



U.S. DEPARTMENT OF
ENERGY

Office of
Science

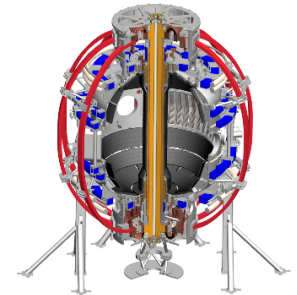


Aspect ratio dependence of tearing stability

Lucas Morton

3DSP Meeting
11/14/2016

ORAU



Multi-device comparison can test aspect ratio dependences in modified Rutherford equation

- Terms vary in different ways

$$1.22^{-1} \frac{\tau_R}{r} \frac{dw}{dt} = \Delta' r + C_R \frac{r D_R}{w} + \left(\epsilon^{1/2} \frac{r L_q}{L_{pe}} \beta_{\theta e} \left[\frac{1}{w} - \frac{w_{small}^2}{3w^3} \right] \right)$$

$$\epsilon = r/R_0$$

$$w_{small}^2 \propto \epsilon ?$$

$$D_R \propto \epsilon^2 \beta_{\theta}$$

La Haye *et al*, PoP **19**, 062506 (2012)

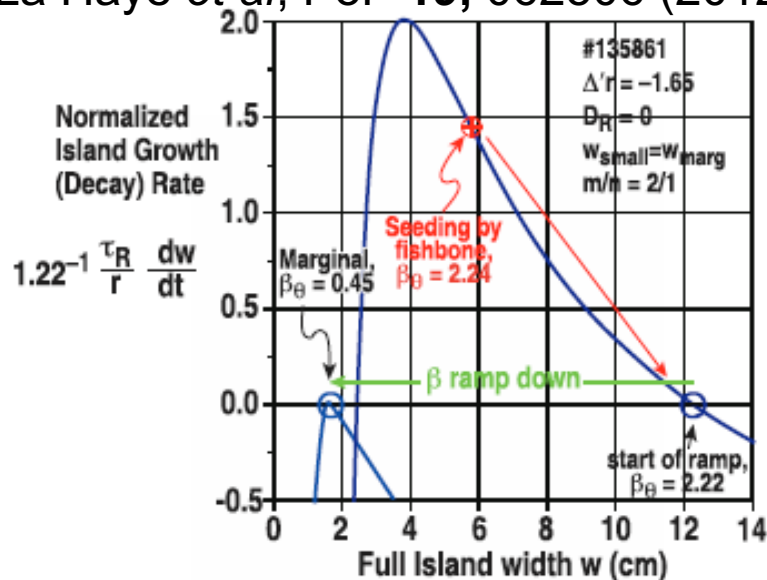


FIG. 9. Evaluation of the MRE for DIII-D $m/n = 2/1$ case of #135861 from Fig. 3. Fitting at the marginal point with the D_R term neglected. The same fitting parameters are used for the saturated mode at the start of the NBI ramp at higher beta (except $w_{small} \propto \beta_{\theta}^{1/2}$ assumed). And in red, the initially growing island width seeded by a fishbone (at similar beta to the start of the ramp) is noted.

The modified Rutherford equation (MRE) defines a model for neoclassical tearing mode evolution

- Adds geometric (curvature) and kinetic (bootstrap) effects to classical instability

Normalized 'growth' rate

Interchange (curvature) stabilization

Neoclassical bootstrap destabilization

$$1.22^{-1} \frac{\tau_R}{r} \frac{dw}{dt} = \Delta' r + C_R \frac{r D_R}{w} + \varepsilon^{1/2} \frac{r L_q}{L_{pe}} \beta_{\theta e} \left[\frac{1}{w} - \frac{w_{small}^2}{3w^3} \right]$$

Classical stability (current gradient drive)

Scales as w^{-1} for large w

Goes to zero at $w = 0$

Neoclassical tearing modes follow MRE: large island limit

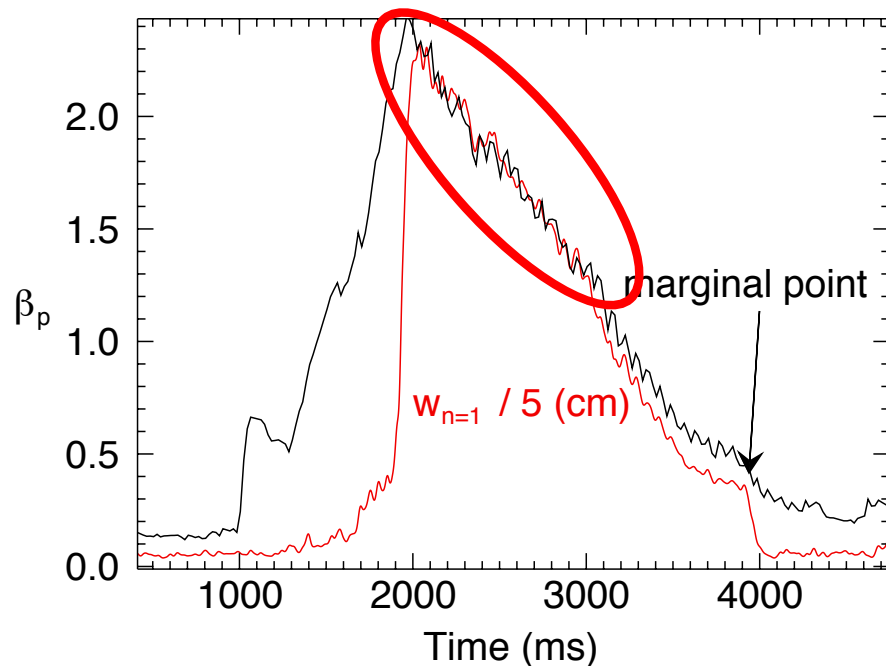
- At large island width, $w \propto \beta / \Delta'$
 - Assuming w 'small'
 - Curvature small in DIII-D

$$1.22^{-1} \frac{\tau_R}{r} \frac{dw}{dt} = \Delta' r + C_R \frac{r D_R}{w} + \varepsilon^{1/2} \frac{r L_q}{L_{pe}} \beta_{\theta e} \left[\frac{1}{w} - \frac{w_{small}^2}{3w^3} \right]$$

Diagrammatic annotations: A blue arrow points to the left-hand side of the equation. A red circle highlights $\Delta' r$. A green arrow points to $r D_R$. A red circle highlights $\beta_{\theta e}$. A black arrow points to the term $\frac{w_{small}^2}{3w^3}$.

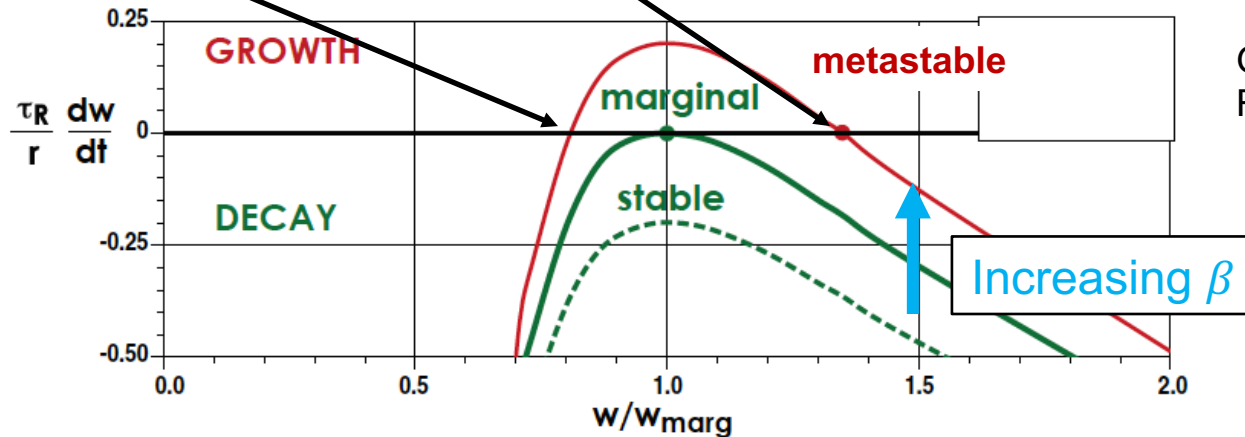
Slow β ramp-down experiment on DIII-D

Shot 135861



MRE has bifurcation in solutions

- Two (non-zero) solutions over w for $\dot{w} = 0$ for large β
 - One repeller, one attractor

