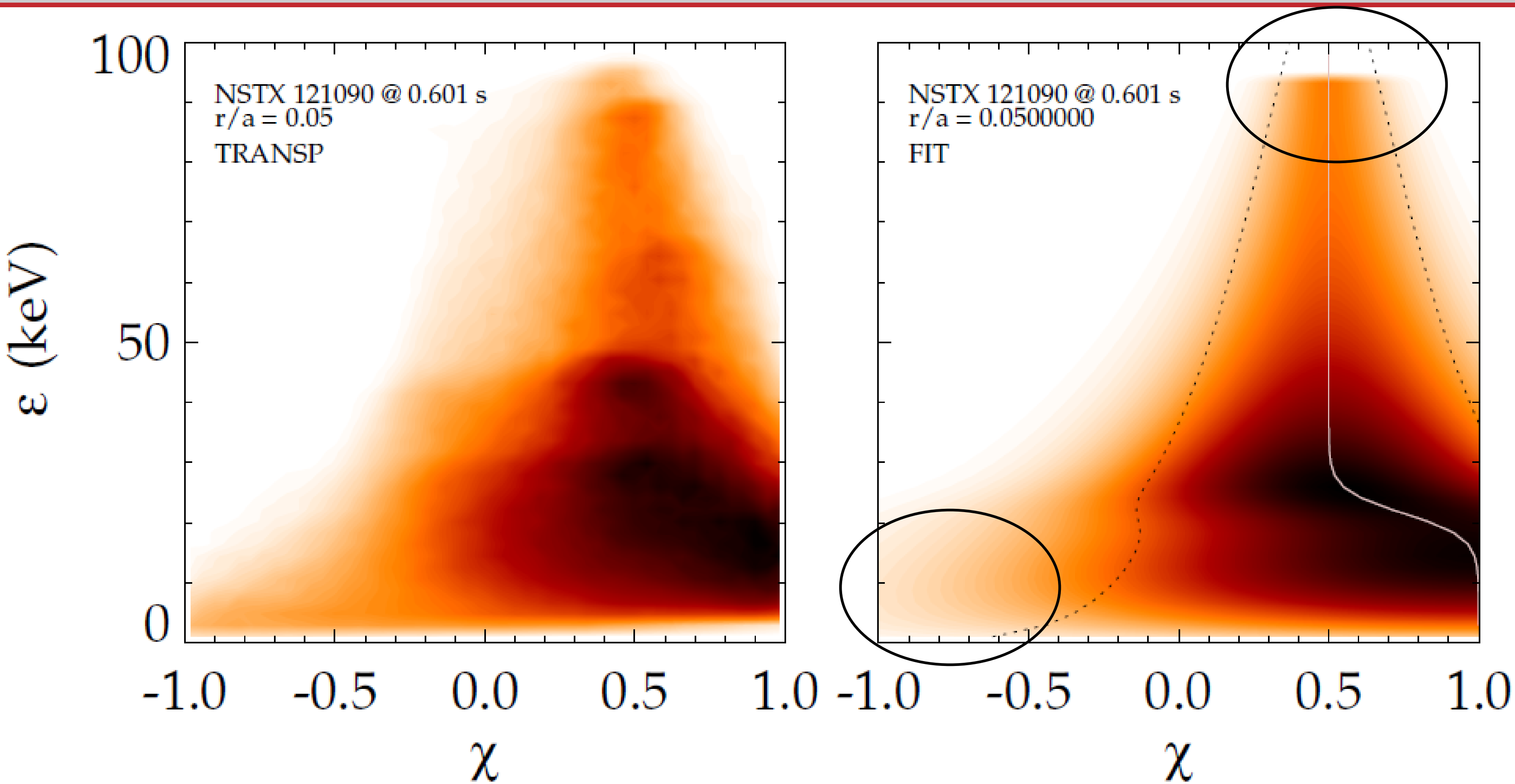
$$\delta\chi^2(\varepsilon,\Psi) = C_8(\Psi) - \frac{1}{3}\ln\left[\frac{\varepsilon^{\frac{3}{2}}}{\varepsilon^{\frac{3}{2}} + C_9(\Psi)}\right] + C_{10}(\Psi)\left|\chi_0(\varepsilon,\Psi) - \chi_0(\varepsilon_{\max},\Psi)\right|$$

Two major changes:

- "magnitude" term is more generalized
- χ_0 is dependent on ε (which causes correction to δχ as well)

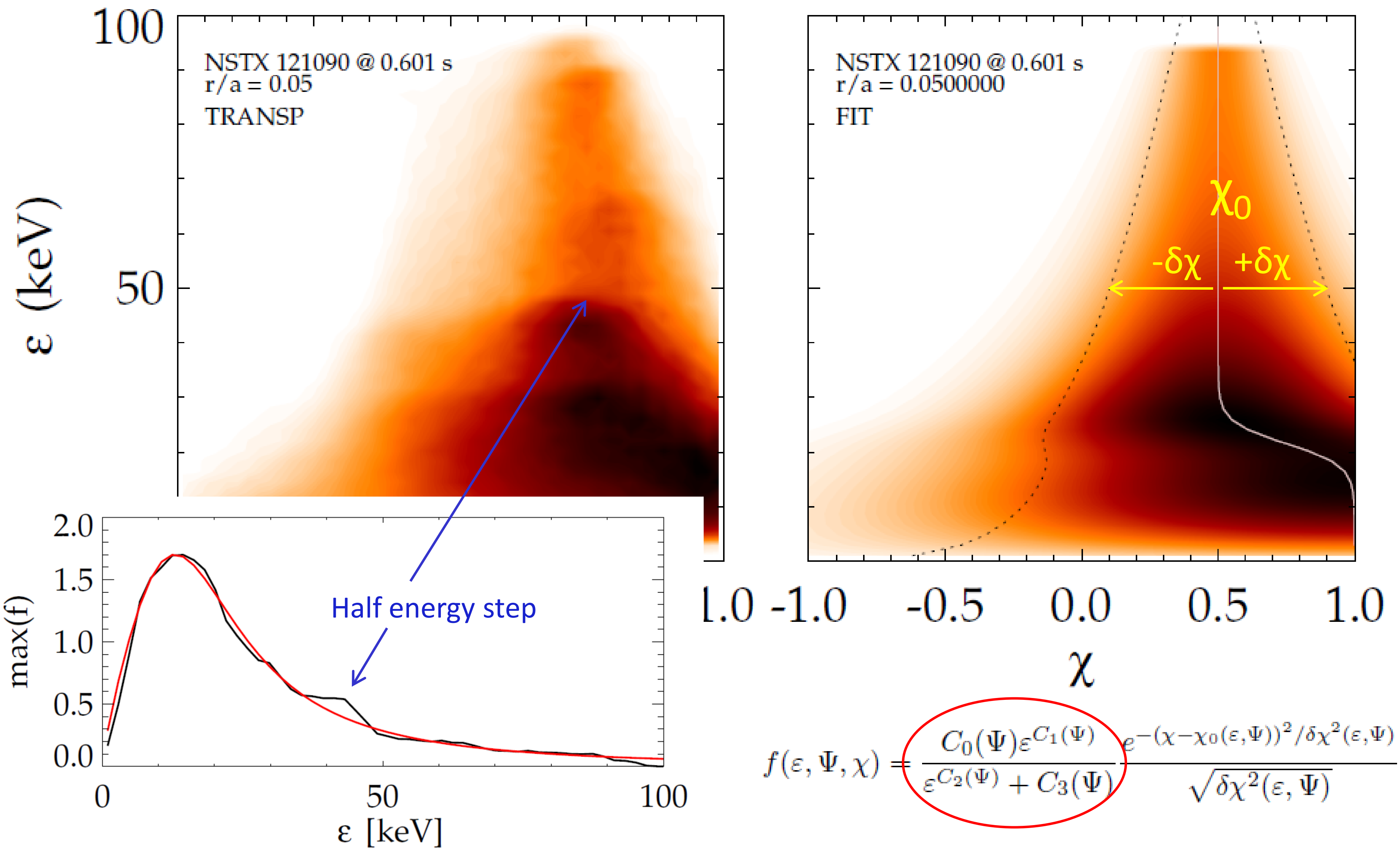
- Note:
  - C_0 is a normalization factor so that: $p_a = \int f_a\left(\frac{2}{3}\varepsilon\right) d^3\mathbf{v}$
  - Otherwise there are ten other C(Ψ) to fit.
  - There is no physics in the constants, they are selected purely to fit the TRANSP output.
  - This is all on the basis of one shot at one time point.

