NSTX-U

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Interpretive model for resonant fast ion transport by Alfvénic instabilities

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Motivation

- Resonant/stochastic fast ion transport may not be well described by *diffusive* processes, Γ~dn/dr
  - Resonances introduce selectivity in phase space (Energy, pitch)
- Fast ion transport models presently implemented in TRANSP/NUBEAM do not include basic physics of wave-particle resonant interaction
- Codes such as M3D-K do treat resonant interaction correctly, but are not designed for 'mass production'

- *New "general/reduced" model is required to complement existing models*
  - *Must capture basic mechanisms of wave-particle interaction*
  - *Formulation must be compatible with TRANSP/NUBEAM*

Outline

- Scope of new model
- Main ingredients
  - Choice of variables
  - Constraints from resonant interaction with *AEs
- General implementation
  - *Probability distribution* to evolve particles' orbits
- Questions, comments, open issues

Backup:

- Example#1: computing the 'Mode Amplitude'
- Example#2: deriving the transport coefficients
- Example#3: evolving F_{nb} in time

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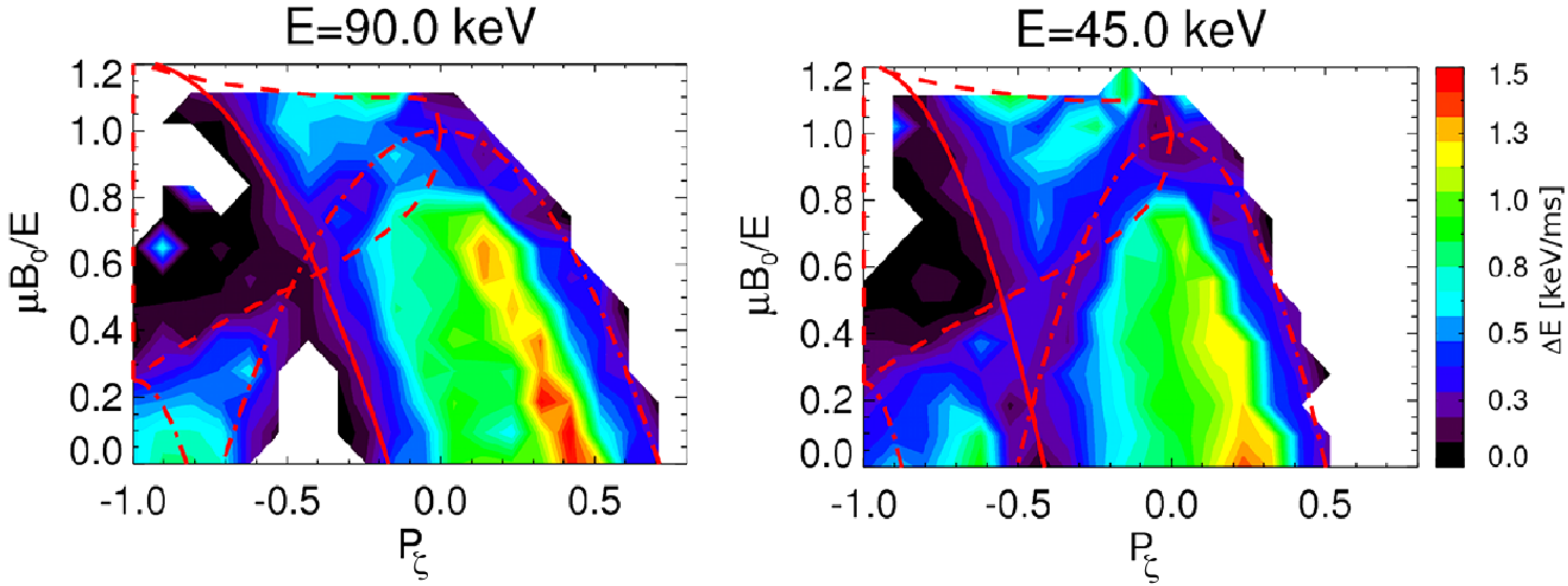
Goal: simulate *long-term* effects on F_{nb} from Alfvén Eigenmodes (*AEs) with arbitrary amplitude evolution $A_{modes}(t)$



- Example: average energy step at fixed energy, A_{modes}
  - ORBIT simulation with 3 TAE modes from NOVA-K
- Regions with significant <u>resonant</u> interaction well identified in (E,P_ζ,μ) space
- *How can these variations be accounted for in general-purpose codes such as TRANSP?*

All fast ion transport models presently implemented in TRANSP have *diffusive* nature; little/no phase space selectivity

- Four models presently implemented in TRANSP for fast ion diffusion coefficient, D_b

[from http://w3.pppl.gov/~pshare/help/transp.htm]

1) $$D_b = k_{ADIFB} \times D_e$$

k_{ADIFB} : *multiplier*
$D_e(x,t)$: electron particle diffusivity

2) $$D_b = k_{ADIFB} \times D_e^{WP}$$

$D_e^{WP}(x,t)$: electron particle diffusivity from Ware-pinch corrected flux

3) $$\Gamma_{fi} = -D_b \nabla n_b + v_b n_b$$

diffusion/convection model

4) $$D_b(E,t,x) = \Sigma_k \alpha_k D_k$$

$D_k(E,x,t)$: diffusivity for deeply trapped, barely trapped, co-barely passing, ...

New model introduces selectivity in phase space, generalizes "diffusive transport" paradigm

- Info on phase space dynamics paramount for Verification&Validation of codes, theory-experiment comparison : *need phase-space representation*
- Steady-state ("fluid") equilibrium does not provide all information we need : *need E, pitch details*
  - E.g. phase space transiently (and selectively) modified by *AE bursts : effects propagate during slowing-down

  ⇒ *Solutions for F_{nb} based on integral quantities (f.i. density, neutrons, E_r/rotation, NB-driven I_p, ...)* ***not unique***
- Time-scale τ_{res} for resonant/stochastic transport can be faster than diffusion time : *implementation?*

$$\tau_{transit} \leq \tau_{res} < [d(\log A_{modes})/dt]^{-1} \quad << \quad \tau_{diff} \ \text{(typically)}$$

- *The new model must be "simple" enough to be included in TRANSP/NUBEAM for routine use*
  - *Retain only minimum required amount of information to represent resonant/stochastic fast ion response to *AEs*
  - *e.g., no attempts to follow "single-particle" orbits*

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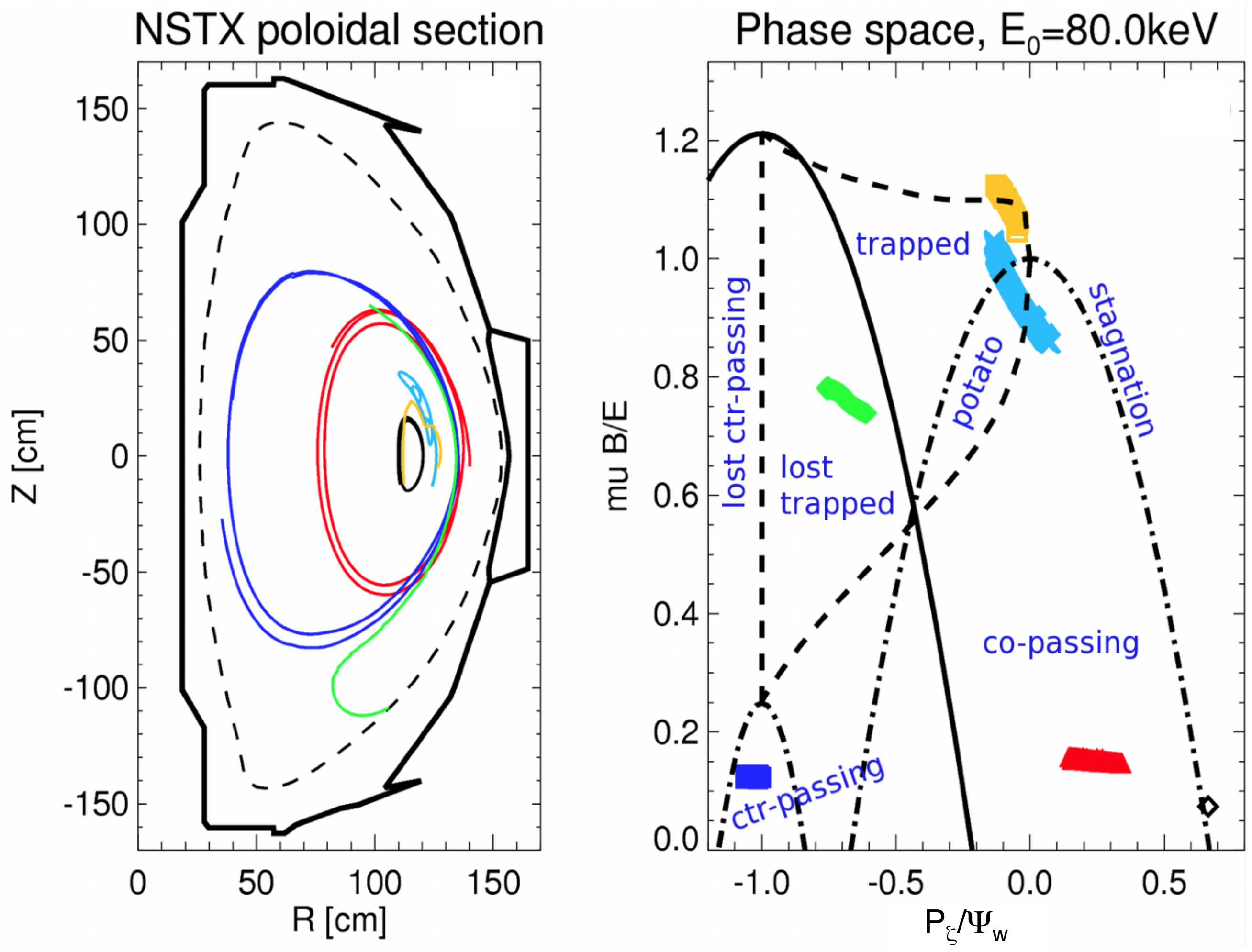
‘Constants of motion’ are the natural variables to describe resonant wave-particle interaction

