Simulation of microtearing turbulence in NSTX and scaling with collisionality

Walter Guttenfelder[1]

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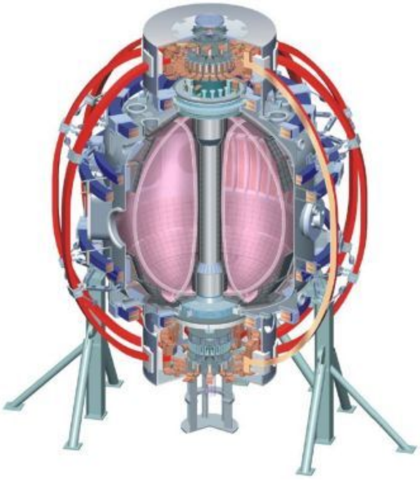

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Overview

- Experimental motivation - favourable $\Omega_i\tau_{E,th} \sim \nu_*^{-(0.8-0.95)}$ dependence in STs
  - Microtearing modes found to be unstable in experimental ν_* scans, $\gamma_{lin} \sim \nu_e$

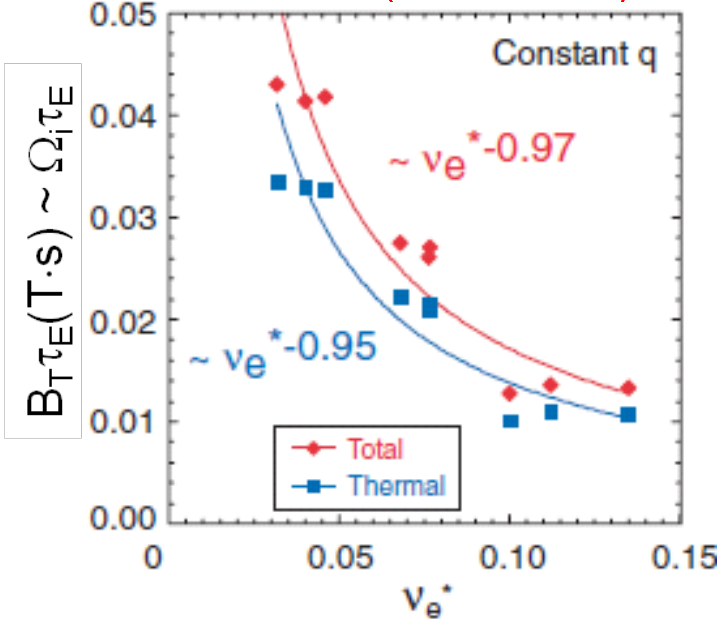
- Linear microtearing properties for high-β NSTX discharges
  - Electromagnetic, electron drift mode with narrow resonant current layer, $\Delta_j<\rho_s$
  - Non-monotonic dependence on $\nu^{e/i}/\omega$, threshold in β_e and ∇T_e
  - Z_{eff} (and s/q) scaling distinct from ETG

- Non-linear simulations
  - Necessary to "resolve" (distinguish) each simulated rational surface
  - Transport is experimentally significant and dominated by magnetic "flutter"

- Scaling of non-linear transport
  - Predicted $\chi_{e,sim} \sim \nu_e^{1.1}$ close to experimental trend
  - "Stiff" with ∇T_e but suppressible by experimental levels of E×B shear

Experimental motivation - strong collisionality scaling in STs

NSTX (no-Lithium)



NSTX* (Kaye et al., Nucl. Fusion 2007)

$$\boxed{\Omega\tau_E^{th} \sim \nu_{*e}^{-0.95}}$$

MAST (Valovič et al., Nucl. Fusion 2011)

$$\boxed{\Omega\tau_E^{th} \sim \nu_{*e}^{-0.82}}$$

ITER (PIPB, Doyle et al., Nucl.Fusion 2007)

$$\boxed{\Omega\tau_E^{th,04(2)} \sim \nu_{*e}^{-0.2}}$$

*Y. Ren, TI-2.2
S. Kaye, PP-9.30 (Thurs. PM)
S. Gerhardt, YI-2.2 (Fri. AM)

- Ion transport is neoclassical, consistent with strong toroidal flow and flow shear
- What is the cause of anomalous electron thermal transport?
- Will favorable τ_E scaling hold at lower ν_* envisioned for next generation ST (high heat flux, CTF, …)?

Experimental motivation - strong collisionality scaling in STs

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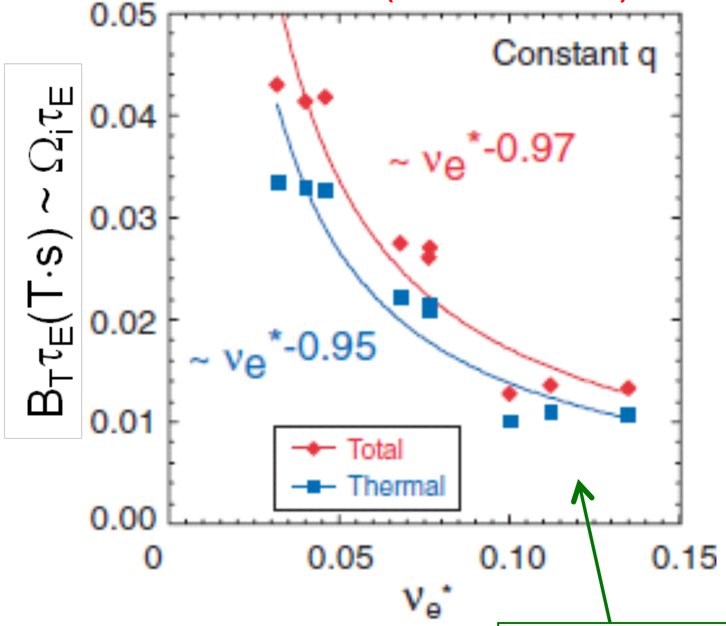

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Following simulations based on a single NSTX high-ν_* discharge
B_T=0.35T, I_p=0.7 MA, P_{NBI}=4 MW, $n_e \approx 6\times10^{19}$ m^{-3}, $T_e(0)$~1 keV

- Ion transport is neoclassical, consistent with strong toroidal flow and flow shear
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- Will favorable τ_E scaling hold at lower ν_* envisioned for next generation ST (high heat flux, CTF, …)?

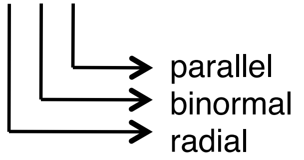
GYRO* used for gyrokinetic simulations

- Eulerian solver of gyrokinetic-Maxwell equations, evolving $\delta f(E,\lambda,r,\alpha,\theta)$

$$\delta f \sim \delta\hat{f}(r,\theta)e^{-in\alpha} \qquad \alpha = \phi + \nu(r,\theta) \approx \phi - q(r)\theta$$

$$k_\theta \doteq \frac{nq}{r}$$

High-aspect ratio, low β limit



- Kinetic ions (D+C) and electrons, general equilibrium
- Fully collisional & electromagnetic ($\delta A_{||}$, $\delta B_{||}$) (both important in NBI heated ST)
- Freedom to include toroidal flow and flow shear (important in NBI heated ST)
- Can use experimental profile variations, T(r), n(r), q(r), etc… (likely important in ST, $\rho_s/a \sim 1/100$, $\rho_s/L \sim 1/40$)

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*J. Candy & R.E. Waltz, Phys. Rev. Lett. **91**, 045001 (2003); J. Comp. Physics **186**, 545 (2003); https://fusion.gat.com/theory/Gyro
J. Candy, Phys. Plasmas Control. Fusion **51**, 105009 (2009); E.A. Belli & J. Candy, Phys. Plasmas **17**, 112314 (2010).

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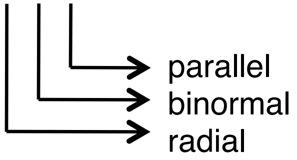

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- *All following linear calculations performed in the local flux-tube limit (periodic BC's) without toroidal flow & shear*

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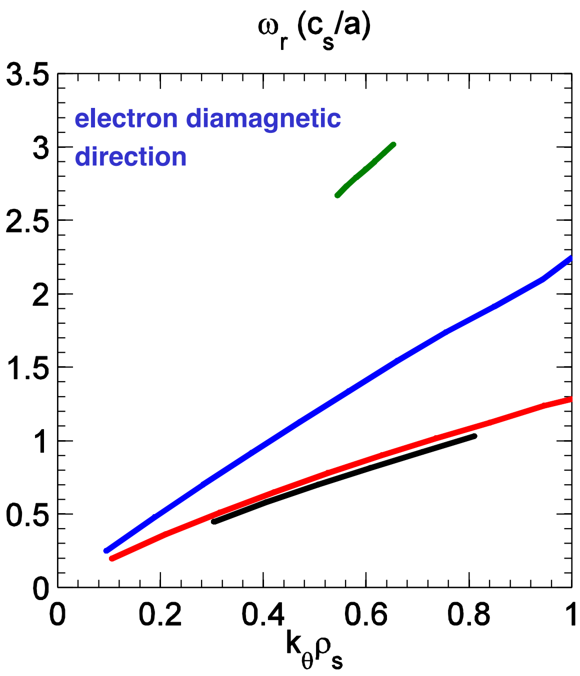
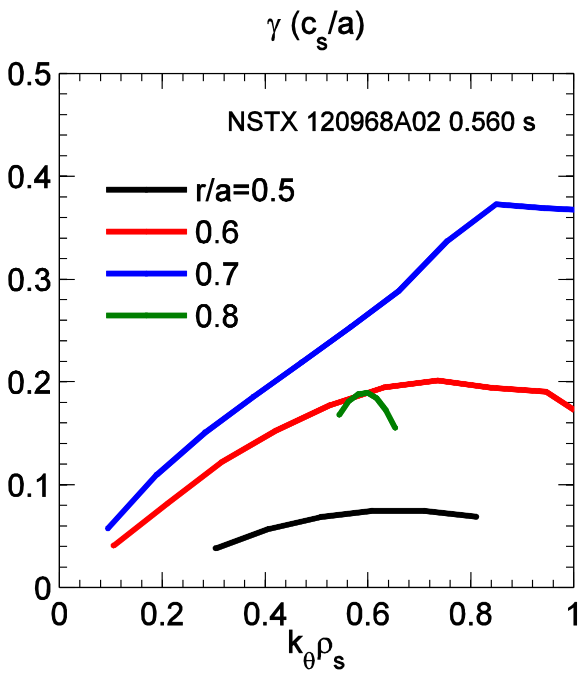
Microtearing modes found to be unstable in many high ν_* discharges

- Microtearing dominates over r/a=0.5-0.8, $k_\theta\rho_s<1$ (n≈5-70)
- Real frequencies in electron diamagnetic direction, $\omega \approx \omega_{*e} = (k_\theta\rho_s)\cdot(a/L_n+a/L_{Te})\cdot(c_s/a)$
- ETG mostly stable due to larger $Z_{eff}\approx 3$, $(R/L_{Te})_{crit,ETG} \sim (1+Z_{eff}T_e/T_i)$