TRANSP output vs. analytic fit



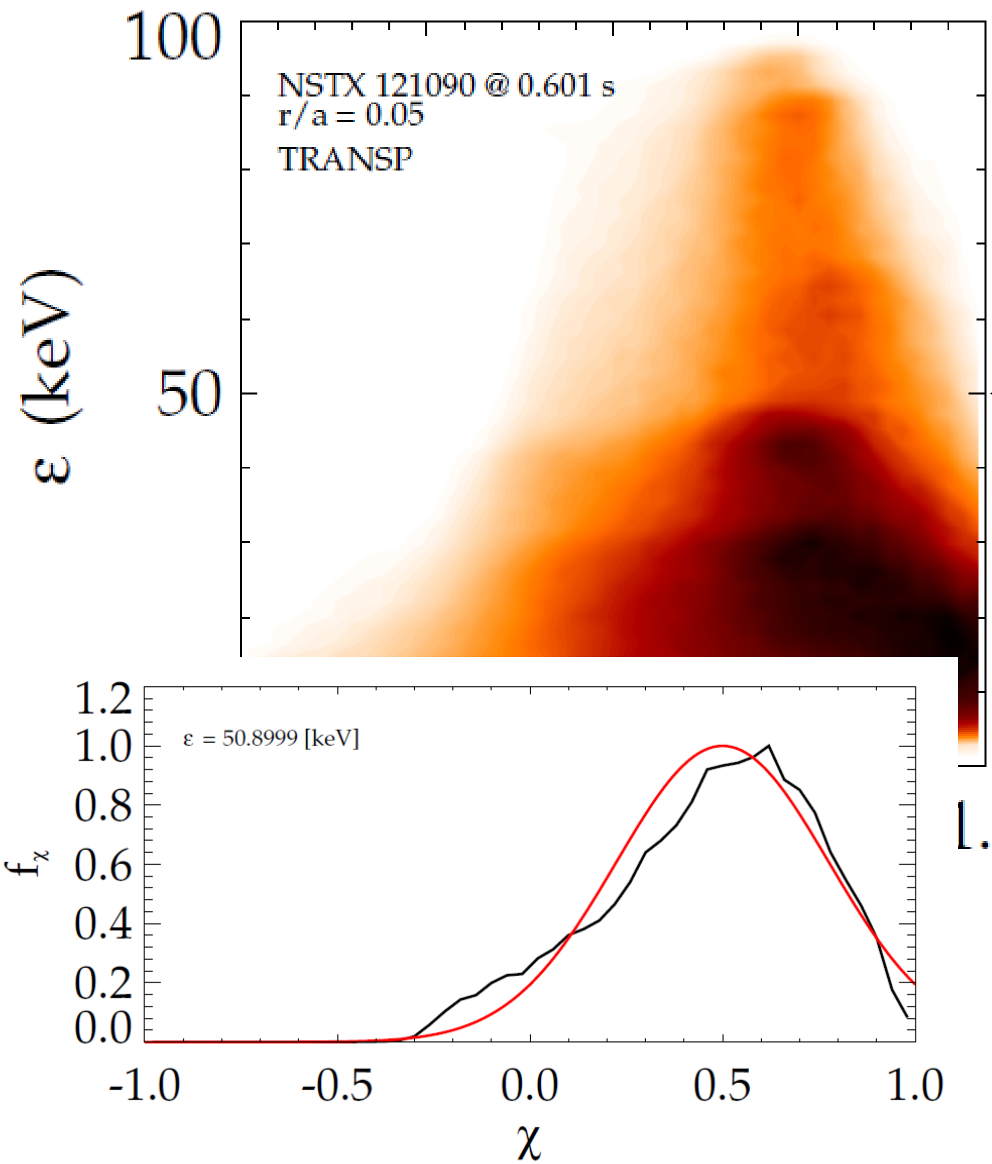
- Does a pretty good job.
  - Not as good at highest and lowest energies.

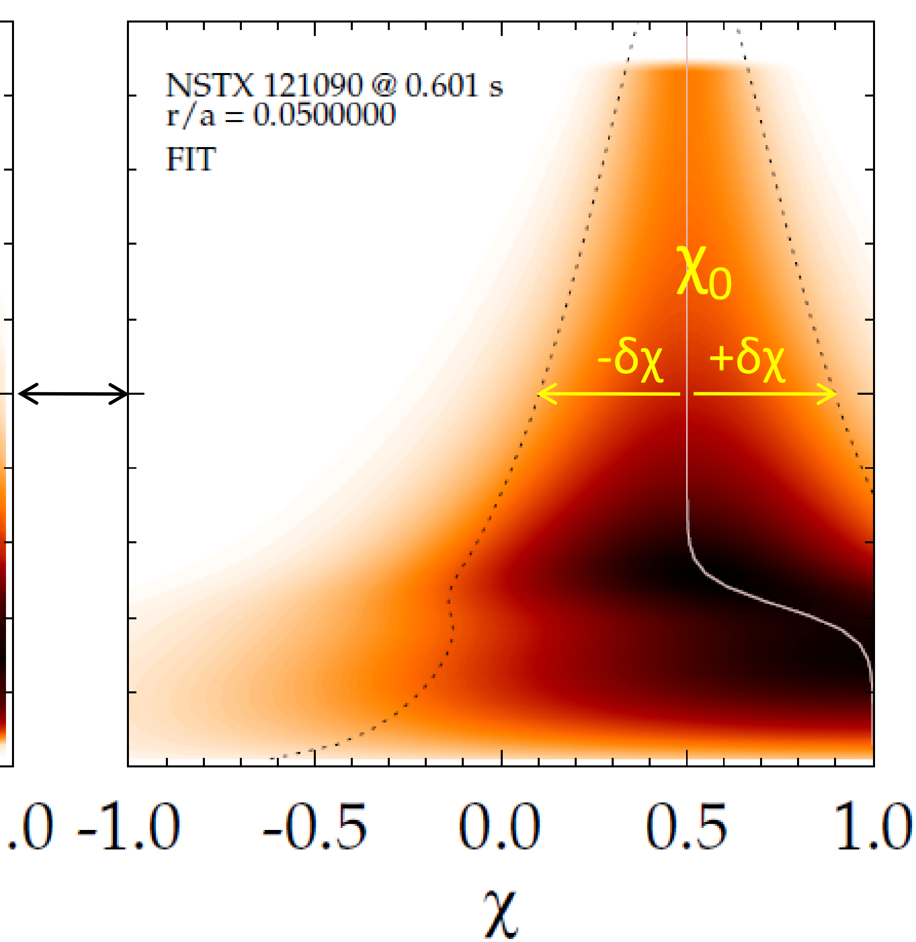
Looking at the "ridge" gives the Gaussian peak magnitude