- Each orbit fully characterized by:
  - E, energy
  - P_ζ, canonical angular momentum
  - μ, magnetic moment

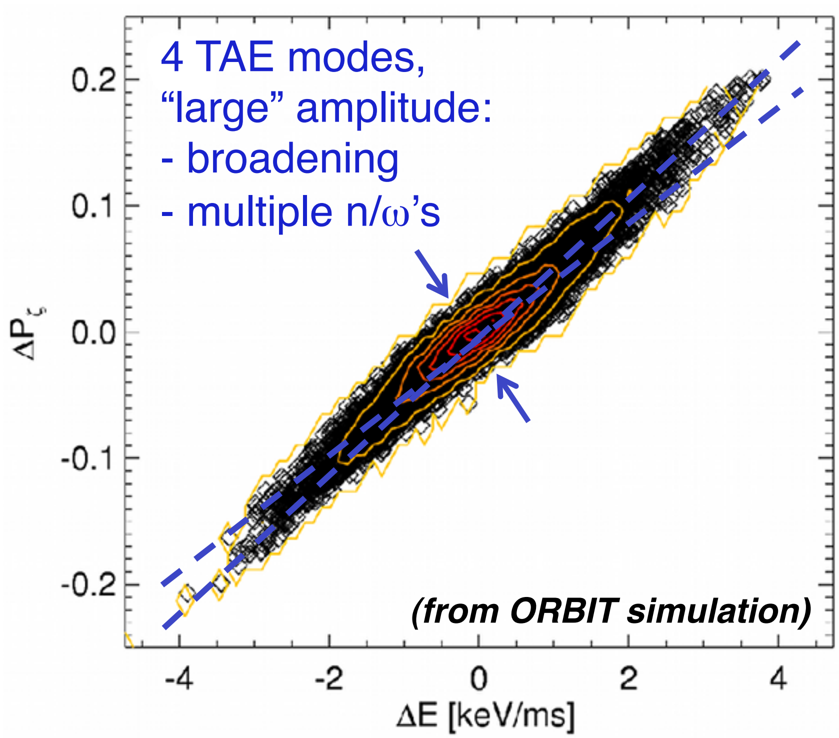


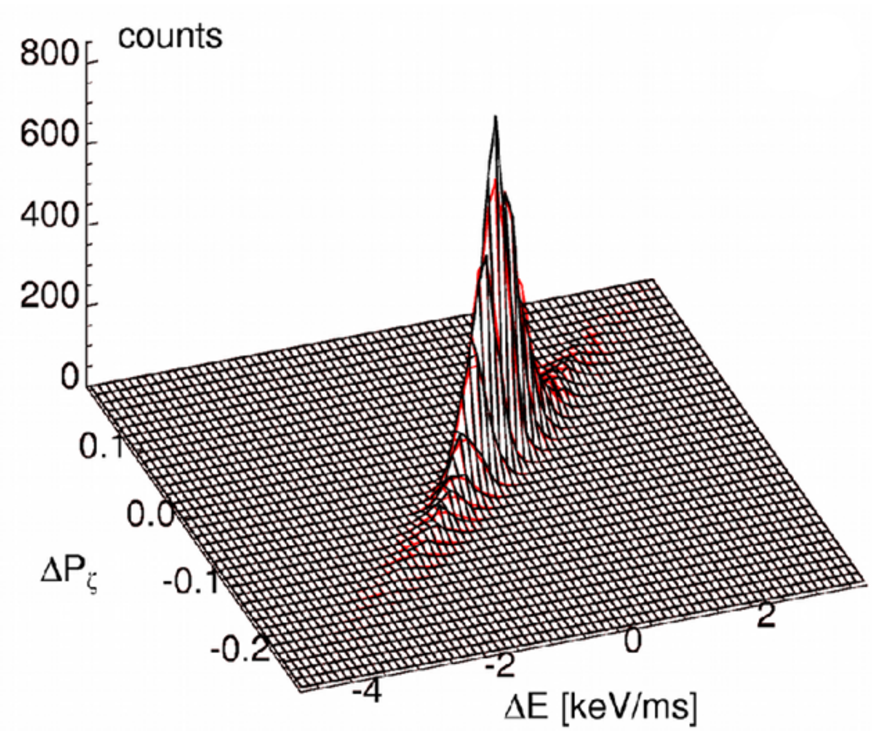
Single (isolated) resonances introduce simple constraints on particle's trajectory in (E,P_ζ,μ)

- From Hamiltonian formulation:

$$\omega P_\zeta - nE = const. \Longrightarrow \Delta P_\zeta / \Delta E = n/\omega$$

$\omega=2\pi f$, mode frequency n, toroidal mode number





Presence of multiple modes/resonances distorts the 'ideal' (linear) relationship

For low-frequency *AEs with $\omega \ll \omega_{ci}$ such as TAEs, magnetic moment μ is conserved (...well, *almost*)

- *In these notes, it is assumed that $\Delta\mu=0$*

- However: $\Delta\mu=0$ hypothesis may break down if
  - ρ_f ~ radial width of the modes
  - ρ_f ~ scale-length of equilibrium profiles
  - ➪ *Both conditions are likely to be met in spherical tokamaks (e.g. NSTX)*

➪ **Proposed model can be generalized to cases where μ is *not* conserved**
  - Also important for ω_{ci}–range instabilities: GAE/CAEs

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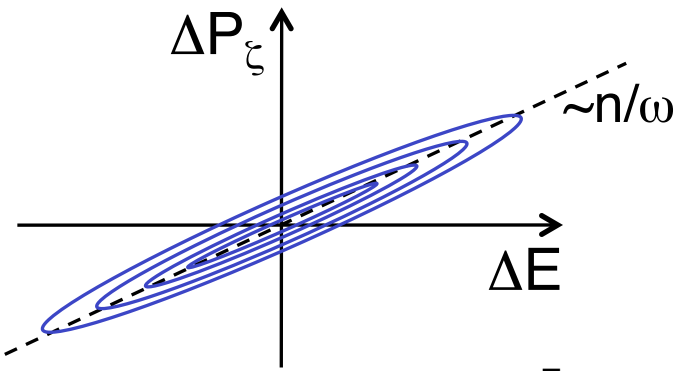
Single-resonance case: introducing the *probability distribution function* for particle transport



