Long-wavelength MHD stability at high pressure required for STs, ITER and other next-step devices

- Motivation
  - The resistive wall mode (RWM) is a primary cause of plasma disruption at high β
  - Understanding passive stabilization physics determining RWM stability is critical to extrapolate stability requirements for future devices
  - Active control of RWM required when profile transients cause instability
- Passive stability: Very brief history
  - Early theory: RWM can be stabilized by sufficient plasma rotation
  - Critical ω_ϕ for passive stability assessed (Ω_{crit})
  - Low levels of Ω_{crit} (< 0.5% Alfven at q =2) suggested
  - RWMs found to be unstable at relatively high ω_ϕ, and stability depends on profile, not simple scalar value – no simple, low Ω_{crit}!
  - Stability model including kinetic effects evaluated (NSTX) - can explain greater complexity of experimental RWM marginal stability

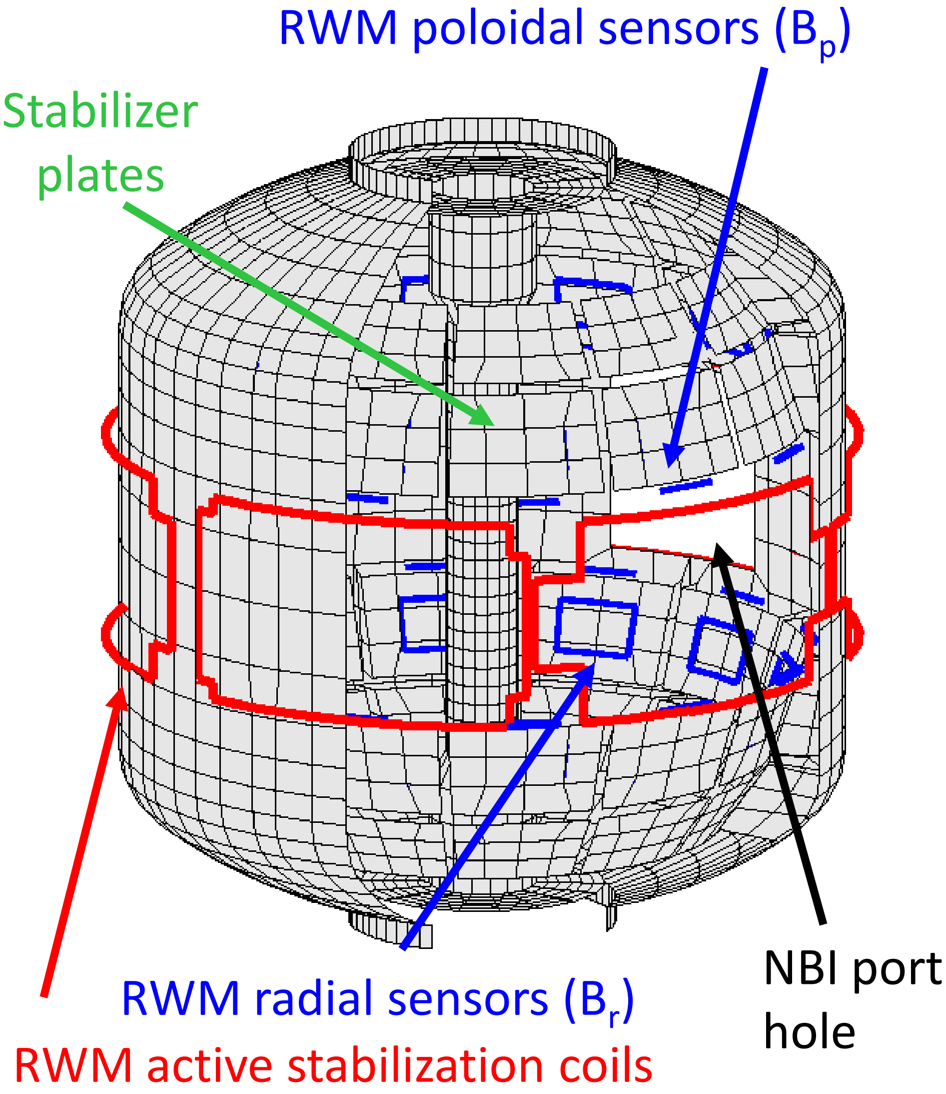
Understanding plasma stability gradients vs. key profiles is essential for continuous, high beta operation

- Current research focuses on:
  - greater understanding of the stabilization physics
  - quantitative comparison to experiment
  - measurement of plasma stability
  - demonstration of improved active control techniques that can reduce resonant field amplification (RFA) or disruptions

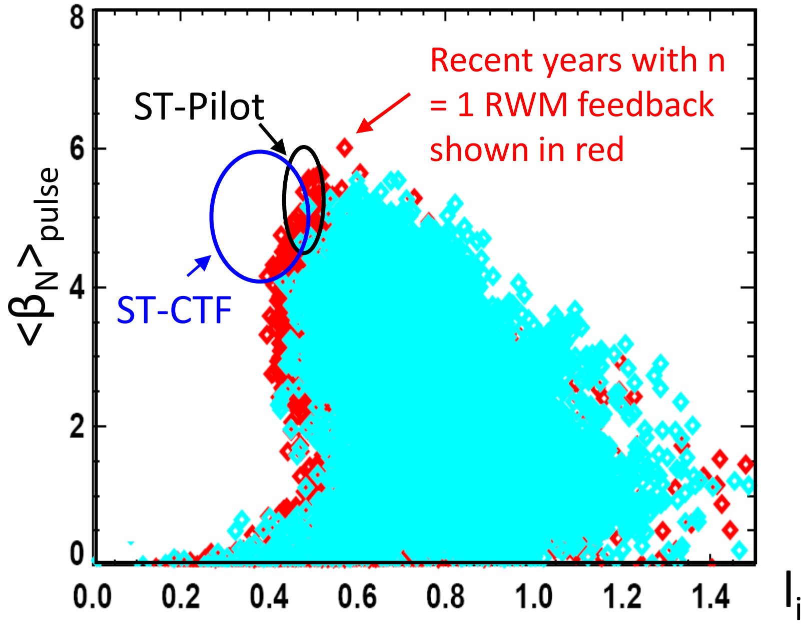
- Outline
  - NSTX active feedback
    - Dual field component active control
    - Model-based state space controller
  - NSTX resonant field amplification experiments
    - Comparisons with kinetic theory: resonances and collisionality
  - ITER analysis with alpha particles and internal transport barriers

NSTX is a spherical torus equipped to study passive and active global MHD control, rotation variation by 3D fields

- High beta, low aspect ratio
  - $R = 0.86$ m, $A > 1.27$
  - $I_p < 1.5$ MA, $B_t = 5.5$ kG
  - $\beta_t < 40\%$, $\beta_N > 7$
- Copper stabilizer plates for kink mode stabilization
- Midplane control coils
  - $n = 1 - 3$ field correction, magnetic braking of ω_ϕ by NTV
  - $n = 1$ RWM control
- Combined sensor sets now used for RWM feedback
  - 48 upper/lower B_p, B_r



NSTX reaches high β_N, low l_i; RWM stability investigated in unstable plasmas, active control applied

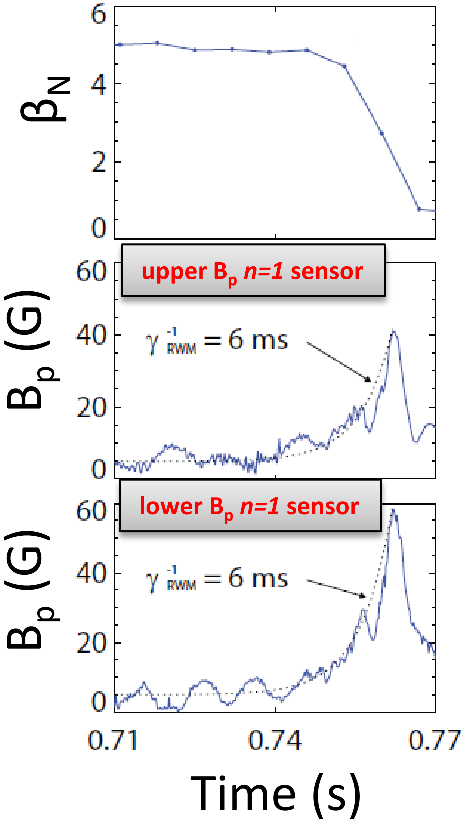


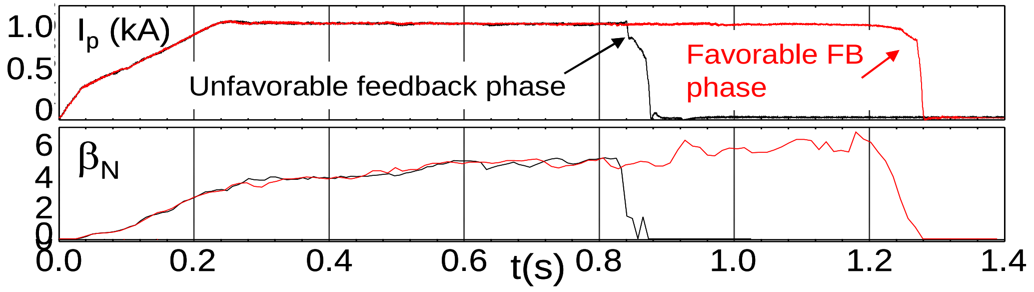
- NSTX plasmas have begun to reach low l_i and high $\langle\beta_N\rangle_{pulse}$ suitable for next-step ST fusion devices
  - Some parameters (e.g. elongation > 3) still need to be reached self-consistently
  - Broad current profile → low $l_i = \langle B_p^2\rangle/\langle B_p\rangle_\psi^2$, has global mode stability implications

