Resonance overlap and Phase Correlations in the NSTX RF-NBI interaction

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53rd Annual Meeting of the APS Division of Plasma Physics

Work supported by US DoE contract DE-AC02--09CH11466

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Abstract

An analytic model for single particle motion in the presence of a wave field and multiple cyclotron harmonics is developed and investigated. The model sheds light on how particle dynamics change as the number of harmonic layers increases. Moreover, it suggests that the quasi-linear theory may not be appropriate for modeling the RF-NBI interaction in NSTX. This point is illustrated with direct calculations of phase space diffusion.

Background

- Experimental studies show that the NBI ions act as a parasitic energy sink for the HHFW
- This has lead to efforts to model the RF-NBI interaction by CompX and M. Choi
- Both have employed the quasi-linear theory (QLT)
- Some quantitative disagreement with experimental data remains
- Harvey et. al. have demonstrated that QLT may break down in C-Mod

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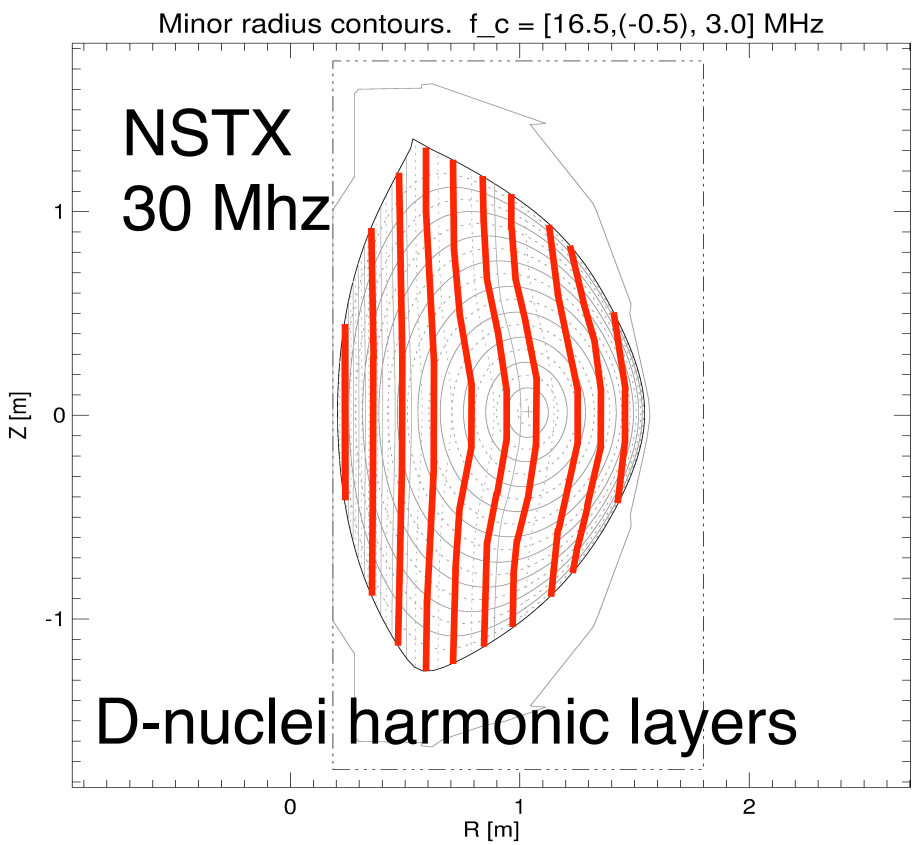
Question

Can the QLT be used to model the RF-NBI interaction in NSTX?

The answer is not obvious.

Beam particles in NSTX experience many harmonic layers



major radius

cyclotron frequency

RF frequency

$$\omega = m \frac{\Omega_o R_o}{R_m}$$

harmonic number

location of m'th layer

$$\Rightarrow \Delta = \frac{\Omega_o}{\omega} R_o$$

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Goals for this poster

1. Examine the effects of multiple harmonic layers on particle orbits
2. Assess the validity of the use of QLT to model the RF-NBI interaction in NSTX

Outline

① Present a simple analytical model for particle dynamics with multiple harmonic layers

② Analyze how harmonic layers change the phase space resonance structure

③ Compare the QLT predicted diffusion rate to the actual model prediction using NSTX relevant parameters

The model fields are as follows:

NSTX

$$
\begin{aligned}
\mathbf{B} &= B_z(x)e_z + B_P e_x \\
B_z(x) &= B_T\left(1 - \epsilon\cos(k_b x)\right) \\
\mathbf{A} &= -\frac{E_o}{\omega}\sin\left(k_\perp x - \omega t\right)e_y
\end{aligned}
$$

These fields are akin to those found in the vicinity of a drift surface along which a compressional Alfven wave propagates