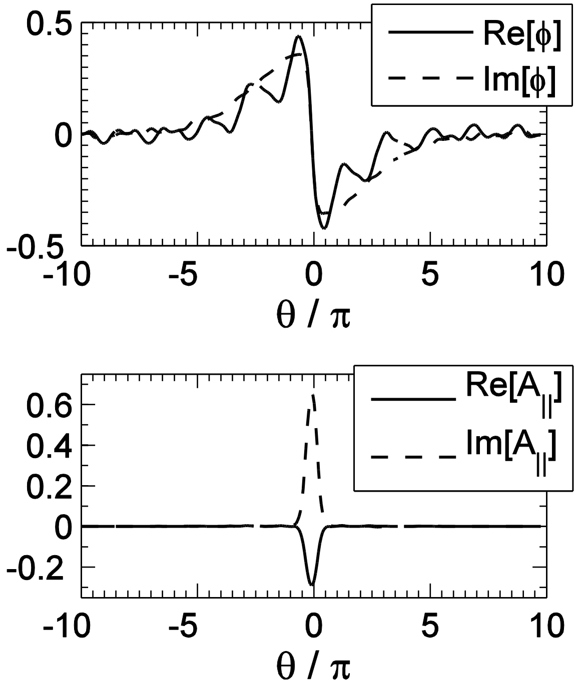
real frequencies



growth rates



eigenfunctions in “ballooning” space



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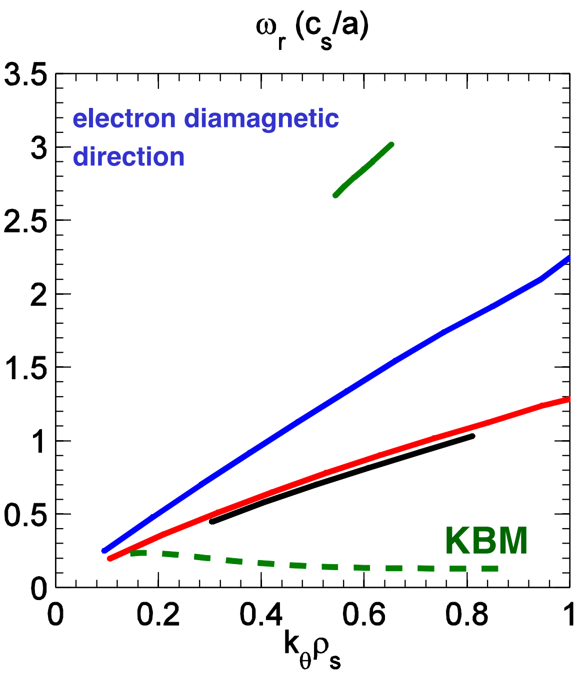
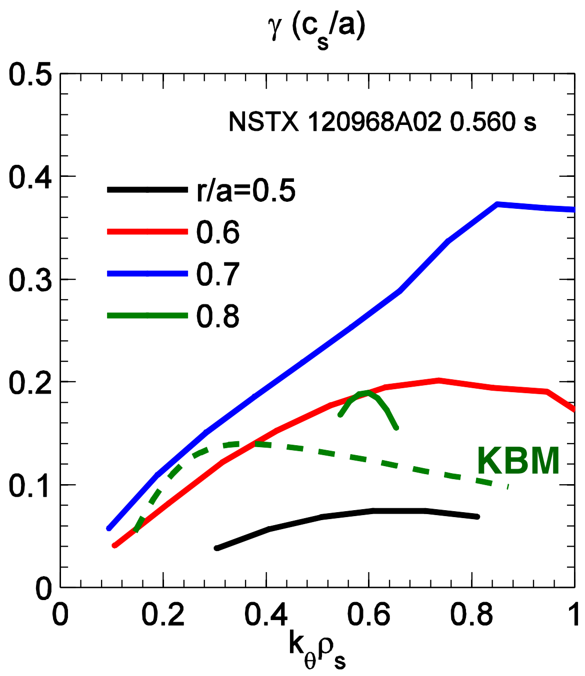
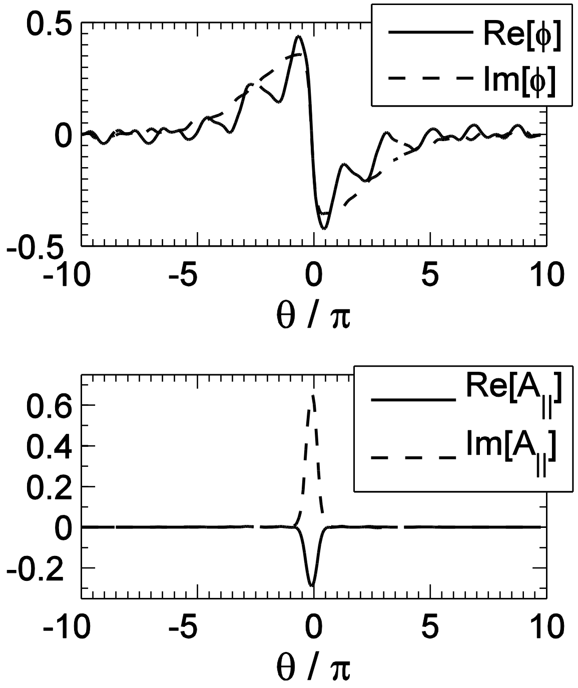
- ETG mostly stable due to larger $Z_{eff}\approx3$, $(R/L_{Te})_{crit,ETG}\sim(1+Z_{eff}T_e/T_i)$
- KBM competes farther out (r/a≥0.8) where $\alpha_{MHD}=-q^2R\beta'$ much larger (larger q, a/L_n)


real frequencies


growth rates


eigenfunctions in “ballooning” space

Following calculations mostly for r/a=0.6

Linear microtearing instability

- High-m tearing mode around a rational $q(r_0)=m/n$ surface ($k_{||}(r_0)=0$)
  (Classical tearing mode stable for large m, $\Delta' \approx -2m/r < 0$)
- Driven by ∇T_e with time-dependent parallel thermal force* $\Rightarrow$ *requires e-i collisions*

Conceptual linear picture

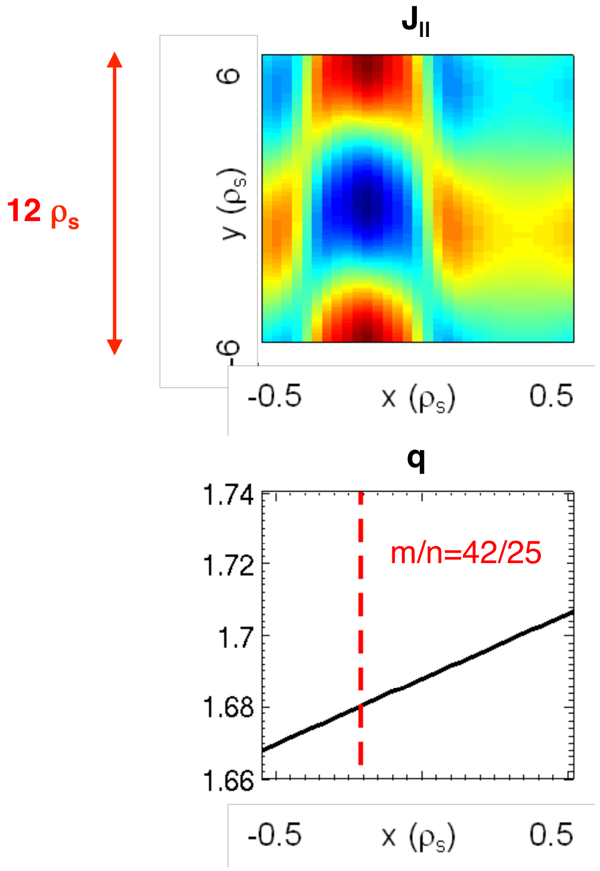
- Imagine helically resonant ($q=m/n$) δB_r perturbation $$\delta B_r \sim \cos(m\theta - n\varphi)$$
- δB_r leads to radially perturbed field line, finite island width $$w = 4\left(\frac{\delta B_r}{B}\frac{rR}{n\hat{s}}\right)^{1/2}$$
- ∇T_e projected onto field line gives parallel gradient $$\nabla_{||} T_{e0} = \frac{\vec{B}\cdot\nabla T_{e0}}{B} = \frac{\delta B_r}{B}\underline{\nabla T_{e0}}$$
- Parallel thermal force ($R_{T||} \sim -\alpha(\omega) n_e \nabla_{||} T_e$) drives parallel electron current that reinforces δB_r via Amperes's law $$k_\perp^2 \rho_s^2 \hat{A}_{||} = \frac{\beta_e}{\underline{2}}\hat{j}_{||} \quad , \quad B_r = i k_\theta A_{||}$$
- Instability requires sufficient ∇T_e, β_e, ν_e, and time dependence (ω) important

*e.g. Hazeltine et al., Phys. Fluids 18, 1778 (1975); Gladd et al., Phys. Fluids 23, 1182 (1980); D'Ippolito et al., Phys. Fluids 23, 771 (1980); M. Rosenberg et al., Phys. Fluids 23, 2022 (1980).

Linear mode structure in perpendicular plane illustrates key microtearing mode features

- Narrow resonant current channel (≈0.3ρ_s≈1.4 mm) centered on rational surface

x-y perpendicular plane (θ=0)

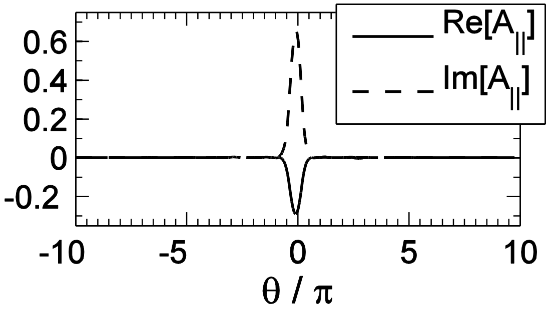


Linear mode structure in perpendicular plane illustrates key microtearing mode features

- Narrow resonant current channel (≈0.3ρ_s≈1.4 mm) centered on rational surface
- Finite $\langle A_{||}\rangle_\theta$ (resonant tearing parity), strongly ballooning

“ballooning” space

$$k_r(\theta) = \hat{s}k_\theta(\theta - \theta_0)$$



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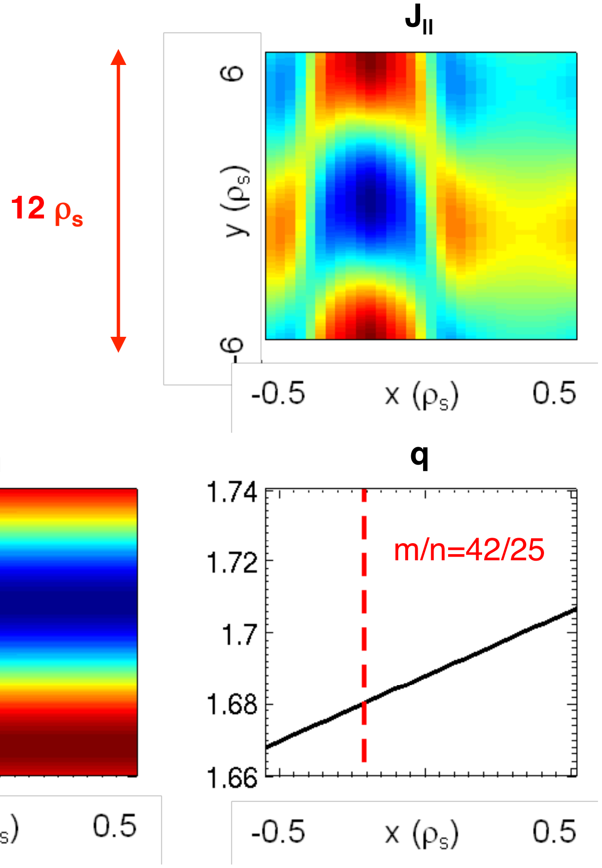