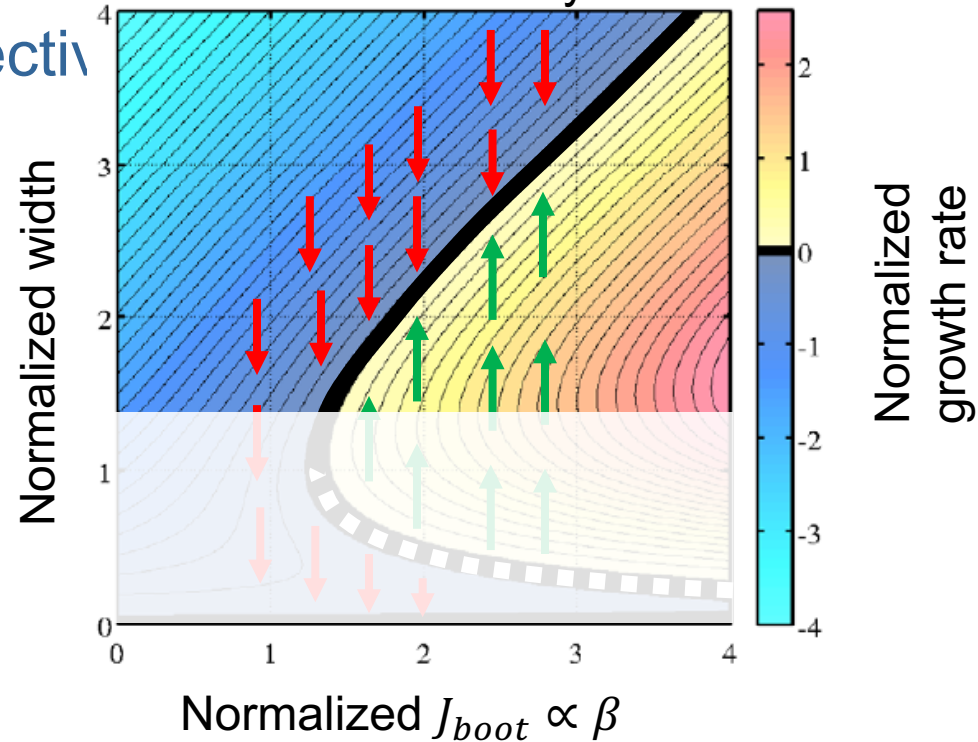


- No (non-zero) solutions at low β
 - Solutions merge, disappear at ‘marginal point’

Solutions to MRE can be represented graphically

- Attractor: $\rightarrow| \leftarrow$
 - ‘Stable’ branch (from perspective of equilibrium)
- Repellor: $\leftarrow| \rightarrow$
 - ‘Unstable’ branch

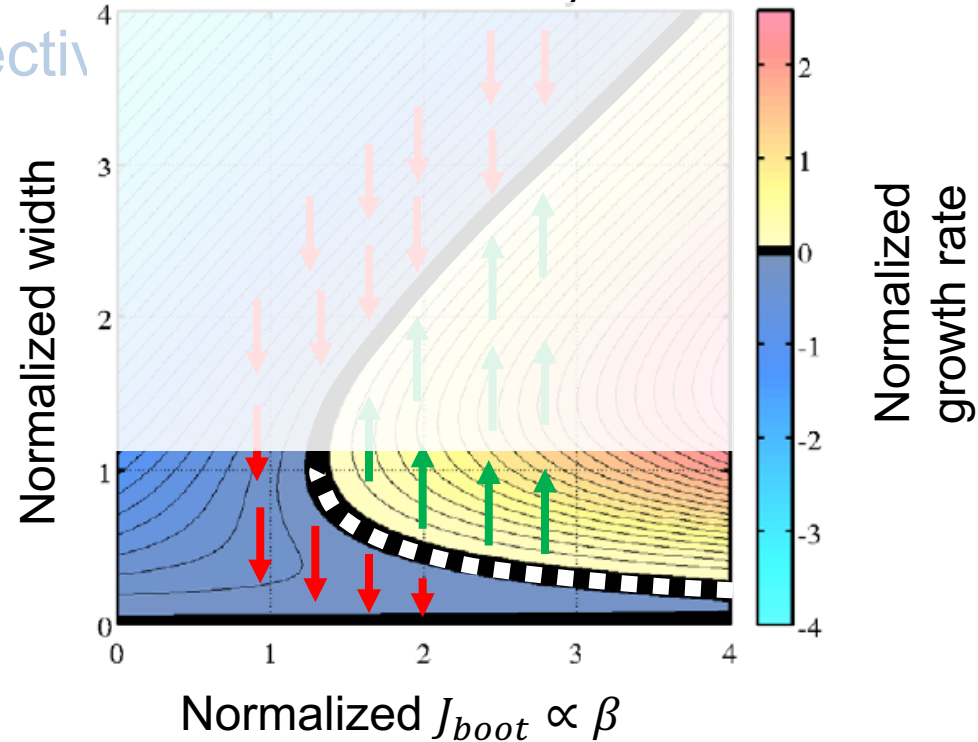
D. L. Rayburn, PhD thesis,
Princeton University 2011



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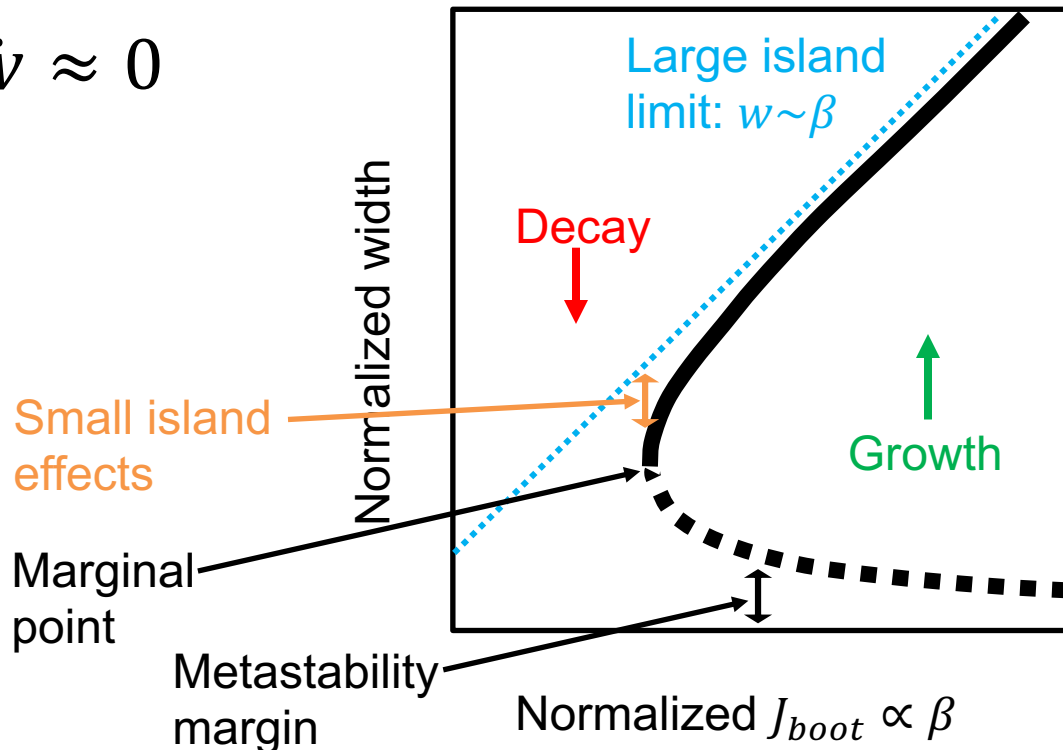
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D. L. Rayburn, PhD thesis,
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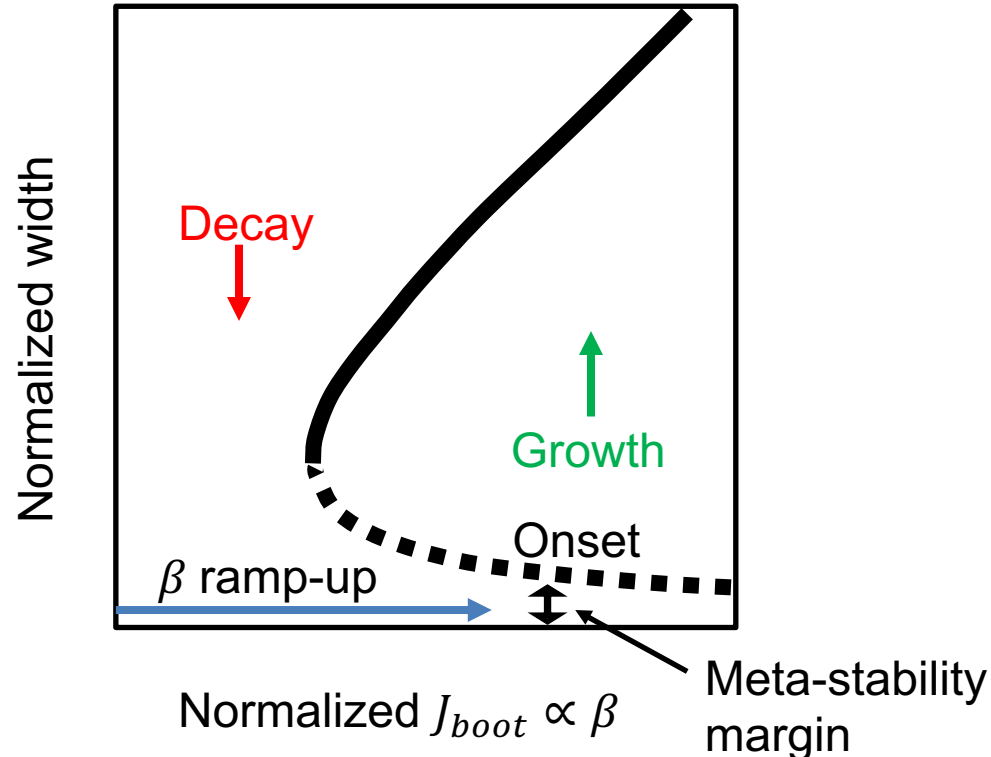
Solutions to MRE can be represented graphically