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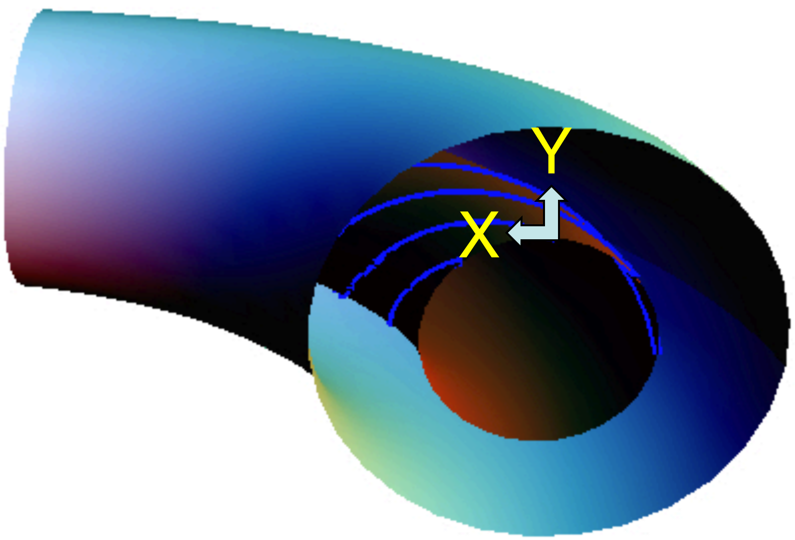

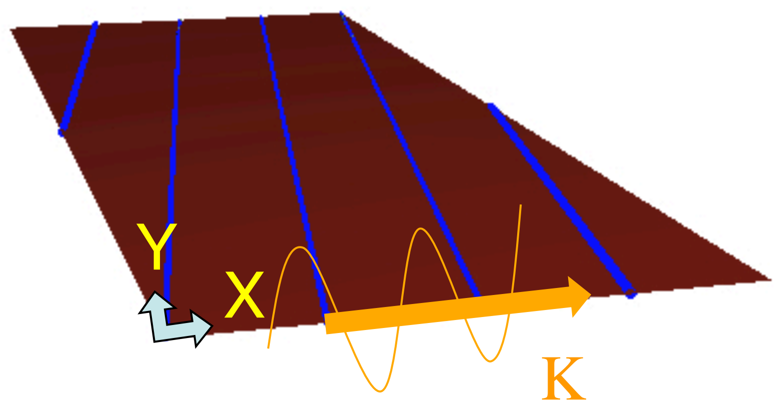

•drift surface in red
•field lines in blue

While these are idealized, they capture important physics

- Important **omissions**:
  - – wave propagation perpendicular to drift surface
  - – multiple Fourier components for the wave

- Important **features:**
  - – multiple harmonic layers which particles drift across
  - – basic mechanism for wave-particle interaction

From the fields, the Hamiltonian determines the dynamics

Invoking the guiding center approximation and symmetry under translations along z, one can deduce the guiding center coordinate change to write the ion Hamiltonian as follows:

$$
\begin{aligned}
h &= \frac{\mu}{\nu}(1-\epsilon\cos X) + \frac{1}{2l^2\nu}\psi^2 + p_\tau \\
&\quad +\alpha\sqrt{2\mu}\sin\theta\sin\left(\kappa X + \sqrt{2\mu}(1-\frac{1}{2}\epsilon\cos X)\sin\theta - \tau\right)
\end{aligned}
$$

zero'th order

wave

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These are the normalizations and definitions

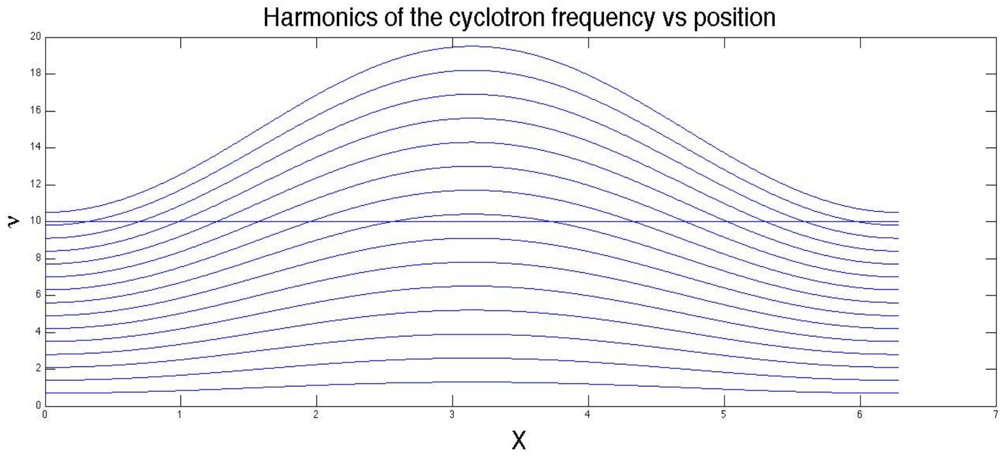
Normalizations:

$$
\begin{aligned}
\mu &= \frac{k_\perp^2}{\Omega_z^2}\frac{1}{2}v_\perp^2 \\
X &= k_b X \\
\psi &= k_\perp^2 Y/k_b + const. \\
\tau &= \omega t
\end{aligned}
$$