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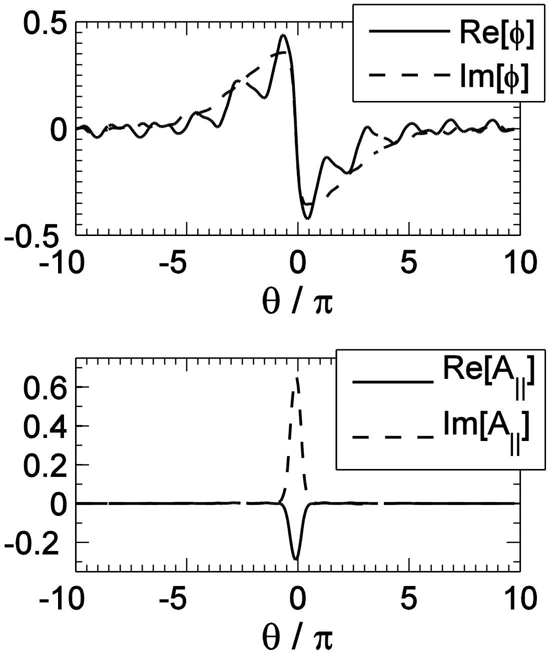
- Narrow resonant current channel (≈$0.3\rho_s$≈1.4 mm) centered on rational surface
- Finite $\langle A_{||}\rangle_\theta$ (resonant tearing parity), strongly ballooning
- Narrow n_e & T_e perturbations
- Nearly unmagnetized/adiabatic ion response $\Rightarrow \dfrac{\tilde{n}}{n_0} \approx -Z_{eff}\left(\dfrac{e\tilde{\varphi}}{T_i}\right)$

"ballooning" space

$$k_r(\theta)=\hat{s}k_\theta(\theta-\theta_0)$$



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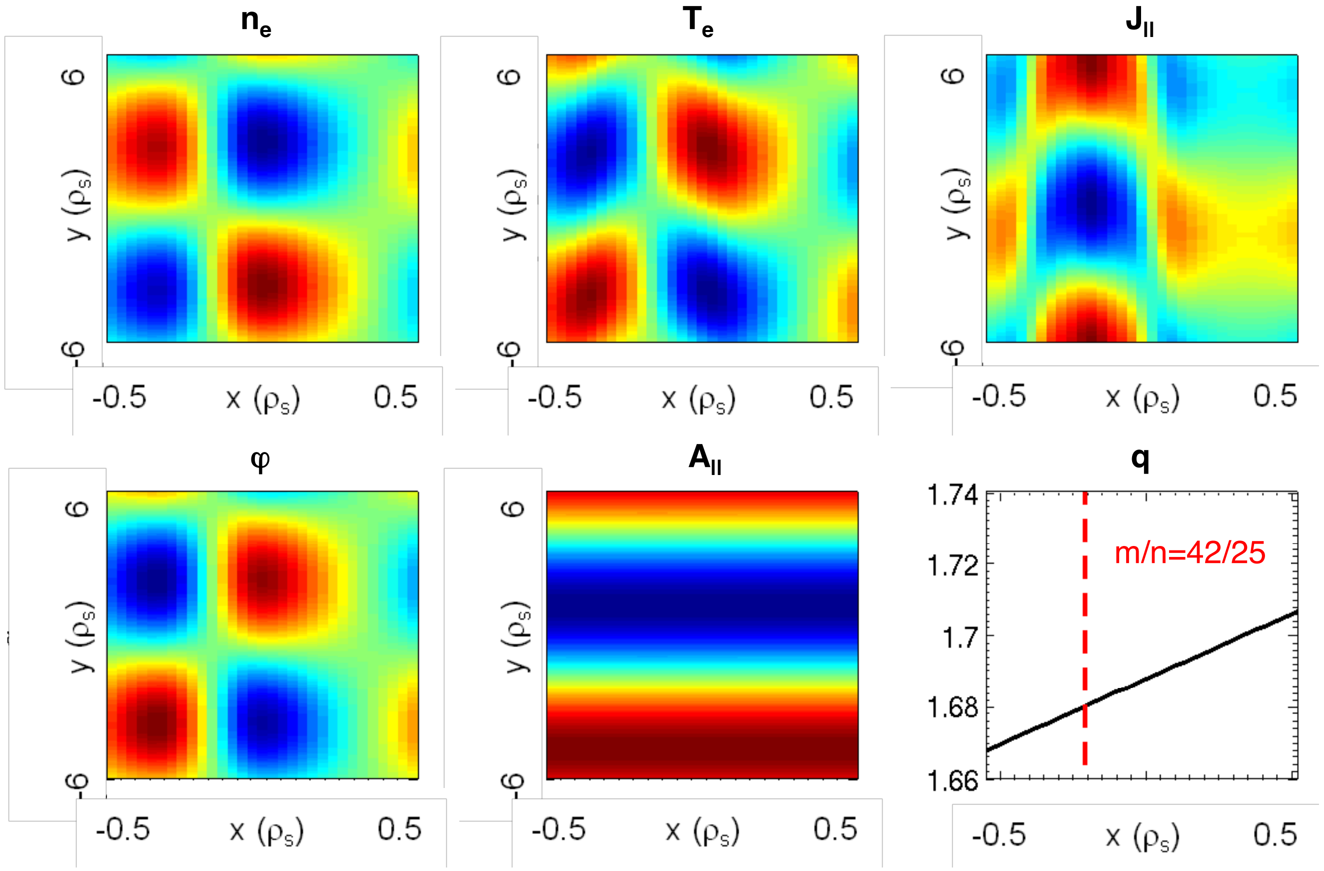
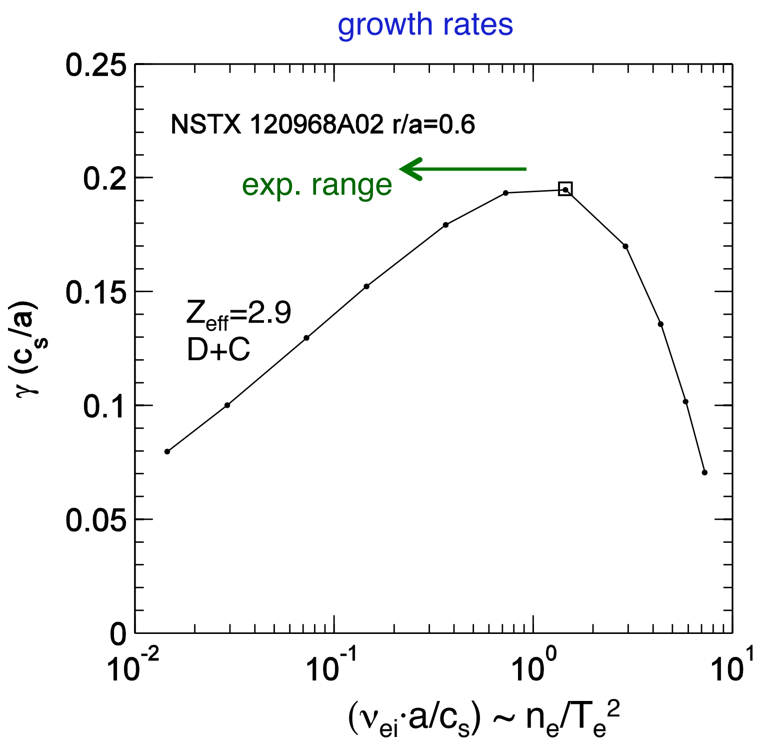


A distinguishing feature of the microtearing mode is the non-monotonic dependence on $\nu^{e/i}/\omega$

- Peak γ occurs for $\nu^{e/i}/\omega = Z_{eff}\cdot\nu_{ei}/\omega \sim 1\text{-}6$, similar to slab calculations (Gladd et al., 1980)
- γ decreases with ν_e in experimental range, qualitatively consistent with confinement scaling



$$\boxed{\nu^{e/i} = Z_{eff}\nu_{ei} \propto Z_{eff}\frac{n_e}{T_e^{3/2}}}$$

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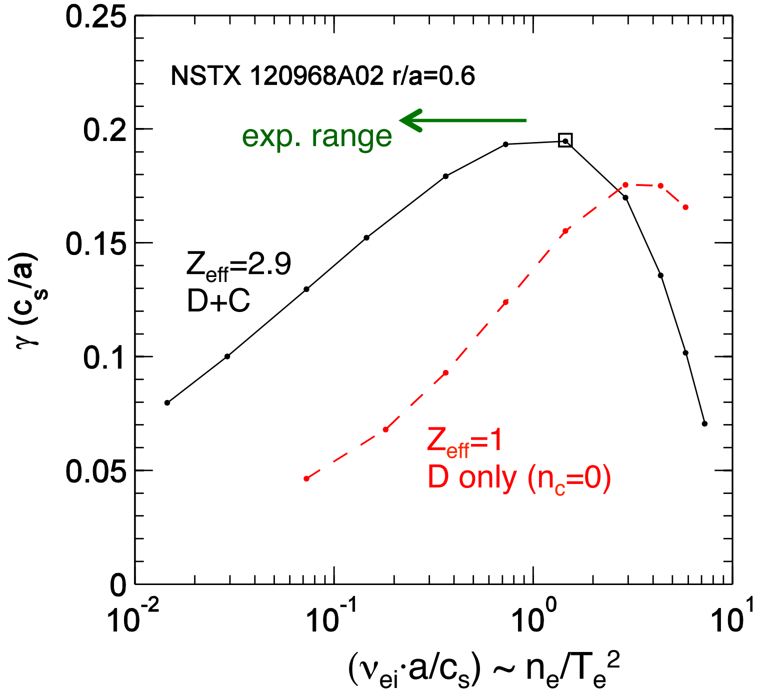
- γ decreases with ν_e in experimental range, qualitatively consistent with confinement scaling
- In addition to shifting peak in $\nu^{e/i}/\omega$, Z_{eff} can enhance instability through shielding potential (from adiabatic ion response, $\delta n_i \sim -Z_{eff}\delta\varphi/T_i$)

growth rates



$$\nu^{e/i} = Z_{eff}\nu_{ei} \propto Z_{eff}\frac{n_e}{T_e^{3/2}}$$

Jenko et al. (2001)

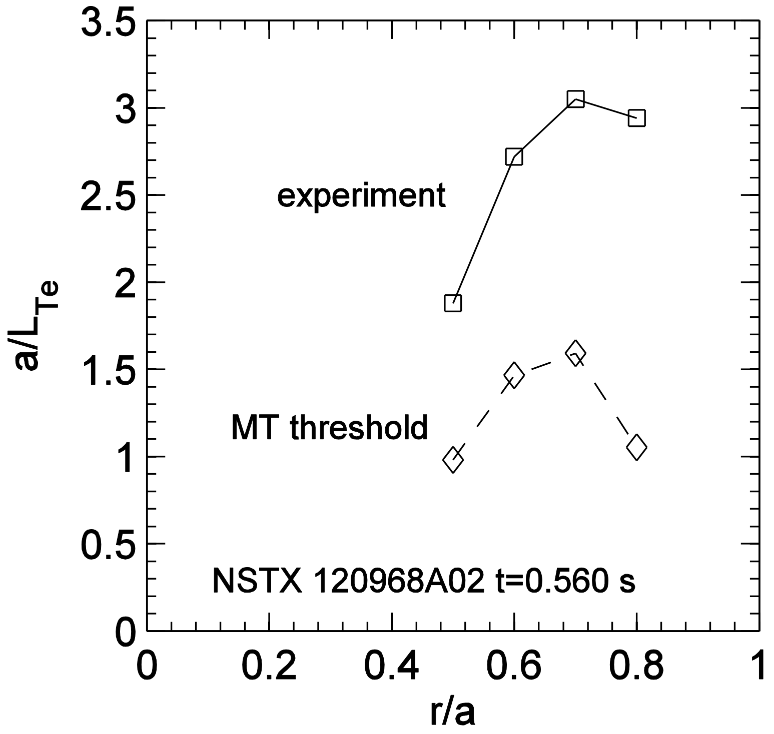
- Z_{eff} (and s/q)* dependence opposite to ETG expectations