$$f(\varepsilon,\Psi,\chi) = \frac{C_0(\Psi)\varepsilon^{C_1(\Psi)}}{\varepsilon^{C_2(\Psi)} + C_3(\Psi)} \frac{e^{-(\chi-\chi_0(\varepsilon,\Psi))^2/\delta\chi^2(\varepsilon,\Psi)}}{\sqrt{\delta\chi^2(\varepsilon,\Psi)}}$$

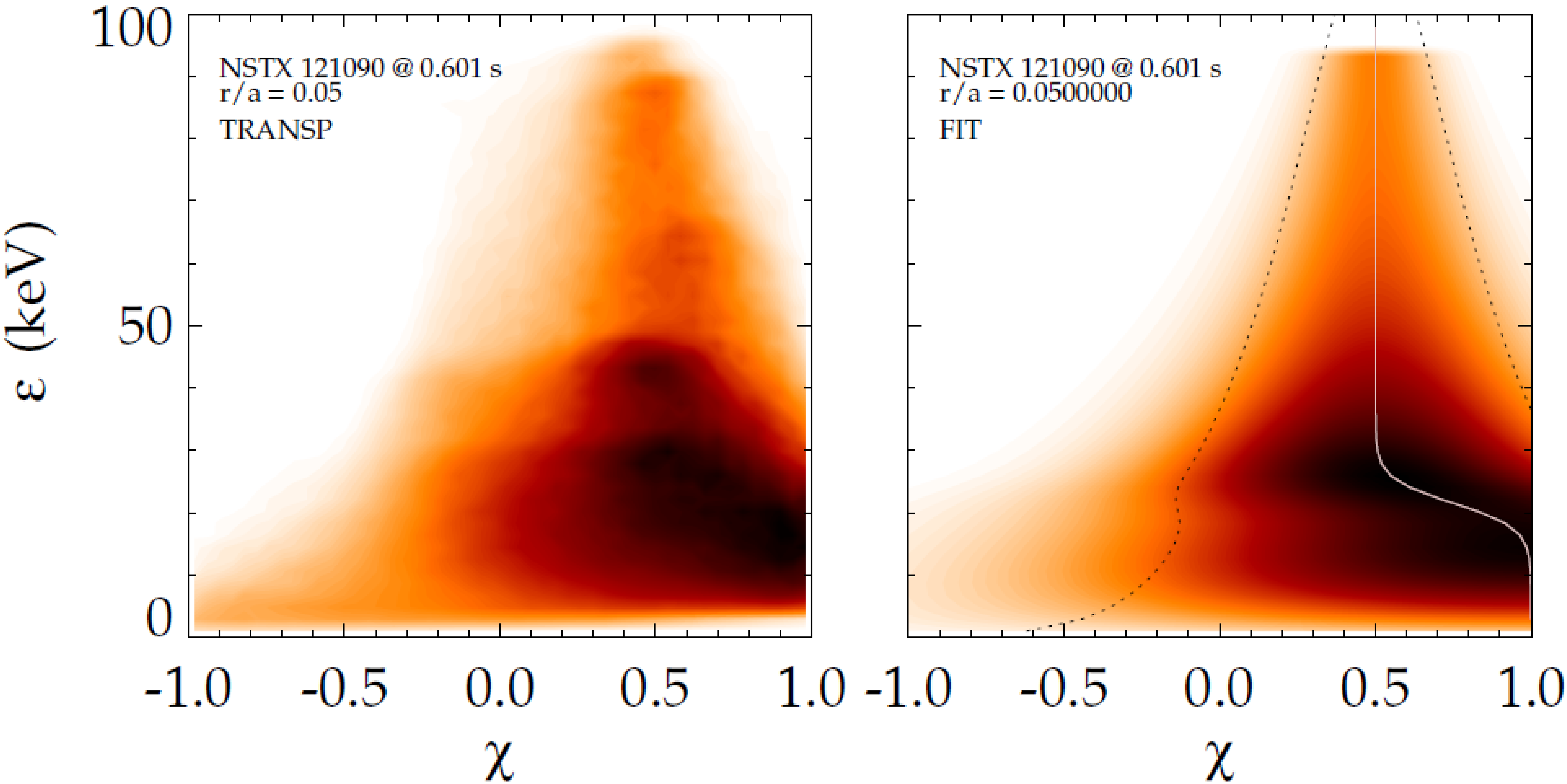
Looking at particular energies shows the Gaussian fits