For each *bin* in (E,P_ζ,μ), steps in ΔE,ΔP_ζ can be described by bivariate $p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu, A)$

$$p = p_0\, e^{-\frac{1}{2(1-\rho)}\left[\frac{(\Delta E - \Delta E_0)^2}{\sigma_E^2} + \frac{(\Delta P_\zeta - \Delta P_{\zeta 0})^2}{\sigma_{P_\zeta}^2} - 2\rho \frac{(\Delta E - \Delta E_0)(\Delta P_\zeta - \Delta P_{\zeta 0})}{\sigma_E\, \sigma_{P_\zeta}}\right]}$$

$$p_0 = \frac{1}{2\pi\, \sigma_E\, \sigma_{P_\zeta}\, \sqrt{1-\rho^2}}$$ *normalization*

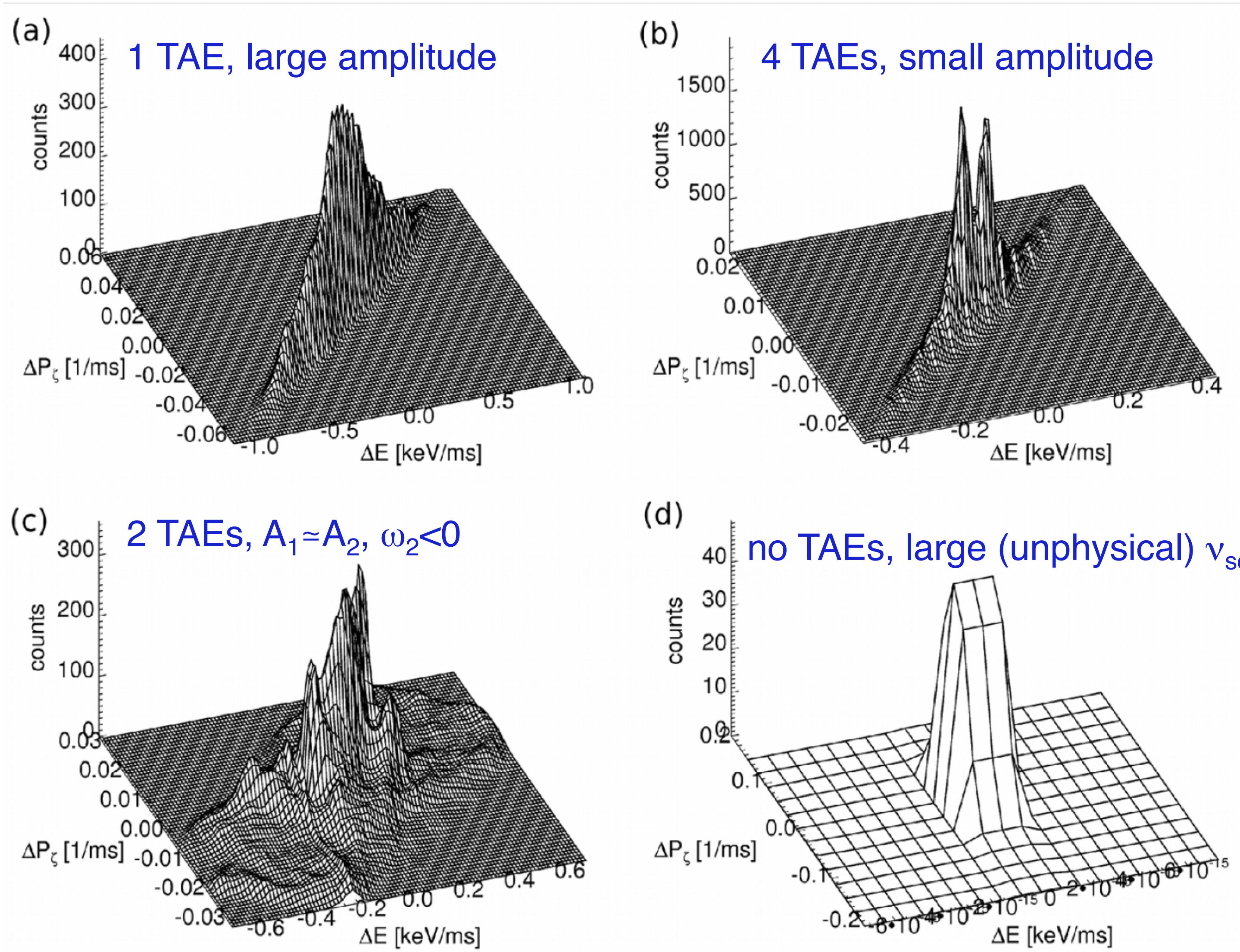
$$\rho = \frac{< (\Delta E - \Delta E_0)(\Delta P_\zeta - \Delta P_\zeta 0) >}{\sigma_E\, \sigma_{P_\zeta}}$$ *correlation parameter,*

such that $\Delta P_\zeta(\Delta E) = \Delta P_{\zeta 0} + sign(\rho) \times \frac{\sigma_{P_\zeta}}{\sigma_E}(\Delta E - \Delta E_0)$

...and all parameters depend, in principle, on the mode amplitude A=A(t)

Analytical formulation *could* be extended to multi-mode case – but it would be quite unpractical

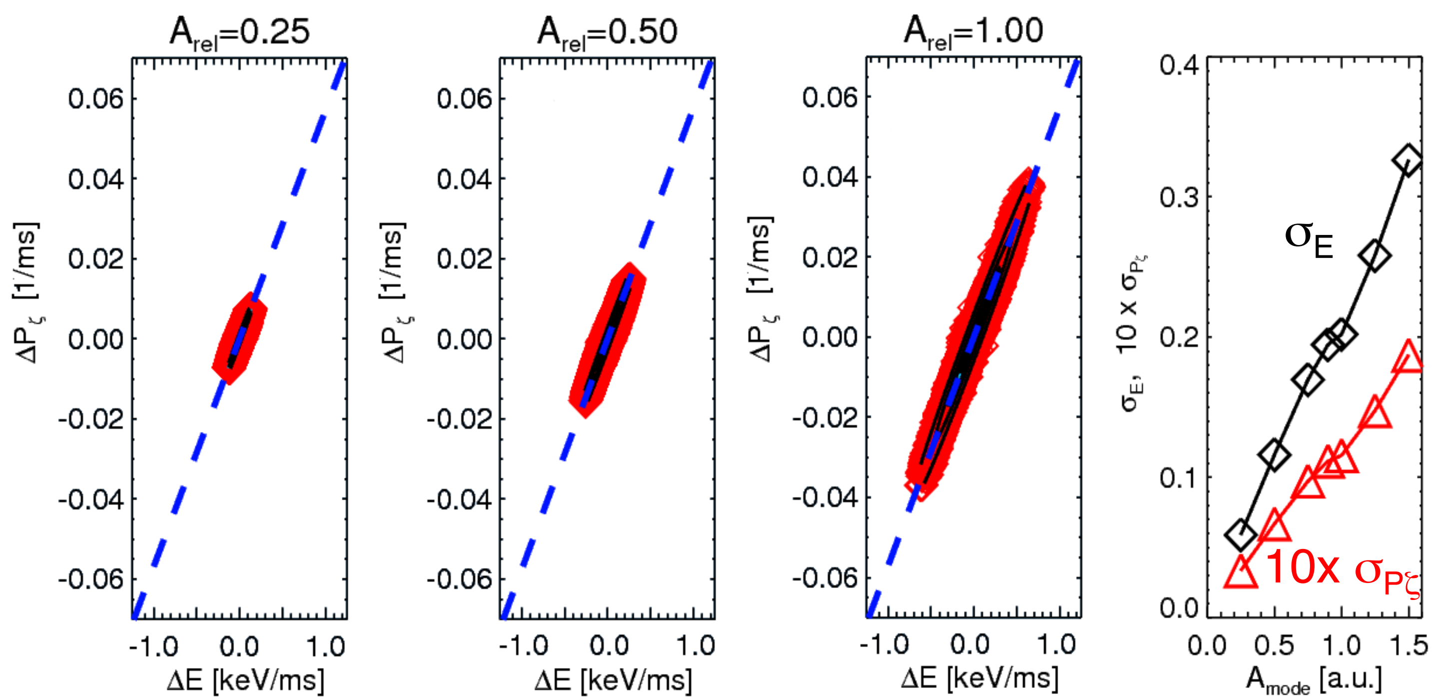
E.g., use: $p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu) = \Sigma_{i=1}^{N} w_i \, p_i(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu)$



In practice, too many parameters to be passed to TRANSP/NUBEAM

General case: reduce number of parameters; dependence on $A=A_{modes}(t)$ can be simplified; *use raw $p(\Delta E,\Delta P_\zeta)$ directly*

- 'Convective' terms ΔE_0, $\Delta P_{\zeta 0}$ are =0 if slowing down is accounted for elsewhere
- Variances $\sigma_E,\sigma_{P\zeta}$ are roughly linear with A_{modes}:
  - Specify $p(\Delta E,\Delta P_\zeta)$ for $A_{modes}=1$ only, then re-scale $\sigma\rightarrow\sigma'=A_{modes}\times\sigma$



➩ *Pass the raw matrix $p(\Delta E,\Delta P_\zeta|P_\zeta,E,\mu)$ to TRANSP/ NUBEAM directly – no analytical form required*

Layout of TRANSP/NUBEAM implementation (as seen from an experimentalist...)

