Simulation of tearing mode in slab geometry*

Edward Startsev and Wei-li Lee
Plasma Physics Laboratory, Princeton University
Princeton, NJ, USA

PPPL NSTX/Theory collaboration meeting
September 27, 2013

*Research supported by the U.S. Department of Energy.

Governing gyrokinetic equations for a finite-β plasma for $k_\perp^2 \rho_i^2 \ll 1$

- Gyrokinetic Vlasov equation

$$\frac{dF_\alpha}{dt} \equiv \frac{\partial F_\alpha}{\partial t} + v_\parallel \mathbf{b} \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}} + \mathbf{E}^L \times \mathbf{b}_0 \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}} + s_\alpha v_{t\alpha}^2 (\mathbf{E}^L \cdot \mathbf{b} + E_\parallel^T) \frac{\partial F_\alpha}{\partial v_\parallel} = 0,$$

$$\mathbf{b} \equiv \widehat{\mathbf{b}}_0 + \frac{\delta \mathbf{B}}{B_0} = \frac{\mathbf{B_0}}{B_0} + \nabla A_\parallel \times \widehat{\mathbf{b}}_0,$$

$$\mathbf{E}^L = -\nabla \phi, \quad E_\parallel^T = -\frac{\partial A_\parallel}{\partial t},$$

- The gyrokinetic Poisson's equation

$$\nabla_\perp^2 \phi = -\int (F_i - F_e) dv_\parallel d\mu,$$

- Ampere's law

$$\nabla^2 A_\parallel = -\beta \int v_\parallel (F_i - F_e) dv_\parallel d\mu,$$

- Here $\tau \equiv T_e/T_i$, $\alpha = \{e, i\}$, $v_{te}^2 = m_i/m_e$, $v_{ti}^2 = 1/\tau$, $s_e = -1$, $s_i = \tau$, $\int F_{0\alpha} dv_\parallel d\mu = 1$ and $\mu \equiv v_\perp^2/2$, $\beta \equiv c_s^2/v_A^2$, $v_A \equiv c\lambda_D/\rho_s$ is the Alfvén speed, and λ_D is the electron Debye length.

Perturbative Simulation Schemes

- Using $F_\alpha = F_{0\alpha} + \delta f_\alpha$ and $\partial F_{0\alpha}/\partial t + v_\parallel \widehat{\mathbf{b}}_0 \cdot \partial F_{0\alpha}/\partial \mathbf{x} = 0$, we obtain

$$\frac{d\delta f_\alpha}{dt} = -\nabla(\phi - v_\parallel A_\parallel) \times \widehat{\mathbf{b}}_0 \cdot \kappa_\alpha F_{0\alpha} - s_\alpha v_\parallel (\nabla\psi + \nabla\phi \times \nabla A_\parallel) \cdot \widehat{\mathbf{b}}_0 F_{0\alpha},$$

- Here

$$\kappa_\alpha \equiv -(\partial F_{0\alpha}/\partial \mathbf{x})/F_{0\alpha} = \kappa_n - \frac{3}{2}\kappa_{T\alpha} + \frac{1}{2}\kappa_{T\alpha}(v_\parallel^2 + v_\perp^2)/v_{t\alpha}^2$$

- $\kappa_n \equiv -d(lnn_0)/d\mathbf{x}$, and $\kappa_{T\alpha} \equiv -d(lnT_{0\alpha})/d\mathbf{x}$. $F_{0\alpha}$ is the background Maxwellian.