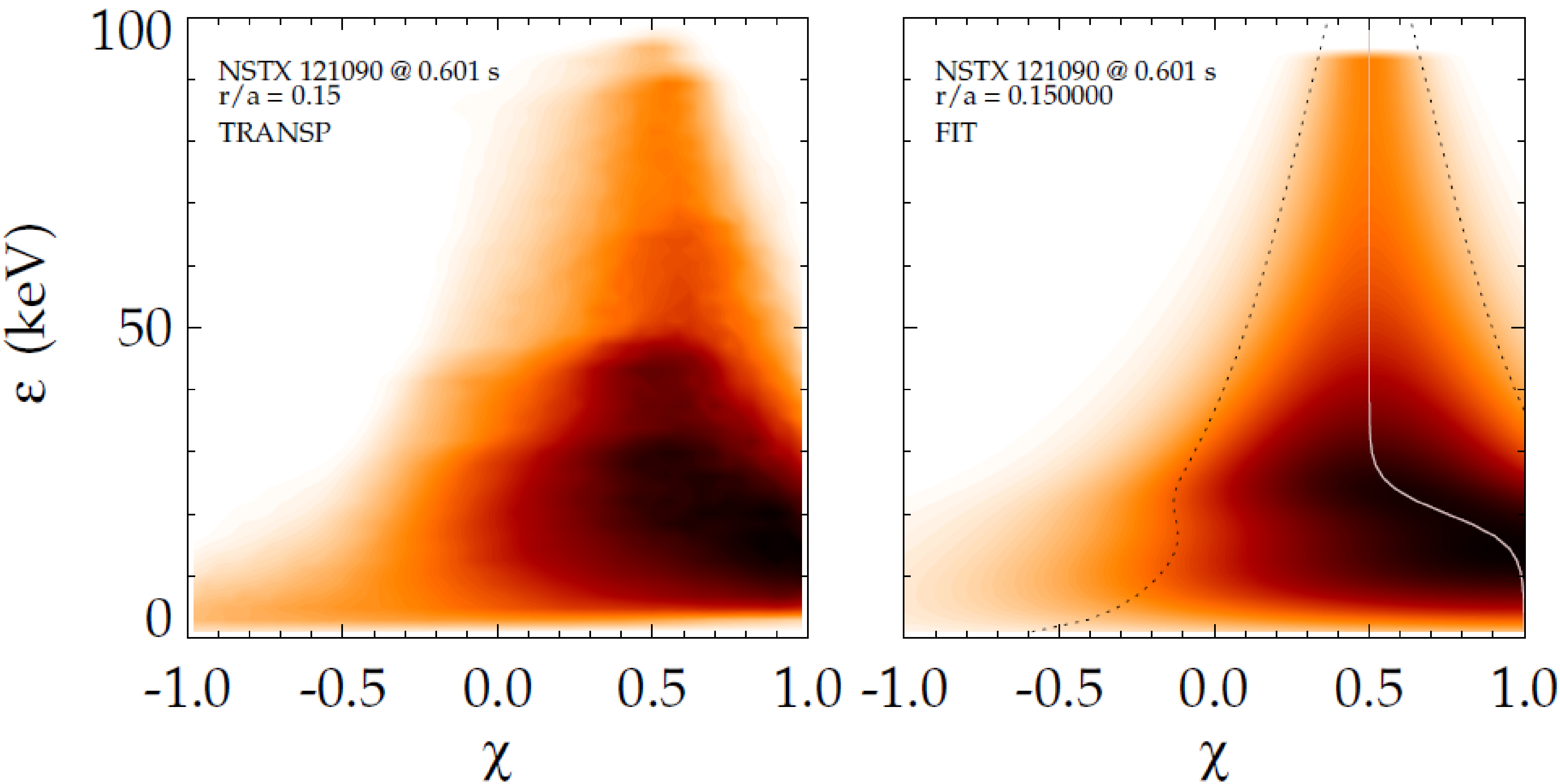


$$f(\varepsilon,\Psi,\chi) = \frac{C_0(\Psi)\varepsilon^{C_1(\Psi)}}{\varepsilon^{C_2(\Psi)} + C_3(\Psi)} \frac{e^{-(\chi-\chi_0(\varepsilon,\Psi))^2/\delta\chi^2(\varepsilon,\Psi)}}{\sqrt{\delta\chi^2(\varepsilon,\Psi)}}$$

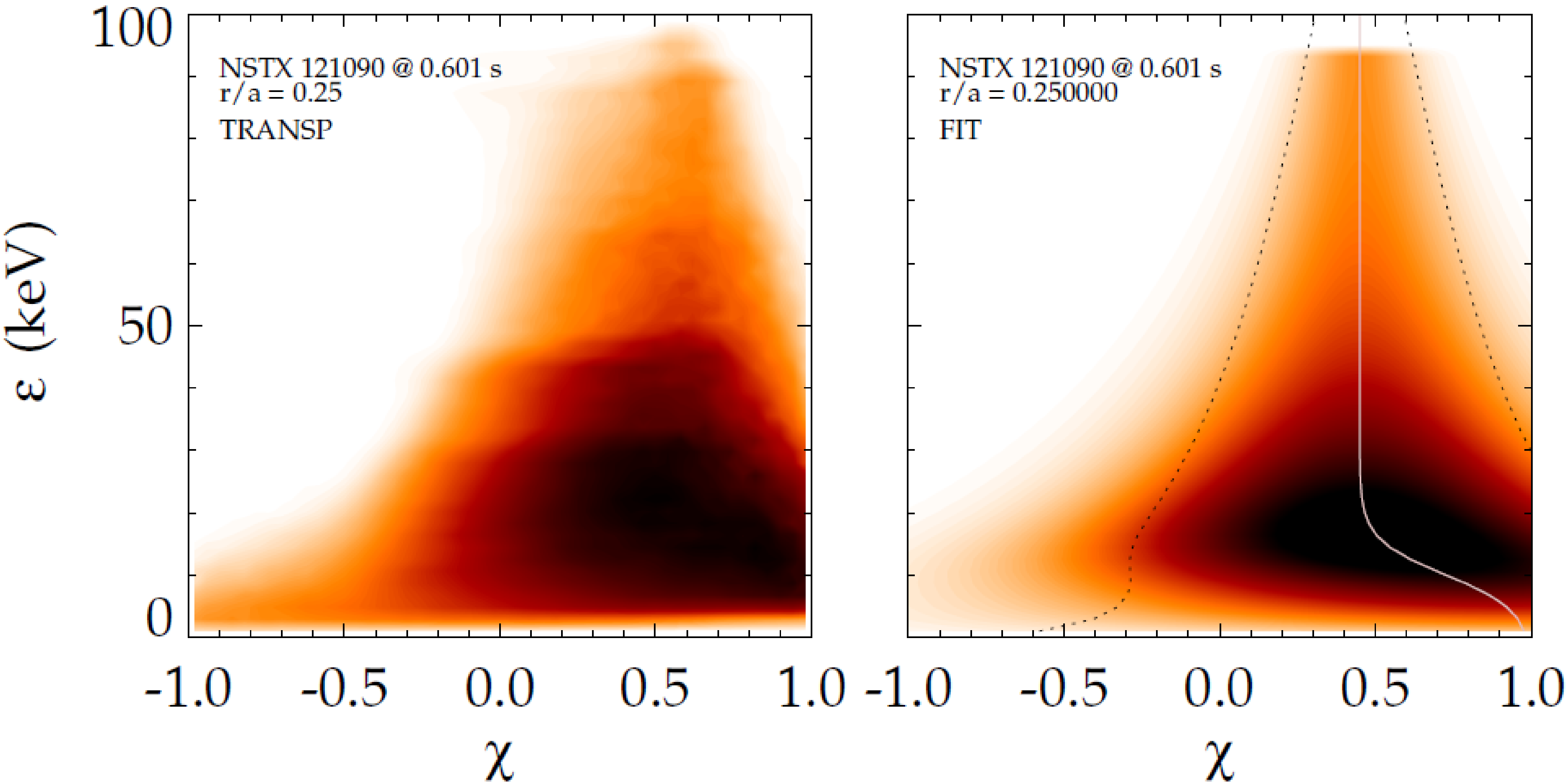
r/a = 0.05


100
50
0
ε (keV)
NSTX 121090 @ 0.601 s
r/a = 0.05
TRANSP
-1.0
-0.5
0.0
0.5
1.0
χ
NSTX 121090 @ 0.601 s
r/a = 0.0500000
FIT
-1.0
-0.5
0.0
0.5
1.0
χ