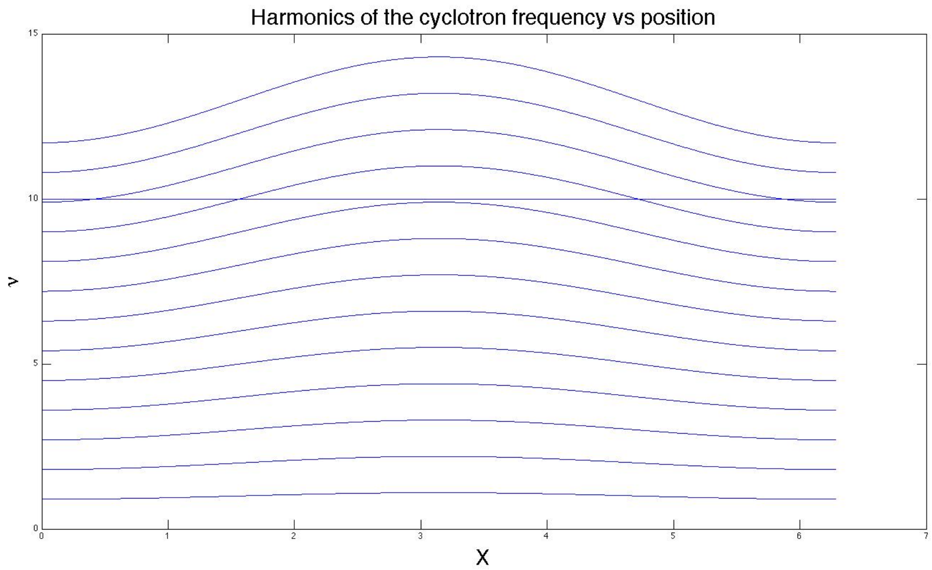
Definitions:

$$
\begin{aligned}
\kappa &= k_\perp/k_b \in \mathbb{Z} \\
\alpha &= \frac{q_i E_o}{m_i \omega}\frac{k_\perp}{\omega} \\
\nu &= \omega/\Omega_T \\
l &= k_\perp R_o q
\end{aligned}
$$

Larger epsilon leads to particles crossing more harmonic layers



$\epsilon = .3$



$\epsilon = .1$

Now we can analyze the dynamics

The approach will be:

1) Determine frequencies of unperturbed motion
2) Calculate the first-order resonance conditions
3) Compute the widths and spacing of the resonances

The result will be:

Conditions for the onset of stochastic motion and therefore stochastic heating

1) Unperturbed frequencies

$$h_o = \frac{\mu}{\nu}(1 - \epsilon \cos X) + \frac{1}{2l^2\nu}\psi^2$$

- Unperturbed motion has 2 first integrals
- Therefore motion occurs on invariant tori with a pair of frequencies that only depend on values of constants of motion

$$\forall q, \quad q(t) = \sum_n a_n e^{in\omega_1 t} + b_n e^{in\omega_2 t}$$

$$\omega_1 = \omega_1(\mu, h) \quad , \quad \omega_2 = \omega_2(\mu, h)$$

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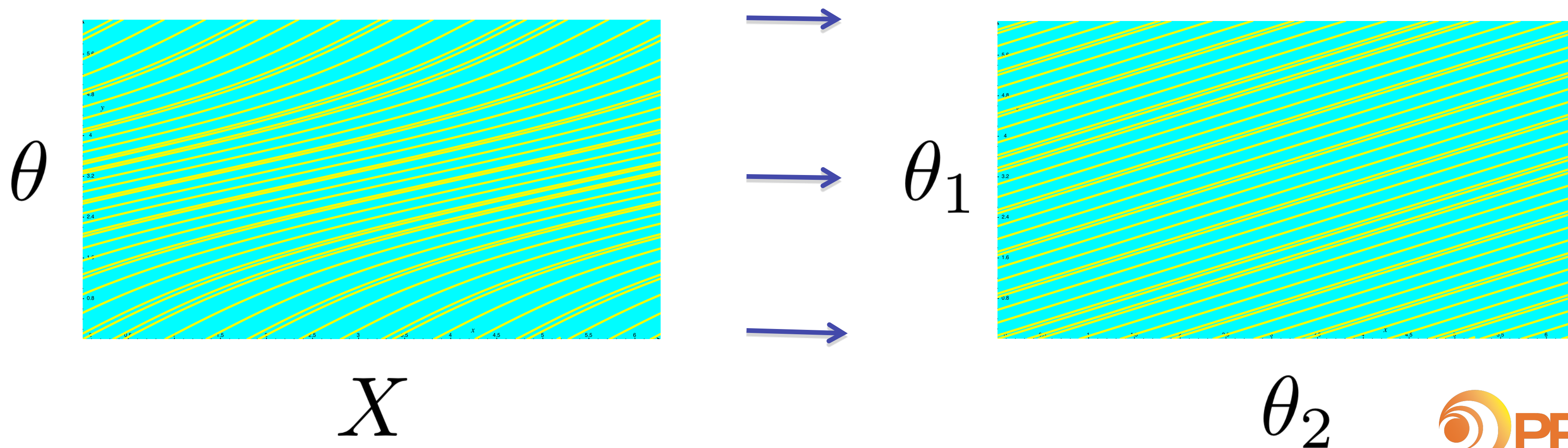
1) Unperturbed freq. (contd.)