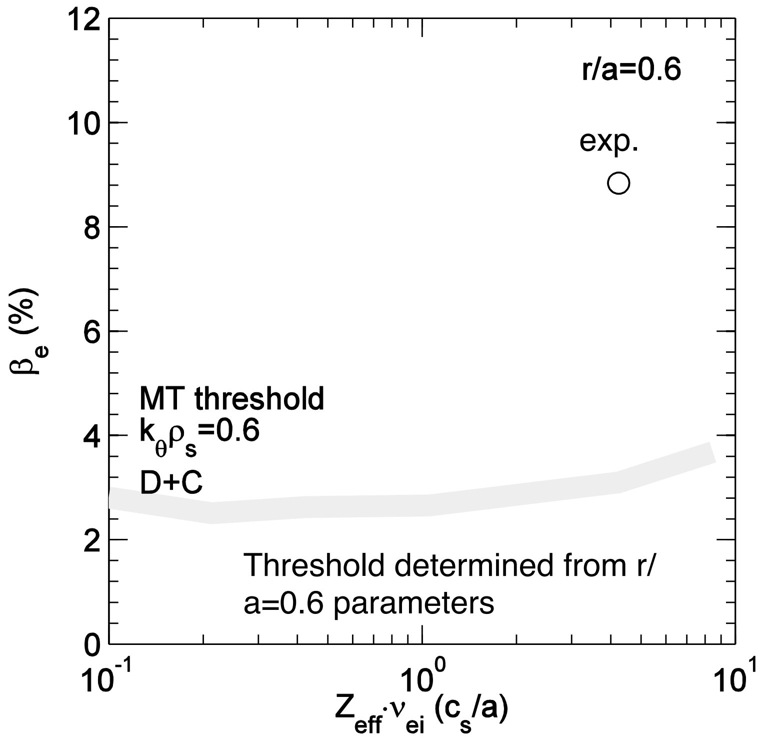
$$\left(\frac{R}{L_{Te}}\right)_{crit}^{ETG} \sim \left(1 + Z_{eff}\frac{T_e}{T_i}\right)\left(1.3 + 1.9\frac{s}{q}\right)(\cdots)$$

* Guttenfelder et al., *Scaling of linear microtearing stability for a high collisionality NSTX discharge*, submitted to Phys. Plasmas (Oct, 2011)

Microtearing instability exhibits thresholds in electron temperature gradient and beta

- In this high-ν_* discharge, a/L_{Te} and β_e are 2-3× larger than linear thresholds





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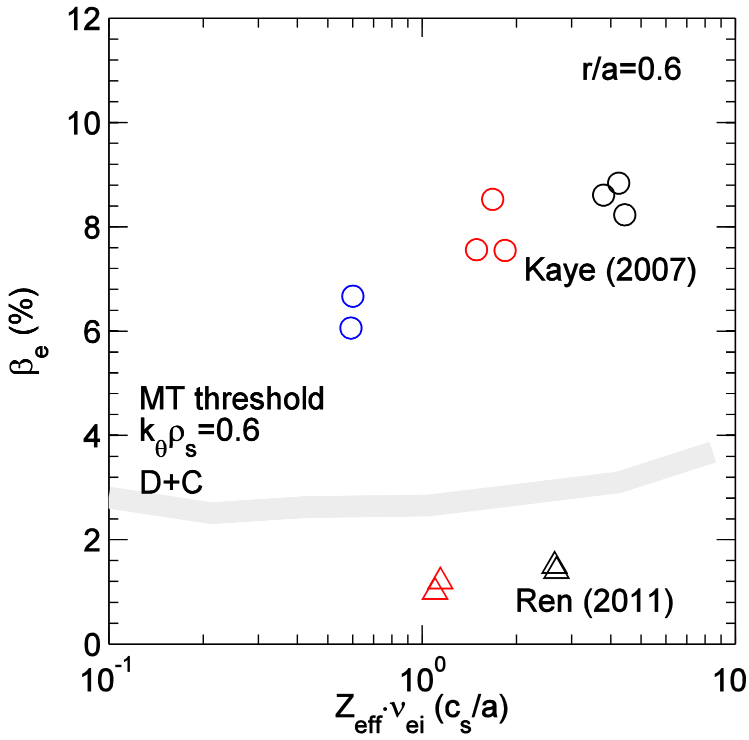
NSTX has studied electron transport for a range of beta and collisionality

- Ren (2011) ν_* experiment (invited talk TI-2.2) performed at lower β_e compared to Kaye (2007) (lower density and NBI power)



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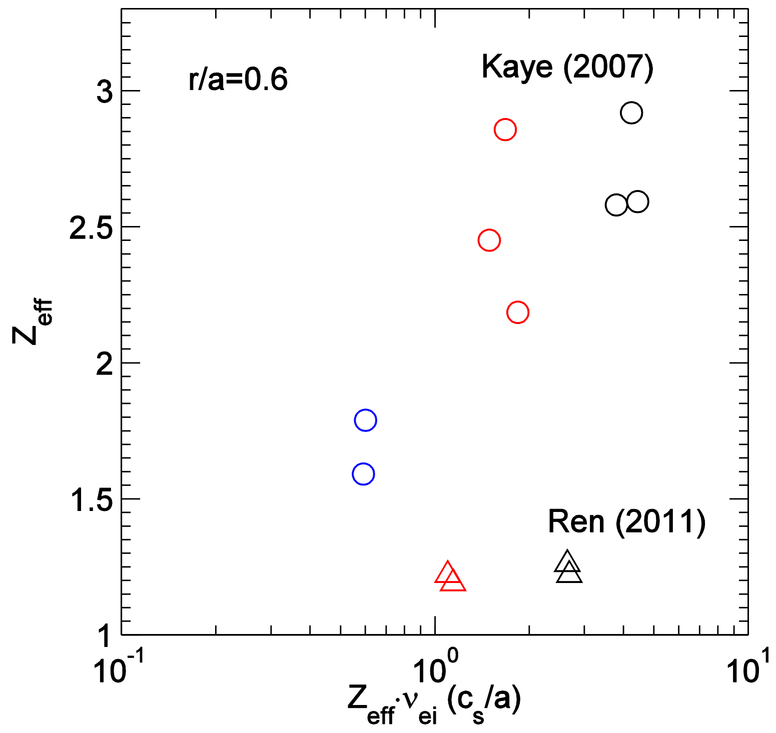

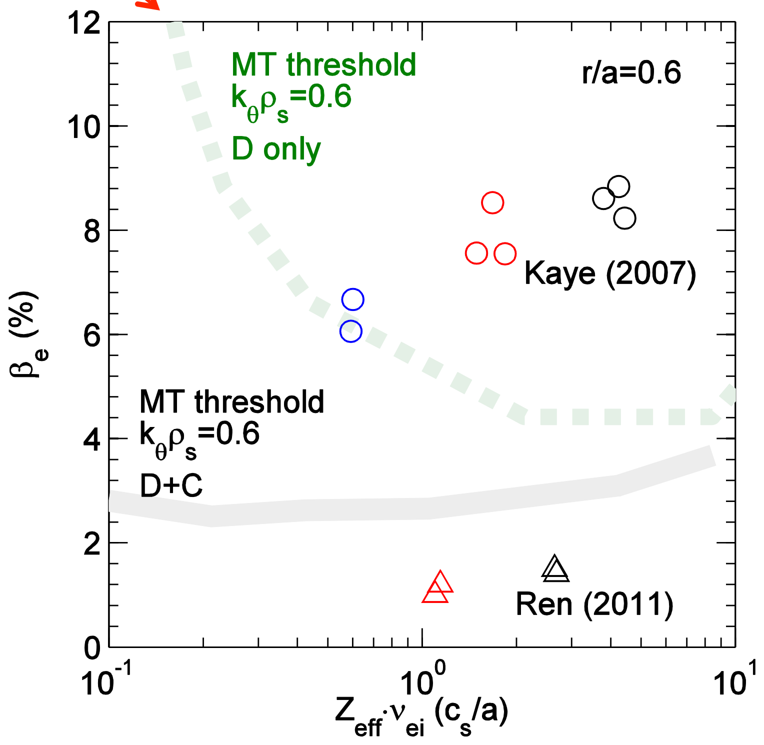

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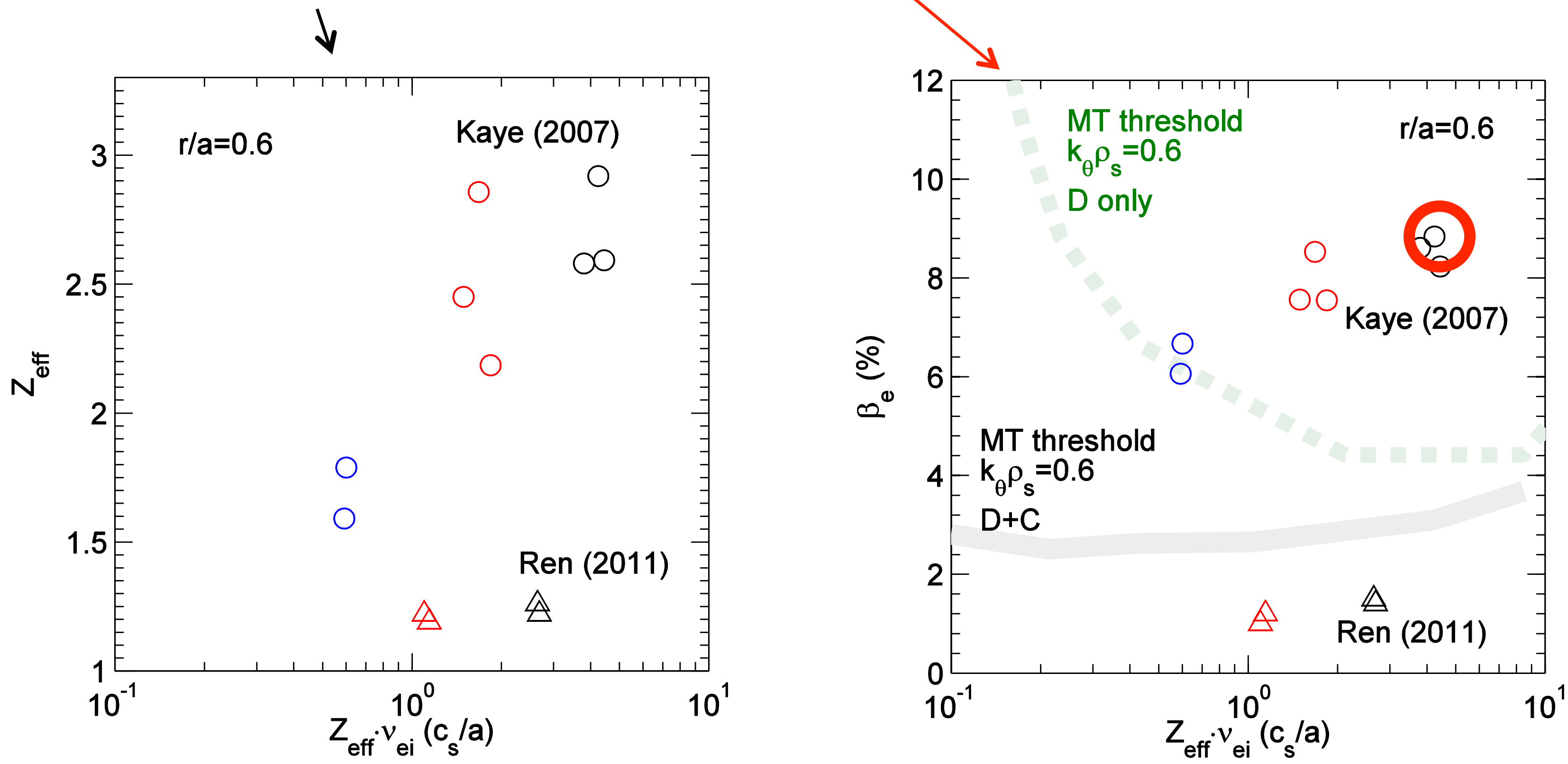