- Unstable plasma
  - Causes β collapse, I_p disruption
  - Correlate marginal stability point with kinetic theory MISK calculations

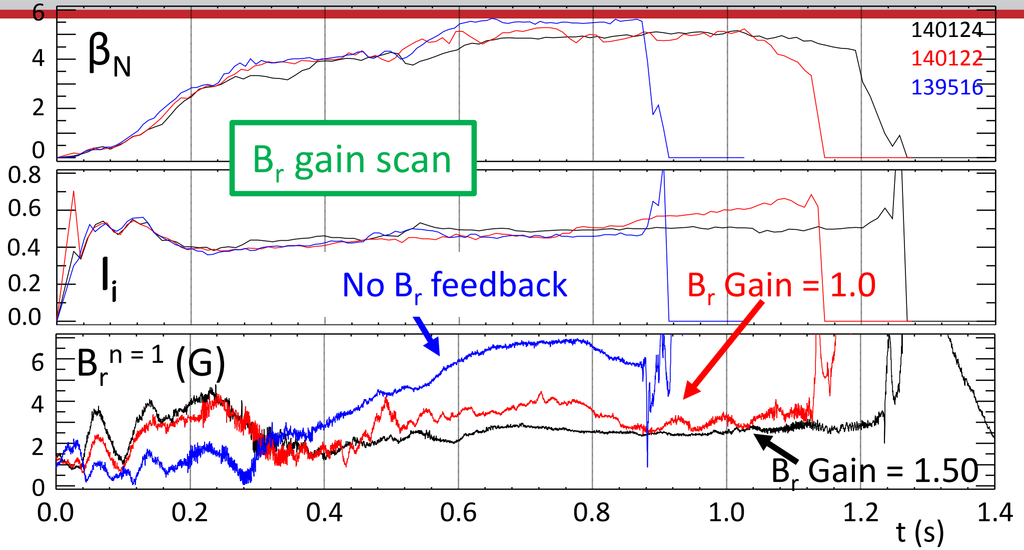
    or

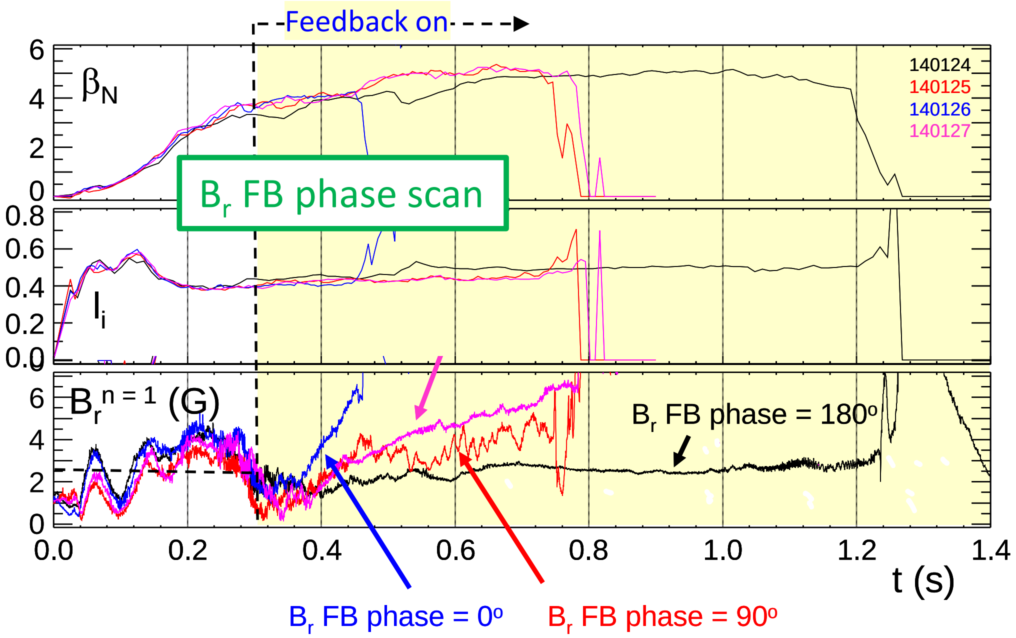
  - Use active control to stabilize



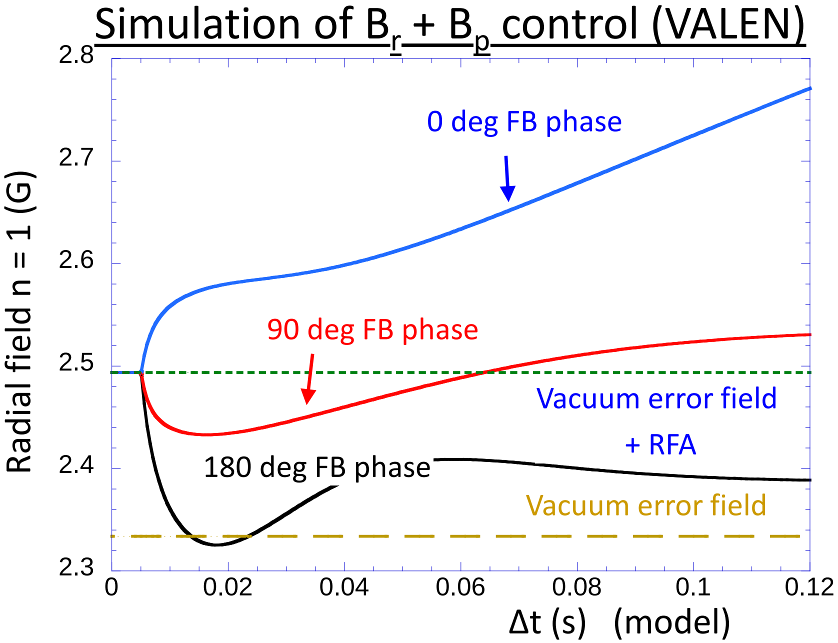


RWM B_r sensor n = 1 feedback phase variation shows superior settings when combined w/B_p sensors; good agreement w/theory so far