- Simplify to stability boundary for qualitative picture
- Keep in mind: $\dot{w} \approx 0$



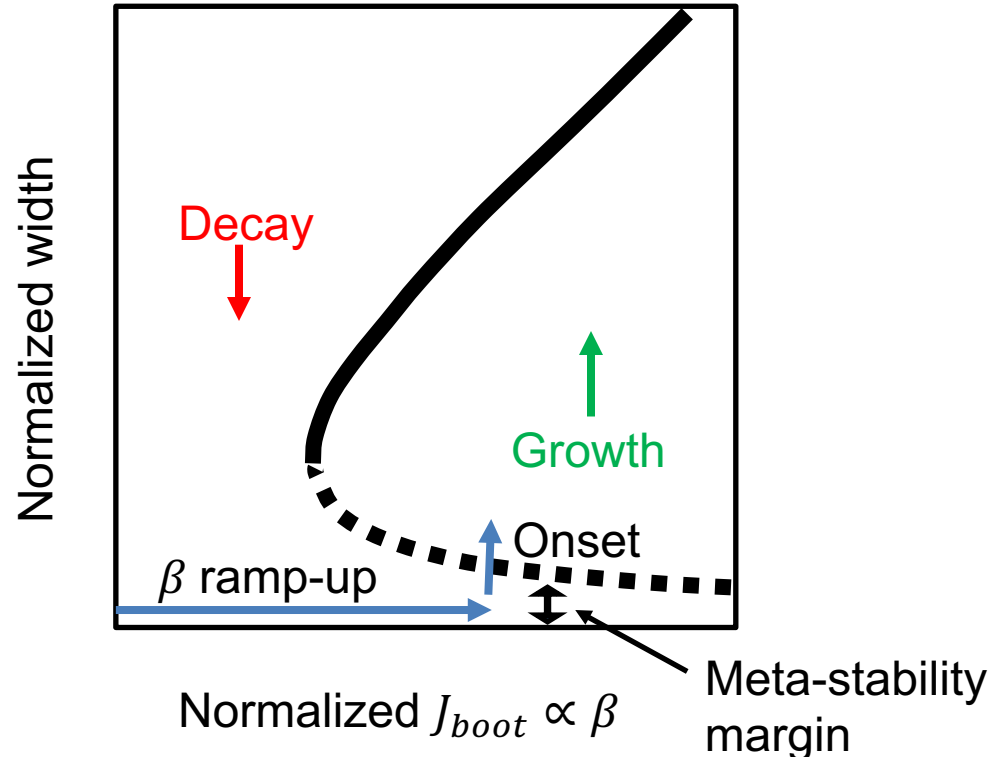
Trajectories during β ramp up/down exhibit hysteresis

1. Ramp-up: $w = 0$



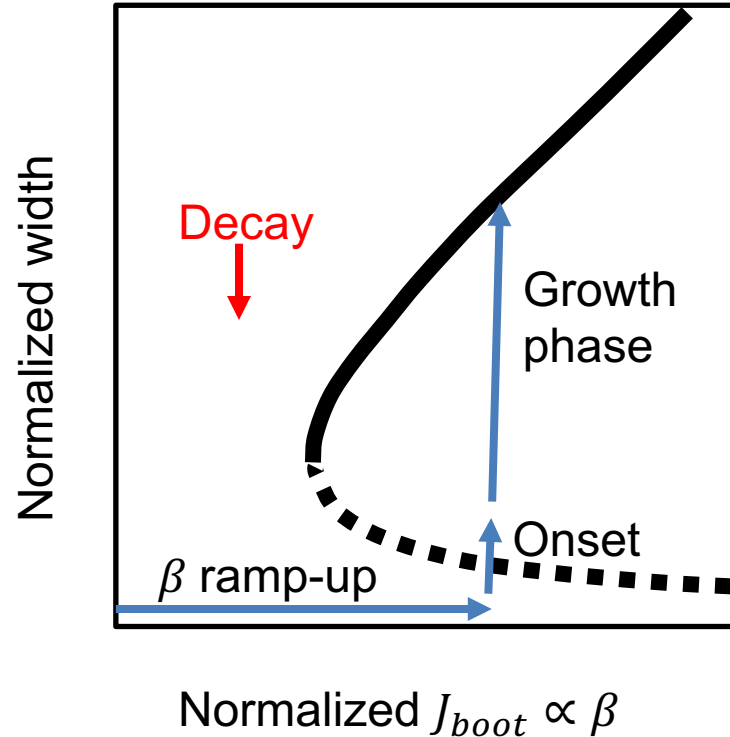
Trajectories during β ramp up/down exhibit hysteresis

1. Ramp-up: $w = 0$
2. Onset: something 'kicks' mode over boundary



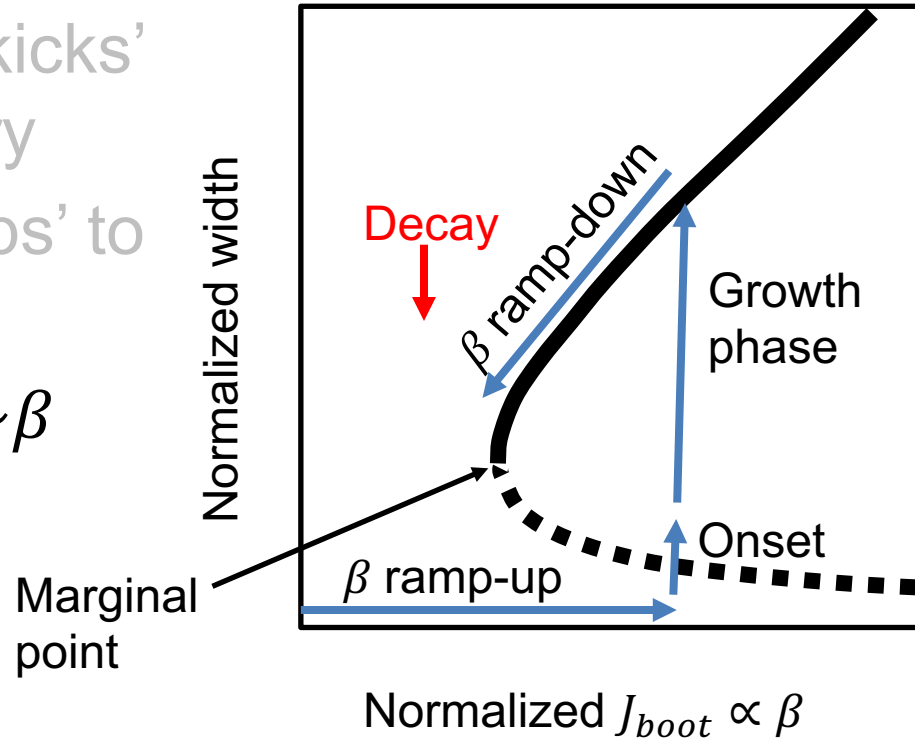
Trajectories during β ramp up/down exhibit hysteresis