- By finding the action-angle variables, the tori and the frequencies can be found all at once
- These coordinates are just “straight field line coordinates” of the dynamical vector field



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1) Unperturbed freq. (final)

- In action-angle coordinates for the passing motion, the Hamiltonian is simply:

$$h = \frac{1}{\nu} I_1 + \frac{1}{2l^2\nu} I_2^2 + O(\epsilon^2)$$

- The frequencies are therefore:

$$\begin{aligned} \omega_1(I_1, I_2) &= \frac{1}{\nu} + O(\epsilon^2) \\ \omega_2(I_1, I_2) &= \frac{1}{l^2\nu} I_2 + O(\epsilon^2) \end{aligned}$$

2) Resonance structure

- The wave perturbation can be expressed as a Fourier series in the angles and time:

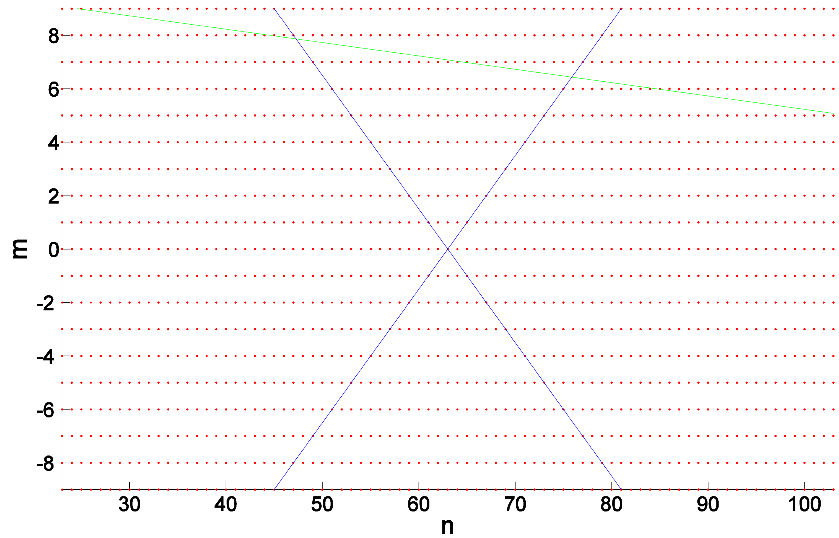
$$h_1 = \sum_{(m,n)} a_{mn}(I_1, I_2) \exp i \left(m\theta_1 + n\theta_2 - \tau \right)$$

- As long as an unperturbed particle samples it's entire invariant torus, each term in the perturbation will approximately 'average out'
- However, if for some (m,n) $m\omega_1 + n\omega_2 - 1 = 0$ then the corresponding term in the perturbation has a large effect

2) Resonance structure (contd.)

- When epsilon is 0, the only non-zero resonant terms have n=κ (Doppler shifted cyclotron harmonic resonances)
- As epsilon increases the number of finite resonant terms increases as illustrated on next slide

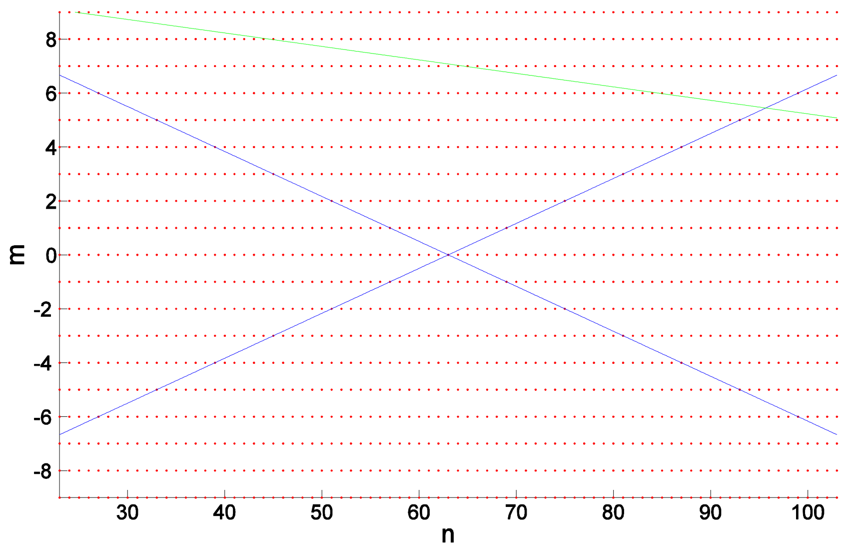
2) Resonance structure (final)



$\epsilon = .1$

Green: resonance line for a fixed I_2

Blue: bounding line



$\epsilon = .3$

- Those m,n inside of the blue 'light cone' correspond to non-zero terms in the perturbation
- increasing epsilon results in a greater number of resonant terms

3) Resonance overlap condition

- The width of the resonance regions may be calculated by dropping the non-resonant terms from the Hamiltonian and examining the resulting dynamics
- The result is that the width in I_2 is given by

$$\delta I_2 = 4l\,(2/\pi)^{1/2}\,(\nu\alpha)^{1/2}(2\mu)^{1/8}J_0\left(\frac{ml^2\epsilon}{I_2}\right)^{1/2}$$

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3) Resonance overlap condition (cond.)