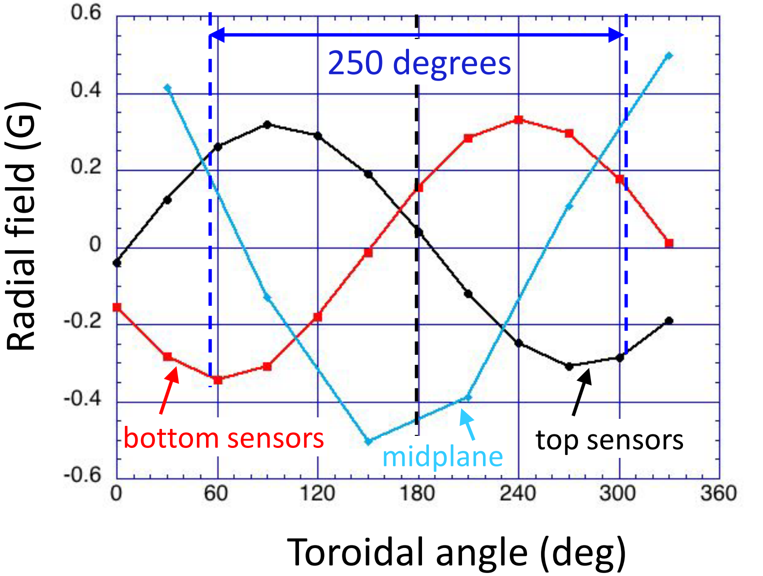


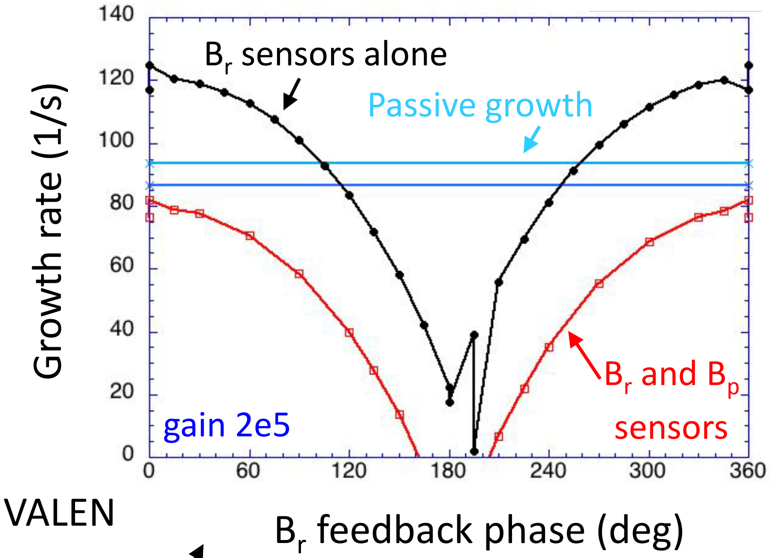
- Favorable B_p + B_r feedback (FB) settings found (low l_i plasmas)
  - — Fast RWM growth ~ 2 - 3 ms control by B_p
  - — B_r FB controls (~10 ms ~ $\tau_{w\text{-radial}}$) n=1 field amplification, modes
- Time-evolved theory simulation of B_r+B_p feedback follows experiment



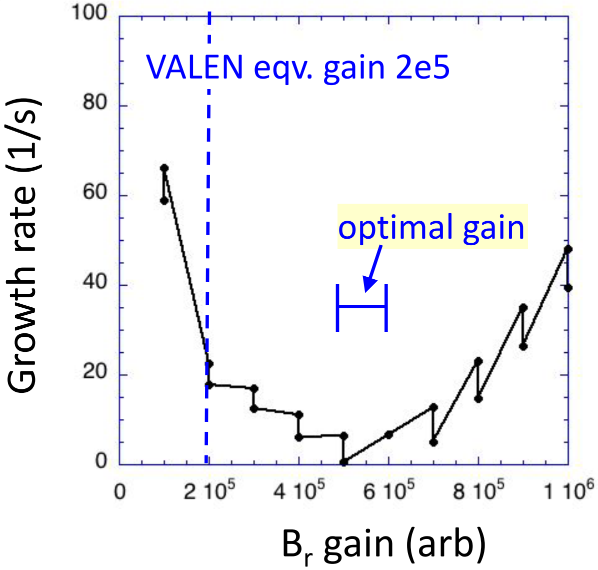
RWM feedback using upper/lower B_p and B_r sensors shows good agreement with B_r feedback phase; gain not optimized

Modeled B_r field at sensors and midplane



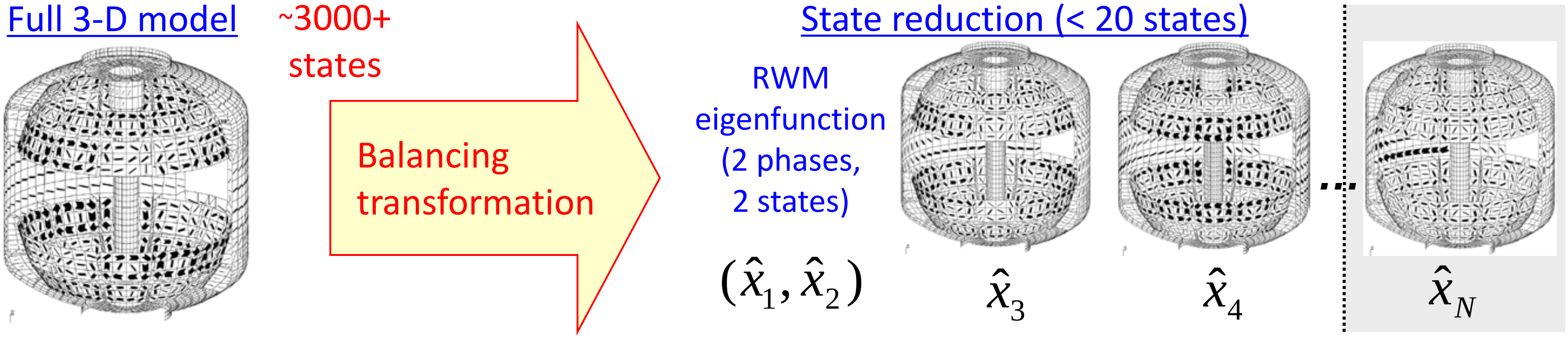


DCON, VALEN codes



- Both B_r, B_p feedback contribute to active control
  - B_r mode structure and optimal feedback phase agrees with parameters used in experiment
  - B_r feedback alone provides stabilization for growth times down to ~ 10 ms with optimal gain
  - Physics of best feedback phase for B_p sensors in low l_i plasmas under investigation

New RWM state space controller implemented to best sustain high β_N



Full 3-D model

~3000+ states

Balancing transformation

State reduction (< 20 states)

RWM eigenfunction (2 phases, 2 states)

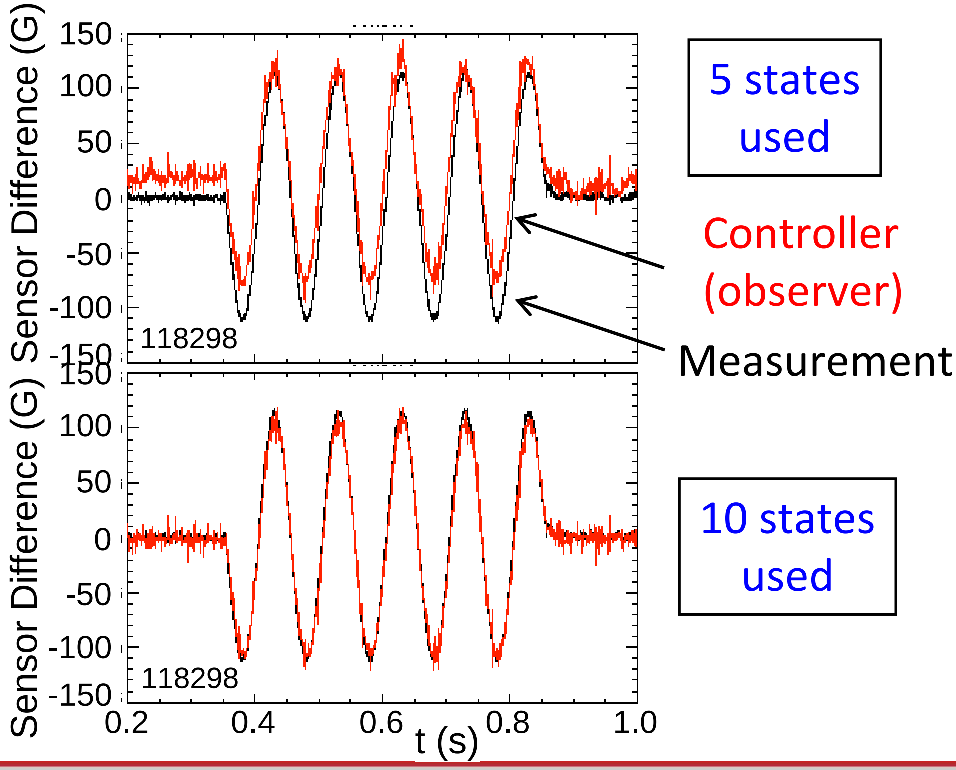
$(\hat{x}_1, \hat{x}_2)$ $\hat{x}_3$ $\hat{x}_4$ … $\hat{x}_N$

- Controller models, can compensate for wall currents
  - Including mode-induced current
- Potential to allow more flexible control coil positioning
  - May allow control coils to be moved further from plasma, and be shielded (e.g. for ITER)

[O. Katsuro-Hopkins *et al.*, Nucl. Fusion **47**, 1157 (2007)]

- Straightforward inclusion of multiple modes (n > 1) in feedback

Controller reproduction of applied n = 1 field



State derivative feedback algorithm used for current control

State equations to advance

$$\dot{\vec{x}} = A\vec{x} + B\vec{u} \qquad \vec{u} = -K_c\vec{x} = \dot{I}_{cc}$$

$$\vec{y} = C\vec{x} + D\vec{u}$$

Control vector, u; controller gain, K_c

Observer est., y; observer gain, K_o ; D = 0

K_c , K_o computed by standard methods (e.g. Kalman filter used for observer)