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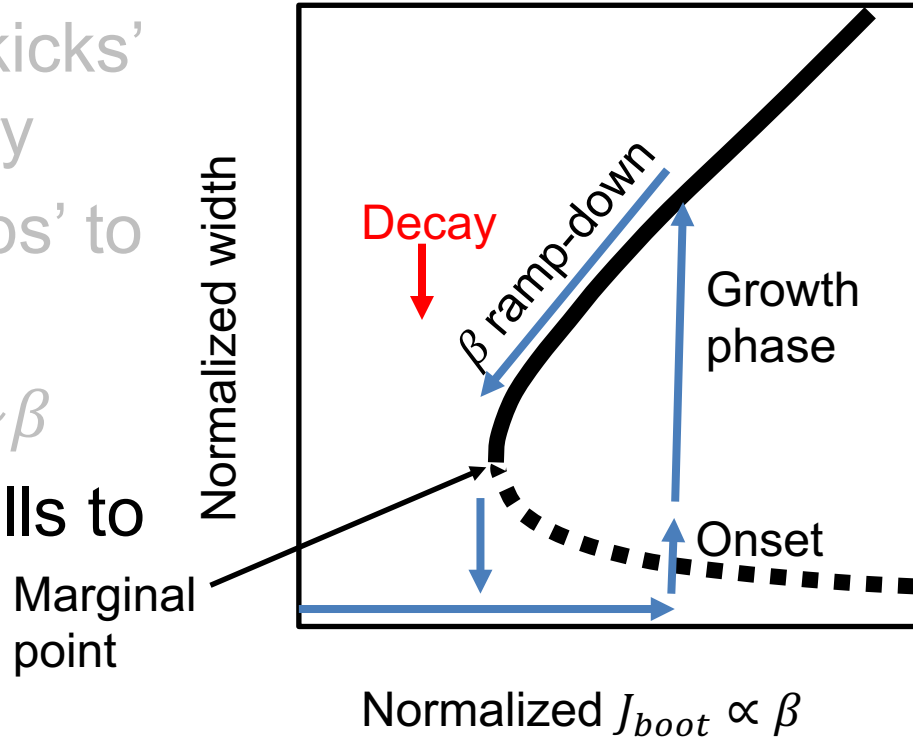
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4. Saturated state: $w \sim \beta$



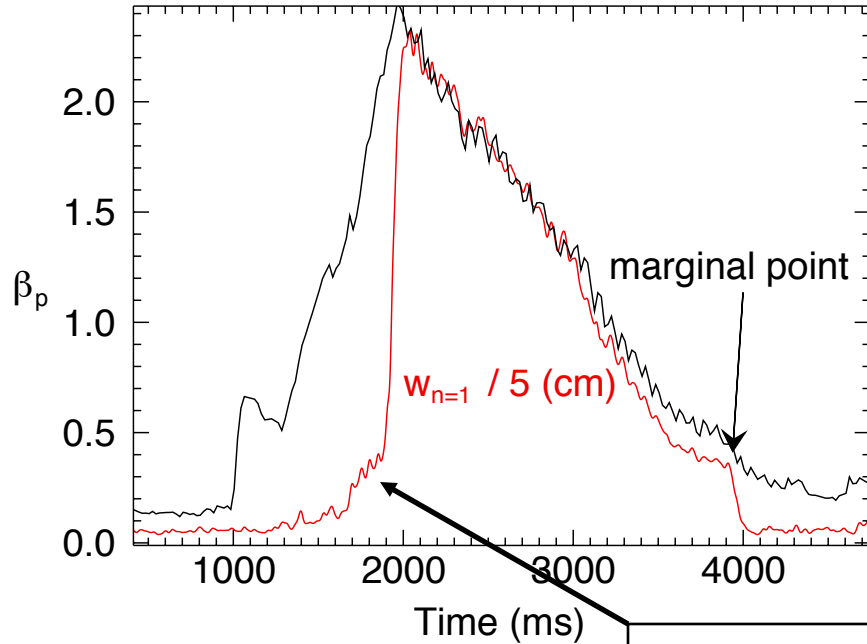
Trajectories during β ramp up/down exhibit hysteresis

1. Ramp-up: $w = 0$
2. Onset: something 'kicks' mode over boundary
3. Growth: mode 'jumps' to upper branch
4. Saturated state: $w \sim \beta$
5. Marginal point: w falls to zero
 - Loss of 'equilibrium'

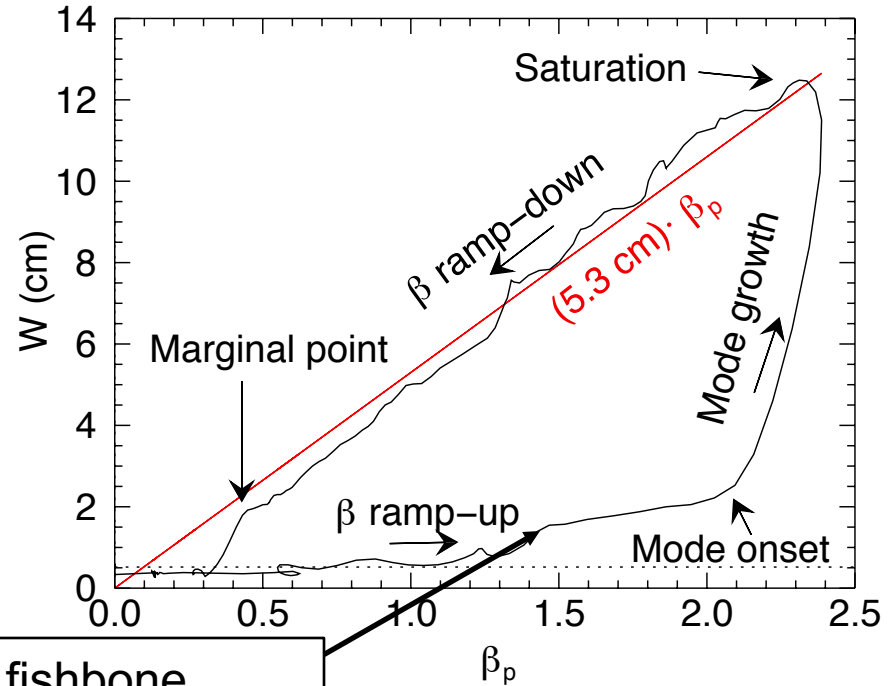


Hysteresis loop seen in DIII-D experiments

Shot 135861



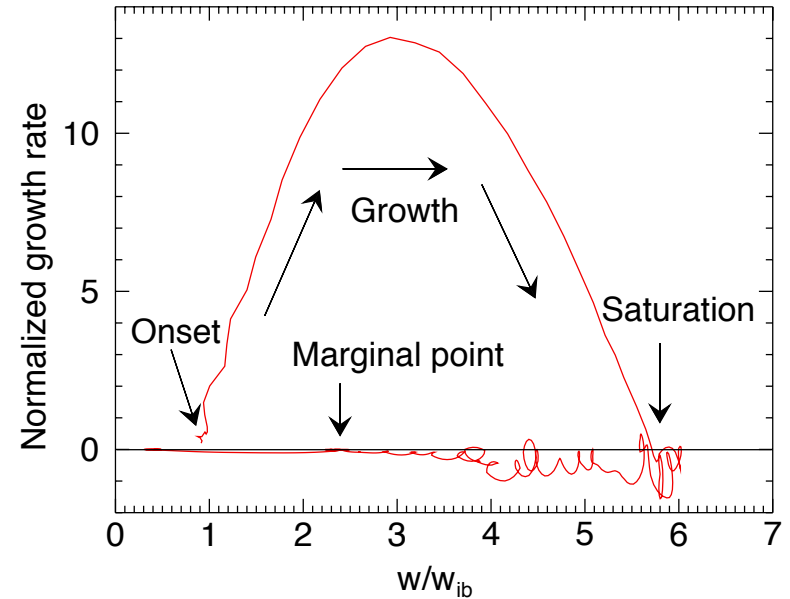
Shot 135861



Caveat: $m=1$ fishbone activity in $n=1$ RMS

Evaluation of MRE using special points

- Maximum, saturated width
 - Relates bootstrap + curvature to Δ'
 - Neglect small-island-width terms
- Peak growth rate
 - Relates bootstrap/curvature to τ_r
 - Need to get τ_r correct
- Marginal point
 - Address small-island-width terms



Multi-device comparison can test aspect ratio dependences in modified Rutherford equation

- Terms vary in different ways

- With ϵ , in addition to β

$$1.22^{-1} \frac{\tau_R}{r} \frac{dw}{dt} = \Delta' r + C_R \frac{r D_R}{w}$$

$D_R \propto \epsilon^2 \beta_\theta$

$$+ \left(\epsilon^{1/2} \frac{r L_q}{L_{pe}} \beta_{\theta e} \left[\frac{1}{w} - \frac{w_{small}^2}{3w^3} \right] \right)$$

$$\epsilon = r/R_0$$

$$w_{small} \propto \epsilon^{-1/2} ?$$

La Haye *et al*, PoP **19**, 062506 (2012)