- The spacing between resonances is approximately

$$\Delta I_2 = \frac{(I_2/l)^2}{\epsilon\nu + \kappa I_2/l^2}(1+\epsilon)$$

- The overlap criterion is therefore

$$\frac{\delta I_2}{\Delta I_2} > 1$$

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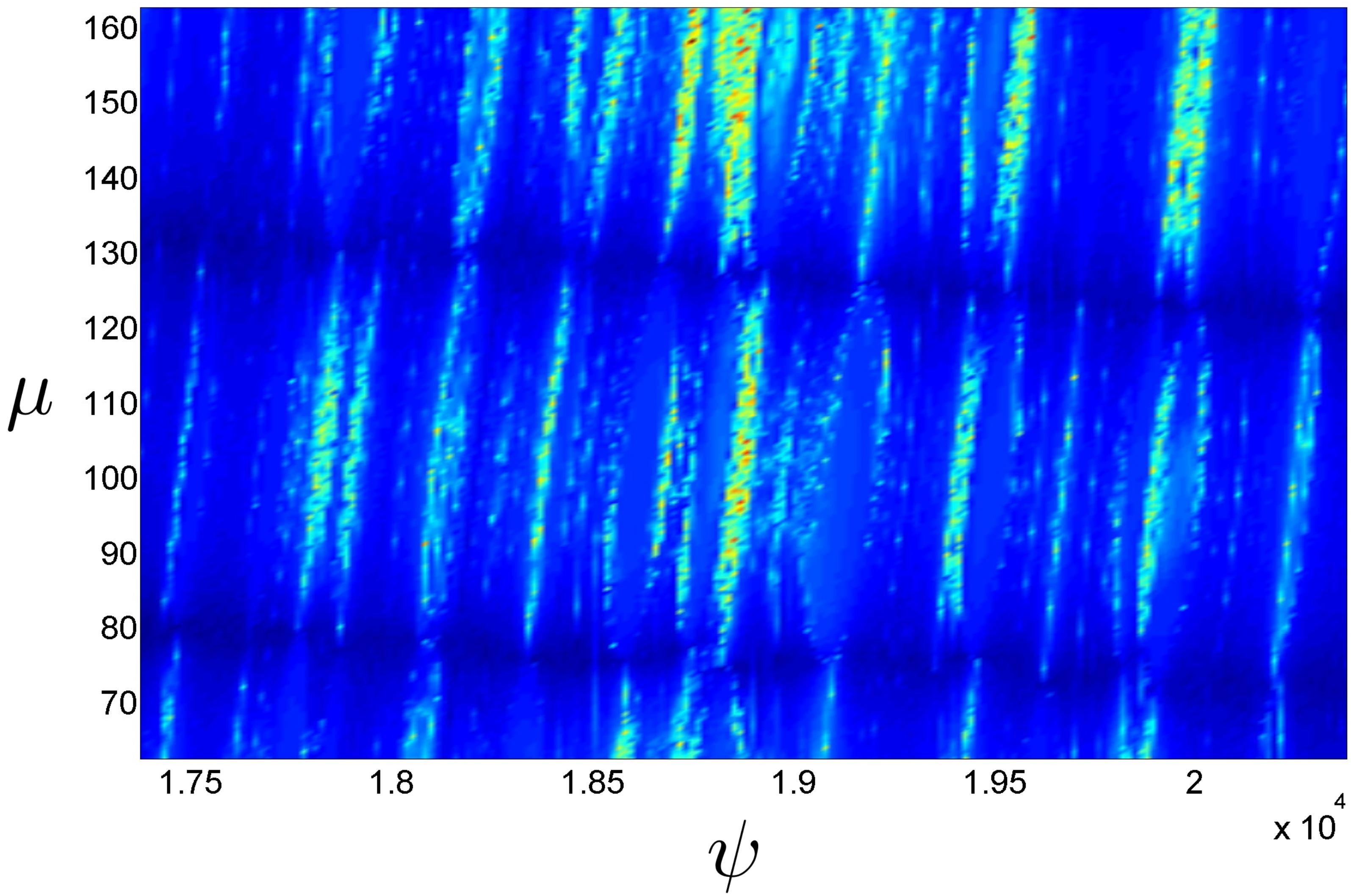
3) Resonance overlap condition (contd.)