Advance discrete state vector

$$\hat{\vec{x}}_t = A\vec{x}_{t-1} + B\vec{u}_{t-1}; \hat{\vec{y}}_t = C\hat{\vec{x}}_t$$

"time update"

$$\vec{x}_{t+1} = \hat{\vec{x}}_t + A^{-1}K_o(\vec{y}_{sensors(t)} - \hat{\vec{y}}_t)$$

"measurement update"

State derivative feedback: superior control approach

$$\dot{\vec{x}} = A\vec{x} + B\vec{u} \qquad \vec{u} = -\hat{K}_c\dot{\vec{x}} \longrightarrow \vec{I}_{cc} = -\hat{K}_c\vec{x}$$

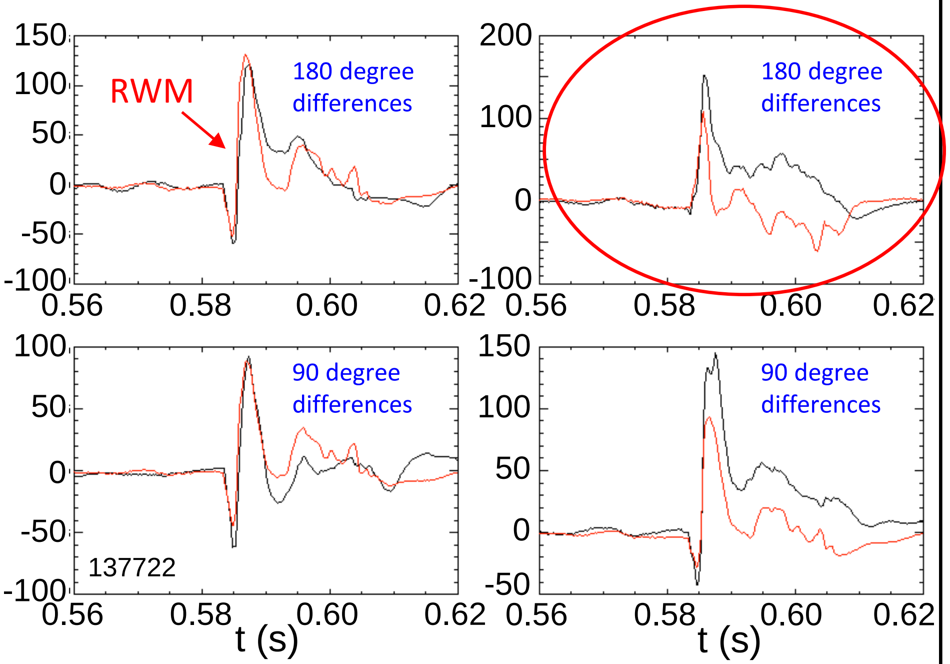
$$\dot{\vec{x}} = ((I + B\hat{K}_c)^{-1}A)\vec{x}$$

new Ricatti equations to solve to derive control matrices – still "standard" solutions in control theory literature

[T.H.S. Abdelaziz, M. Valasek, Proc. of 16th IFAC World Congress (2005)]

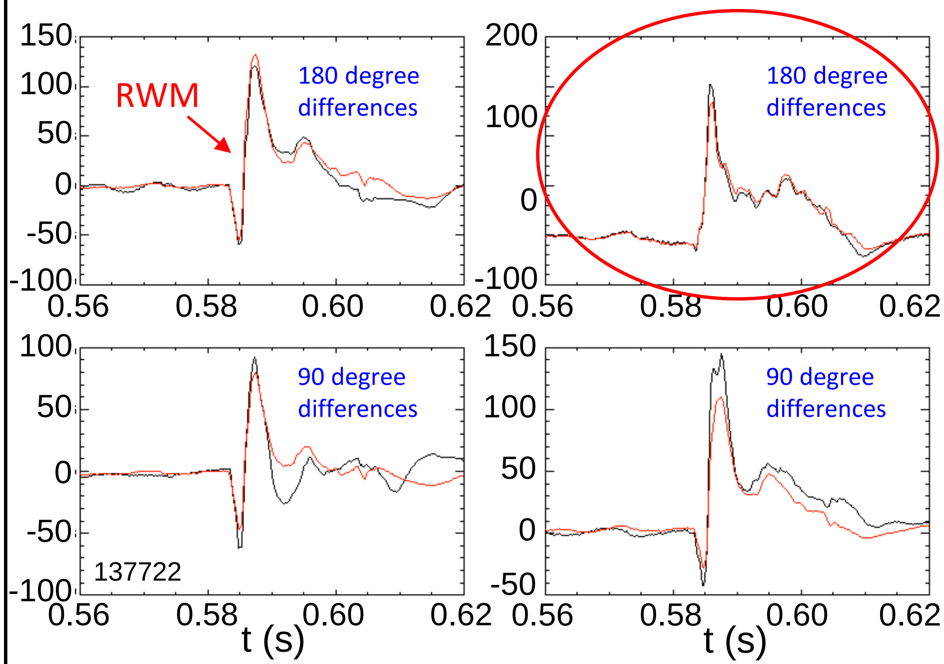
Increased number of states in RWM state space controller improves match to sensors over entire mode evolution

RWM Upper B_p Sensor Differences (G) – 2 States



- Reasonable match to all B_p sensors during RWM onset, large differences later in evolution

RWM Upper B_p Sensor Differences (G) – 7 States

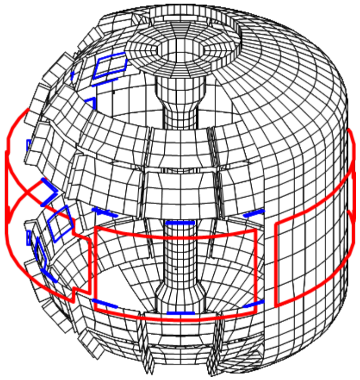


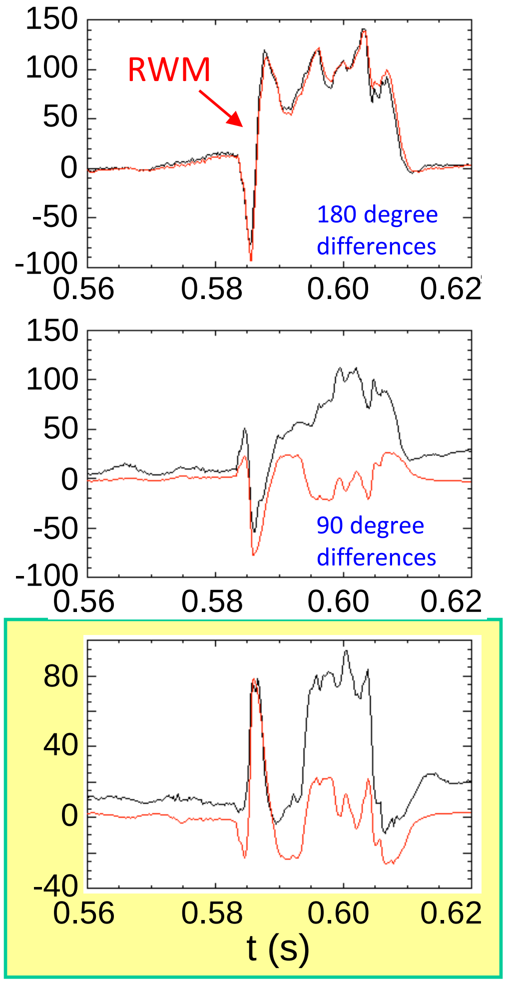
- Some 90 degree differences not as well matched
  - May indicate the need for an n = 2 eigenfunction state

Black: experiment Red: offline RWM state space controller

3-D conducting structure detail can improve RWM state space controller match to sensors

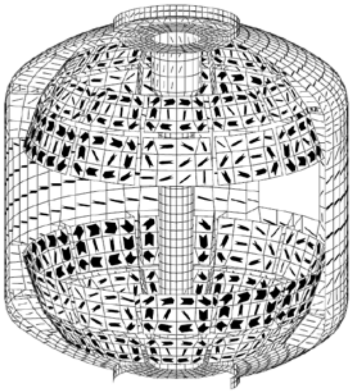
RWM Lower B_p Sensor Differences (G) – NO PORT

