Anisotropy

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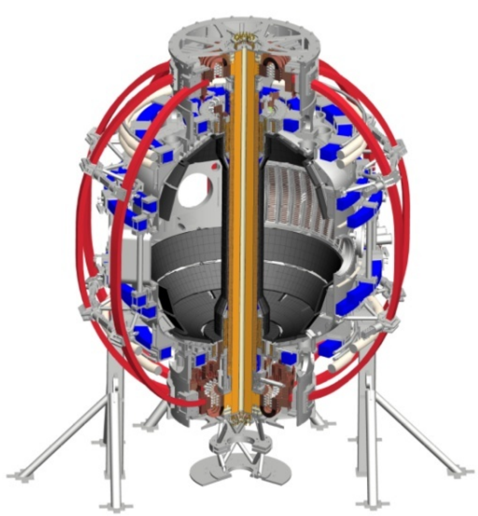

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Pressure anisotropy leads to a modification of the Energy Principle

$$-i\omega\tau_w = -\frac{\delta W_V^\infty + \delta W_F + \delta W_A + \delta W_K}{\delta W_V^b + \delta W_F + \delta W_A + \delta W_K}$$

δW_V : usual changes in vacuum potential energy without a wall, and with an ideal wall

δW_F : usual isotropic fluid term

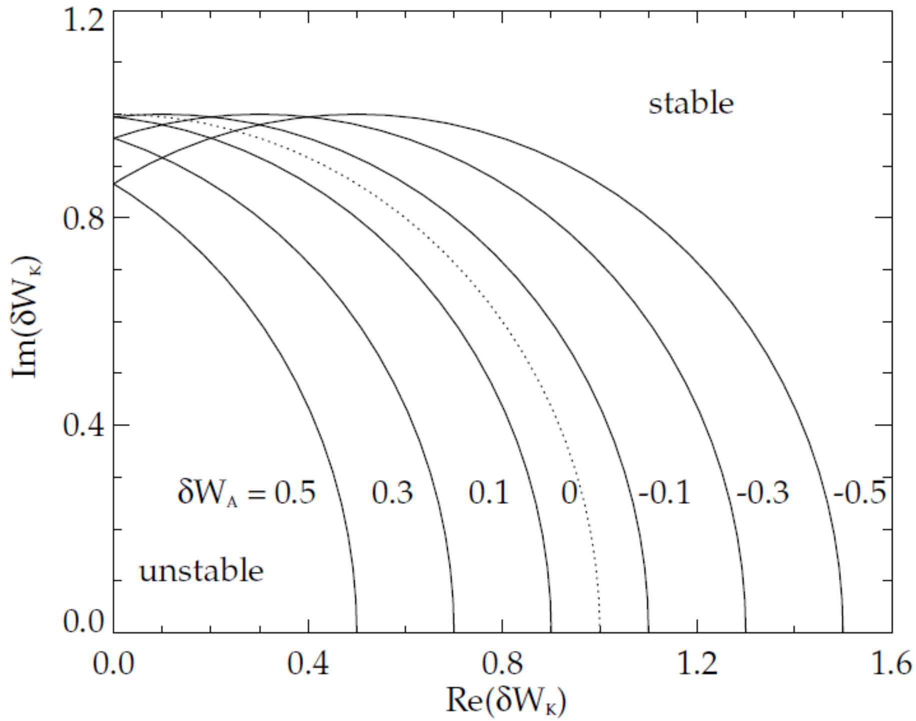
δW_A : anisotropic fluid correction

δW_K : kinetic term (also modified by anisotropy)

$$\omega = \omega_r + i\gamma$$

ω_r : real mode rotation frequency

γ : RWM growth rate



Stability diagram showing contours of $\gamma\tau_w = 0$ on Re(δW_K) vs. Im(δW_K) with $\delta W_\infty = -1$ and $\delta W_b = 1$ [arb.].

Positive δW_A shifts the unstable region to the left, negative δW_A to the right.

The effect of anisotropy on the plasma equilibrium must be small for the perturbative approach to stability calculation

Define an anisotropy parameter: $$\sigma = 1 + \frac{\mu_0 \left(p_\perp - p_\parallel\right)}{B^2}$$

Then the plasma equilibrium in the perpendicular direction is:

$$\boldsymbol{\nabla}_\perp \left(\frac{\mathbf{B}^2}{2\mu_0} + \frac{p_\parallel + p_\perp}{2\sigma} \right) = \boldsymbol{\kappa} \frac{B^2}{\mu_0}$$

Now define a corrected magnetic field: $$\mathbf{D} = \mathbf{B}\sqrt{\sigma}$$

Then: $$\boldsymbol{\nabla}_\perp \left(\frac{\mathbf{D}^2}{2\mu_0} + p_{\text{avg}} \right) = \boldsymbol{\kappa} \frac{D^2}{\mu_0}$$

So, the plasma equilibrium can be considered to first order the isotropic equilibrium, and then having an anisotropic correction of the second order. The perturbative approach is valid as long as $\sigma \approx 1$.