r/a = 0.15

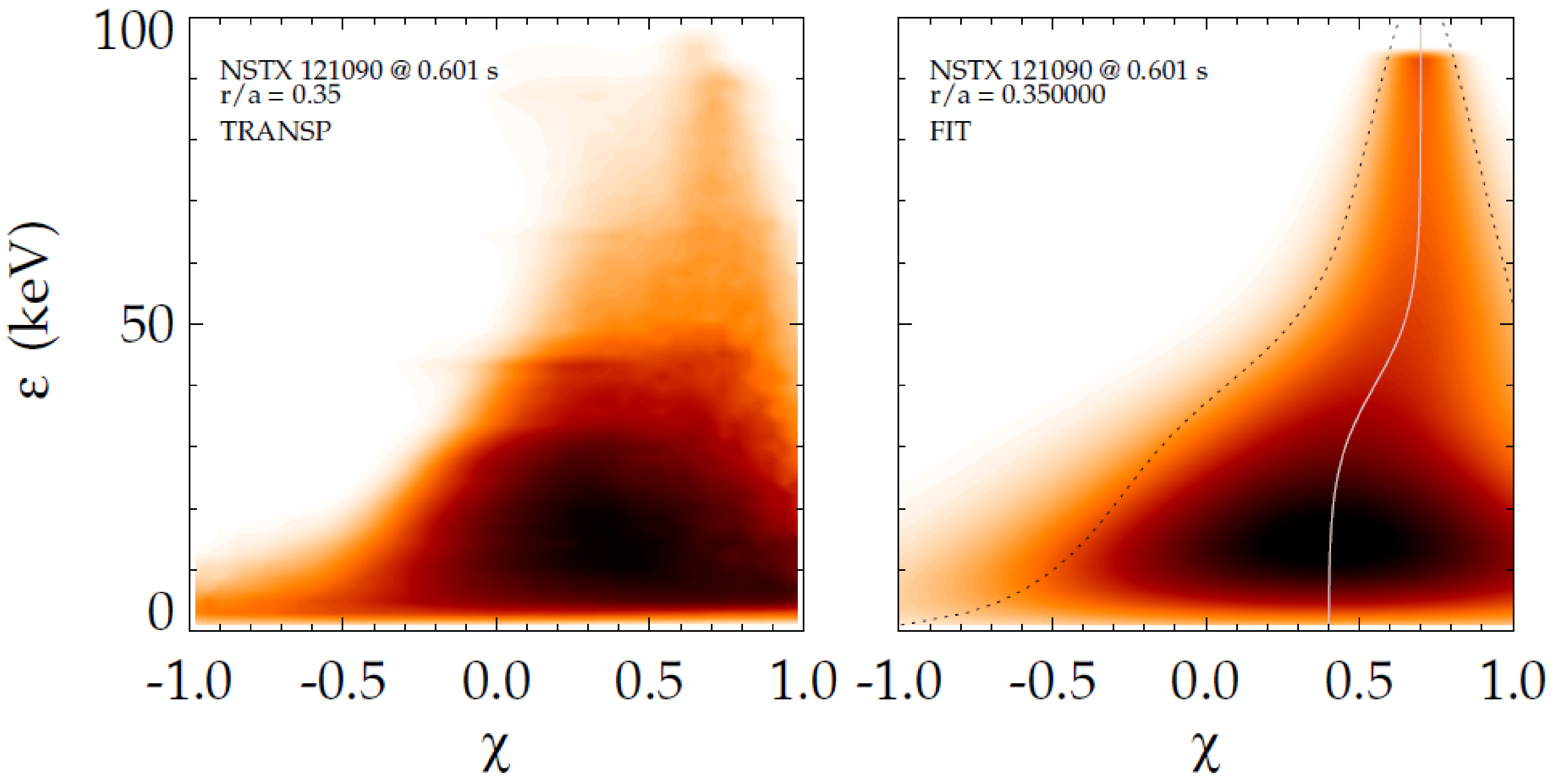

NSTX 121090 @ 0.601 s
r/a = 0.15
TRANSP
NSTX 121090 @ 0.601 s
r/a = 0.150000
FIT
ε (keV)
100
50
0
-1.0
-0.5
0.0
0.5
1.0
χ

r/a = 0.25



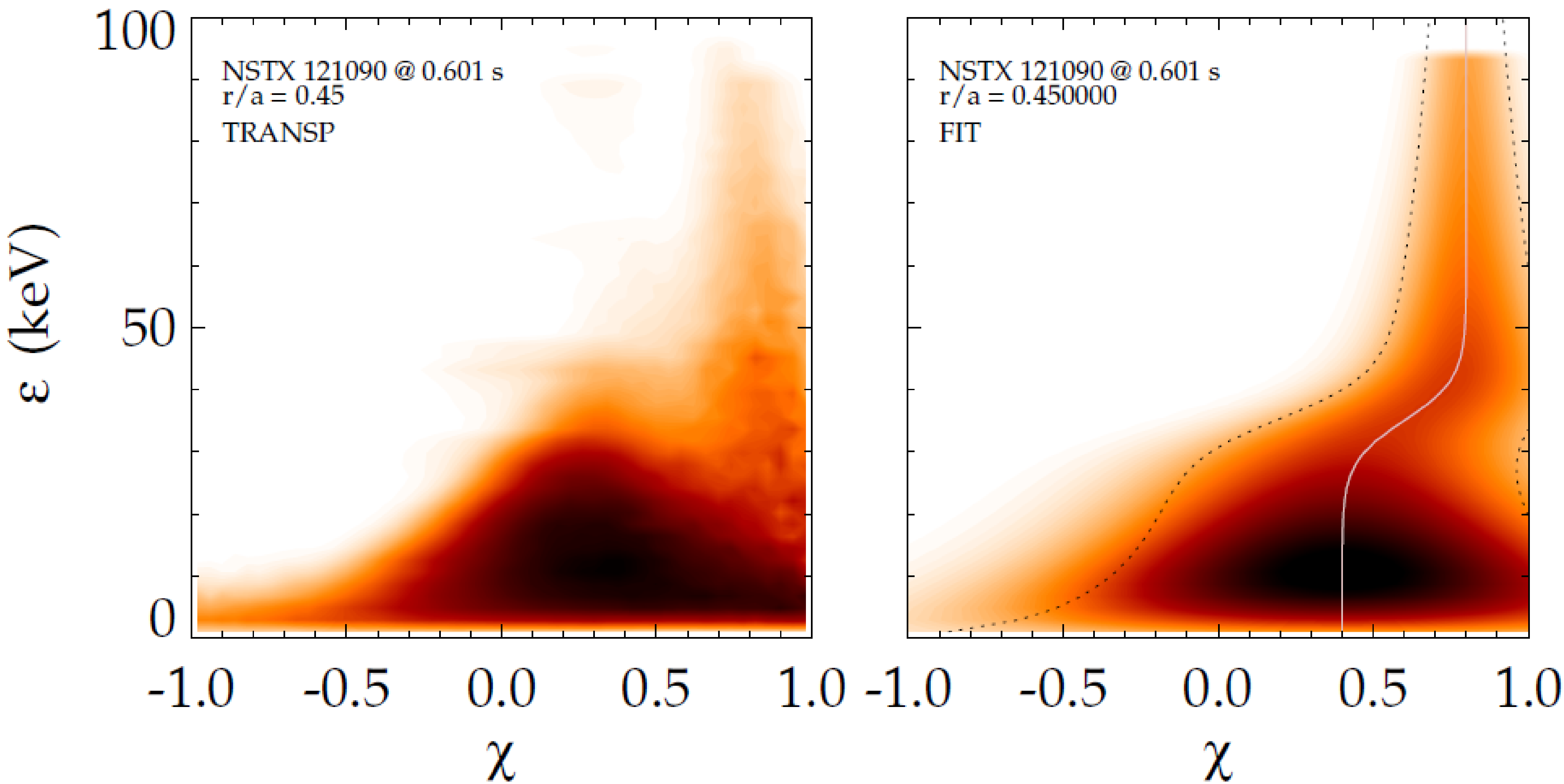
NSTX 121090 @ 0.601 s
r/a = 0.25
TRANSP
NSTX 121090 @ 0.601 s
r/a = 0.250000
FIT
ε (keV)
100
50
0
-1.0
-0.5
0.0
0.5
1.0
χ