$\epsilon = .1$

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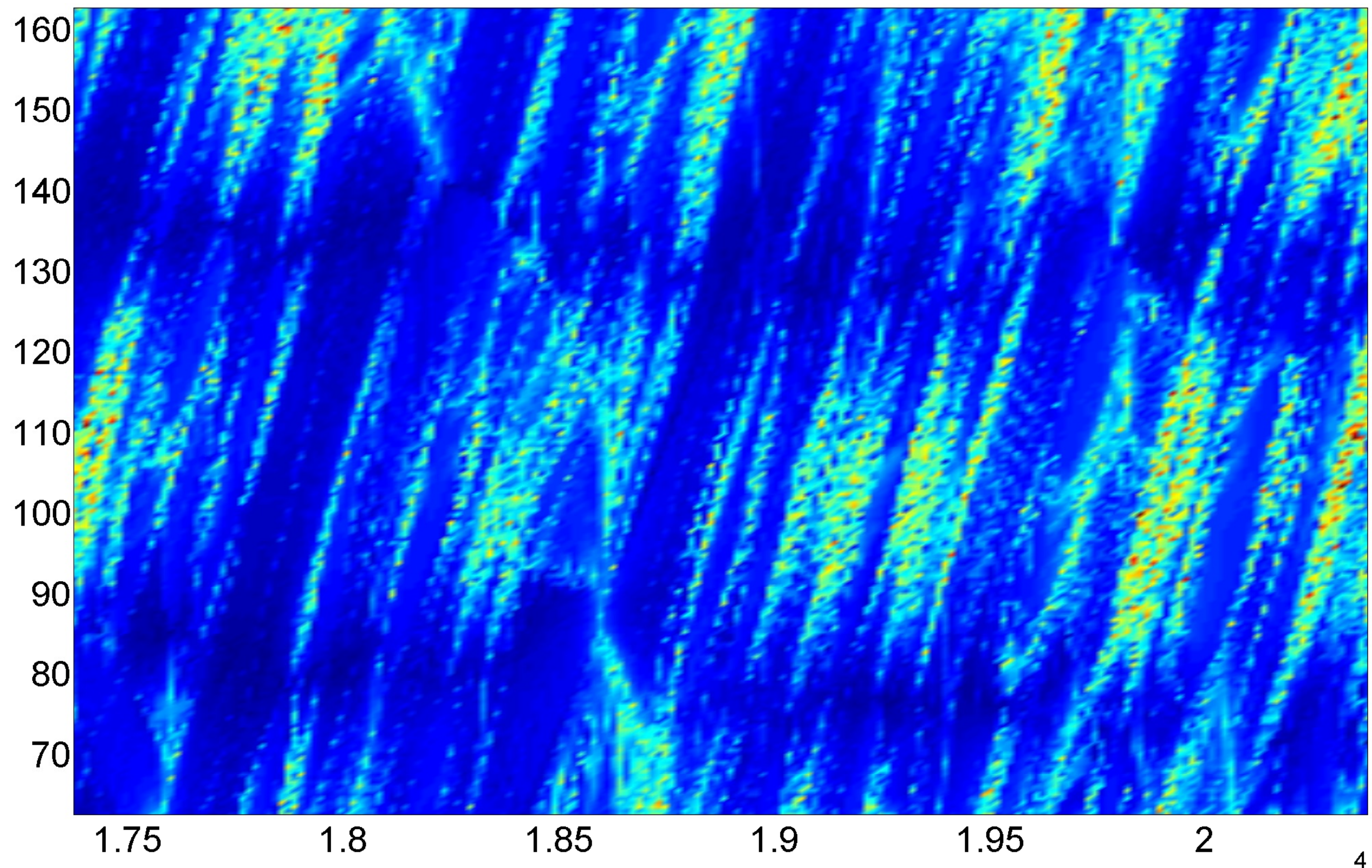
3) Resonance overlap condition (final)



μ

ψ

$\epsilon = .3$

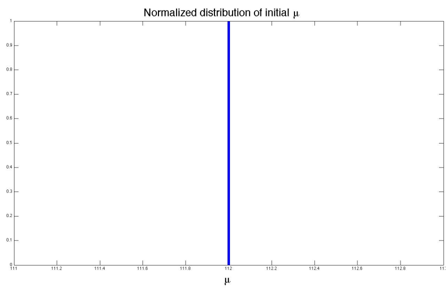
The behavior of chaotic trajectories is important

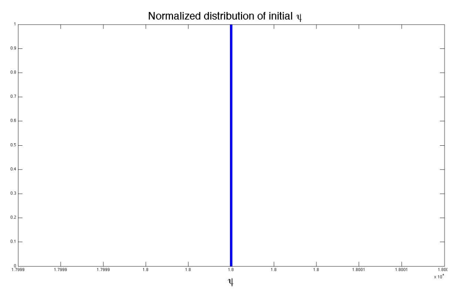
The rate of diffusion will be partially responsible for the heating rate of the chaotic ions:

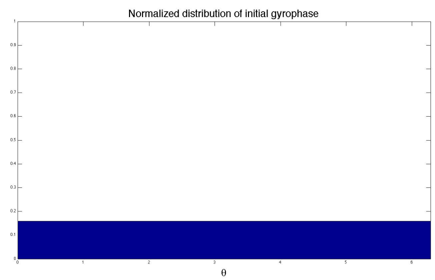
- A population of chaotic beam ions will diffuse in energy
- The rate of that diffusion will determine how quickly ions give or take energy from the wave