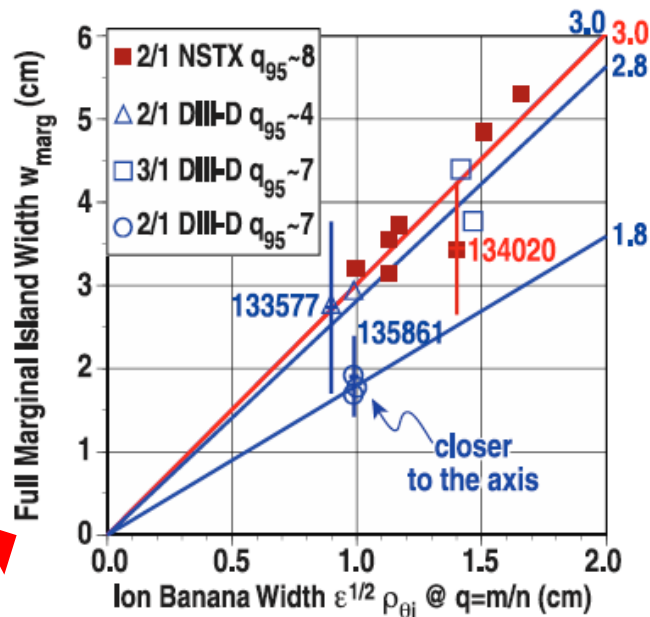
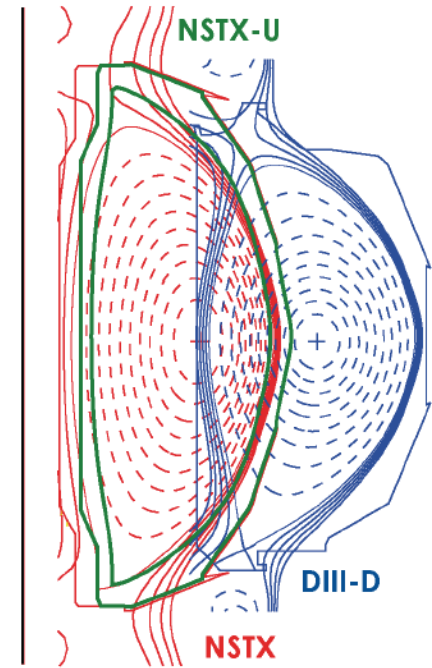


FIG. 5. Full marginal island width at outboard midplane $q = m/n$ versus local ion banana width. NSTX and two of three DIII-D cases have a ratio of about 3 while the one of three DIII-D cases that is closer to the magnetic axis (smaller local inverse aspect ratio magnetic field variation) is closer to a ratio of 2.

Dedicated DIII-D experiments can match NSTX-U tearing discharges already run

- Match shaping (except R/a), $q_{95} \approx 7$, $I_p \approx 0.9$ MA
 - $B_T \approx 1.44$ T (DIII-D) vs. 0.65 (NSTX-U) for q_{95} matching
- Get growth, saturation, marginal point
- Tearing stability is high priority topic in MSG
 - Exp's on DIII-D will provide complimentary data for NSTX-U



Aspect Ratio $R/a = 1.4, 1.7, 2.7$

'Leapfrog' NSTX-U MSG XP1544

- XP1544 approved but not run
 - “Make contact with NSTX for n=1 tearing stability”
- PPPL / GA collaboration on core stability
 - Previously published study

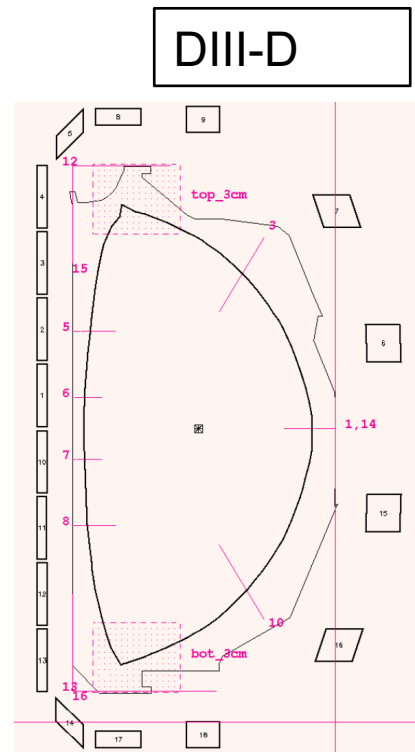
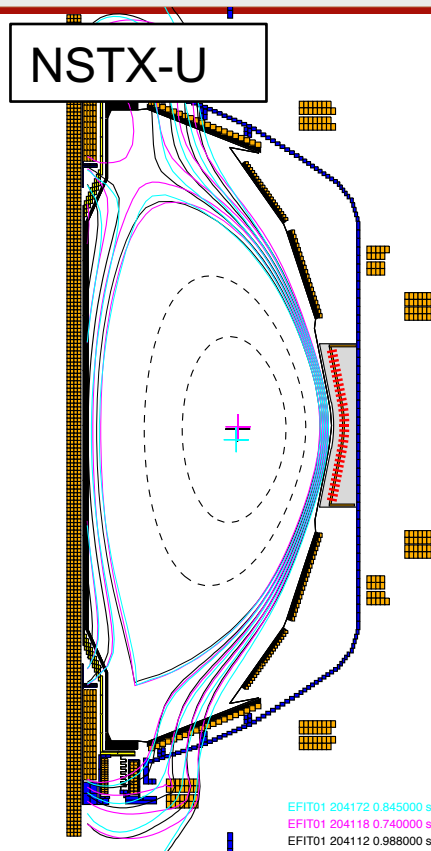
R. J. La Haye, R. J. Buttery, S. P. Gerhardt, S. A. Sabbagh, D. P. Brennan, *“Aspect ratio effects on neoclassical tearing modes from comparison between DIII-D and National Spherical Torus Experiment”*
PoP **19**, 062506 (2012)

	DIII-D	NSTX-U	NSTX
R/a	3.1	1.7	1.4
Growth rate	!!!	✓	✓
Saturated size	!!!	✓	✓
Marginal point	!!!	✗	✓

XP 1544

DIII-D shot developed from target 135861

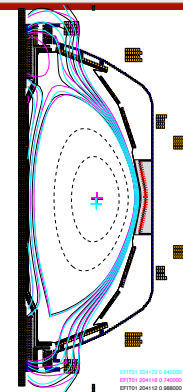
- Near q_{95}, I_p targets
- Improving shape matching
 - Increased inner gap at midplane
 - Increases elongation
 - Adjusted B_T to match q_{95}



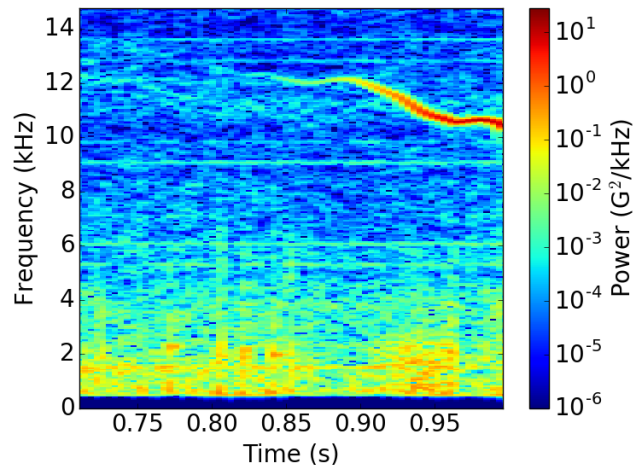
Courtesy R. La Haye

Tearing mode activity observed in NSTX-U initial operations

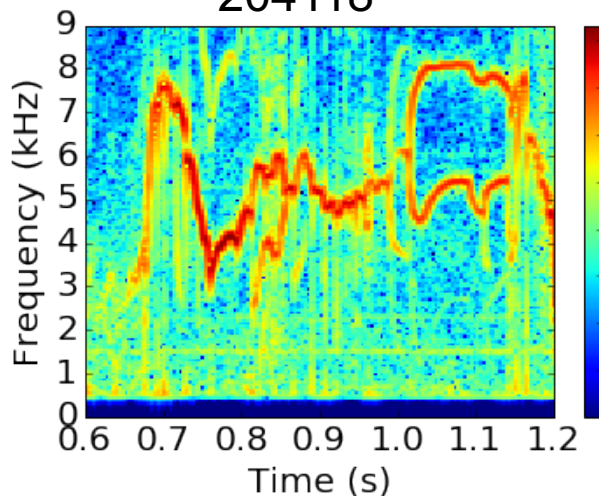
- H-mode
- All high β_N
- All growing n=1 tearing during flattop
- Little/no CHERS
- No MSE
- **Did have Thomson**



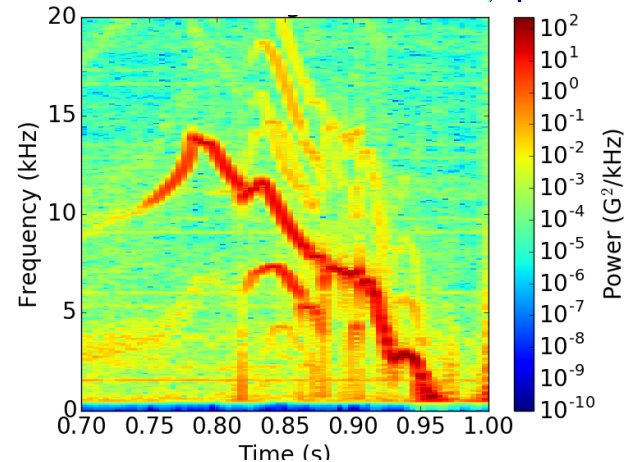
204112



204118



204172



Step 1: Toroidal mode number analysis

- Use cross-phase, amplitude to auto-calibrate coils
 - Assume signal is $B(t, \phi_j) = C_j(B_n e^{i\omega t})$ with $C_j = |C_j|e^{in\phi_j}$
 - Check that nominal \approx actual phase (correct n)