r/a = 0.35

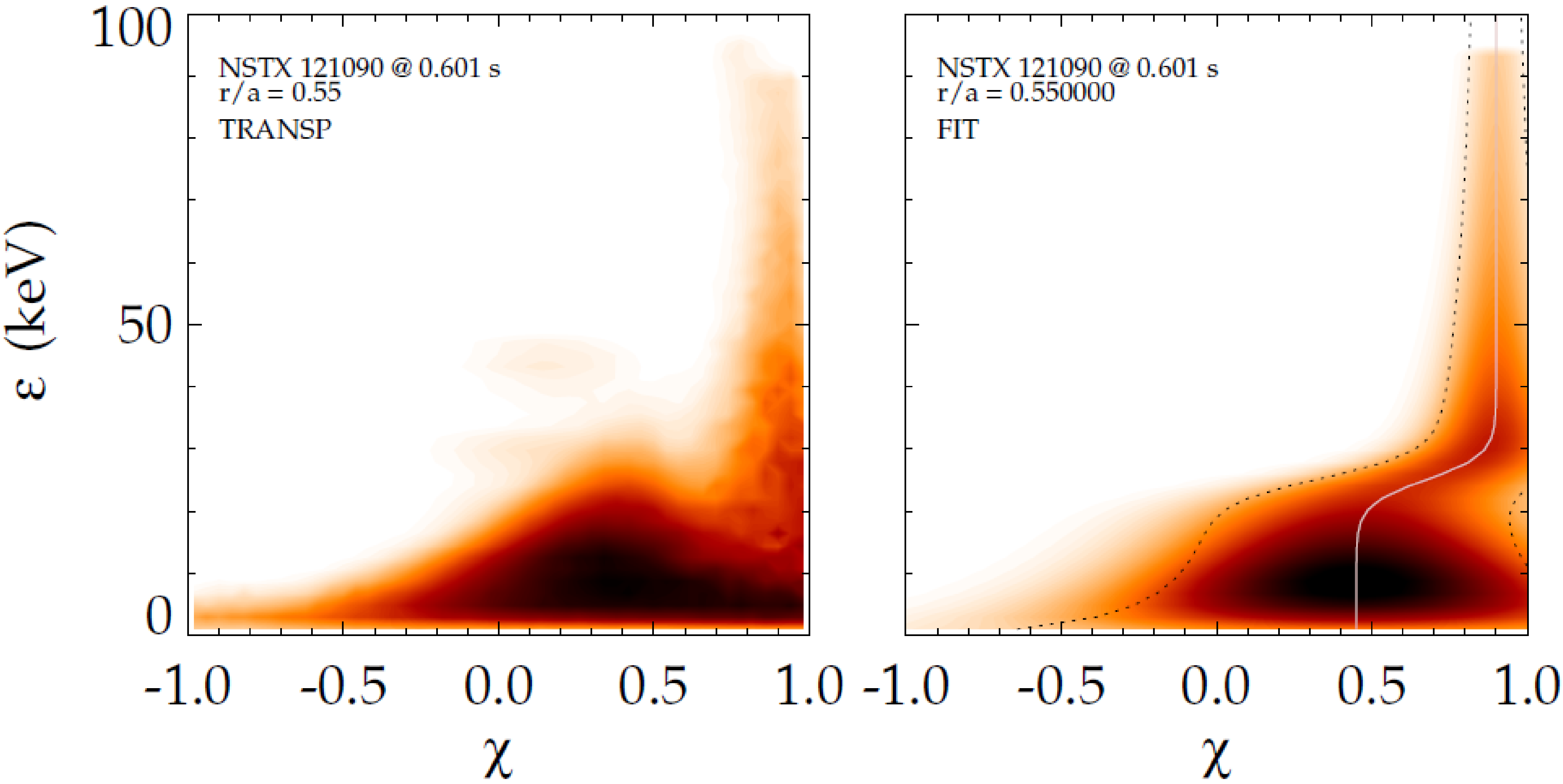


100
50
0
ε (keV)
NSTX 121090 @ 0.601 s
r/a = 0.35
TRANSP
-1.0
-0.5
0.0
0.5
1.0
χ
NSTX 121090 @ 0.601 s
r/a = 0.350000
FIT
-1.0
-0.5
0.0
0.5
1.0
χ