Two anisotropic distributions are considered: bi-Maxwellian for thermal particles, slowing-down for energetic particles

A bi-Maxwellian distribution with different temperatures ⊥ and ∥ to the magnetic field:

$$f_j^{bM}(\varepsilon,\Psi,\chi) = n_j \left(\frac{m_j}{2\pi}\right)^{\frac{3}{2}} \frac{1}{T_{j\perp} T_{j\parallel}^{\frac{1}{2}}} e^{-\varepsilon\chi^2/T_{j\parallel}} e^{-\varepsilon(1-\chi^2)/T_{j\perp}}$$

A slowing down distribution function with a Gaussian distribution of particles in χ:

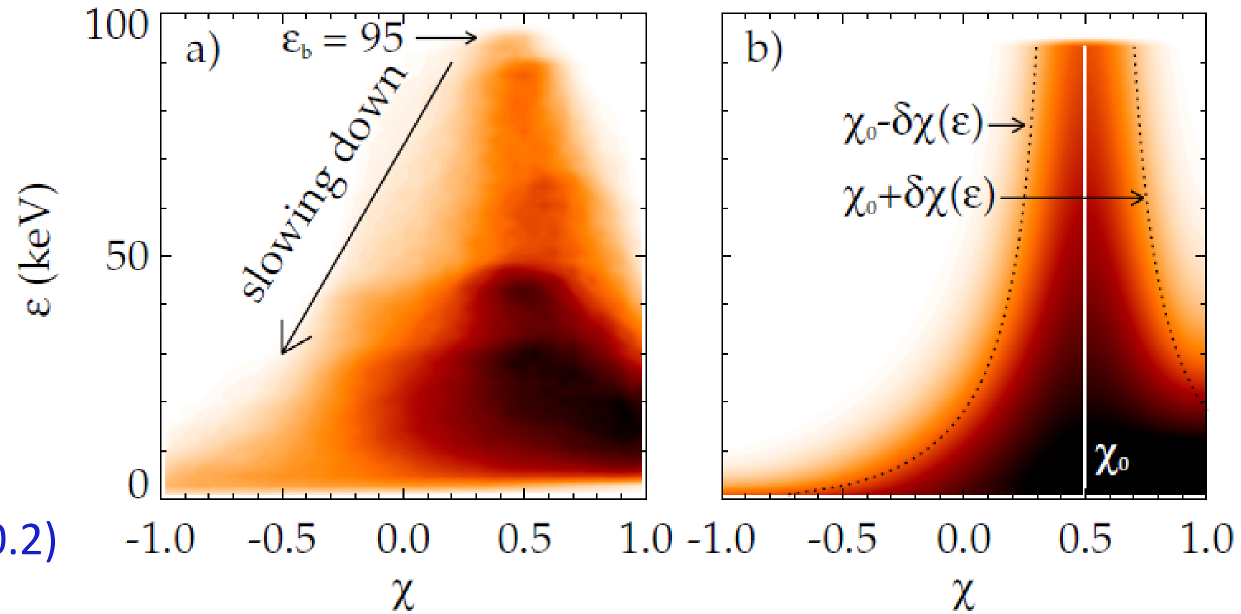
$$f_j^{b}(\varepsilon,\Psi,\chi) = n_j A_b \left(\frac{m_j}{\varepsilon_b}\right)^{\frac{3}{2}} \frac{1}{\hat{\varepsilon}^{\frac{3}{2}} + \hat{\varepsilon}_c^{\frac{3}{2}}} \frac{1}{\delta\chi} \left(\exp\left[\frac{-(\chi-\chi_0)^2}{\delta\chi^2}\right] + \exp\left[\frac{-(\chi+2+\chi_0)^2}{\delta\chi^2}\right] + \exp\left[\frac{-(\chi-2+\chi_0)^2}{\delta\chi^2}\right] \right)$$

Beam ions are injected with injection pitch angle χ_0 and initial spread $\delta\chi_0$

$f(\varepsilon,\chi)$ at a particular Ψ for an NSTX equilibrium from

a) TRANSP

b) model (with $\chi_0 = 0.5$ $\delta\chi_0 = 0.2$)



The kinetic approach is used to obtain δW_K; Anisotropic distribution functions affect the kinetic terms

δW terms are calculated starting from a plasma force balance:

$$\delta W = \frac{1}{2}\int \xi_\perp^* \cdot \left[\mathbf{j}_0 \times \tilde{\mathbf{B}} + \tilde{\mathbf{j}} \times \mathbf{B}_0 - \nabla \cdot \tilde{\mathbb{P}}\right] d\mathbf{V}$$

with: $$\mathbb{P} = p_\parallel \hat{\mathbf{b}}\hat{\mathbf{b}} + p_\perp \left(\hat{\mathbf{I}} - \hat{\mathbf{b}}\hat{\mathbf{b}}\right)$$