- Get best estimate of fluctuation:

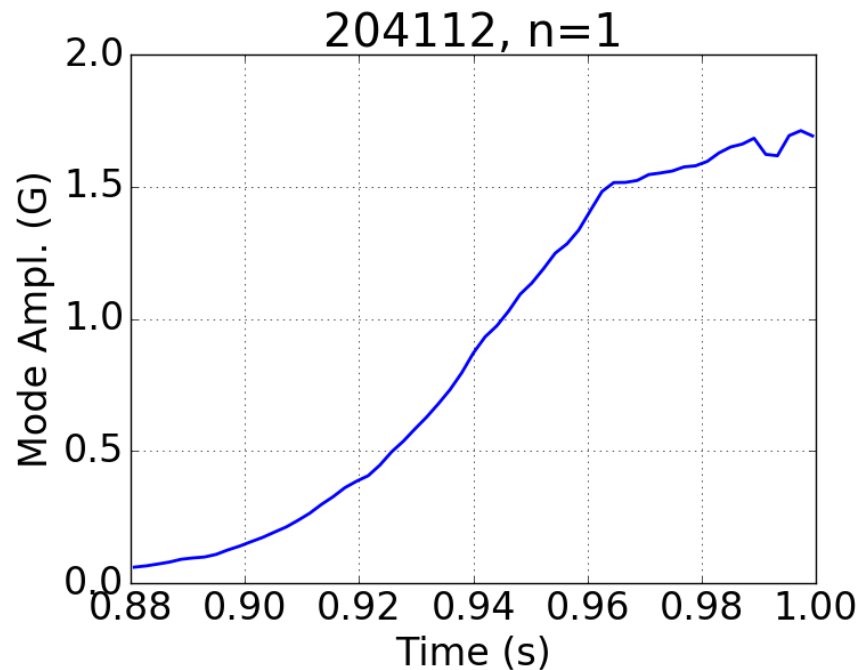
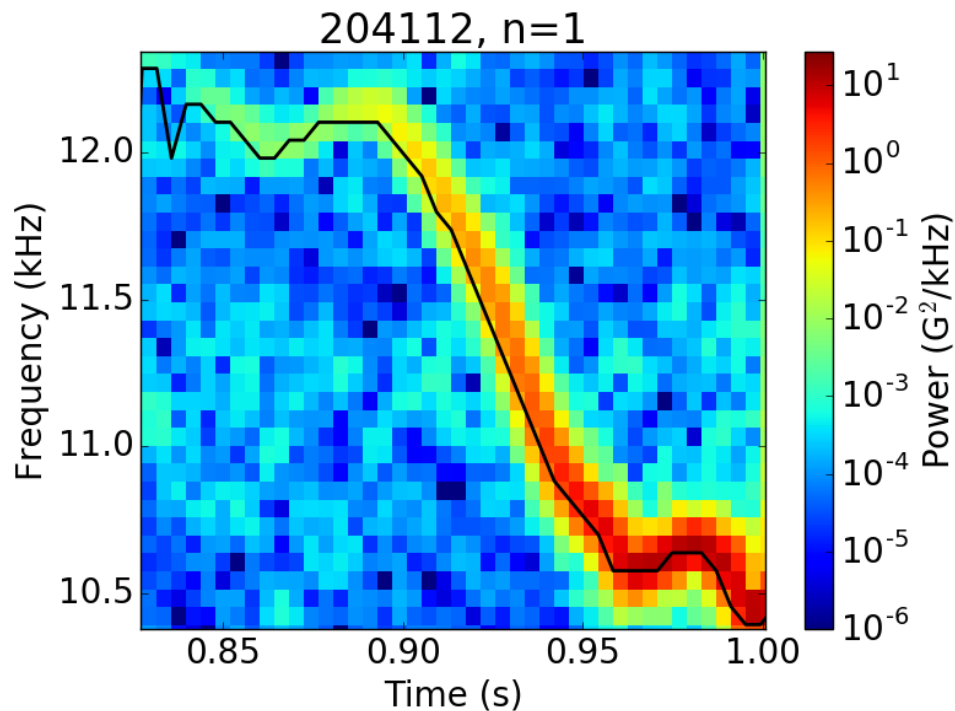
$$- \hat{B}_n(t) = FFT^{-1} \left(\sum_j \frac{B(\omega, \phi_j)}{C_j} \right)$$

- Calculate ‘coherence’ spectrogram:

$$- \chi(t, \omega) = \frac{|\sum_j B(t, \omega, \phi_j)|}{\sum_j |B(t, \omega, \phi_j)|}$$

Step 2: Identify mode frequency, amplitude

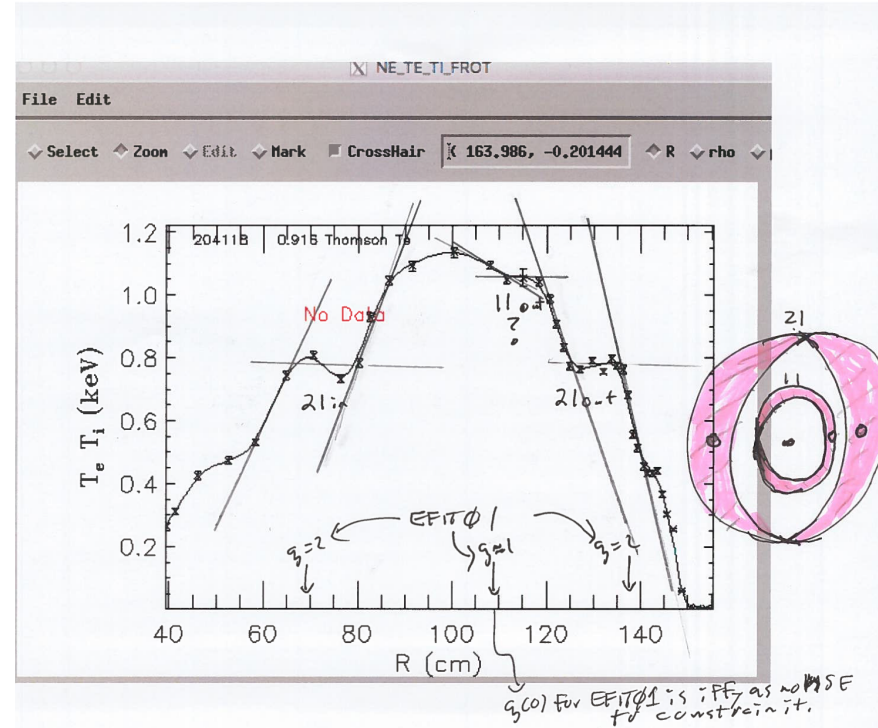
- Fit Gaussian profile in frequency space for total power



Step 3: Find island width from amplitude

- Use TS (NSTX-U), CHERS (NSTX) as available
- Find width of flat spot
- Calibrate mode amplitude:

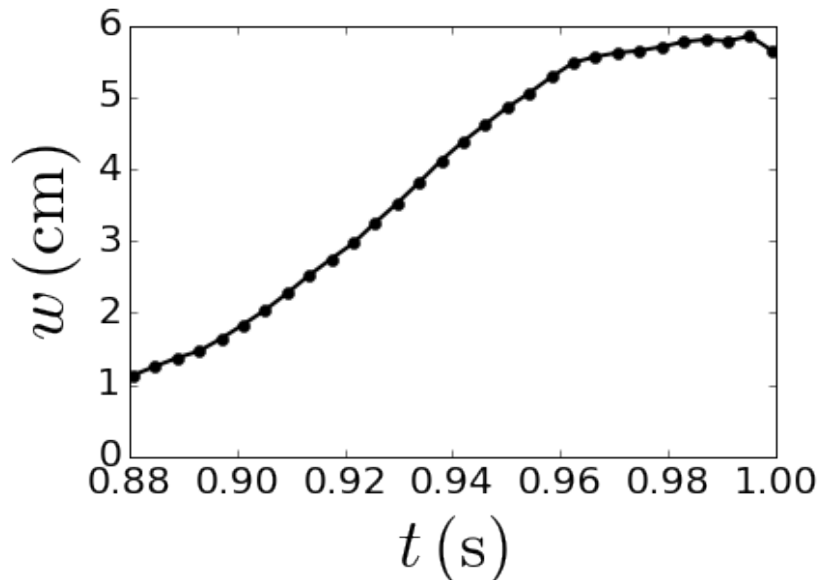
$$-w \propto \sqrt{\frac{\tilde{b}}{B_0}} \rightarrow w = C\sqrt{\tilde{b}}$$