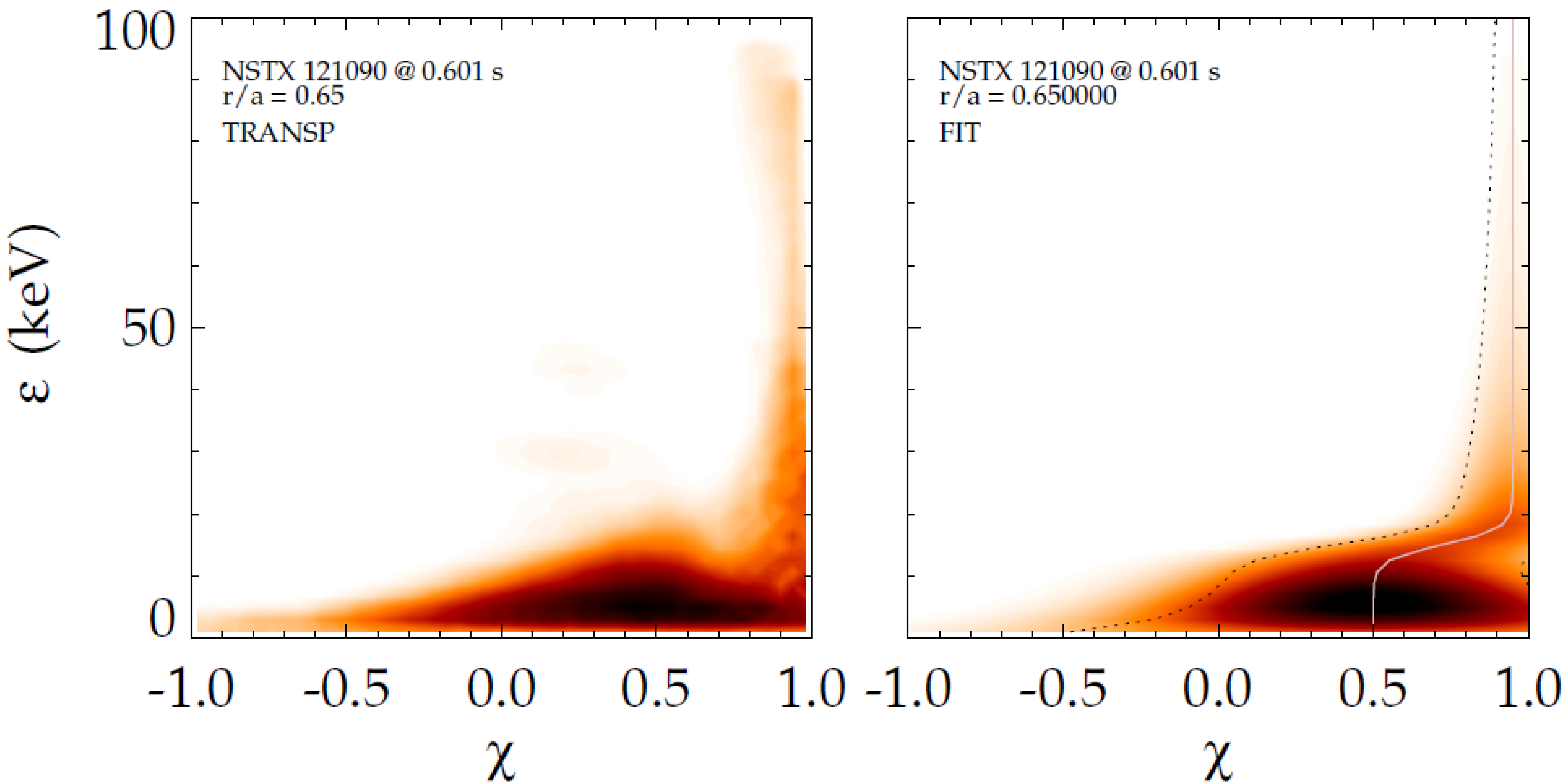
r/a = 0.45


NSTX 121090 @ 0.601 s
r/a = 0.45
TRANSP
NSTX 121090 @ 0.601 s
r/a = 0.450000
FIT
ε (keV)
100
50
0
-1.0
-0.5
0.0
0.5
1.0
χ

r/a = 0.55

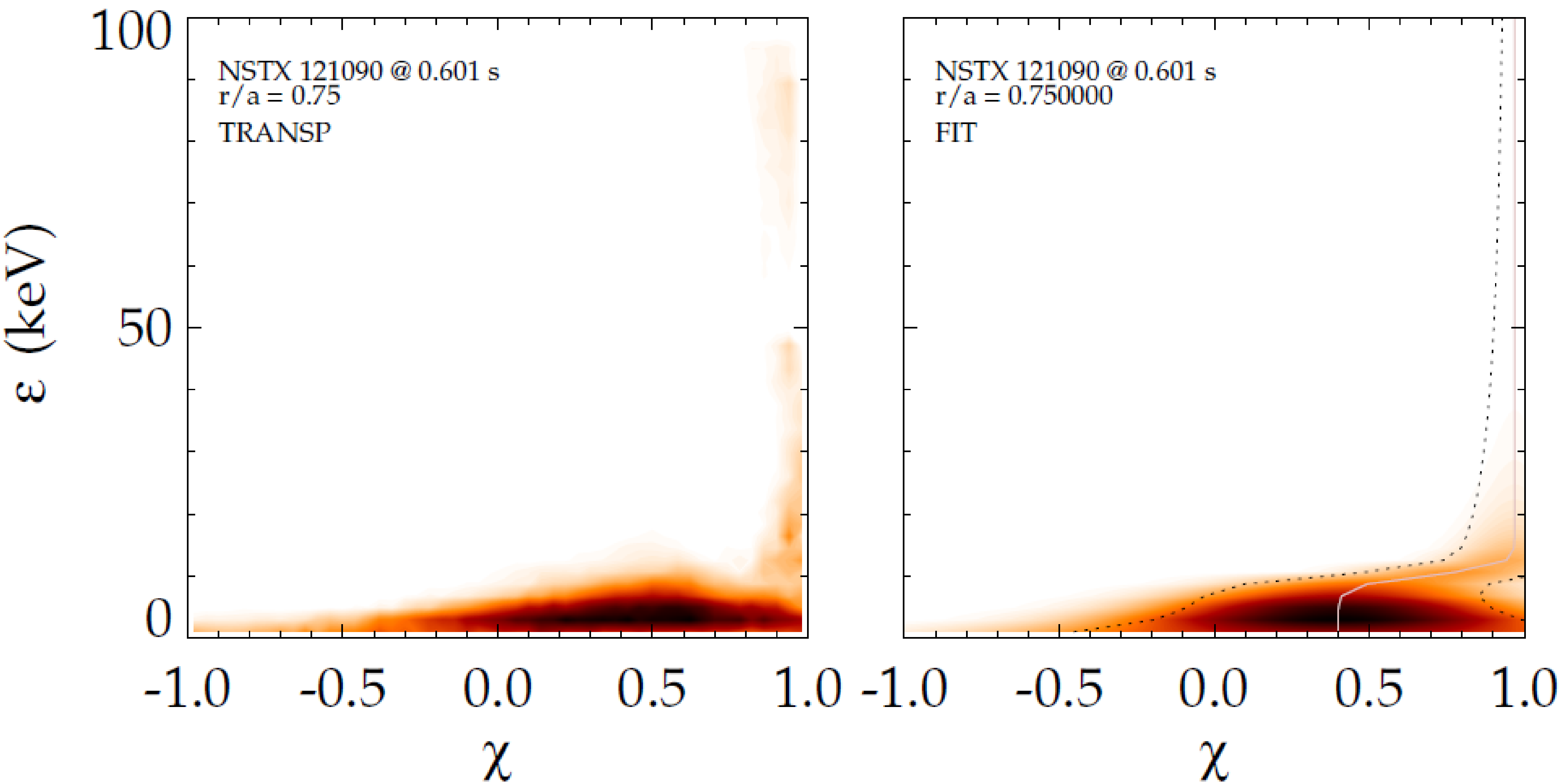

NSTX 121090 @ 0.601 s
r/a = 0.55
TRANSP
NSTX 121090 @ 0.601 s
r/a = 0.550000
FIT
100
50
0
ε (keV)
-1.0
-0.5
0.0
0.5
1.0
χ