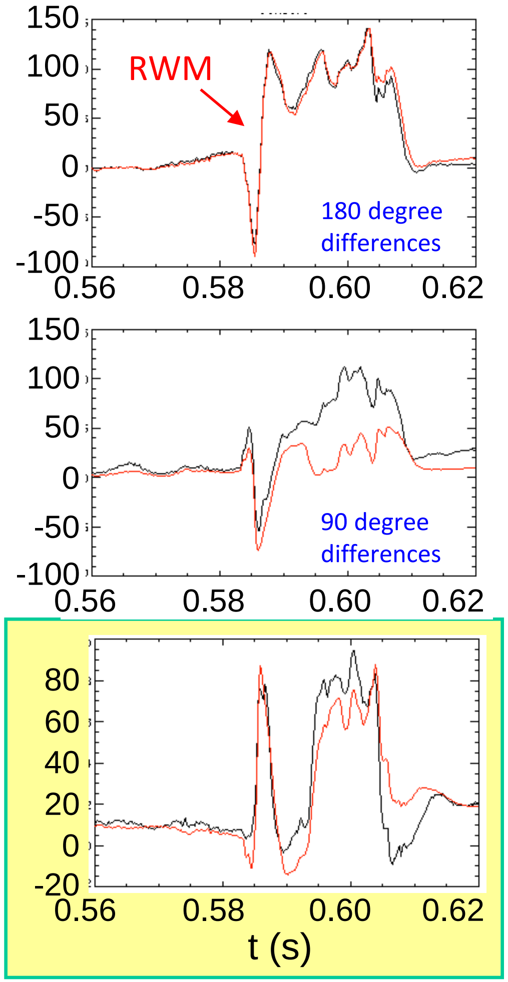




- Some 90 degree differences not well matched

RWM Lower B_p Sensor Differences (G) – NBI PORT



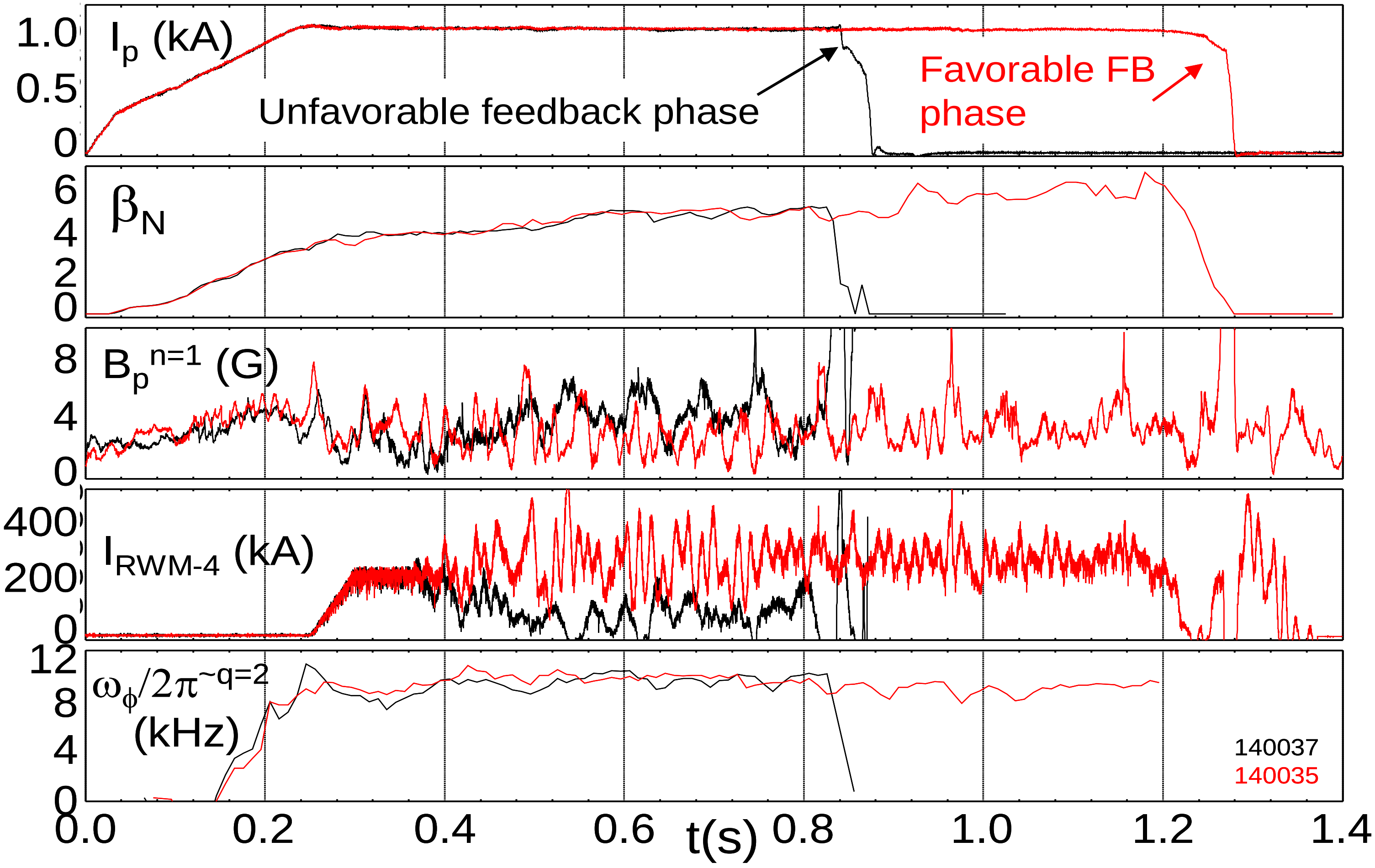


- Adding NBI port leads to greater match on some sensors

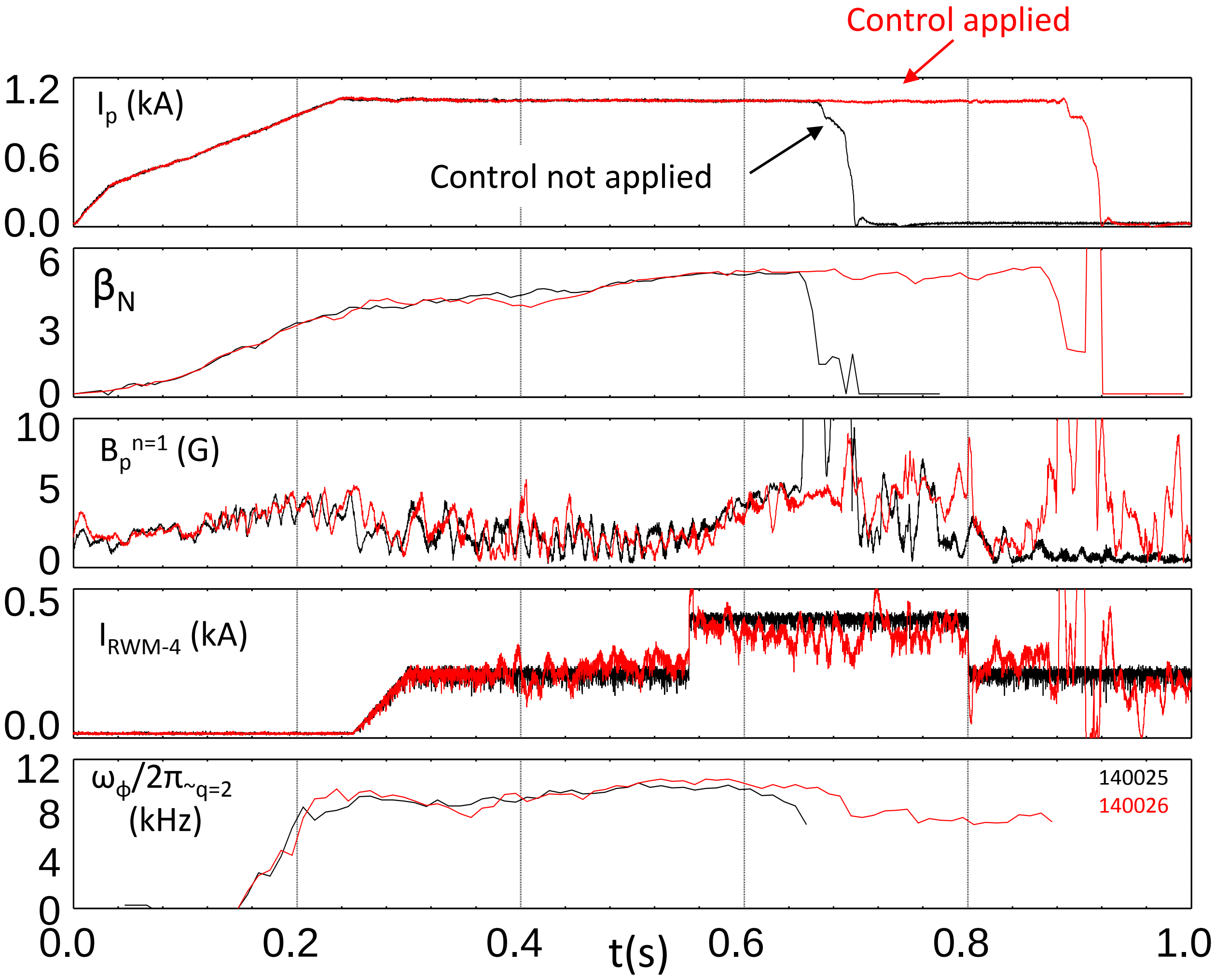
Black: experiment Red: offline RWM state space controller