- Compute $p(\Delta E,\Delta P_\zeta|P_\zeta,E,\mu)$
  - ORBIT, SPIRAL, theory
  - *Generalize to 6D matrix $p(\Delta E,\Delta P_\zeta,\boldsymbol{\Delta\mu}|P_\zeta,E,\mu)$ if μ is not conserved
- Pass 'Ufiles' to TRANSP/NUBEAM
  - 2 files: 5D matrix* for $p(\Delta E,\Delta P_\zeta,\Delta\mu|P_\zeta,E,\mu)$, vector for $A_{modes}(t)$
- Evolve F_{nb} at each step:
  I. Re-normalize bins (P_ζ,E,μ) based on q-profile, fields, ...
  II. Identify 'bin' in (P_ζ,E,μ) for current 'particle' (i.e. orbit)
  III. Extract $\sigma_E,\sigma_{P\zeta}$ (σ_μ) from multivariate $p(\Delta E,\Delta P_\zeta,\Delta\mu)$
  IV. Calculate step magnitude, based on $A_{modes}(t)$
  V. Compute slowing down, scattering
  VI. Advance particle's trajectory in phase space

  Loop over particles (steps II–VI)

  (steps II-VI actually divided in sub-steps)

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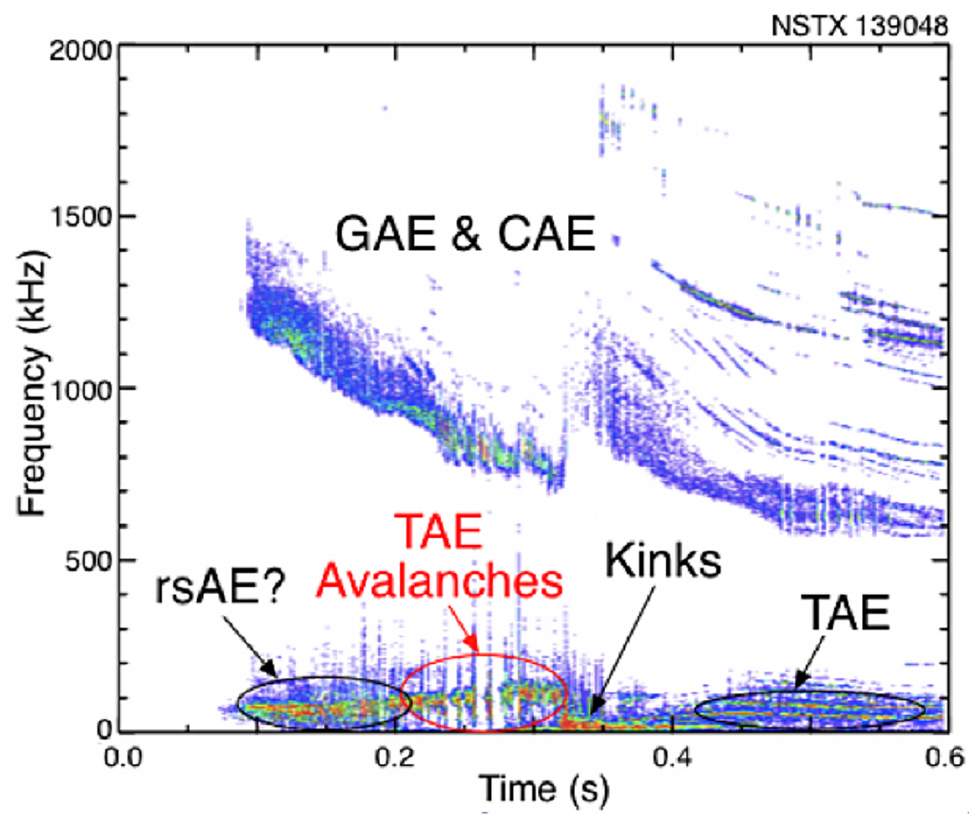
Generalize to multiple MHD classes?

- Scenarios with presence of more than one MHD type of modes are quite common
  - NSTX: kink + GAE/CAEs + TAEs
- Formulation discussed so far can only deal with one class at the time
- However, this general case is of great interest
  - More realistic F_{nb} evolution when modes have comparable amplitude
  - Can mock-up scenarios such as 'fast ion channeling'



- To extend the model to multi-class case, each class can be represented by its $p_k(\Delta E,\Delta P_\zeta,\Delta\mu)$
  - MonteCarlo scheme becomes more complicated: how to decide which class is 'active' for a specific bin (P_ζ,E,μ)?
  - Is the effect of different classes simply additive?
  - *Is this something worth pursing from the beginning?*

Use the model in 'predictive' mode?

- As it is, the model would be OK to analyze 'real' discharges
  - Need mode structure to calculate $p(\Delta E, \Delta P_\zeta)$, e.g. from NOVA-K and reflectometers' data
  - Need data (Mirnovs, neutrons, etc.) to infer $A_{modes}(t)$

- Two possibilities for 'predictive' runs:
  - Find reasonable guess for unstable modes (e.g. NOVA-K)
  - Explore different scenarios w/ scan of $A_{modes}(t)$: weak *AE activity, bursts/avalanches, etc.

  or:

  - Have an additional module to compute $A_{modes}(t)$ self-consistently
  - Still requires mode structure, probably estimates for γ_{drive}, γ_{damp}
  - Let $A_{modes}(t)$ evolve according to F_{nb} evolution – how?
  - *Couple to Quasi-Linear model? Use Q-L instead?*

Additional questions, comments, open issues

Scope/approach:

- Which particular aspects targeted by new model are not included in existing models?
- Are there too many (T)AE resonances that can be accounted for by the model?
- Alfvénic time scale too fast for diffusion, look for steady-state solution?
- Approach looks OK, but is it providing too much detailed info?

Implementation:

- Should $\Delta\mu \neq 0$ case be included by default?
- What new code is required in TRANSP/NUBEAM? Need to include parts from ORBIT (SPIRAL, else)?
- How to optimize sub-steps for orbit calculation? Take A(t) time-scale as reference?
- Is 'normalization' of $p(\Delta E, \Delta P_\zeta)$ vs. time using B0, Ψ_w, ... OK?
- Is simple re-scaling $\sigma_E' = A_{mode} x \sigma_E$ OK, or need to specify a separate scaling function $\sigma_E' = f(A_{mode}) x \sigma_E$?

Test/check-out:

- May need to start with "simple" case, e.g. internal kink w/ $\Delta E=0$, $\Delta P_\zeta \neq 0$ and no energy dependence for given resonance.