r/a = 0.65

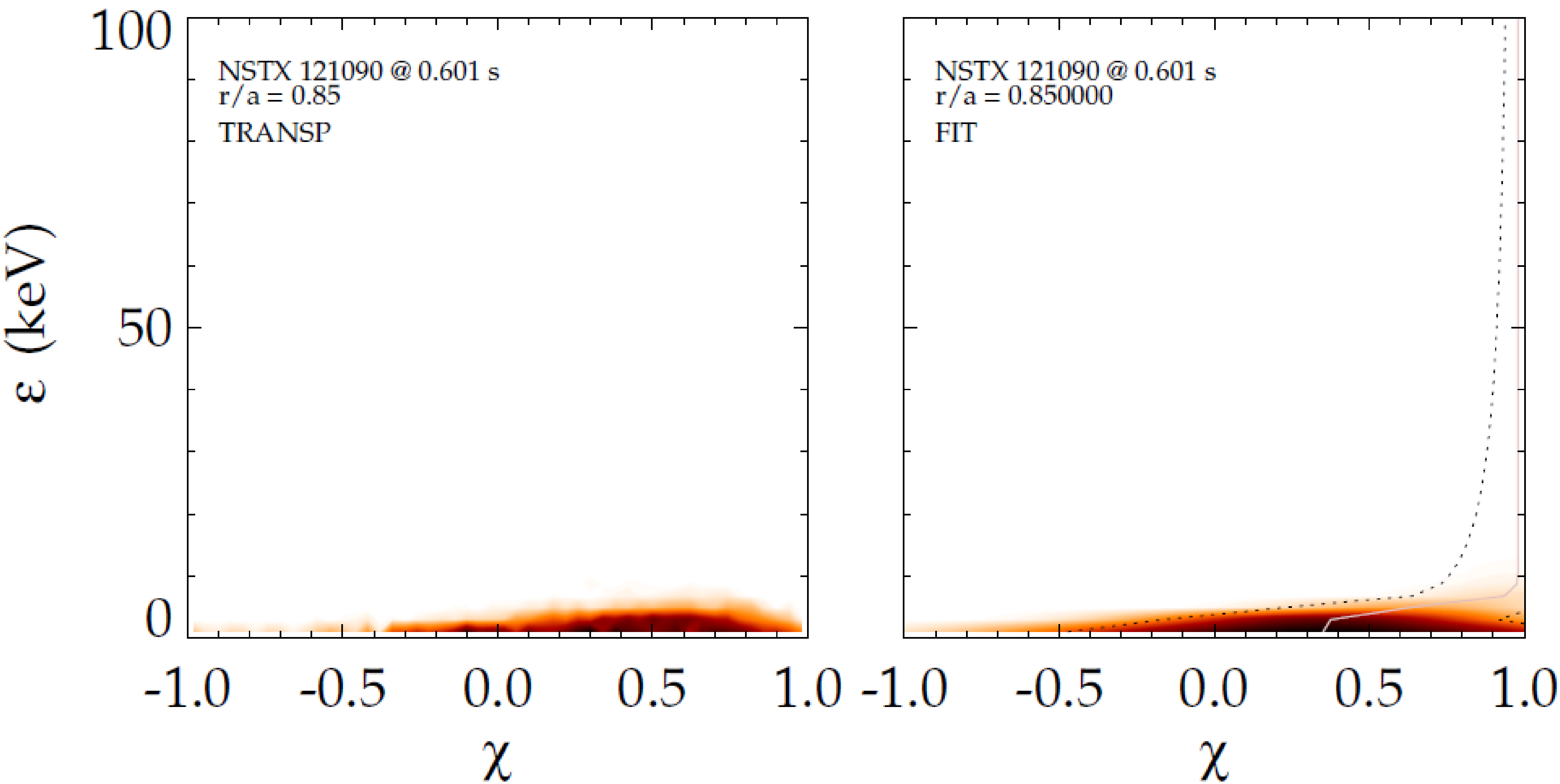


100
50
0
ε (keV)
NSTX 121090 @ 0.601 s
r/a = 0.65
TRANSP
-1.0
-0.5
0.0
0.5
1.0
χ
NSTX 121090 @ 0.601 s
r/a = 0.650000
FIT
-1.0
-0.5
0.0
0.5
1.0
χ

r/a = 0.75

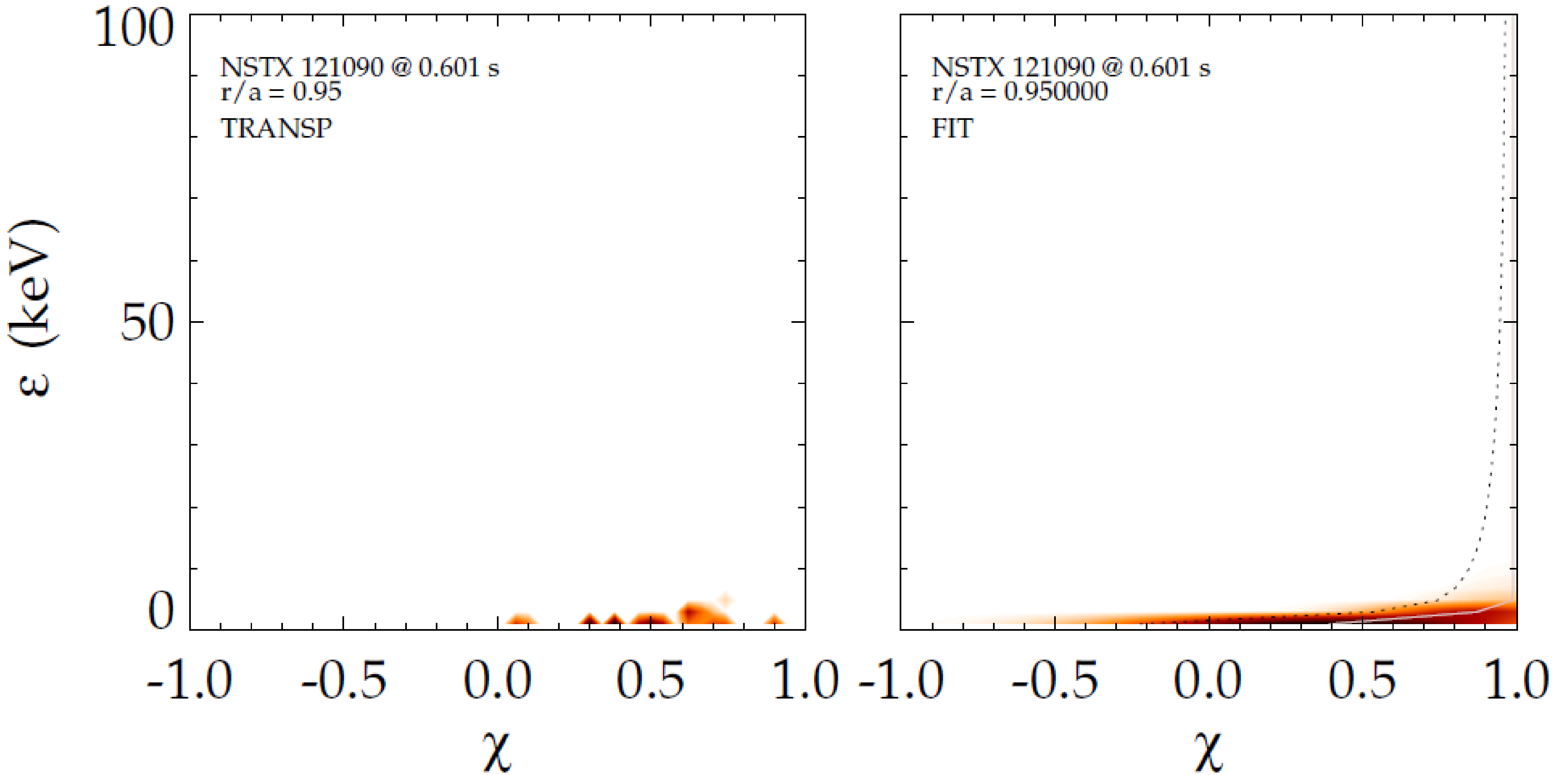

100
50
0
ε (keV)
NSTX 121090 @ 0.601 s
r/a = 0.75
TRANSP
NSTX 121090 @ 0.601 s
r/a = 0.750000
FIT
-1.0
-0.5
0.0
0.5
1.0
χ

r/a = 0.85



NSTX 121090 @ 0.601 s
r/a = 0.85
TRANSP
NSTX 121090 @ 0.601 s
r/a = 0.850000
FIT
ε (keV)
100
50
0
-1.0
-0.5
0.0
0.5
1.0
χ

r/a = 0.95



100
50
0
ε (keV)
NSTX 121090 @ 0.601 s
r/a = 0.95
TRANSP
-1.0
-0.5
0.0
0.5
1.0
χ
NSTX 121090 @ 0.601 s
r/a = 0.950000
FIT
-1.0
-0.5
0.0
0.5
1.0
χ