New RWM state space controller sustains high β_N, low l_i plasma

- RWM state space (12 states) feedback phase scan
  - Best feedback phase produced long pulse, $\beta_N = 6.4$, $\beta_N/l_i = 13$



RWM state space controller sustains otherwise disrupted plasma caused by DC n = 1 applied field



- n = 1 DC applied field
  - Simple method to generate resonant field amplification
  - Can lead to mode onset, disruption
- RWM state space controller sustains discharge
  - With control, plasma survives n = 1 pulse
  - n = 1 DC field reduced
  - Transients controlled and do not lead to disruption
  - NOTE: initial run – gains NOT optimized

Kinetic terms in the RWM dispersion relation enable stabilization; theory consistent with experimental results

Dissipation (Im(δW_K)) and restoring force (Re(δW_K)) from kinetic term enables stabilization of the RWM:

$$(\gamma - i\omega_r)\,\tau_w = -\frac{\delta W_\infty + \delta W_K}{\delta W_b + \delta W_K}$$

[B. Hu *et al.*, Phys. Plasmas **12**, 057301 (2005)]

$$\delta W_K = \sum_j \sum_{l=-\infty}^{\infty} 2\sqrt{2}\pi^2 \int\!\!\int\!\!\int \left[|\langle H/\hat{\varepsilon}\rangle|^2 \frac{(\omega - n\omega_E)\frac{\partial f_j}{\partial \varepsilon} - \frac{n}{Z_j e}\frac{\partial f_j}{\partial \Psi}}{n\langle\omega_D^j\rangle + l\omega_b^j - i\nu_{\mathrm{eff}}^j + n\omega_E - \omega}\right] \frac{\hat{\tau}}{m_j^{\frac{3}{2}} B}|\chi|\hat{\varepsilon}^{\frac{5}{2}}\, d\hat{\varepsilon}\, d\chi\, d\mathbf{\Psi}, \qquad \chi = v_\parallel/v$$

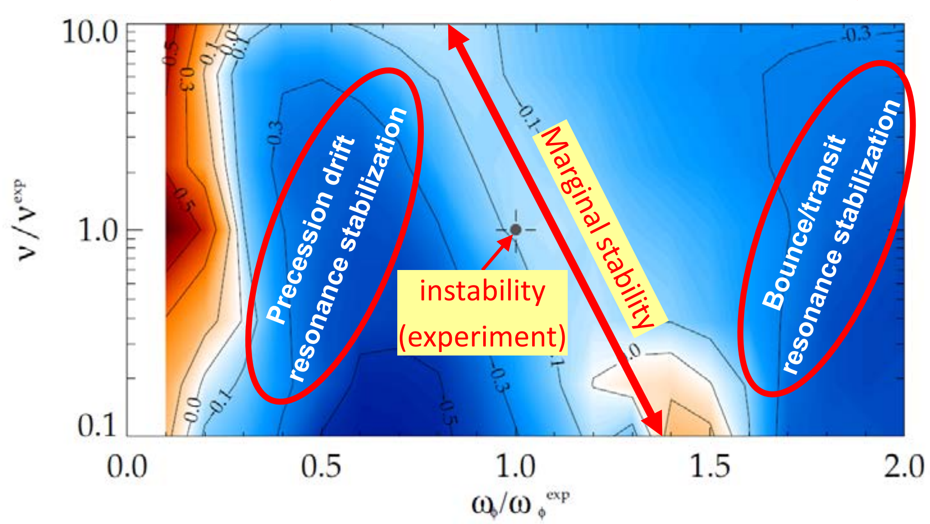
Precession Drift

Bounce

Collisionality

~ Plasma Rotation: $\omega_\phi \approx \omega_E + \omega_{*i}$

γτ$_w$ contours vs. ν and ω_ϕ



[S. Sabbagh *et al.*, IAEA 2010, EXS/5-5]

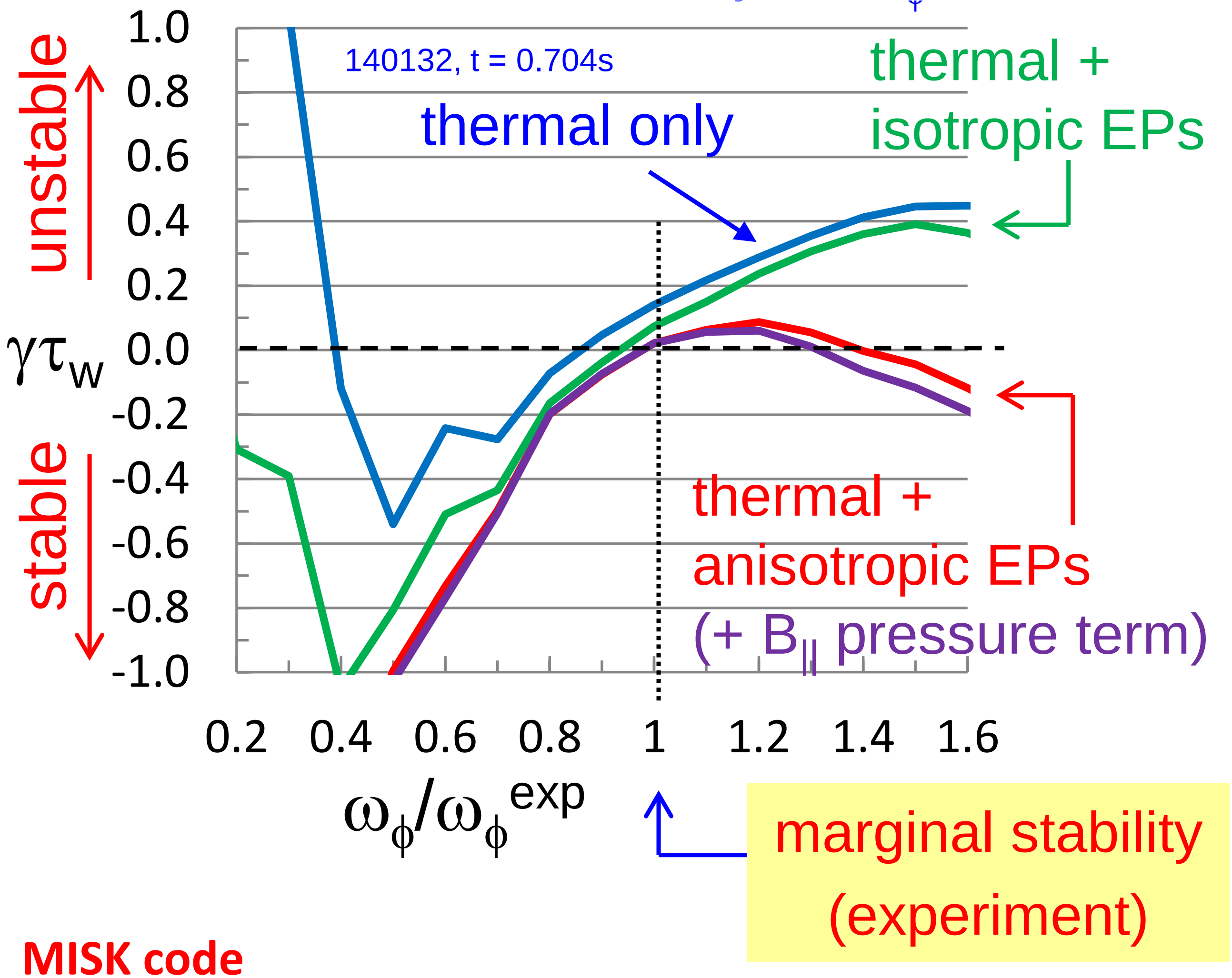
MISK Code

- MISK calculations are consistent with RWM instability at intermediate plasma rotation in NSTX
- Instability appears between precession drift resonance at low ω_ϕ, bounce/transit resonance at high ω_ϕ

[S. Sabbagh *et al.*, Nucl. Fusion **50**, 025020 (2010)]
[J. Berkery *et al.*, Phys. Rev. Lett. **104**, 035003 (2010)]

Improvements to the MISK model continue, including refinement of energetic particle modeling