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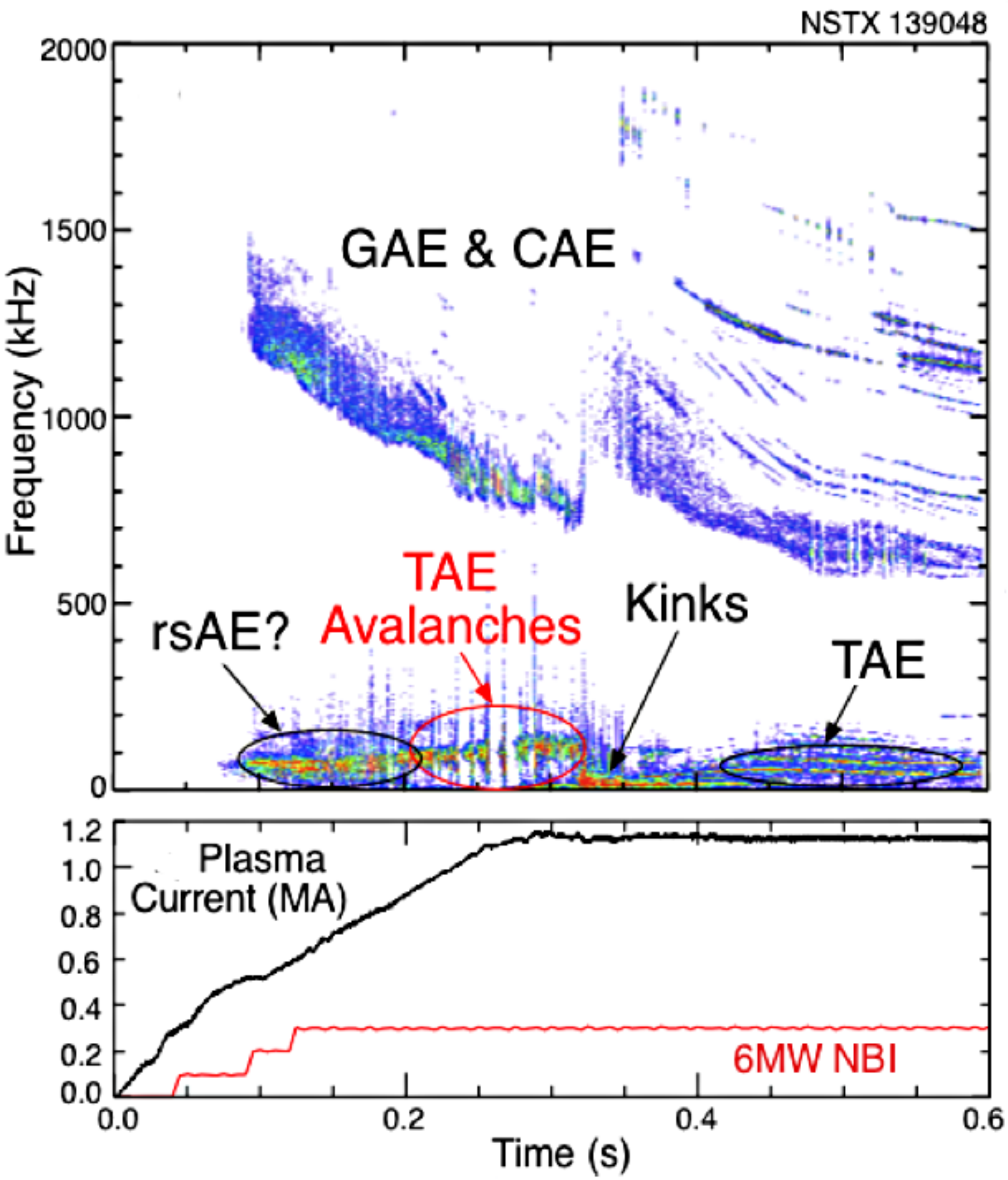
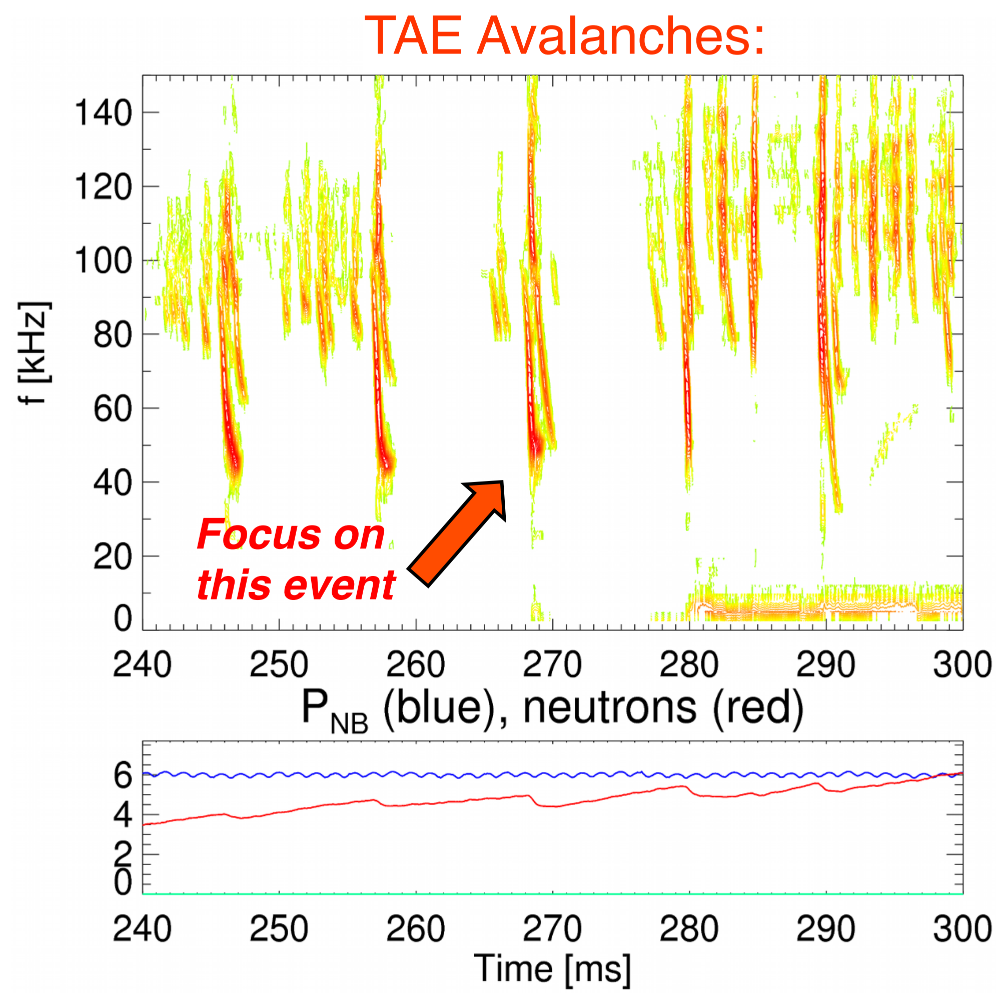
Experimental scenario for tests presented in these notes: NSTX H-mode plasma with bursts of TAE activity

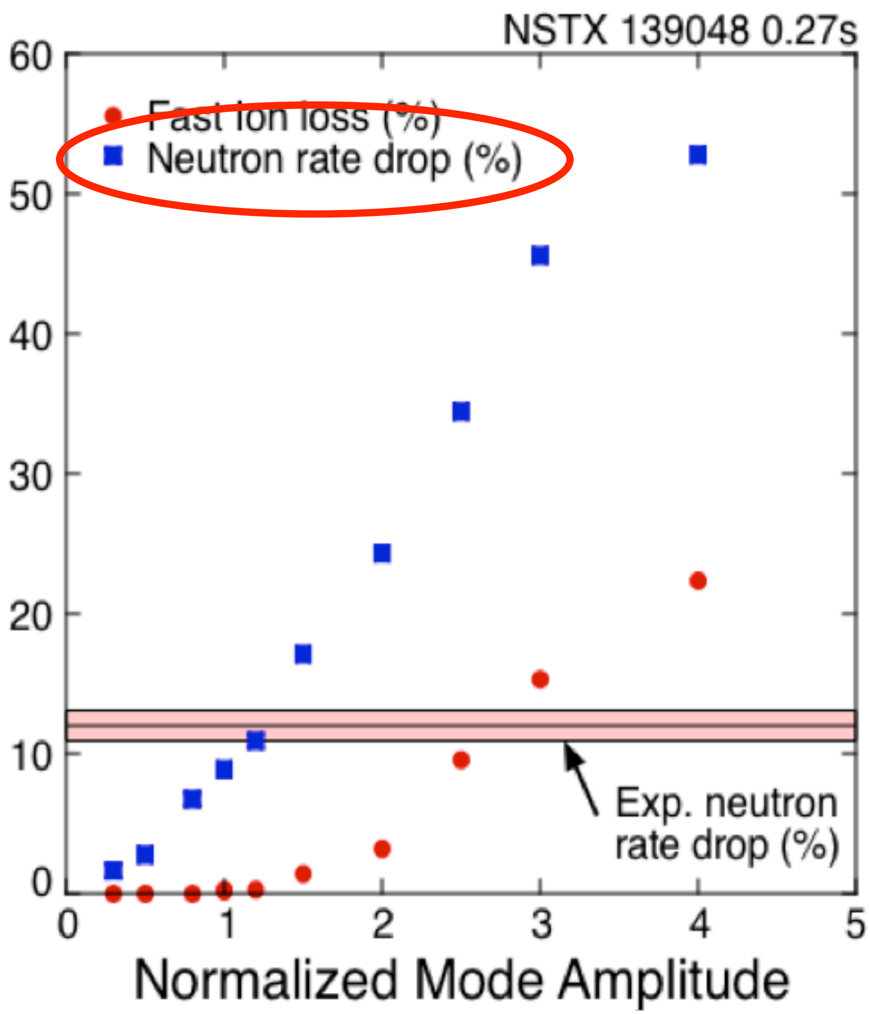




E. Fredrickson et al., Nucl. Fusion (2012, submitted)

Ex#1: compute the 'mode amplitude', A(t) /1

- First step is to obtain the relation between mode amplitude and 'transport':