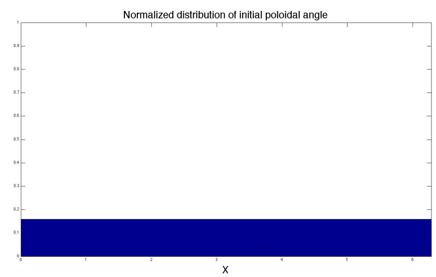
So we will see what diffusion the model predicts…

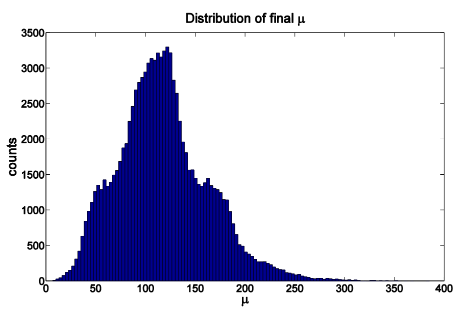
Simply numerically integrate the equations of motion for an ensemble of initial conditions

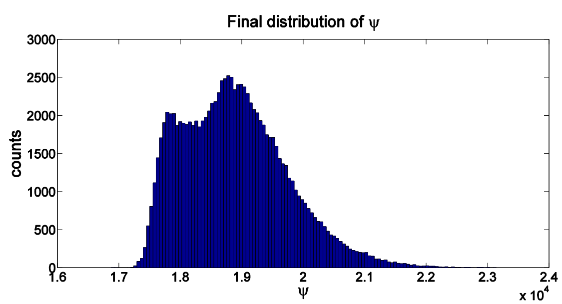












μ ψ θ X

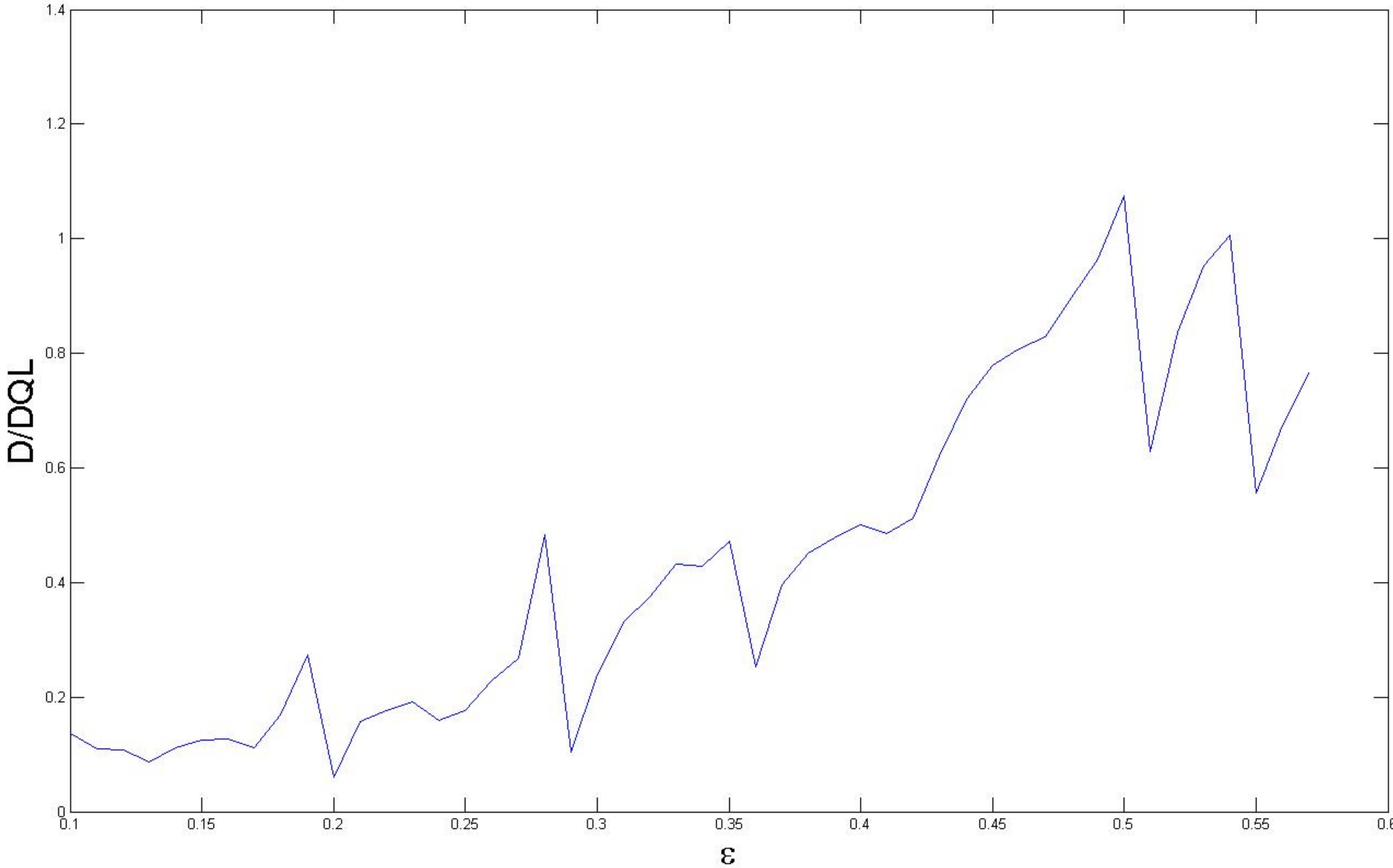
…and compare the results to the quasi-linear prediction