- The field equations become

$$\nabla_\perp^2 \phi = -\int (\delta f_i - \delta f_e) dv_\parallel d\mu,$$

$$\nabla^2 A_\parallel = -\beta \int v_\parallel (\delta f_i - \delta f_e) dv_\parallel d\mu.$$

$$\widehat{\mathbf{b}}_0 \cdot \nabla\psi \equiv \widehat{\mathbf{b}}_0 \cdot \nabla\phi + \frac{\partial A_\parallel}{\partial t}.$$

Separation of adiabatic response

$$\delta f_\alpha = -s_\alpha F_{0\alpha}\psi + F_{0\alpha}\int dx_{||}\kappa_\alpha \cdot (\nabla A_{||} \times \widehat{\mathbf{b}}_0) + g_\alpha,$$

- Adiabatic part produces density gradients transverse to magnetic field

$$\mathbf{b}\cdot\nabla\left[1 + \int dx_{||}\kappa_\alpha \cdot (\nabla A_{||} \times \widehat{\mathbf{b}}_0)\right] F_{0\alpha} \approx 0,$$

  where $\mathbf{b} = \widehat{\mathbf{b}}_0 + \delta\mathbf{B}/B_0$.

- Vlasov equations become

$$\frac{dg_\alpha}{dt} = \left[s_\alpha \frac{\partial\psi}{\partial t} - \nabla\psi \times \widehat{\mathbf{b}}_0 \cdot \kappa_\alpha\right] F_{0\alpha},$$

2-D equations

- Background magnetic field with shear generated by current layer

  $\mathbf{B}_0 = B_0\mathbf{e}_z + B_{0y}(x)\mathbf{e}_y, \quad \mathbf{j}_0 = -en_e u_0(x)\mathbf{e}_z, \quad \theta(x) \equiv B_{0y}(x)/B_0, \quad u_0(x) = \theta'(x)/\beta$

- Only perturbations with $k_z = 0$, so that $\widehat{\mathbf{b}}_0 \cdot \nabla = \theta(x)\partial/\partial y$.

- In 2-d Vlasov equations become:

$$\frac{\partial g_\alpha}{\partial t} = -\theta(x)v_\parallel \frac{\partial}{\partial y} g_\alpha + \left[s_\alpha \psi_t - \kappa_\alpha(x)\frac{\partial}{\partial y}\psi\right] F_{0\alpha},$$

  where $F_{0\alpha}$ is shifted (by $u_0(x)$ for electrons) Maxwellian.

- Fields equations for $\phi(x,y)$ and $\psi(x,y)$

$$\nabla^2\phi - (1+\tau)\psi = \int (g_e - g_i),$$

$$\nabla^2[\theta(x)(\phi - \psi)] - \beta(u_0'(x) + \kappa_n u_0(x))\phi = \beta\theta(x)\int v_\parallel^2 (g_e - g_i)$$

Additional Field equations

- and its time derivative $\phi_t(x,y)$ and $\psi_t(x,y)$ (with use of Vlasov equation to eliminate $\partial_t g_\alpha$)

$$\nabla^2 \phi_t = -\theta(x) \frac{\partial}{\partial y} \int v_\parallel (g_e - g_i) \quad (1)$$