CGL pressures are akin to assumption of fast rotating mode (*not the RWM*), and will not be used. Instead in the kinetic approach, the perturbed pressures are calculated rigorously from the perturbed distribution function:

$$\tilde{f}_j = -\boldsymbol{\xi}_\perp \cdot \nabla f_j + Z_j e \frac{\partial f_j}{\partial \varepsilon}\tilde{\boldsymbol{\Phi}} + i m_j \left(\omega \frac{\partial f_j}{\partial \varepsilon} - n \frac{\partial f_j}{\partial P_\phi}\right)(\mathbf{v}\cdot\boldsymbol{\xi}_\perp - \tilde{s}_j) - \frac{m_j}{B}\frac{\partial f_j}{\partial \mu}\left(-i\omega \boldsymbol{\xi}_\perp \cdot \mathbf{v}_\perp + \frac{\mu}{m_j}\tilde{\mathbf{B}}_\parallel + \frac{v_\parallel}{B}\mathbf{v}_\perp \cdot \tilde{\mathbf{B}}\right)$$

Kinetic effects are modified by inclusion of bi-Maxwellian or anisotropic slowing-down distributions in frequency resonance fraction calculation

$$\delta W_K = \sum_j \sum_{l=-\infty}^{\infty} 2\sqrt{2}\pi^2 \int\int\int \left[|\langle H/\hat{\varepsilon}\rangle|^2 \frac{(\omega - n\omega_E)\frac{\partial f_j}{\partial \varepsilon} - \frac{n}{Z_j e}\frac{\partial f_j}{\partial \Psi}}{n\langle\omega_D^j\rangle + l\omega_b^j - i\nu_{\text{eff}}^j + n\omega_E - \omega}\right] \frac{\hat{\tau}}{m_j^{\frac{3}{2}} B} |\chi| \hat{\varepsilon}^{\frac{5}{2}} d\hat{\varepsilon}\, d\chi\, d\boldsymbol{\Psi},$$

Additionally, an anisotropic correction to the fluid term principally modifies the ballooning destabilization term

$$\delta W_F = \frac{1}{2}\int\Bigg\{ \underbrace{\Bigg(-\frac{|\tilde{\boldsymbol{B}}_\perp|^2}{\mu_0}}_{\text{shear Alfvén}} \underbrace{-\frac{B^2}{\mu_0}|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}_\perp + 2\boldsymbol{\xi}_\perp\cdot\boldsymbol{\kappa}|^2}_{\text{fast magneto-acoustic}} \underbrace{+ j_\parallel\left(\boldsymbol{\xi}_\perp^*\times\hat{\boldsymbol{b}}\right)\cdot\tilde{\boldsymbol{B}}_\perp}_{\text{kink}\ (1)}\Bigg) \underbrace{+ 2\left(\boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp^*\right)\left(\boldsymbol{\xi}_\perp\cdot\boldsymbol{\nabla}p_{\text{avg}}\right)}_{\text{ballooning}\ (2)}\Bigg\} d\mathbf{V},$$

$$\delta W_A = \frac{1}{2}\int\Bigg\{ (\sigma-1)\Bigg(-\frac{|\tilde{\boldsymbol{B}}_\perp|^2}{\mu_0} - \frac{B^2}{\mu_0}|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}_\perp + 2\boldsymbol{\xi}_\perp\cdot\boldsymbol{\kappa}|^2 + j_\parallel\left(\boldsymbol{\xi}_\perp^*\times\hat{\boldsymbol{b}}\right)\cdot\tilde{\boldsymbol{B}}_\perp\ (3)\Bigg) - 2B\,|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}_\perp + \boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp|^2\frac{\partial p_{\text{avg}}}{\partial B}\ (4)\Bigg\} d\mathbf{V}$$

These will be small due to σ ≈ 1.

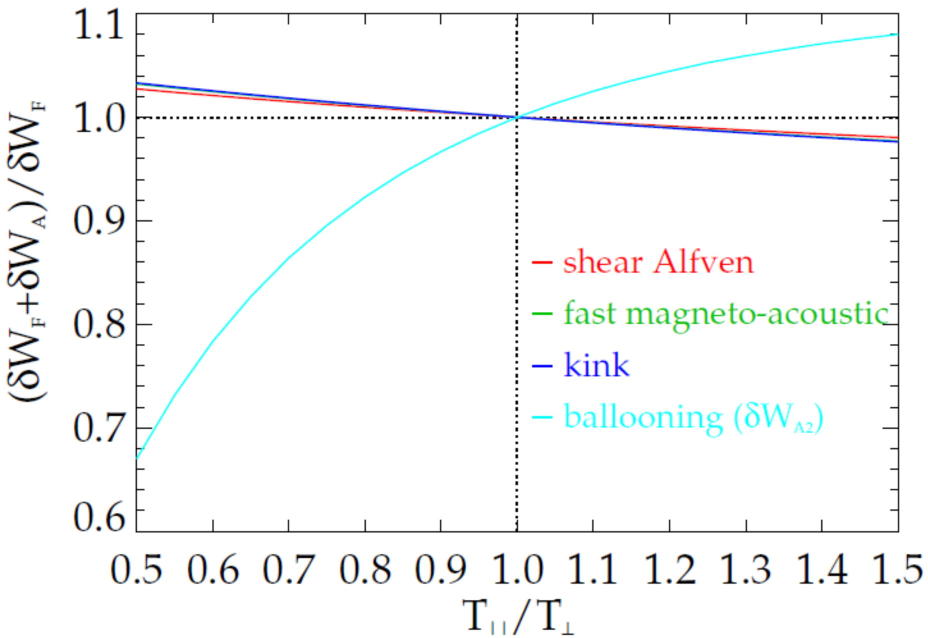
Note: terms 1, 2, and 3 together can be shown to be self-adjoint, so that $\delta W_F + \delta W_A$ is self-adjoint