- Nonlinear simulations run for high-β_e, high-ν_e where only microtearing unstable

First nonlinear microtearing simulations in NSTX*

- Simulations where only microtearing unstable, no ETG (NSTX 120968, r/a=0.6)
  - Electromagnetic (φ, $A_{||}$) and collisional (ν_e)
  - Varying E×B shear (mostly γ_E=0)
  - Deuterium only (but Z_{eff} in collision operator)
  - "Local" → no profile variation in equilibrium quantities
  - $k_\theta\rho_s$=[0,0.105,0.21,…], same as n=[0,5,10,…]

$L_x \times L_y = 80 \times 60\rho_s$

$n_x \times n_y = 400 \times 8$ ($\Delta x = 0.2\ \rho_s$)

n_θ=14 (parallel mesh points)

n_E=8, n_λ=12 × 2 (velocity space)

120968 r/a=0.6 surface

a/L_{Te}=2.73	a/L_n=-0.83
q=1.69	s=1.75
κ=1.7	δ=0.13
T_e/T_i=1.05	Z_{eff}=2.9
β_e=8.8%	ν_{ei}=1.46 c_s/a
($\beta_{e,unit}$=2.5%)	

Acknowledgements: **NERSC & OLCF (INCITE award FUS023)**

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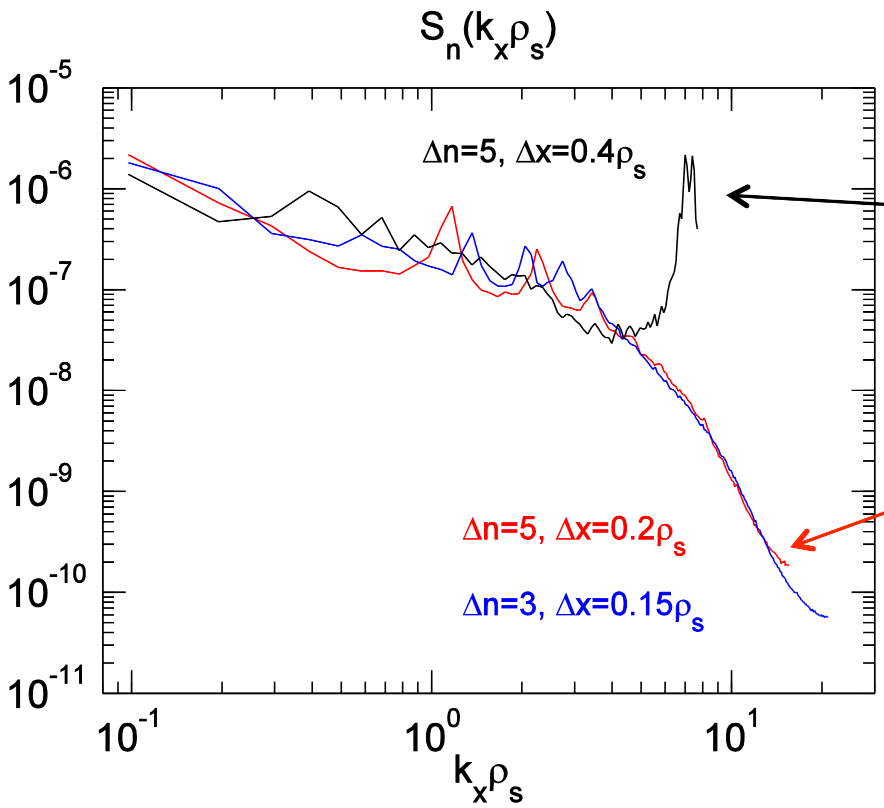
* W. Guttenfelder et al., Phys. Rev. Lett. **106**, 155004 (2011)

Fine radial resolution required to obtain decaying nonlinear spectra

- Unphysical pile-up at high-k with insufficient resolution ($\Delta x=0.4\ \rho_s$)
- Smoothly decaying turbulent spectrum with better resolution ($\Delta x=0.2\ \rho_s$, $\Delta x=0.15\ \rho_s$)



Poor resolution – unphysical pile-up at high-k

Better resolution – smoothly decaying turbulent spectrum

- Similar high-k pile-up observed in first careful attempts of GS2 MAST simulations – for more discussion see Applegate Ph. D. thesis (2007, Imperial College London)

Fine radial resolution required to distinguish *linear* resonant layers of fastest growing mode

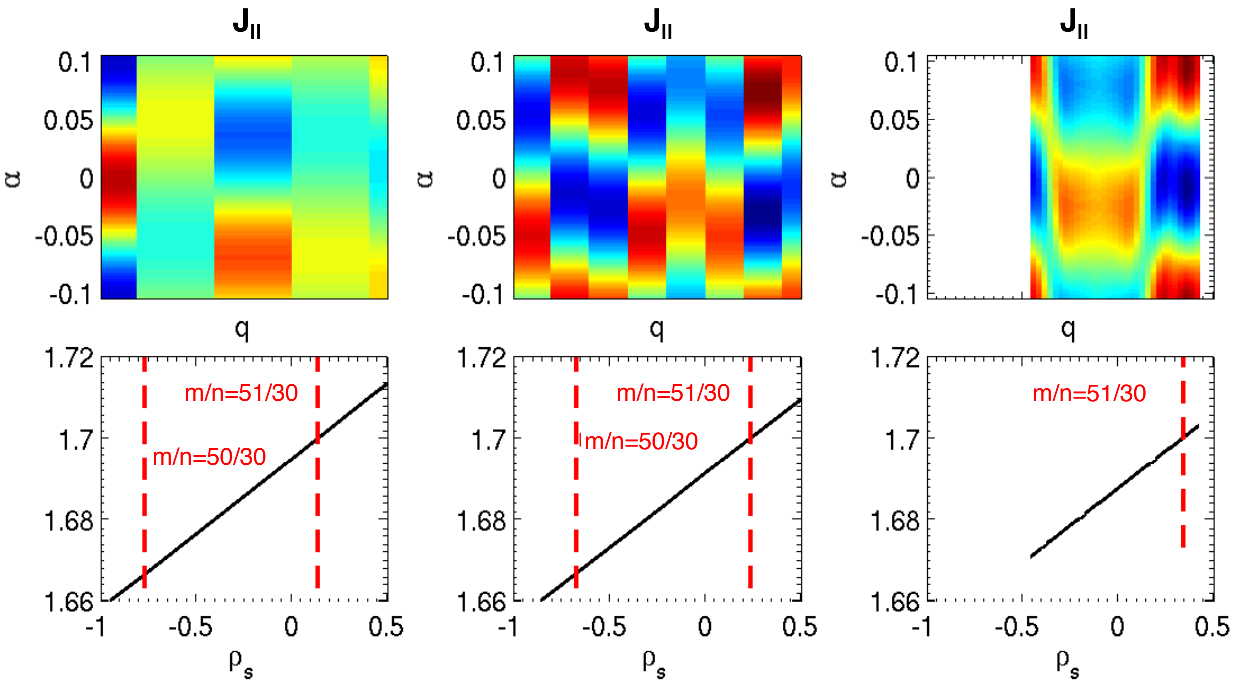
Linear calculation using box width and resolution of nonlinear simulations

"Typical" linear flux-tube calculation

$L_x = 80\ \rho_s = 88\ \Delta r_{rat}$
nx = 200
$\Delta x = 0.4\ \rho_s$

$L_x = 80\ \rho_s = 88\ \Delta r_{rat}$
nx = 400
$\Delta x = 0.2\ \rho_s$

$L_x = 0.9\ \rho_s = \Delta r_{rat}$
nx = 32
$\Delta x = 0.03\ \rho_s$

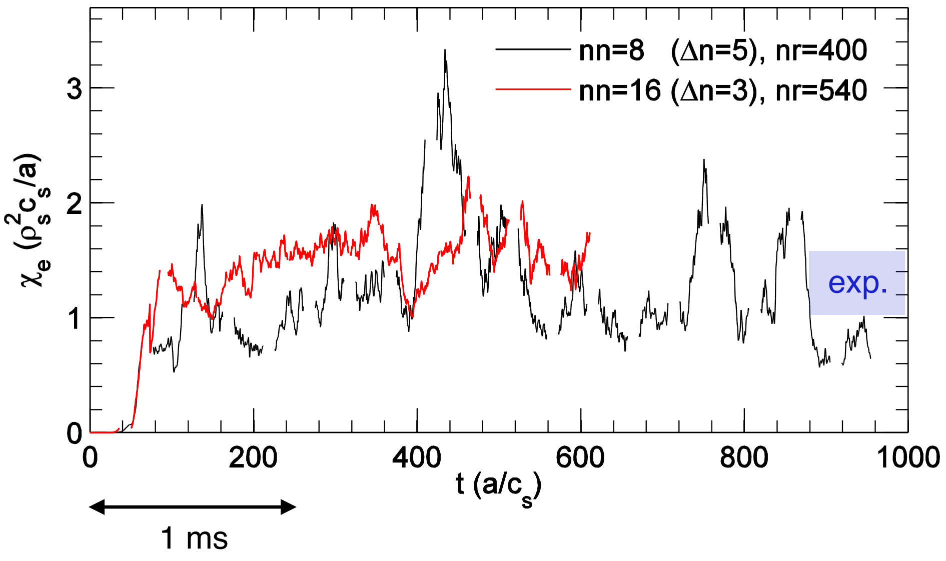


Rough rule-of-thumb: $\Delta x \le \min[\Delta r_{rat}]/4$ **$\Delta x/\rho_s \le 1/(4\cdot\max[k_\theta\rho_s]\cdot s)$**

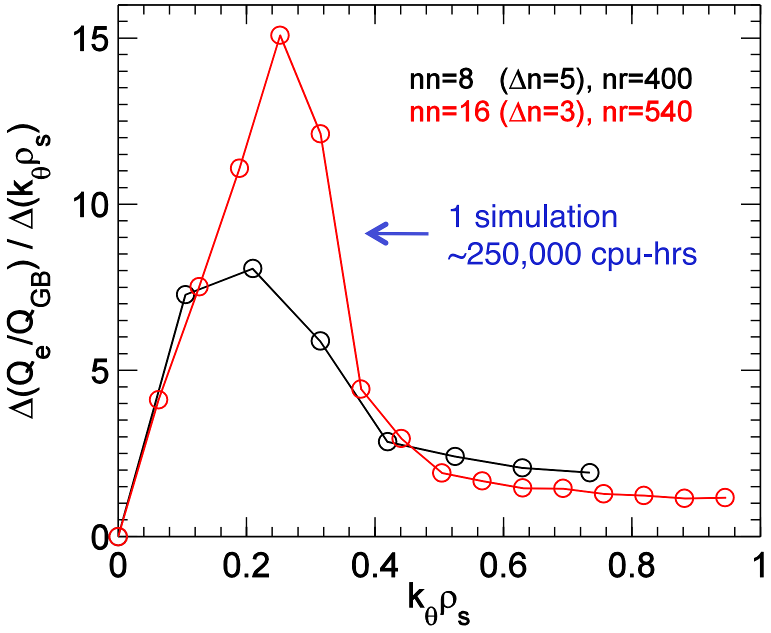
Predicted electron thermal transport comparable to experiment

- Simulated transport (1.2 $\rho_s^2 c_s/a$, 6 m^2/s) comparable to experimental transport (1.0-1.6 $\rho_s^2 c_s/a$)
- Well defined peak in transport spectra ($k_\theta \rho_s \approx 0.2$), downshifted from maximum γ_{lin} ($k_\theta \rho_s \approx 0.6$)
- Slowly decaying tail - predicted transport increases ~25% with higher resolution