RWM stability vs. ω_ϕ



MISK code

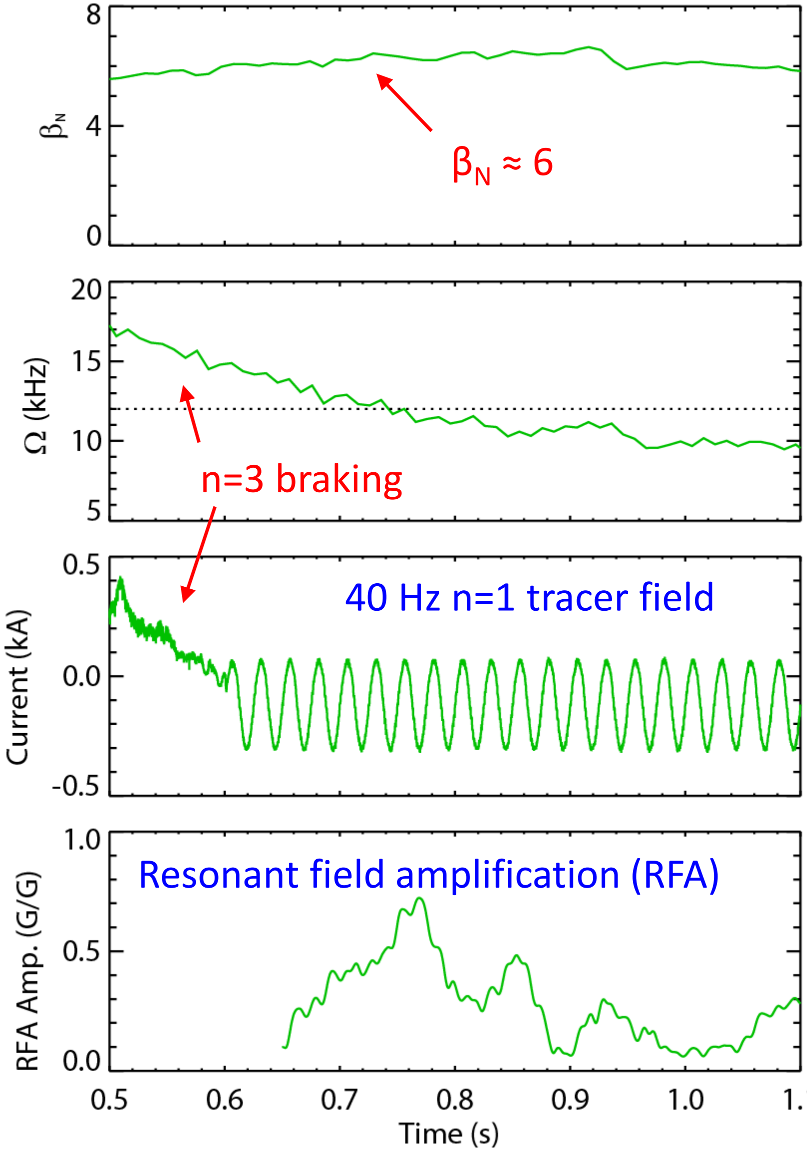
- Improvements to physics model
  - – Anisotropy effects
  - – Testing terms thought small
    - Already good agreement between theory and experiment of marginal stability point improved

Active MHD spectroscopy is used to probe plasma stability

- Active MHD spectroscopy used as a proxy for RWM stability when modes are stable
  - Resonant field amplification (RFA) of an n=1 applied AC field is measured.
  - Increased RFA indicates decreased stability

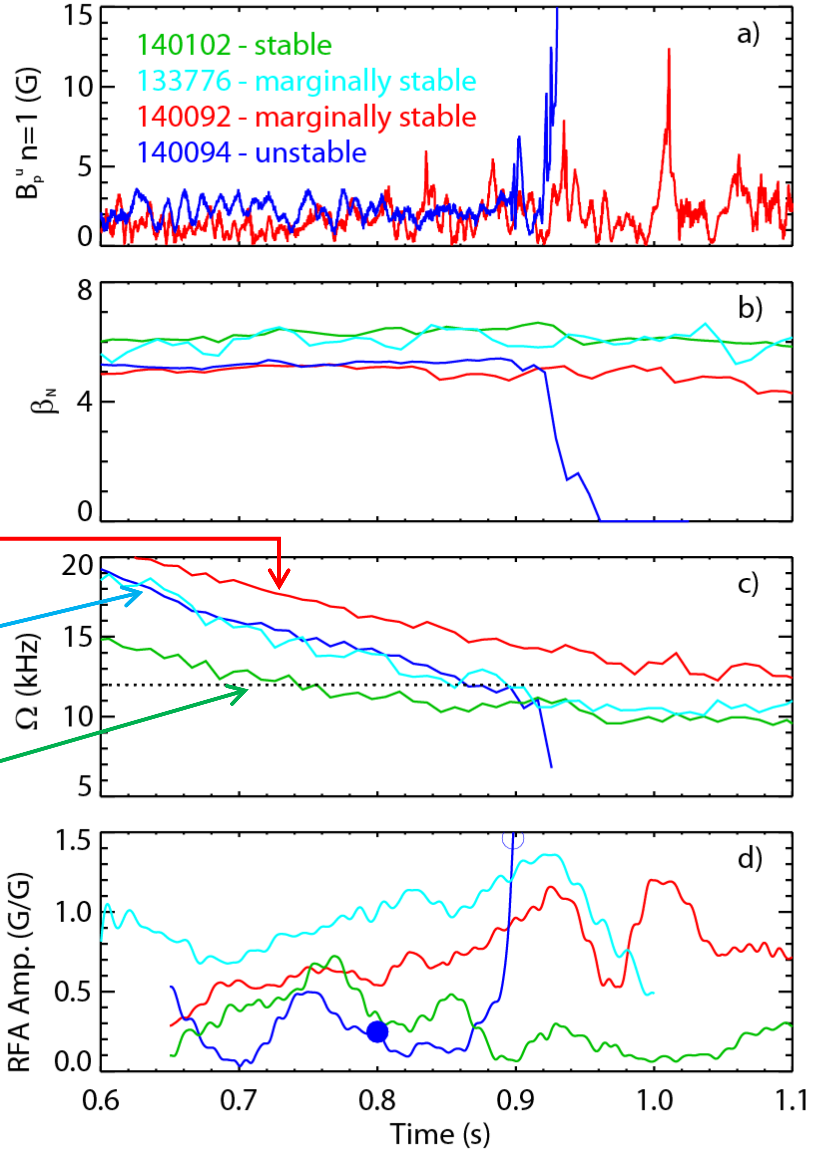
$$RFA = \frac{B_{plasma}}{B_{applied}}$$

[H. Reimerdes *et al.*, Phys. Rev. Lett. **93**, 135002 (2004)]