(4) ⟶ $$\delta W_{A2} = \sqrt{2}\pi^2 \int\!\!\int\!\!\int \frac{1}{m_j^{\frac{3}{2}}}|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}_\perp + \boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp|^2\left(\chi^4-1\right)\frac{\partial f_j}{\partial\chi}\frac{\hat{\tau}}{B}\varepsilon^{\frac{3}{2}}d\varepsilon d\chi d\Psi.$$

The anisotropic correction to the ballooning term depends upon the derivative of the distribution function with respect to pitch angle, $\chi = v_\parallel/v$.

The anisotropic ballooning correction can make a significant impact on the calculated RWM growth rate

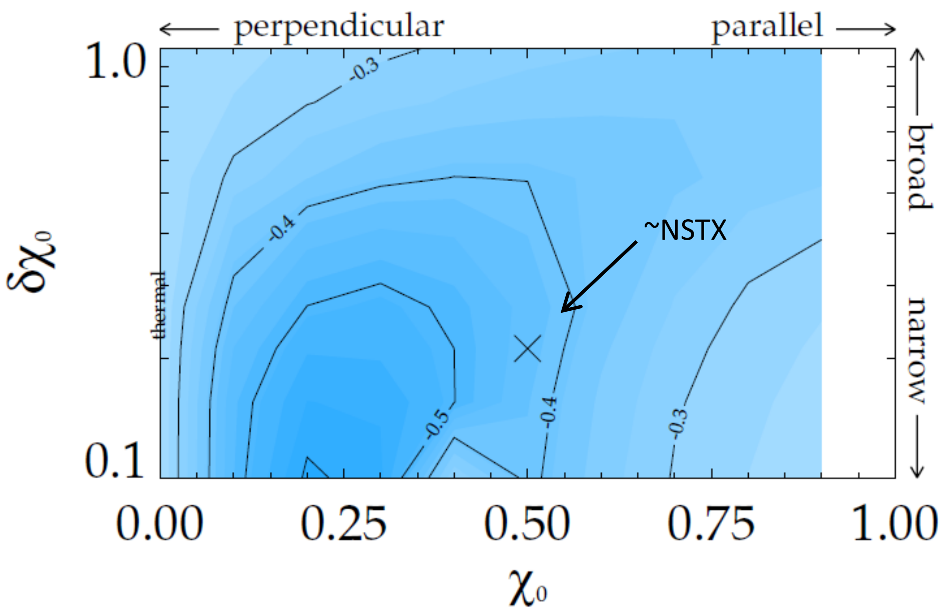
Anisotropic thermal particles



- As $T_{\|}/T_\perp$ is reduced, the destabilizing ballooning term is reduced.
- Up to 30% reduction in RWM fluid growth rate in a test case.

Anisotropic energetic particles

(and isotropic thermal particles)

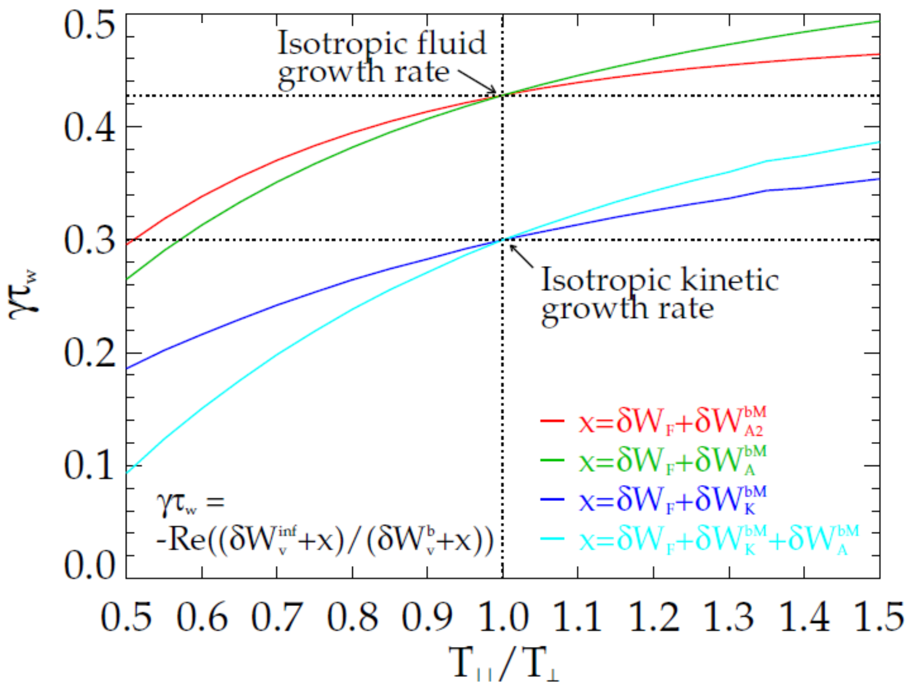


Contours of $\gamma\tau_w$ with δW_{A2} included

- With thermal particles only $\gamma\tau_w$ = -0.24.
- With approx. NSTX values of χ_0 = 0.5 and $\delta\chi_0$=0.2, stability is improved by almost a factor of two!