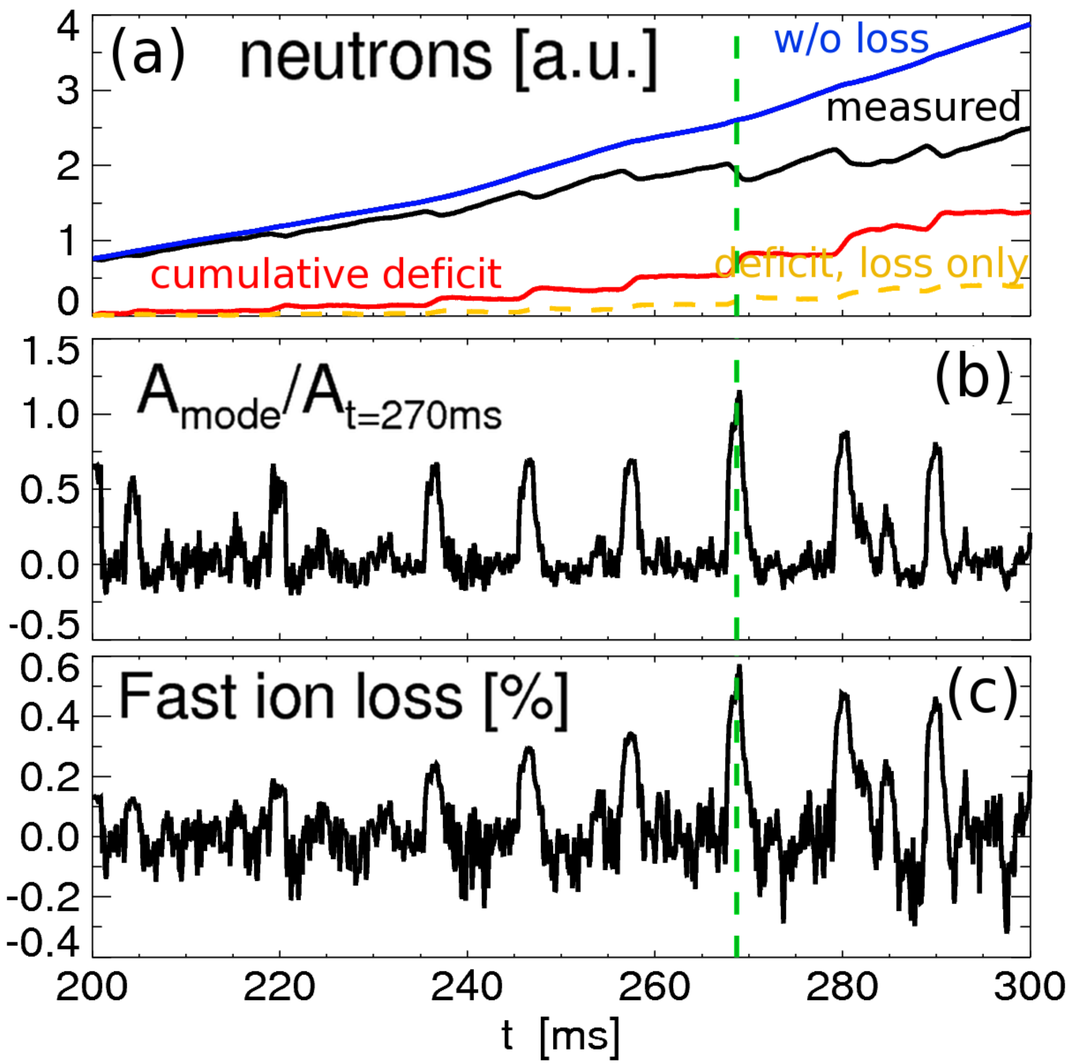


E. Fredrickson et al., Nucl. Fusion (2012, submitted)

- Ideal modes from NOVA-K
- Rescale modes based on comparison with reflectometers
- Compute transport, expected neutron drop with ORBIT
- Scan mode amplitude w.r.t. experimental one, A=1

Ex#1: compute the 'mode amplitude', A(t) /2

- Get A(t) from measured neutrons + 'table look-up':



– Compute fractional R_n drops vs. time

– Use figure from previous slide to find corresponding (normalized) mode amplitude

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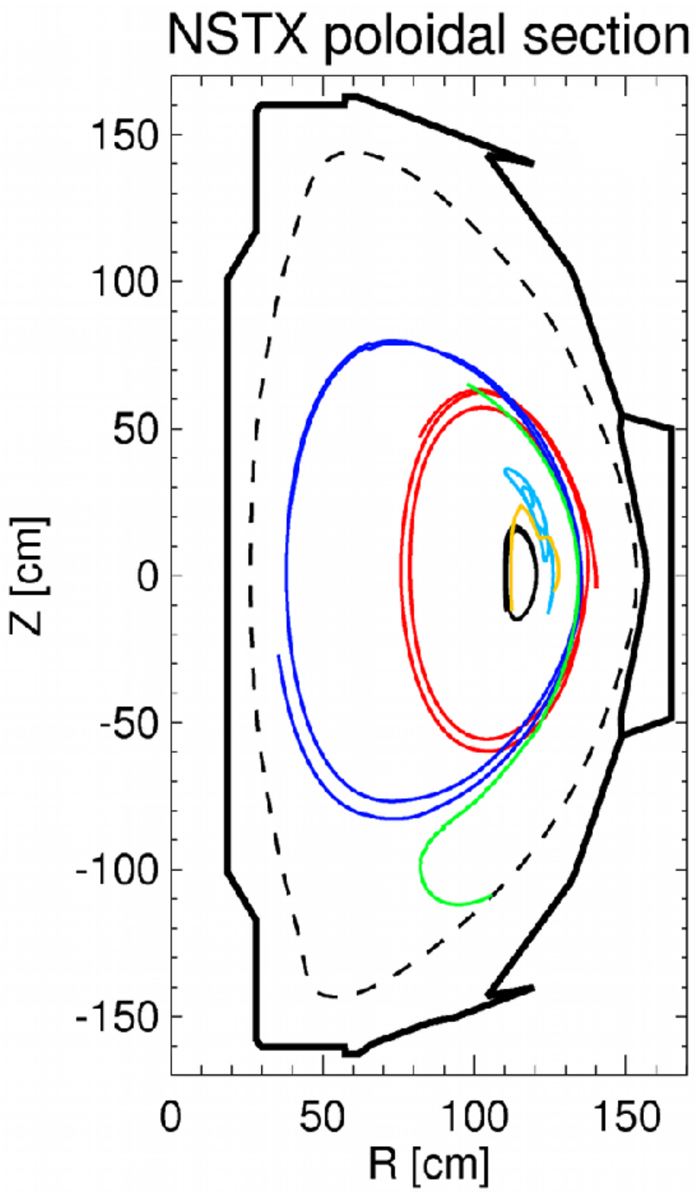
Ex#2: deriving the transport coefficients



- Run ORBIT with A=1, constant
- Simulation time long enough (~1ms) to capture *AE effects
- Track (P_ζ,E,μ) in time for each particle, steps δt_{sim}~50μs
- Compute ΔE,ΔP_ζ,Dμ
- Re-bin over (P_ζ,E,μ) space
- Get p(ΔE,ΔP_ζ|P_ζ,E,μ) for each bin