The QLT assumes particles will loose their phase memory between crossings of the harmonic layers

- This leads to modeling the RF interaction as a sequence of random kicks given at the harmonic layers
- In the context of this model, the magnetic moment diffusion constant is therefore

$$D_{ql} = \frac{\psi / l^2}{2\pi\nu} \sum_i 2 \left\langle \Delta\mu^2 \right\rangle_i$$

The results indicate the presence of anomalous diffusion



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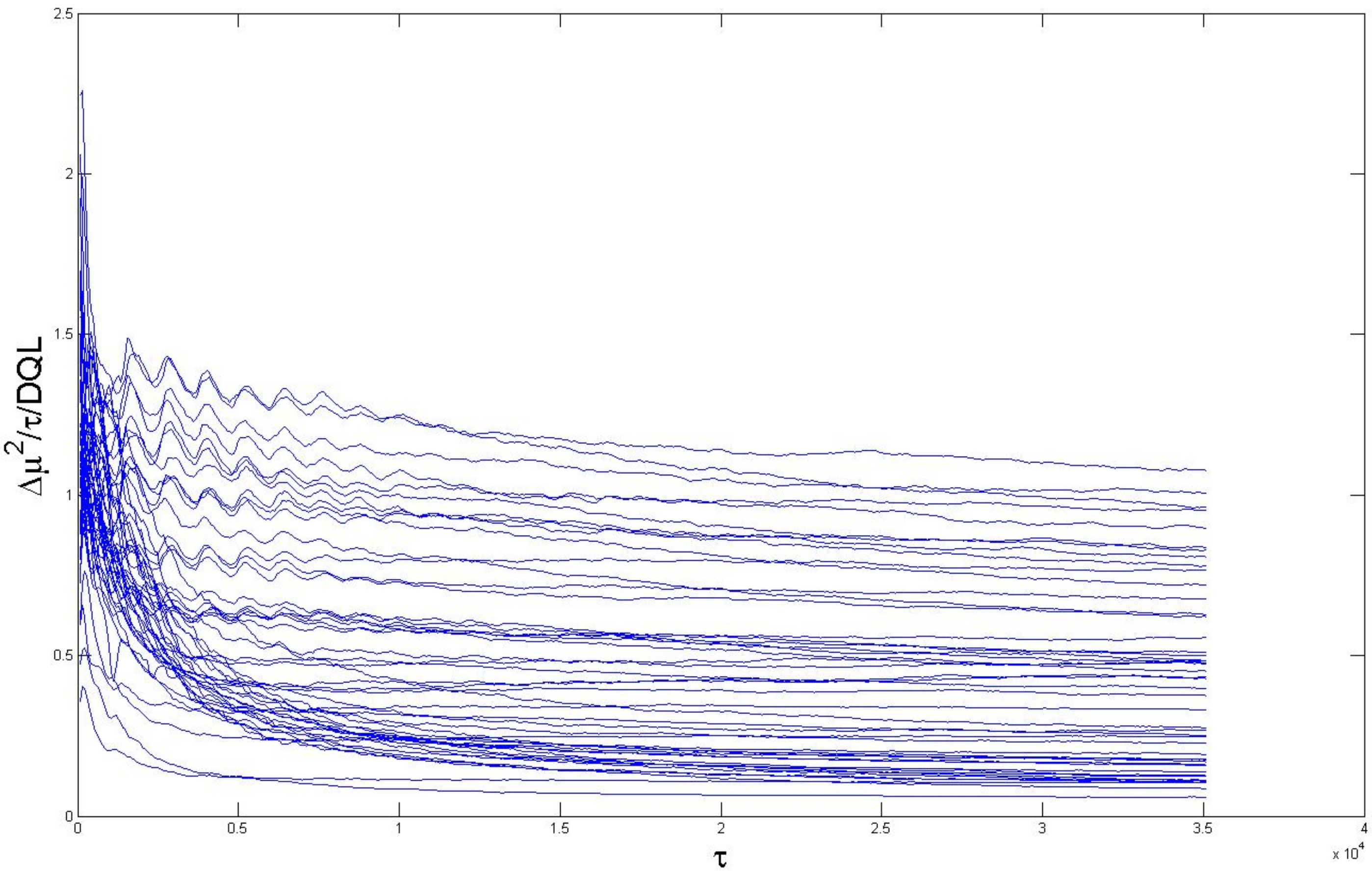

Integration time is 30 circulation periods

Conclusions

- Harmonic layers correspond to resonances in phase space. These can overlap or intersect. When either occurs, chaotic ion dynamics ensue.
- The validity of using the QLT to model the RF-NBI is questionable. The validity needs to be examined for realistic fields. A more careful modeling approach may be necessary.

Future Work

- Calculate particle diffusion in realistic fields using G. J. K.'s full orbit solver SPIRAL to assess validity of QLT more carefully.

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