Transport time series



Fractional transport spectra



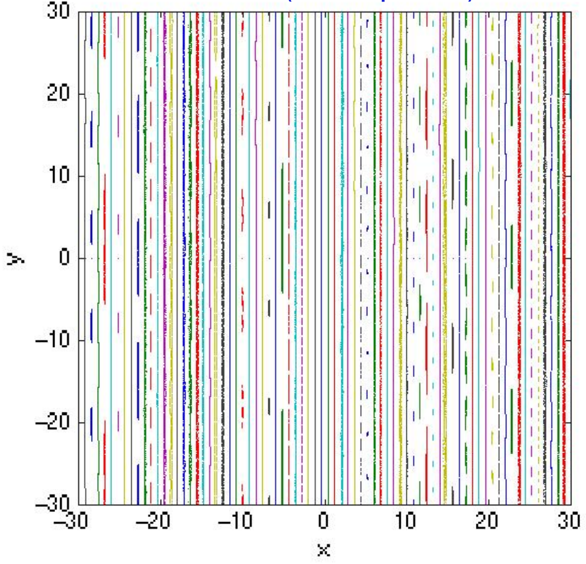
1 simulation
~250,000 cpu-hrs

- Negligible particle, momentum, or ion thermal transport

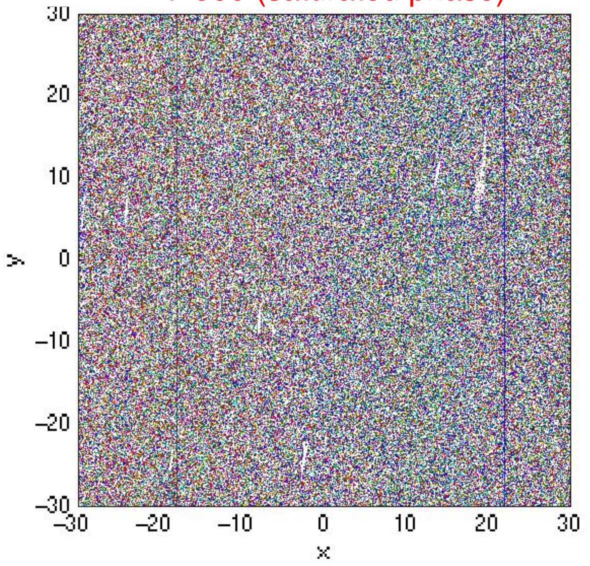
~98% of transport due to magnetic "flutter" contribution

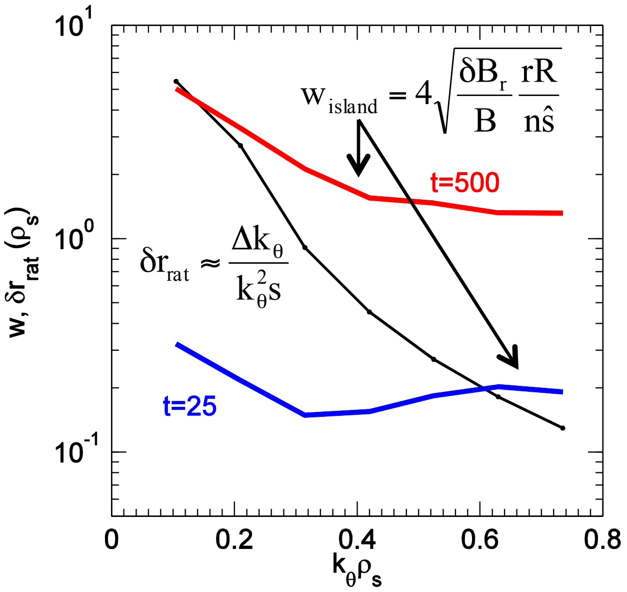
- Flux surfaces become distorted in linear phase (t=25)
- Globally stochastic* in saturated phase, complete island overlap $w_{island}(n) > \delta r_{rat}(n)$

t=25 (linear phase)



t=500 (saturated phase)





- $\chi_{e,EM}$ close to *collisionless* Rechester-Rosenbluth* (λ_{mfp}=12 m, L_c≈2.5 m)

$$D_{st} = \lim_{s\to\infty}\frac{\langle[r_i(s)-r_i(0)]^2\rangle}{2s} \qquad \chi^{RR} \approx 2\left(\frac{2}{\pi}\right)^{1/2} D_{st} v_{Te} f_p \approx 0.9\left(\frac{\rho_s^2 c_s}{a}\right)$$

f_p≈63% passing particles

*Wang et al., Phys. Plasmas (2011); Nevins et al., Phys. Rev. Lett. (2011);
Rechester & Rosenbluth, Phys. Rev. Lett. (1978); Harvey et al., Phys. Rev. Lett. (1981)

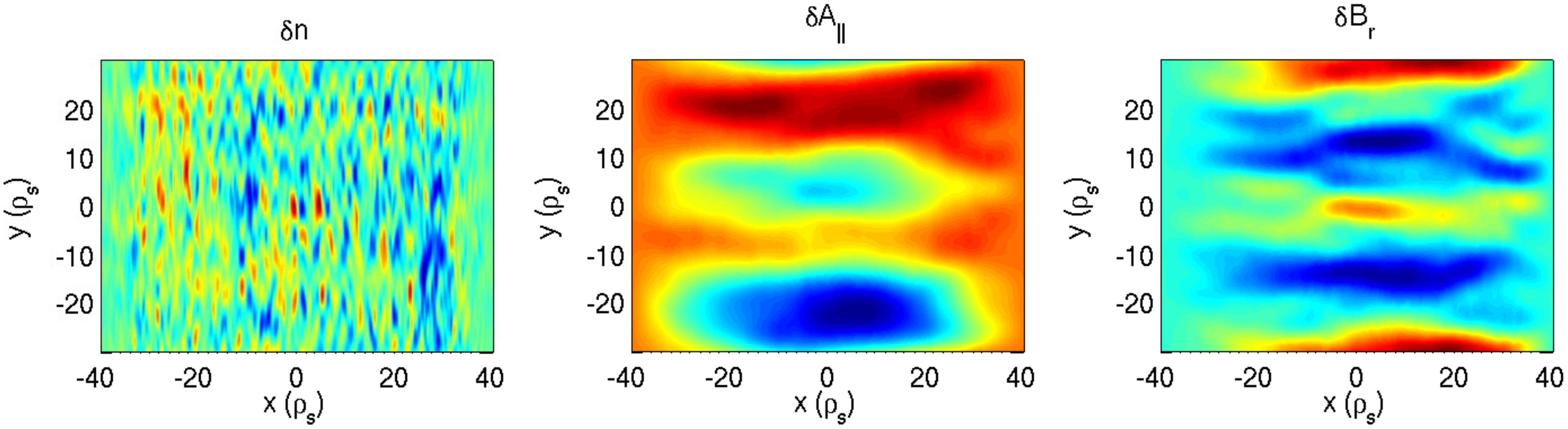
Narrow density perturbations remain in nonlinear simulations

- Narrow radial n, φ, $j_{||}$ structures need to be resolved but $A_{||}$ very broad
- $\delta B_r/B \sim 0.15\% \sim \rho_e/L_{Te} = 0.065\%$
- $\delta B_r/B \sim \rho_e/L_{Te}$ analytic approximation from Drake et al. PRL 1980; used for NSTX in Wong et al. PRL 2007

$\delta n/n \approx 0.5\%$

$\delta A_{||}/\rho_s B \approx 0.8\%$

$\delta B_r/B \approx 0.15\%$

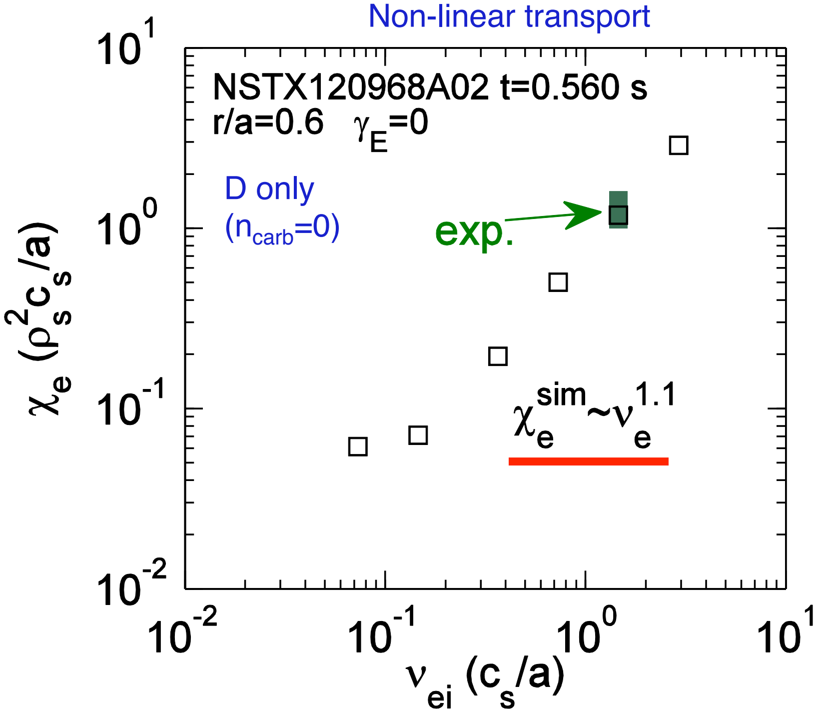


$\delta T_e/T_e \approx 2\%$

$\delta v_{e,||}/c_s \approx 6\%$

Near linear scaling of transport with ν_e consistent with experimental scaling

Non-linear transport



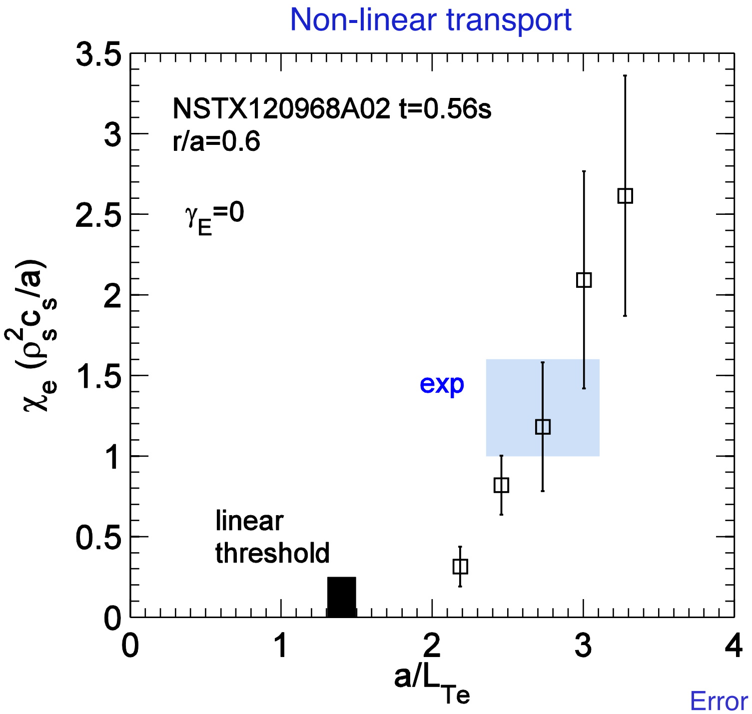
NSTX experimental scaling (Kaye et al., 2007)