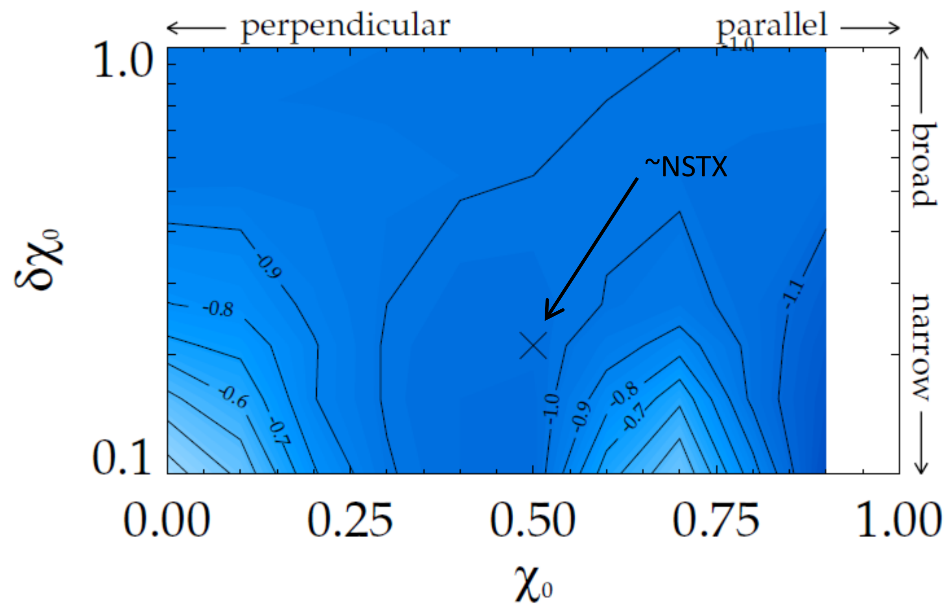
The kinetic effects of anisotropic particles also make a significant impact on the calculated RWM growth rate

Anisotropic thermal particles



- As $T_{\|}/T_{\perp}$ is reduced, the stabilizing kinetic term is increased.
- Up to 66% reduction in RWM kinetic growth rate in a test case with both kinetic effects and anisotropic fluid corrections (δW_A).

Anisotropic energetic particles (and isotropic thermal particles)



Contours of $\gamma\tau_w$ with δW_K for beam ions and δW_{A2} included

- Large increase in RWM kinetic stability for an NSTX case from kinetic effects of anisotropic EPs and anisotropic fluid corrections (δW_A).

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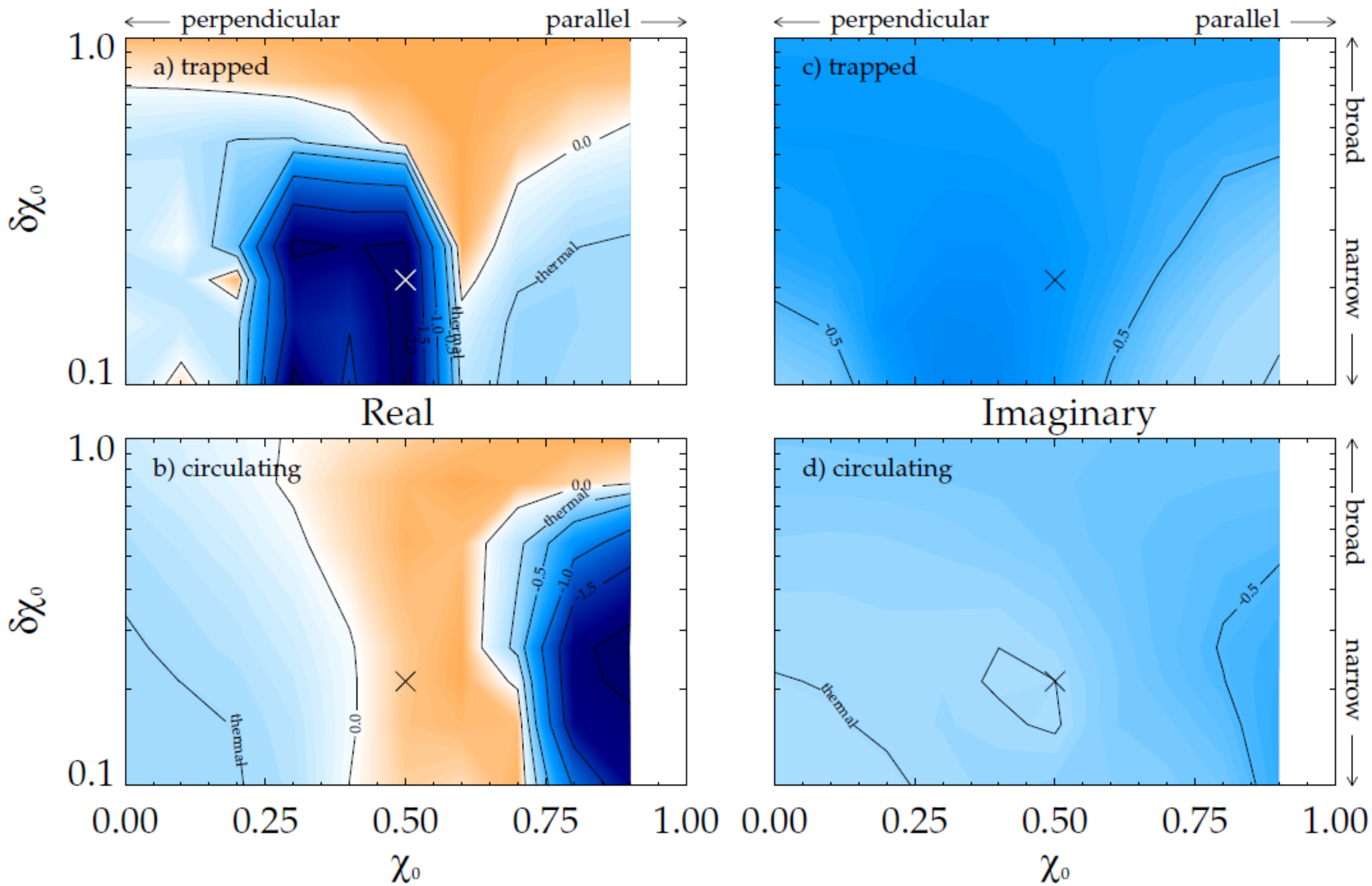