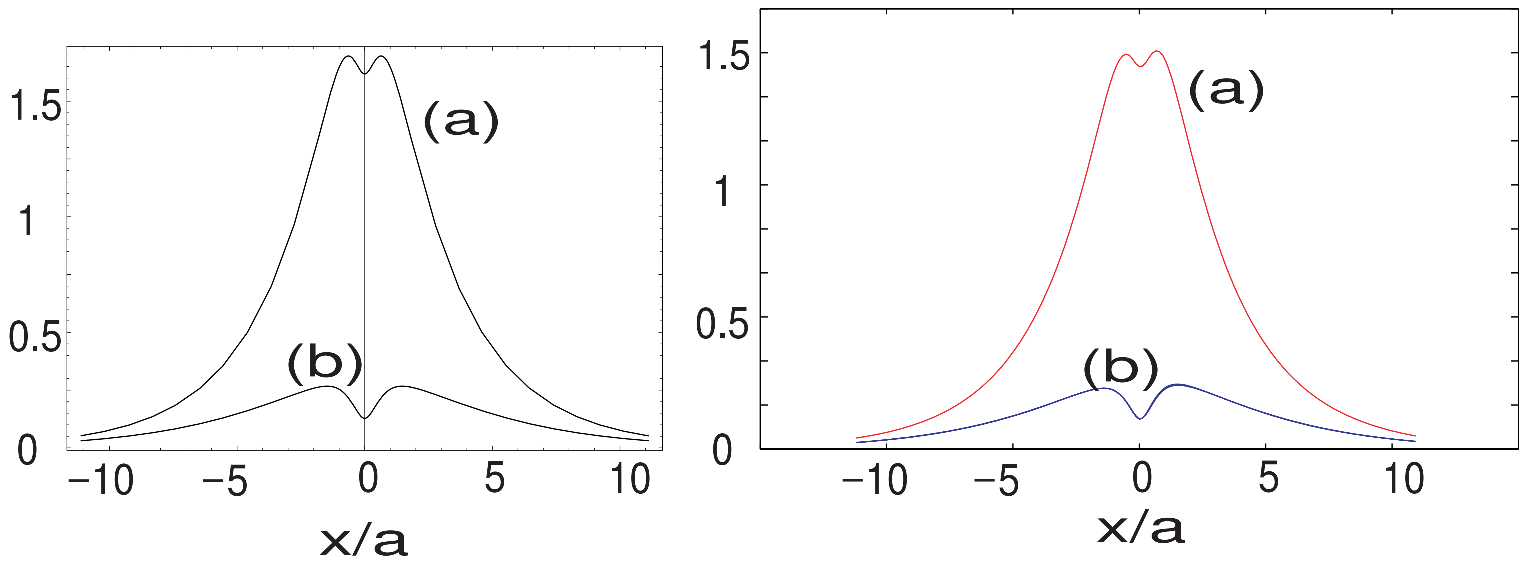
and

$$\left[\nabla^2 - \beta(1 + u_0^2(x) + \frac{m_i}{m_e})\right] \theta(x)\psi_t = \nabla^2[\theta(x)\phi_t] - \beta[u_0'(x) + \kappa_n u_0(x)]\phi_t$$
$$+\beta \left[\frac{m_i}{m_e}(\boldsymbol{\kappa}_n + \boldsymbol{\kappa}_{Te}) - \frac{1}{\tau}(\boldsymbol{\kappa}_n + \boldsymbol{\kappa}_{Ti}) + u_0^2(x)\kappa_n + (u_0^2(x))'\right] \frac{\partial}{\partial y}[\theta(x)\psi]$$
$$+\beta\theta^2(x) \frac{\partial}{\partial y} \int v_\parallel^3 (g_e - g_i) \quad (2)$$

- To avoid resolving δ_e in y direction and insure correct cancellation, replace

$$\frac{m_i}{m_e} \rightarrow \frac{m_i}{m_e}\left(1 + \frac{1}{6}\Delta y^2 \frac{d^2}{dy^2}\right), \quad u_0^2 \rightarrow u_0^2 \left(1 + \frac{1}{6}\Delta y^2 \frac{d^2}{dy^2}\right)$$

Tearing mode simulation results



- Magnetic field shear profile $\theta(x) = \theta_m \tanh(x/a)$.
- Simulation parameters: $n_p = 46368$, $\Delta t \omega_{ci} = 0.05$, $dy/\rho_s = 0.4$,
- $k_y \rho_s = 1$, $a/\rho_s = 0.3$, $L_x/\rho_s = 10$, $\theta_m = 0.01285$, $T_e/T_i = 1$, $\beta = 0.001$, $m_i/m_e = 1836$.
- Code $\omega/\omega_{ci} = i \cdot 0.6$, eigenmode solution $\omega/\omega_{ci} = i \cdot 0.6$.
- left plot-eigenmode solution, right plot- PIC code simulation result.
- (a) - absolute value of $A_{\|}(x)$, (b)- absolute value of $\phi(x)$.