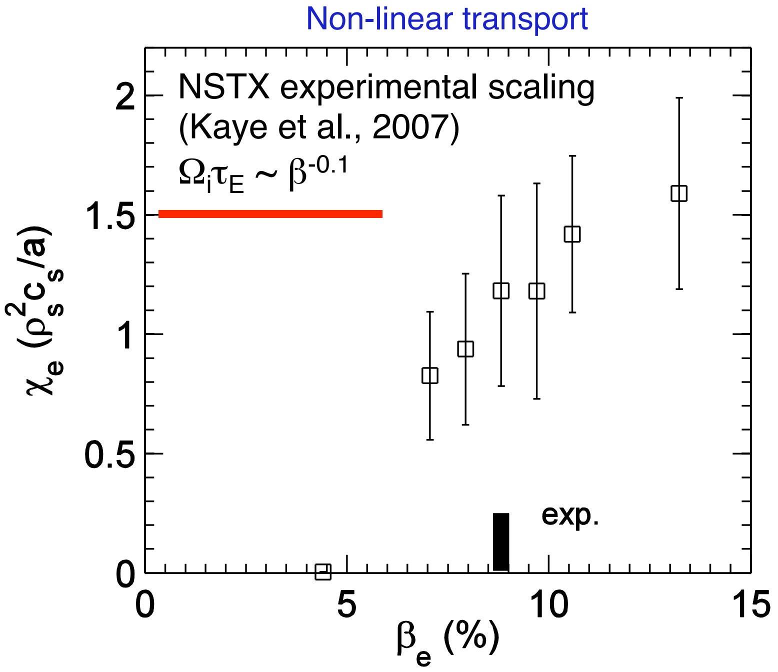
$\Omega_i\tau_E \sim \nu_*^{-0.95}$

- As transport drops, a/L_{Te} will increase (for fixed heat flux), at some point ETG (TEM?) should become important
- This transition likely to determine limit of “favorable” ν_* scaling
- Also likely to depend on ionic charge (Z_{eff}) – above with D only (n_c=0)

Predicted transport "stiff" with ∇T_e, increases with β_e

- Complicates simple interpretation from $\chi_{e,sim} \sim \nu_e^{1.1}$ scaling
- Useful to characterize scaling of threshold gradient (work in progress)



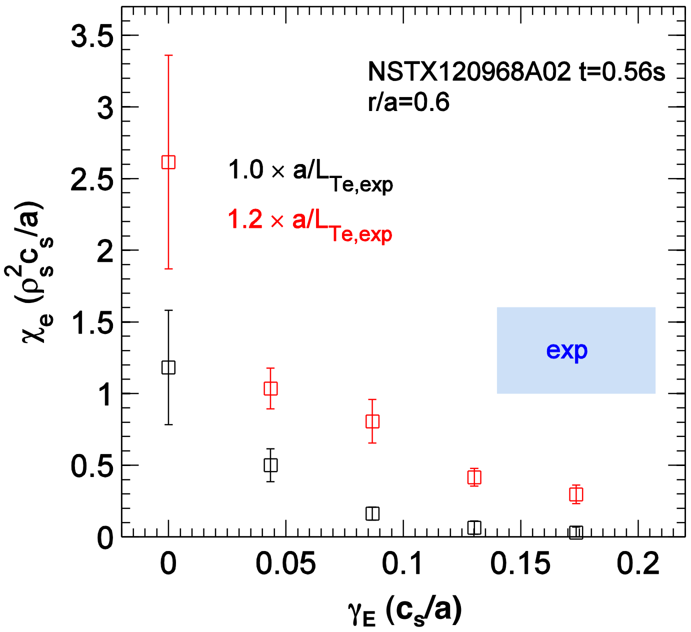


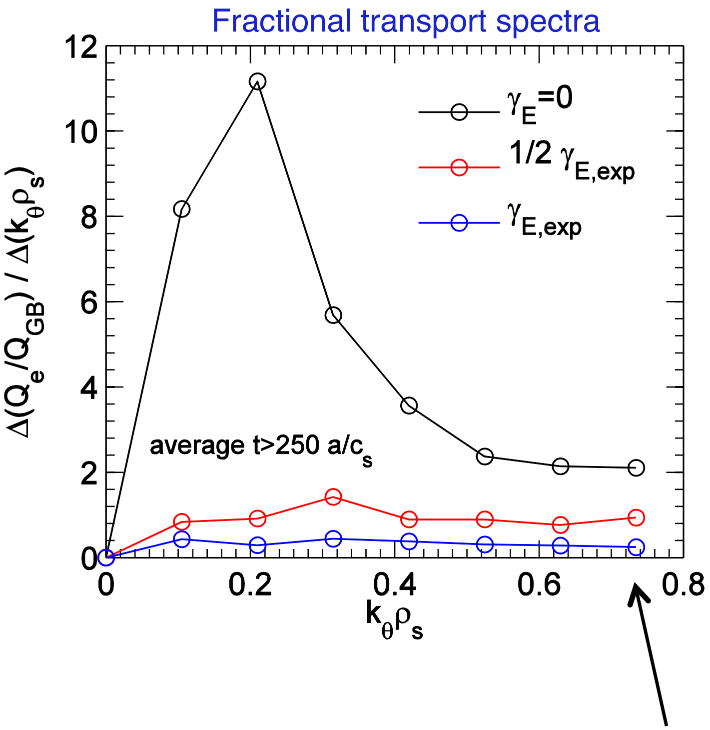
Error bars represent rms variation in transport

- Non-linear threshold ~40% bigger than linear threshold -- possible influence from limited numerical resolution

Nonlinear microtearing transport sensitive to γ_E/γ_{lin}

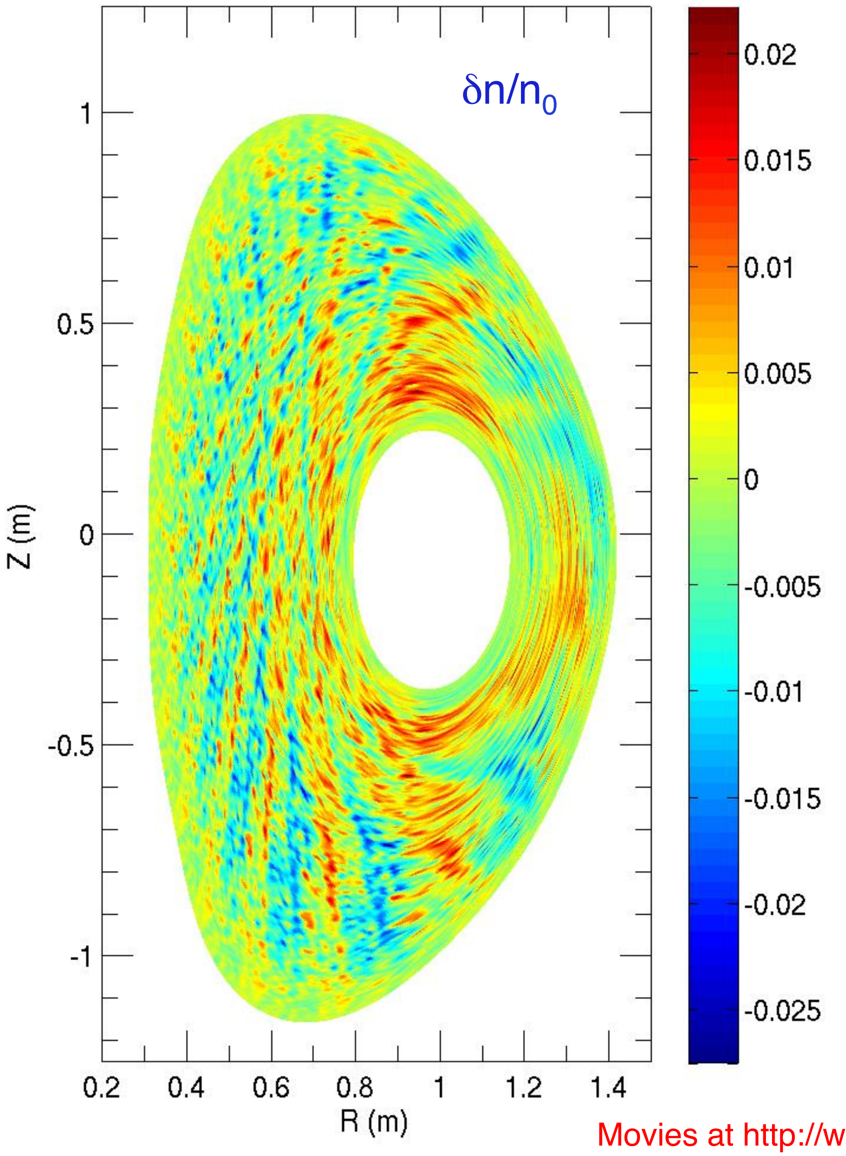
- Transport reduced when increasing γ_E to local experimental value ($\gamma_{E,exp} \sim \gamma_{lin,max} \sim 0.17\ c_s/a$)
- Transport partially recovered with increase in ∇T_e

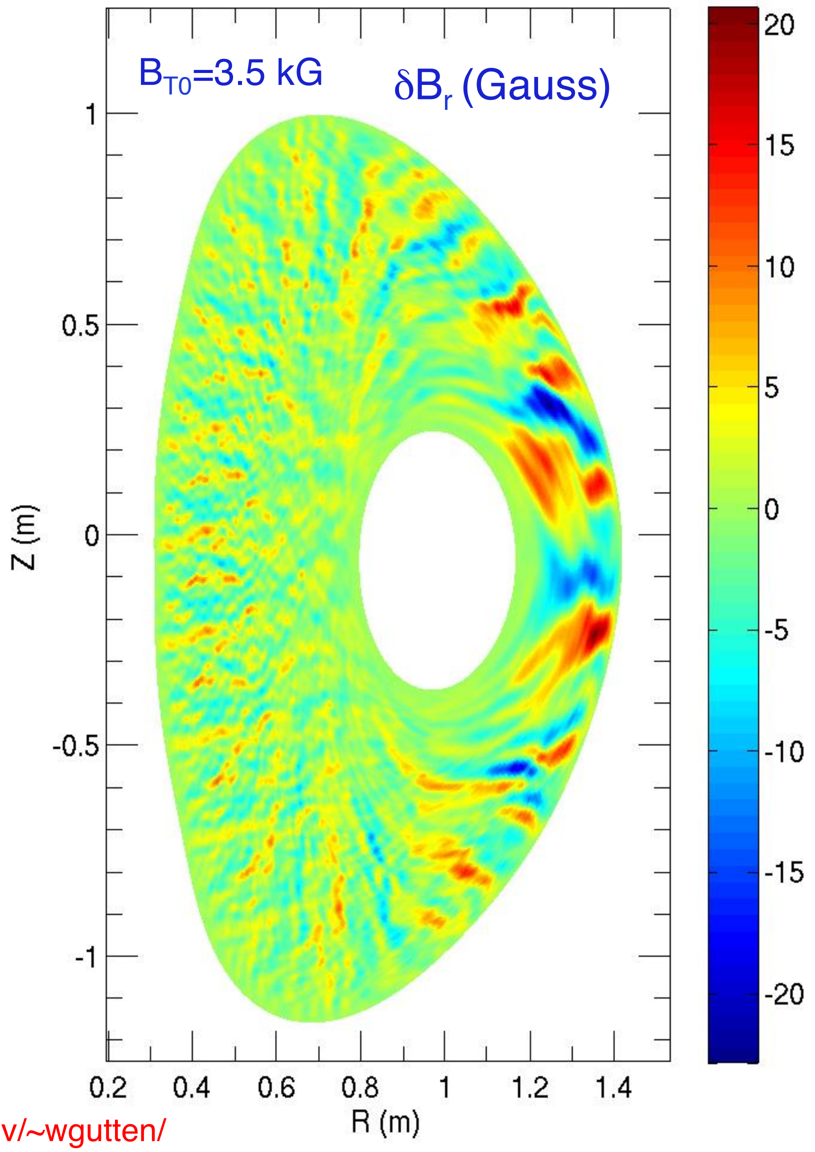




- Higher ionic charge (Z_{eff}>1, through adiabatic response) and improved resolution (binormal and radial) could increase transport
- Profile (non-local) effects could also matter - $\rho_s/a \approx 1/100$ & edge more strongly driven