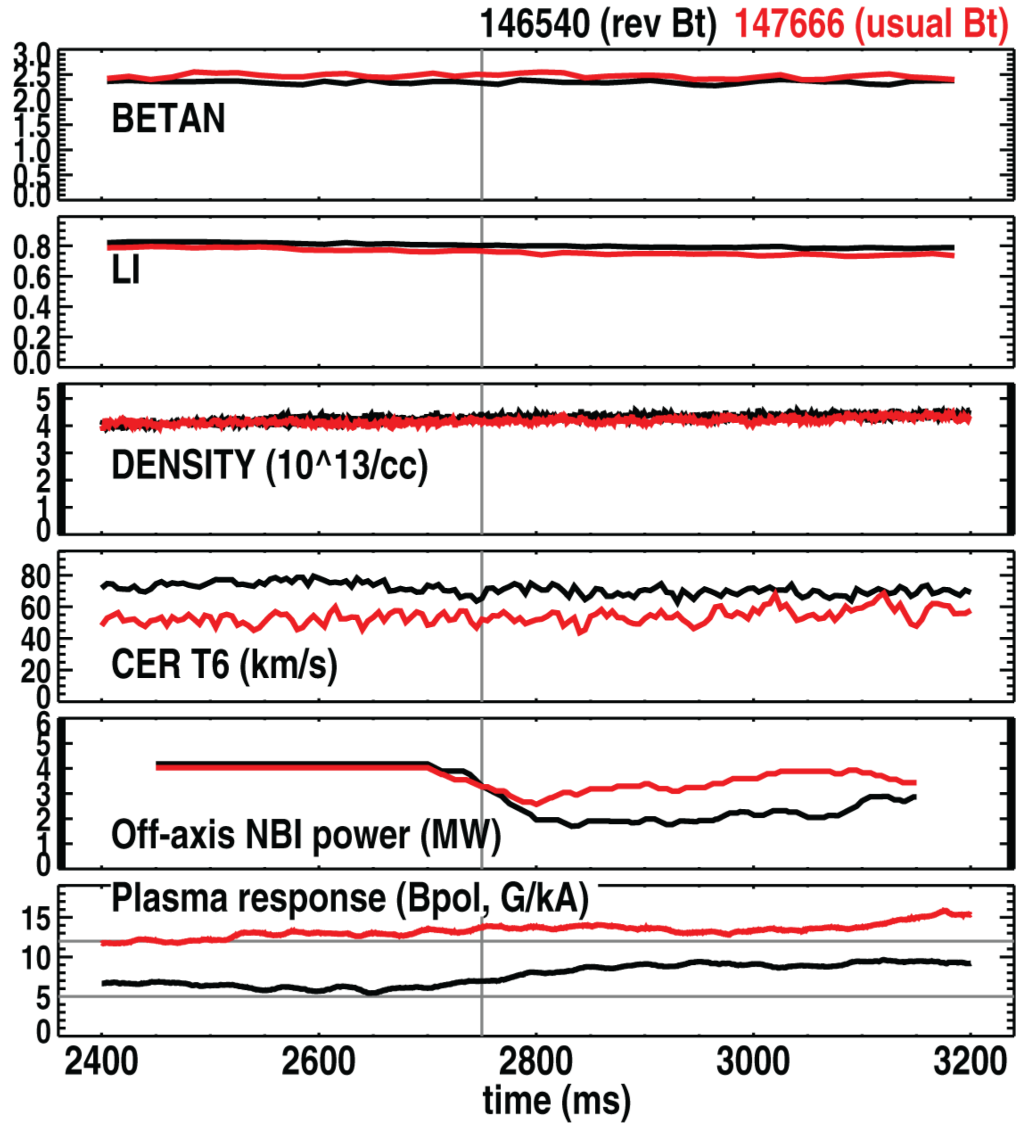
FIG. 7. (Color online) Contours of $\gamma\tau_w$ vs. scaled constant χ_0 and $\delta\chi_0$, where $\gamma\tau_w$ includes δW_K^M and a) $Re(\delta W_K^b)$ for trapped energetic particles, b) $Re(\delta W_K^b)$ for circulating energetic particles, c) $Im(\delta W_K^b)$ for trapped energetic particles, and d) $Im(\delta W_K^b)$ for circulating energetic particles in the NSTX equilibrium. The approximate NSTX experimental values from Fig. 5 are marked with a $\times$ symbol. A contour of $(\gamma\tau_w)_{th} = -0.24$ for the thermal particle only case is marked for comparison.