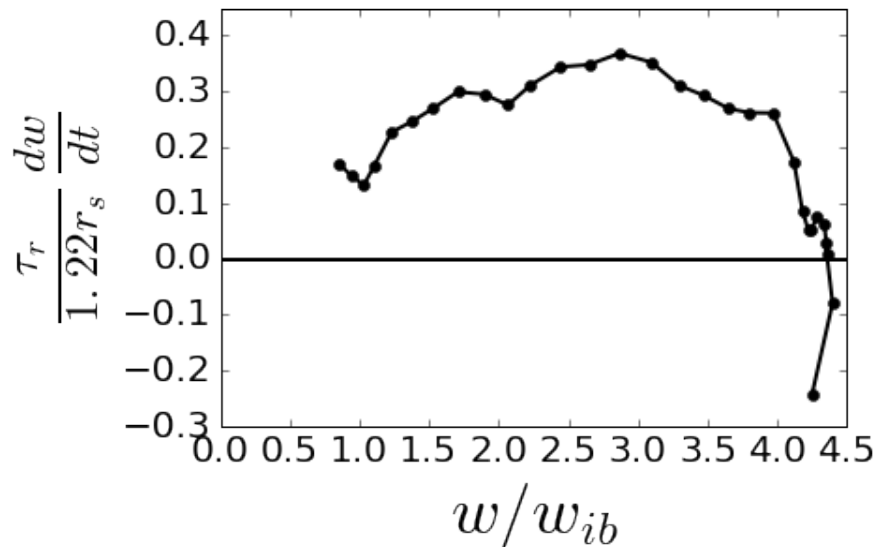
Step 4: Normalized phase diagram

- Growth rate as a function of width
- Normalize to ion banana width, resistive time

NSTX-U

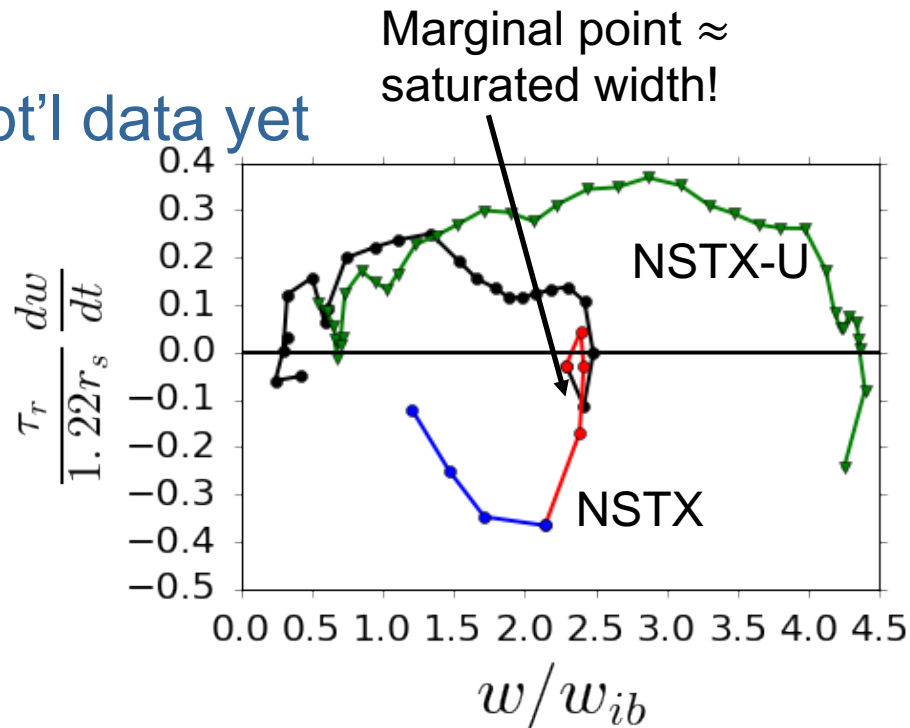
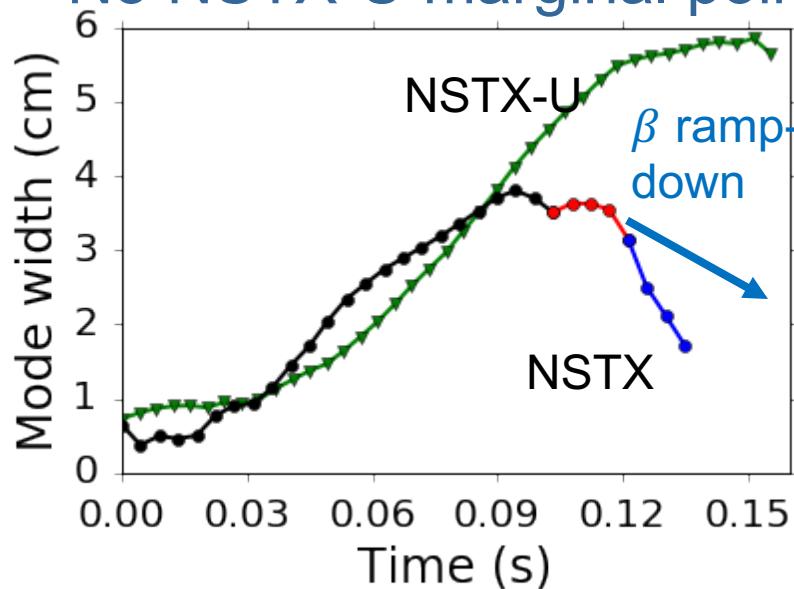


Shot 204112: growth and saturation



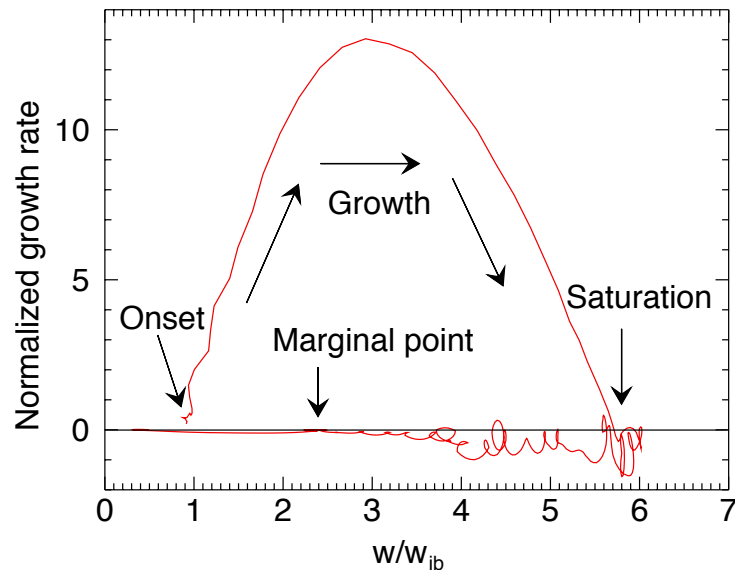
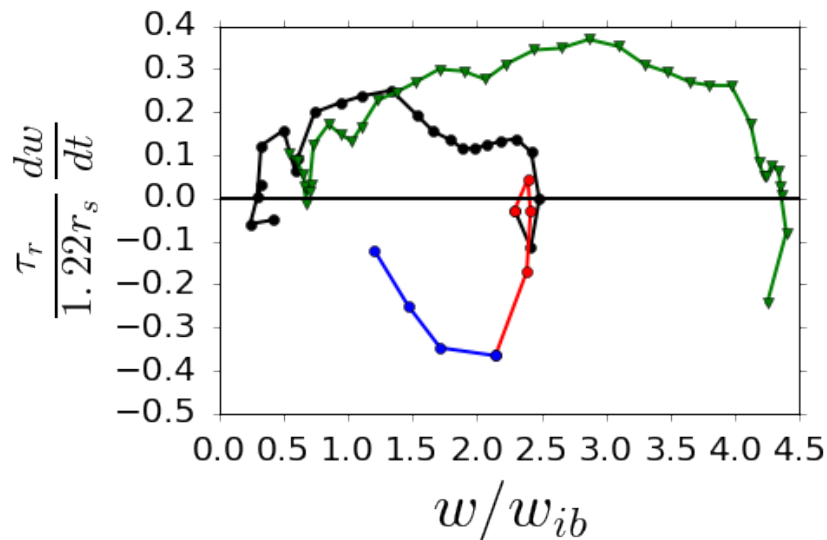
Early results from NSTX/NSTX-U comparison

- NSTX-U sees larger, faster-growing $n = 1$ mode
 - Higher B_T & $T_e \rightarrow$ longer τ_r
 - No NSTX-U marginal point expt'l data yet



Initial comparison of NSTX(-U), DIII-D

- Trend to larger, faster islands with smaller ϵ
- DIII-D peak growth rate very large
 - Resistive time?