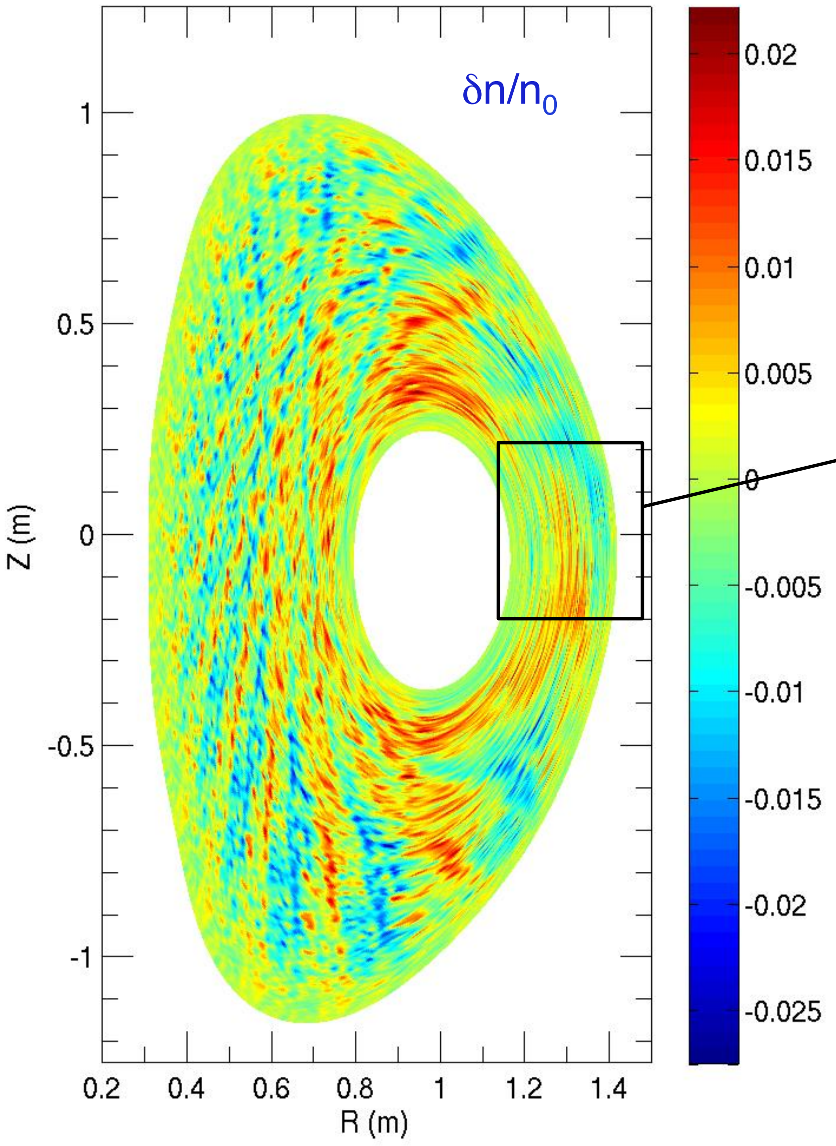
What hope is there to experimentally identify microtearing modes?

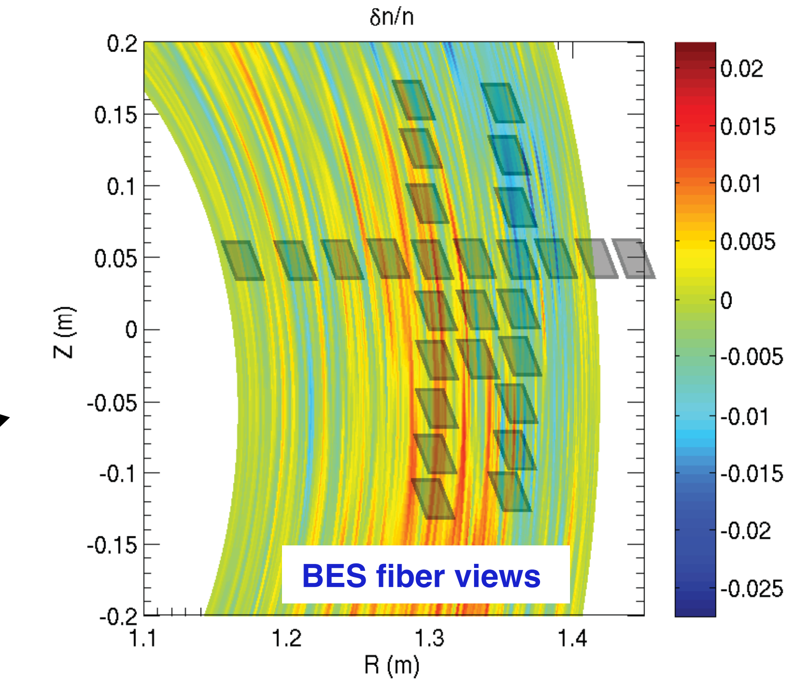




Movies at http://www.pppl.gov/~wgutten/

BES for density fluctuations

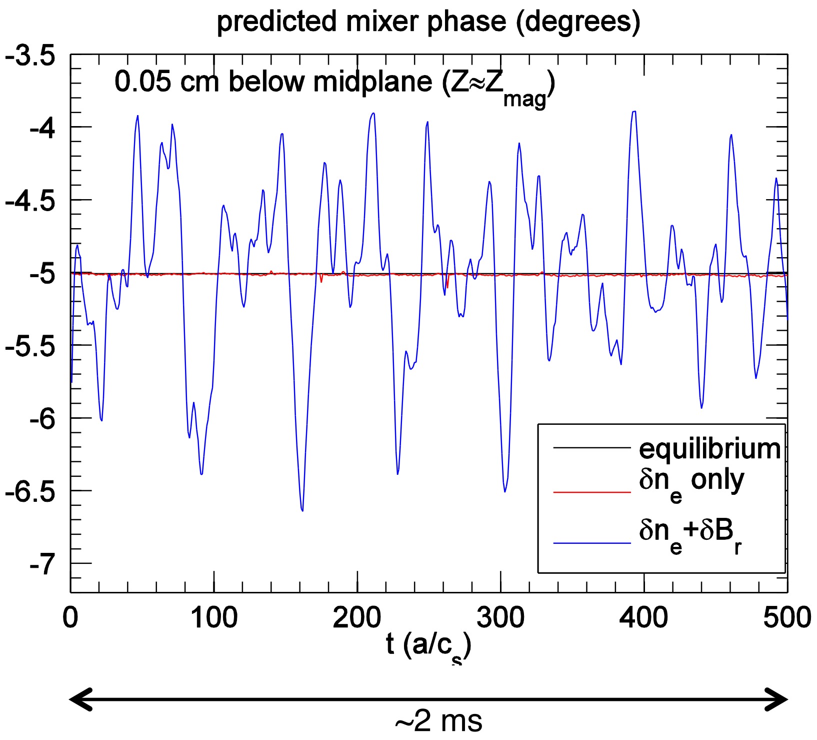


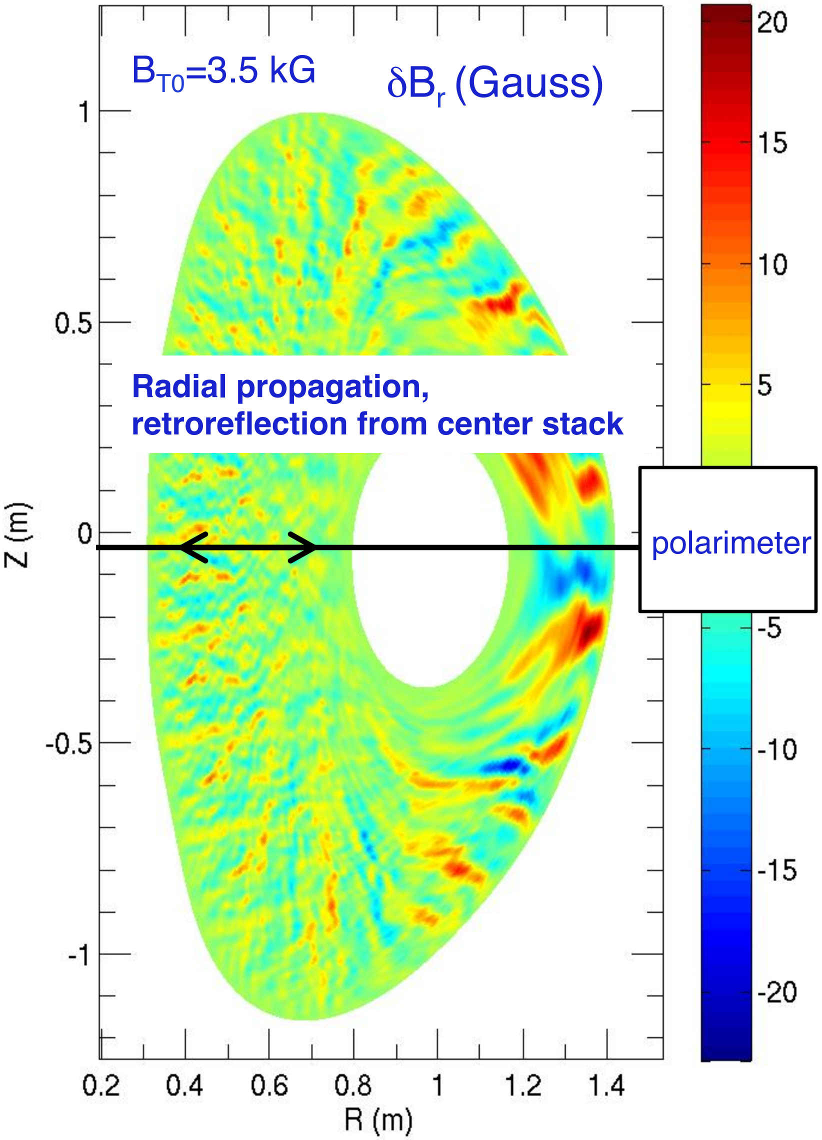


- BES suitable for long poloidal scale (U-Wisconsin, Smith et al., RSI 2010)
- May average over narrow radial scale – requires synthetic diagnostic and instrument function (D. Smith, BO4.2)

Polarimetry for magnetic field fluctuations

- New UCLA polarimetry system (J. Zhang, PP9.71)
- Simulations suggest $(\delta B/B)_{internal} \leq 0.1\%$ may be detectable ($1\text{-}2^0$ or $\sim 0.3^0$ rms mixer phase)





Radial propagation, retroreflection from center stack

Summary

- Microtearing modes found to be unstable in experimental ν_* scans
  - Scaling of linear growth rates $\gamma_{lin} \sim \nu_e$ – potential candidate to explain experimental confinement trend
  - Linear thresholds exist in a/L_{Te} & β_e
  - Ionic charge (Z_{eff}) can enhance instability (opposite to ETG expectations)
- First non-linear microtearing simulations in NSTX
  - Require relatively fine radial resolution ($\Delta x \approx 0.2\rho_s$, nx=400) to capture physics
  - Transport dominated by electromagnetic contribution ($\delta A_{||}$) → stochastic field lines
  - **Predicted $\chi_{e,sim} \sim \nu_e^{1.1}$ close to experimental scaling**
  - "Stiff" with ∇T_e but suppressible by experimental levels of $E \times B$ shear

Acknowledgements

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