Nov 2011 Joint Experiment - Off-axis NBI appears to have a smaller impact on RWM stability in usual B_t

- **In *reversed B_t*, (Sept. 2011) observed reduction in plasma response with off-axis NBI**
  - Consistent with increased RWM stability

- **In *usual B_t*, modulating the off-axis beam leads to smaller impact on plasma response**

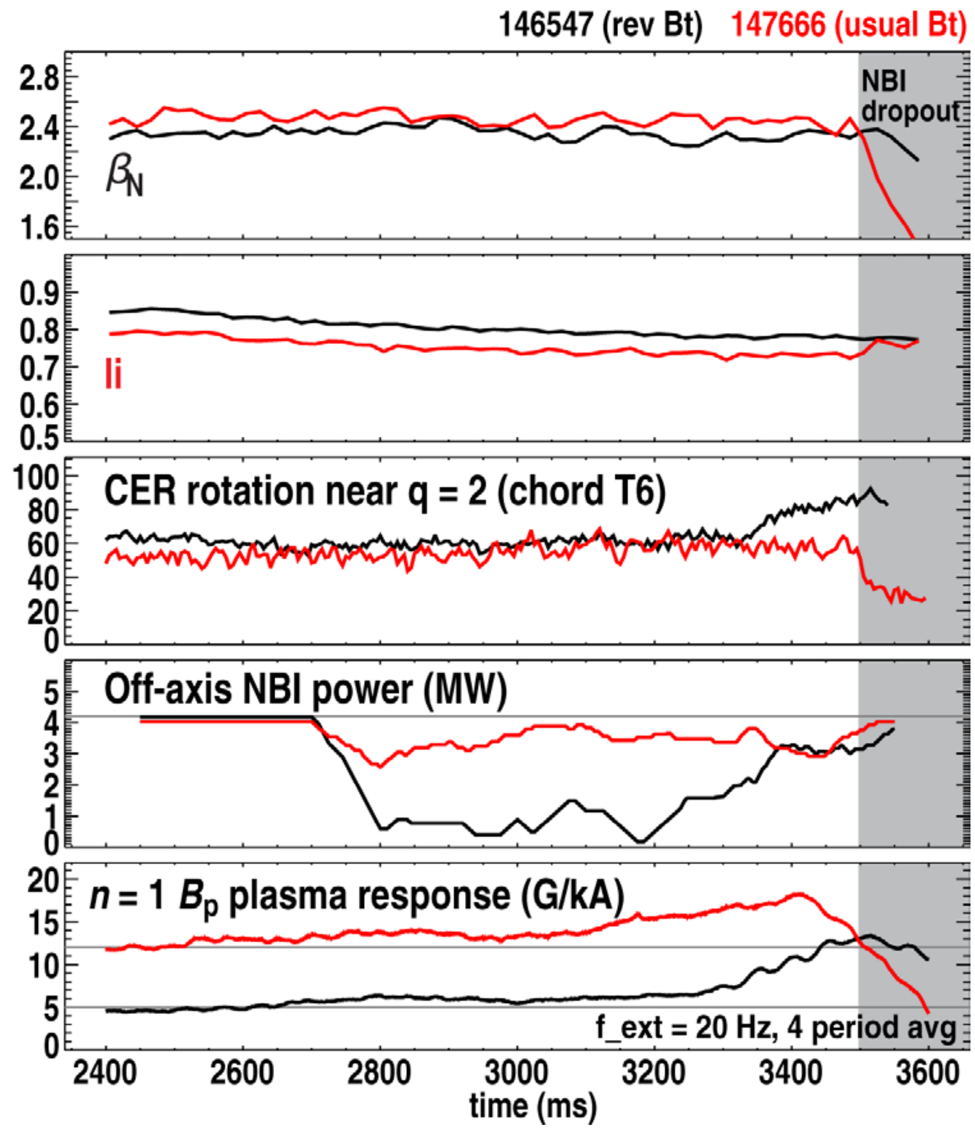
- **Note:**
  - Slightly differing levels on β_N, ℓ_i, rotation between the two shots
  - Couldn't modulate off-axis power more than ~25% at constant β (Nov 2011)
  - Ideal, kinetic stability calculations planned to understand results