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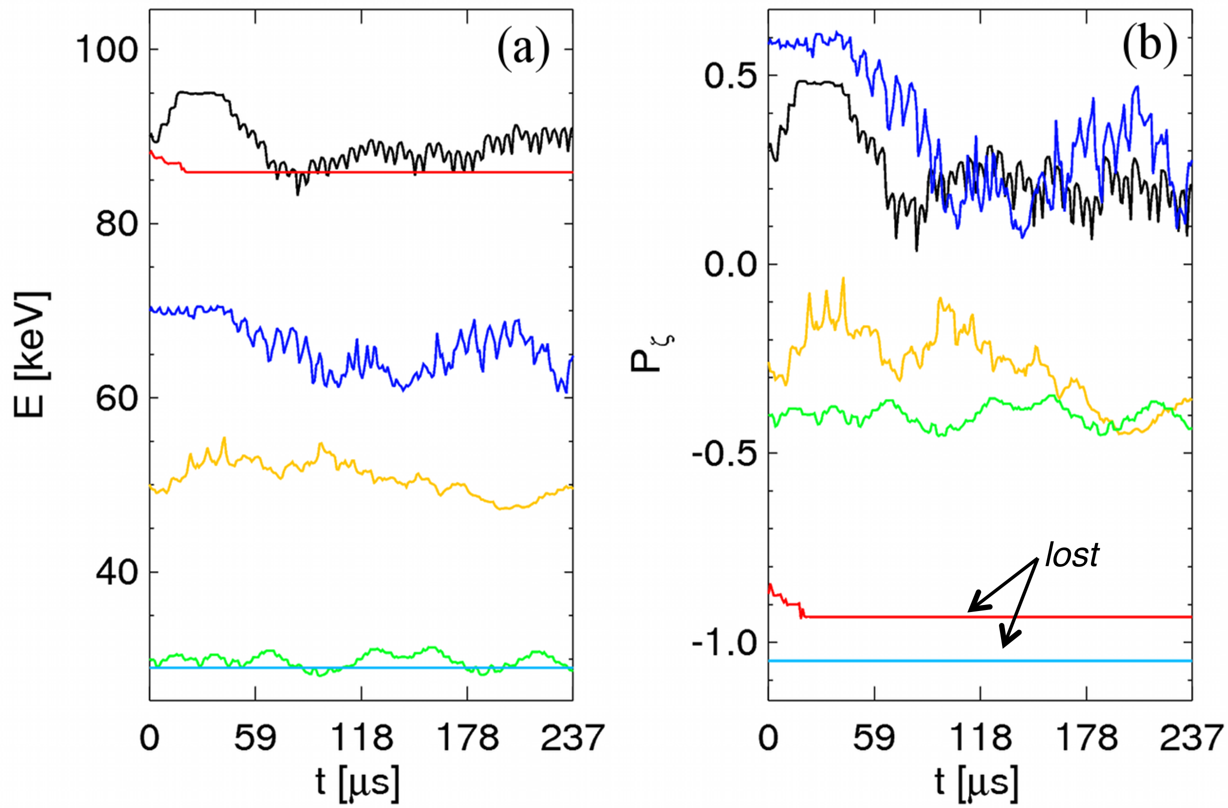
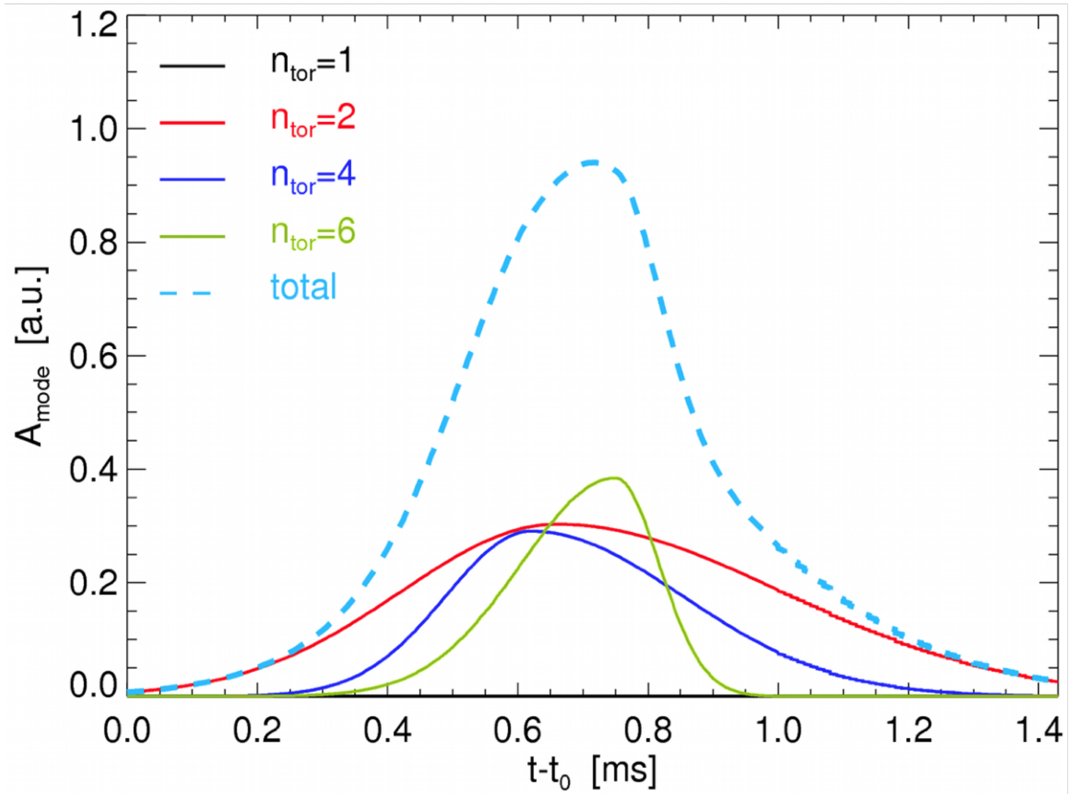
Ex#3: evolving F_{nb} in time

- Particle's motion is characterized by different time-scales:
  - Oscillation in wave field – *neglected*
  - 'Jumps' $\Delta E, \Delta P_\zeta$ around instantaneous energy, P_ζ
  - Slow drift from initial energy, P_ζ



Mode amplitude can evolve on time-scales shorter than typical TRANSP steps ~5-10 ms

- Need to represent F_{nb} evolution as a sequence of sub-step
  - Duration δt_{step} sufficiently shorter than time-scale of mode evolution
  - Examples here have δt_{step}~25-50 μs
  - ***Is this compatible with NUBEAM scheme?***



Discrete bins in (P_ζ,E,μ) can contain both *resonant* and *non-resonant* particles

- 'Resonant' particles undergo large ΔE, ΔP_ζ variations
- 'Non-resonant' particles have small fluctuations around initial energy, P_ζ

- To keep track of particle's class:
  - Sample steps σ_E, $\sigma_{P\zeta}$ at first step only
  - exception: particle move to a different bin -> re-sample

$p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu)$ can be skewed to positive/negative $\Delta E, \Delta P_\zeta$, causing overall drift of $F_{nb}(P_\zeta, E, \mu)$

- Introduce 'random sign' for *i*–th step in MC procedure, $S_{r,i}$
- For each particle (e.g. pair of correlated steps σ_E, $\sigma_{P\zeta}$), calculate $S_{r,i}$ from probability of positive vs. negative steps
  - From $p(\Delta E, \Delta P_\zeta)$ compute

$$p_+ \doteq p(\sigma_{E,i}, \sigma_{P_\zeta,i}) \quad ; \quad p_- \doteq p(-\sigma_{E,i}, -\sigma_{P_\zeta,i})$$

  - Then define f_{sign}:

$$f_{sign} = \frac{p_+}{p_+ + p_-}$$

  - Finally, use $0<f_{sign}<1$ to bias random extraction of $S_{r,i}=+1,-1$

Putting all together

- At each 'macroscopic' TRANSP step:

Loop over particles (steps II–VIII)

I. Re-normalize bins (P_ζ,E,μ) based on q-profile, fields, ...
II. Identify 'bin' in (P_ζ,E,μ) for current 'particle' (i.e. orbit)
III. Extract steps σ_E,$\sigma_{P\zeta}$ (σ_μ) from multivariate p(ΔE,ΔP_ζ,$\Delta\mu$)
IV. Compute sign $S_{r,i}$ from p(ΔE,ΔP_ζ)
V. Rescale steps based on A_{modes}(t):
VI. Advance E, P_ζ (μ):