Small aspect ratio \rightarrow small hysteresis

- Larger $\epsilon \rightarrow$

- More bootstrap

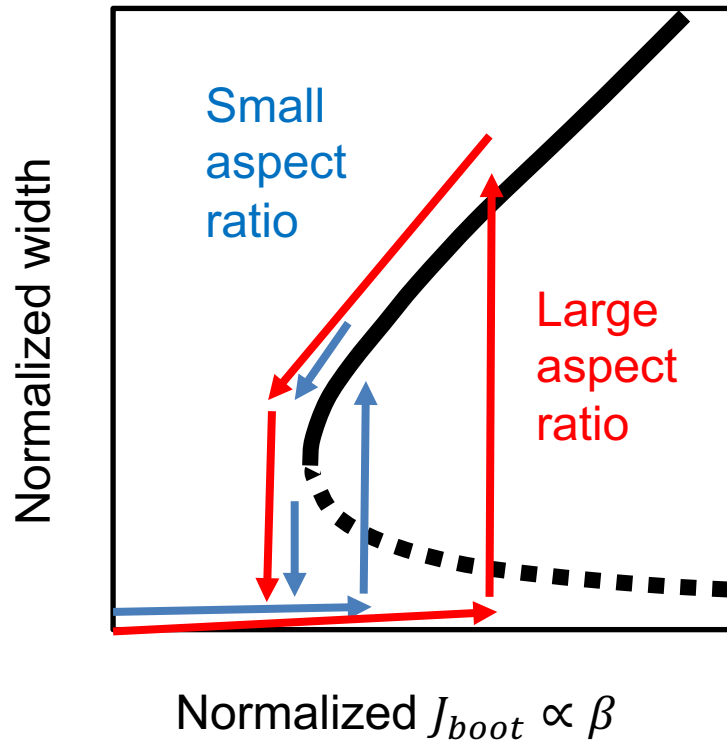
- Smaller ‘small island’ limit

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$D_R \propto \epsilon^2 \beta_{\theta}$

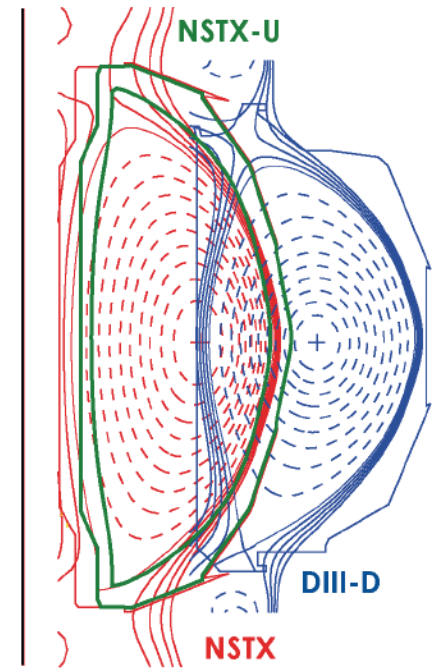
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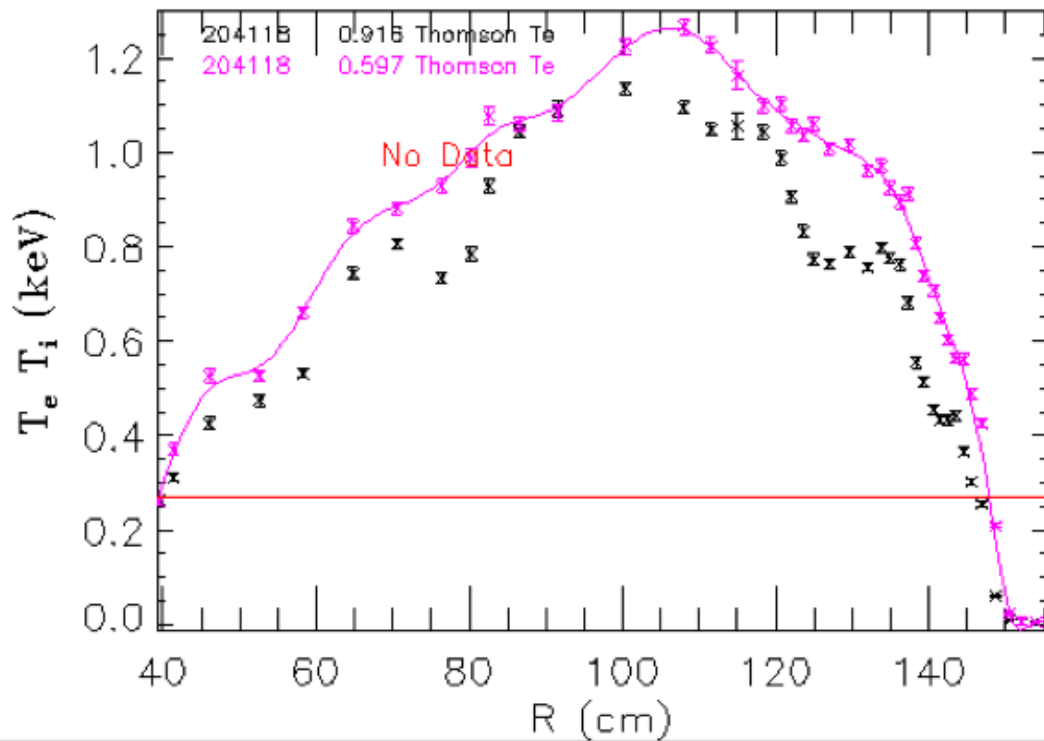
Dedicated DIII-D experiments for the national campaign can match NSTX-U tearing discharges already run

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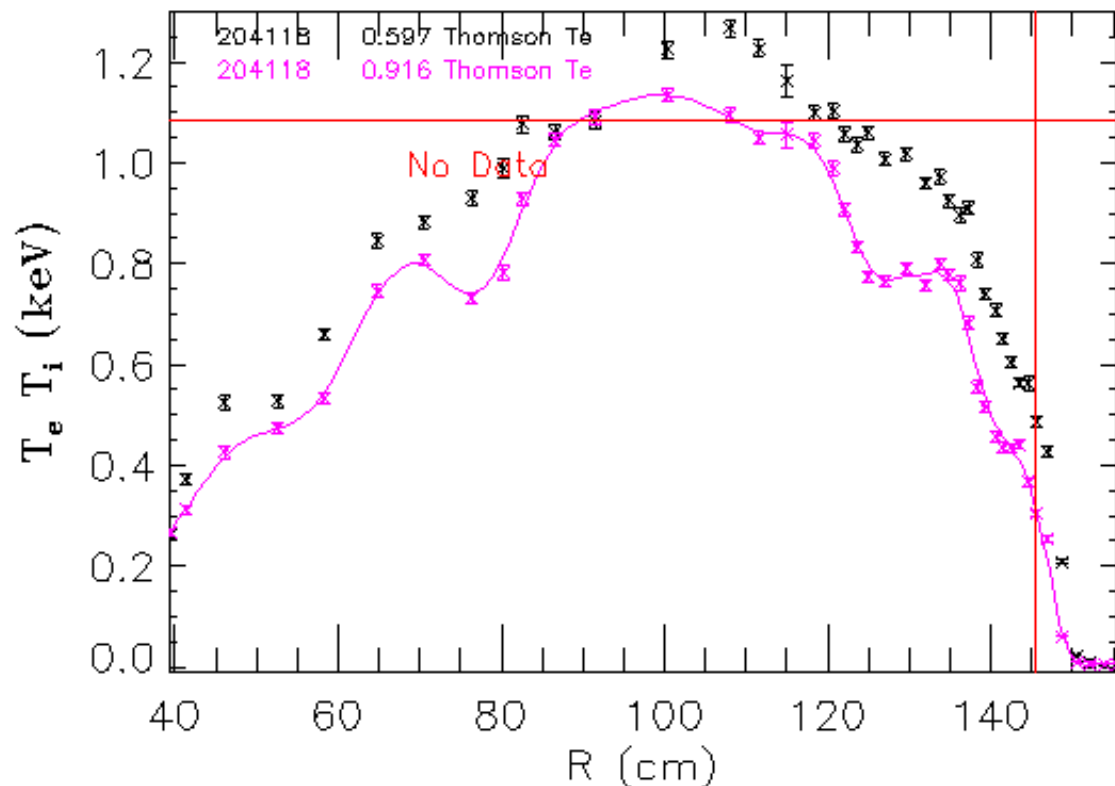


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Reserve



◆ Select ◆ Zoom ◆ Edit ◆ Mark CrossHair (145.437, 1.08411) ◆ R ◆ rho ◆ psi_n



Select Zoom Edit Mark CrossHair (1.55595, 2.47272) R rho psi_n