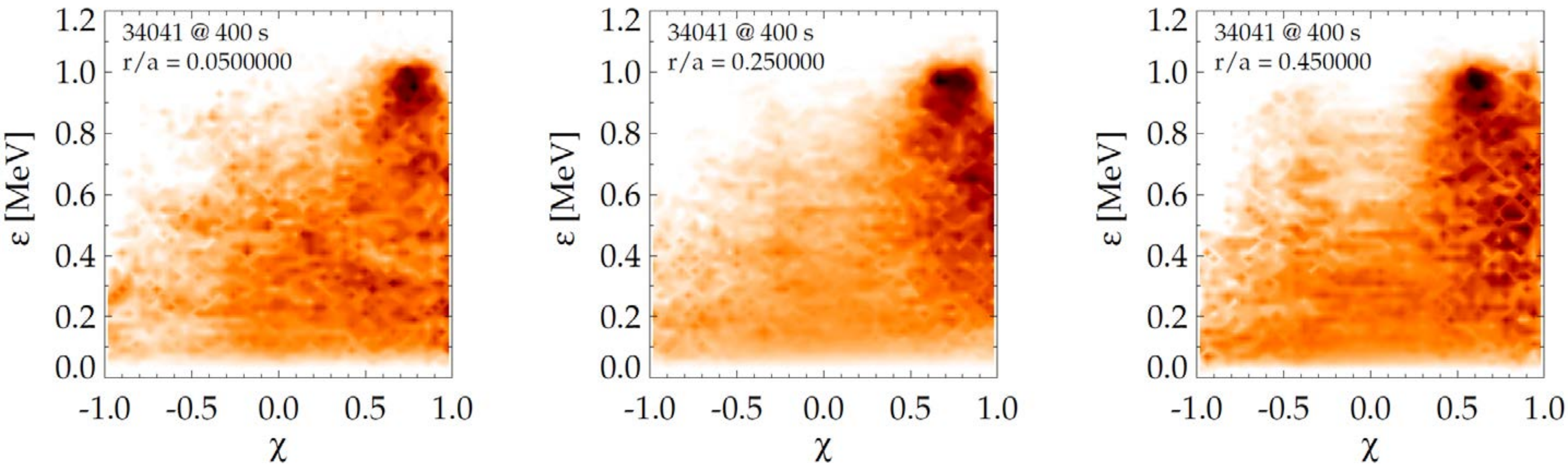
Nov 2011 Joint Experiment – Shot taken with lowest RWM stability observed in this experiment to date

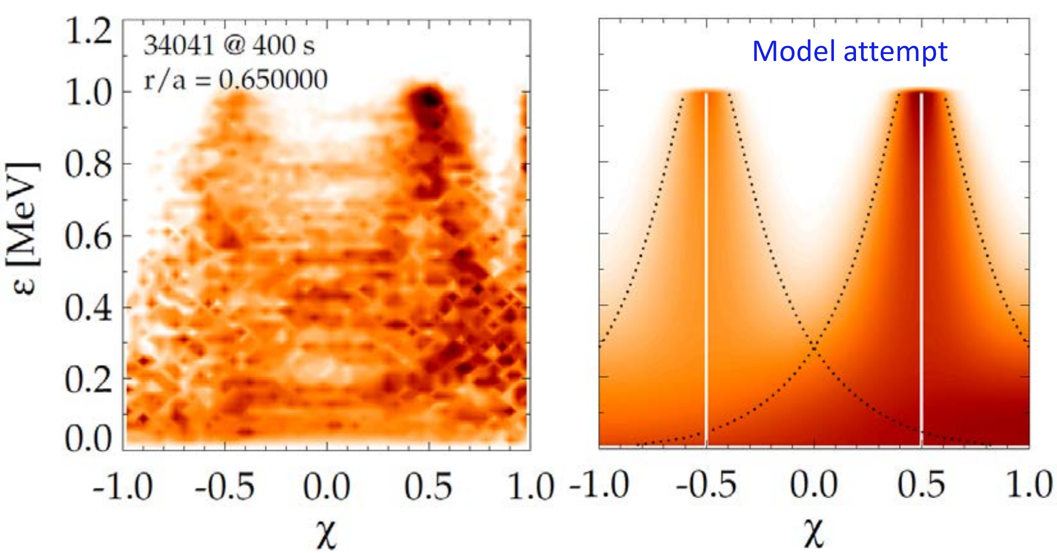
- **Part of experiment aimed to minimize plasma stability**
  - Choose parameters from previous scans that give largest plasma resonant field amplification
  - ***1 shot: Maximized β_N/ℓ_i at intermediate NBI torque -> maximum plasma response***

- **Associated goal: demonstrate reduced stability most clearly by driving RWM unstable**
  - By changing NBI sources, plasma rotation
  - Did not occur during this run at slightly reduced τ_E, reduced NBI
  - Modified experimental idea submitted for 2012



Energetic particle distribution function 34041 @ 400s





- EPDF from TRANSP is complex
  - Multiple heating sources
  - Can attempt to model it with Gaussian distributions