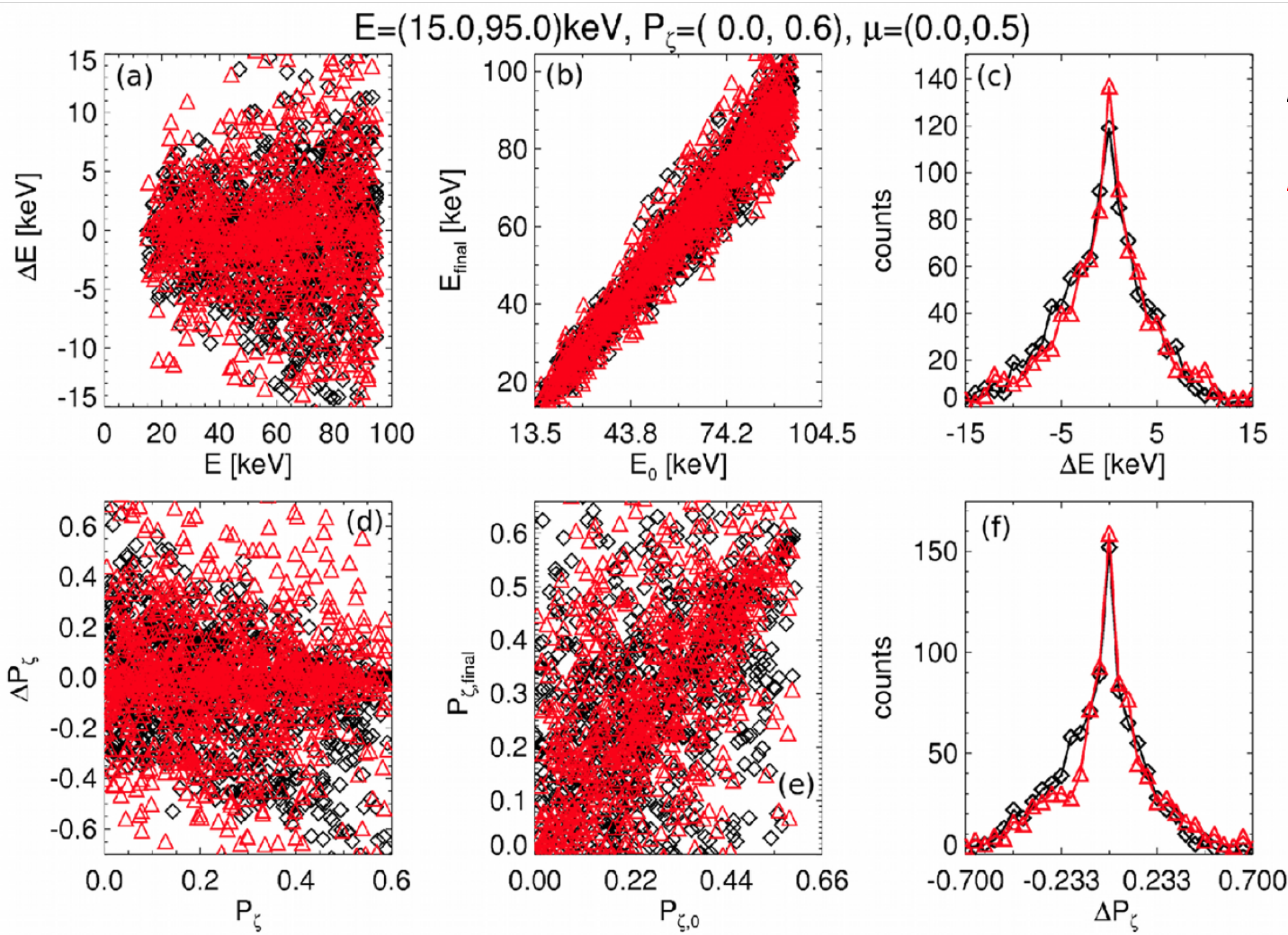
$$\overline{\Delta E}_i = S_{r,i} \times A_{mode}(t=\bar{t}) \times \sigma_{E,i}$$

$$E_i = E_i + \overline{\Delta E}_i$$

VII. Compute slowing down, scattering
VIII. Advance particle's trajectory in phase space

Steps III–VII divided in sub-steps for each particle

Example: evolving F_{nb} over 270 μs in 5 sub-steps

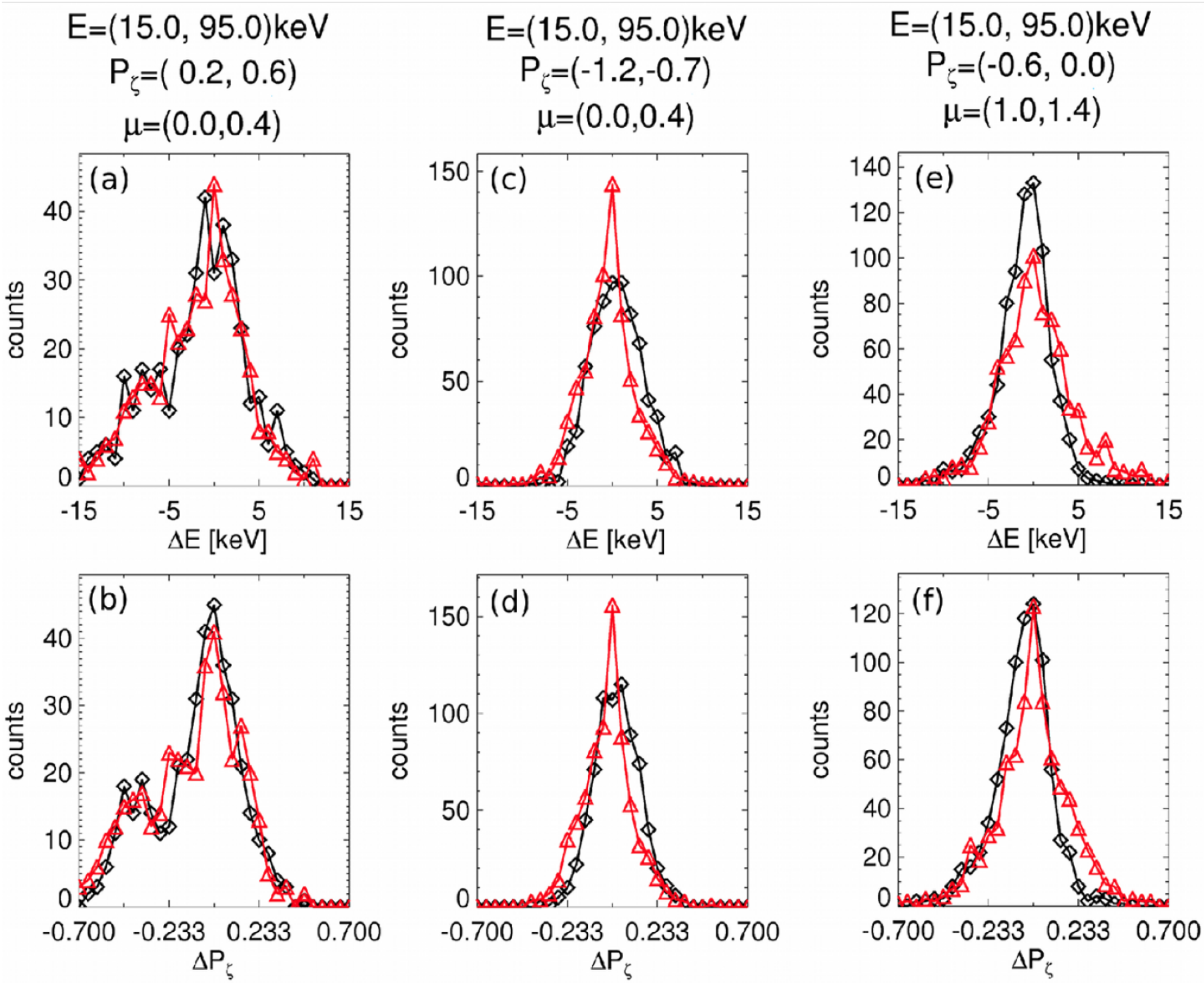


black: ORBIT

red: re-constructed

Reconstruction works (decently) for different classes: co–, counter–, trapped

co-passing | *counter-passing* | *trapped*



black: ORBIT

red: re-constructed

Reconstruction works (decently) at different sub-steps

black: ORBIT ***red: re-constructed***