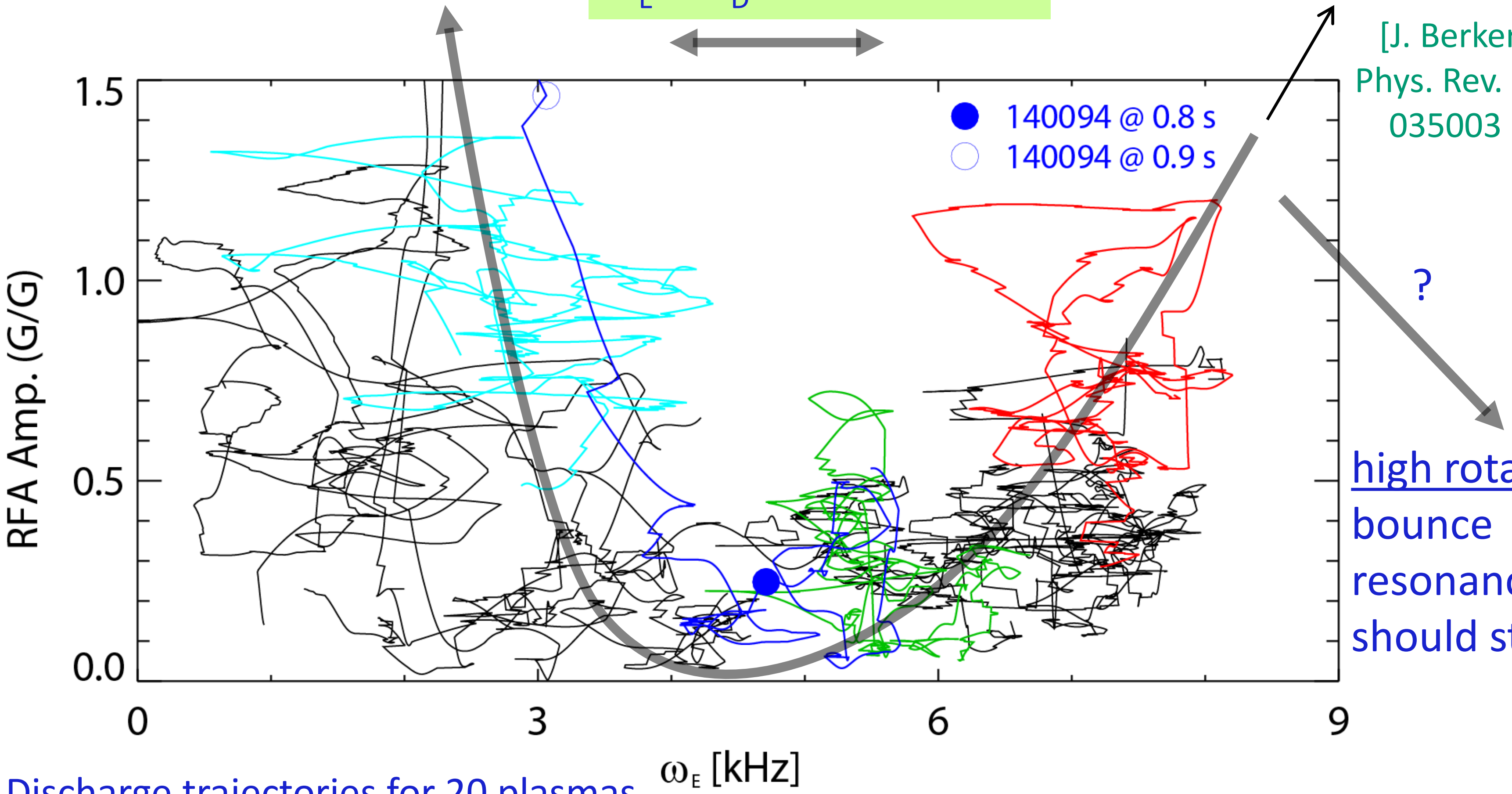


Resonant field amplification experiments in NSTX gauge the stability of plasmas to compare to kinetic stabilty theory

- Experiments in NSTX measured RFA of high beta plasmas with rotation slowed by n=3 magnetic braking.
  - unstable at 0.9 s
  - same β, higher rotation: marginally stable
  - higher β, same rotation: marginally stable
  - higher β, lower rotation: but stable! Counter-intuitive without invoking kinetic effects



RFA measurements add additional support to established theory of RWM stability through kinetic resonances



low rotation

unstable RWMs

precession drift resonance stabilization

$\omega_E - \omega_D \approx 0$

intermediate rotation

unstable RWMs

(not in present data set)

[J. Berkery *et al.*, Phys. Rev. Lett. **104**, 035003 (2010)]

high rotation

bounce resonances should stabilize

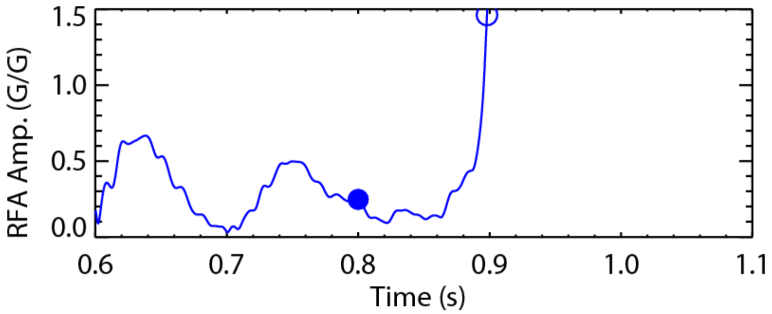
Discharge trajectories for 20 plasmas

Experimental instability can be explained by kinetic theory and MISK calculation

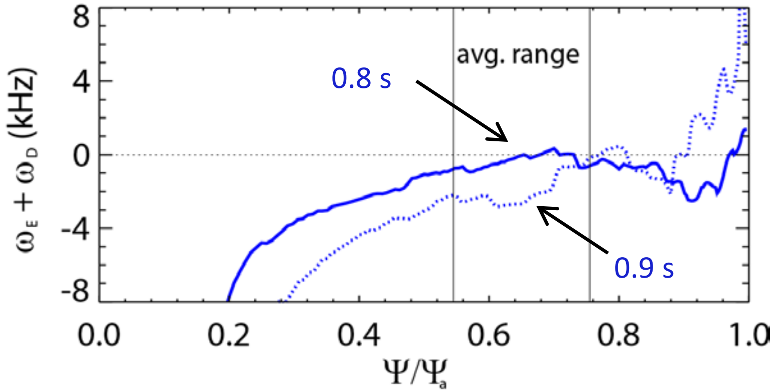
- Earlier time (0.8 s,●):
  $\omega_E + \omega_D \approx 0$
  - Experiment: stable
  - Theory: stabilizing
  - Calculation: stable
- Later time (0.9 s,○):
  $\omega_E + \omega_D < 0$
  - Experiment: unstable
  - Theory: destabilizing
  - Calculation: unstable at 10% lower rotation than marginal point

Effects of EPs still being evaluated

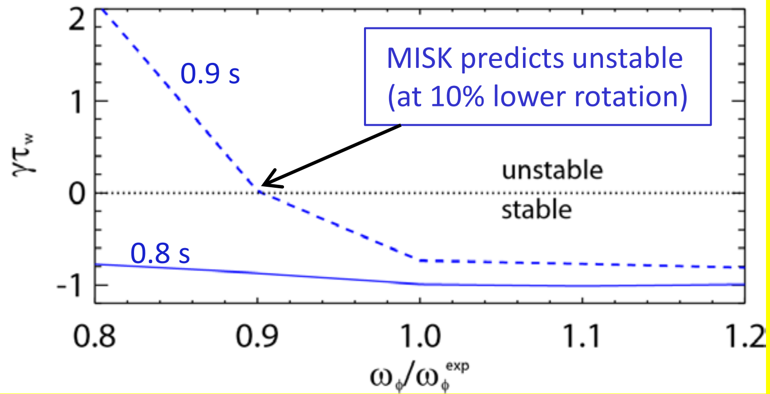
Experiment (RFA)



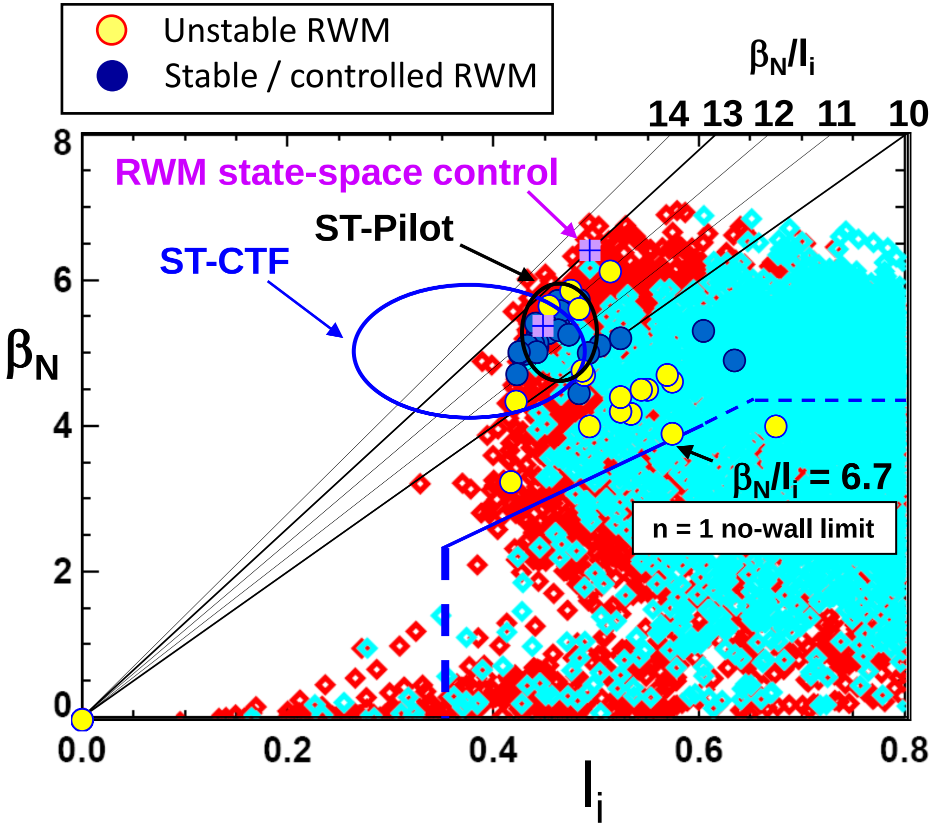
Theory (Profiles)

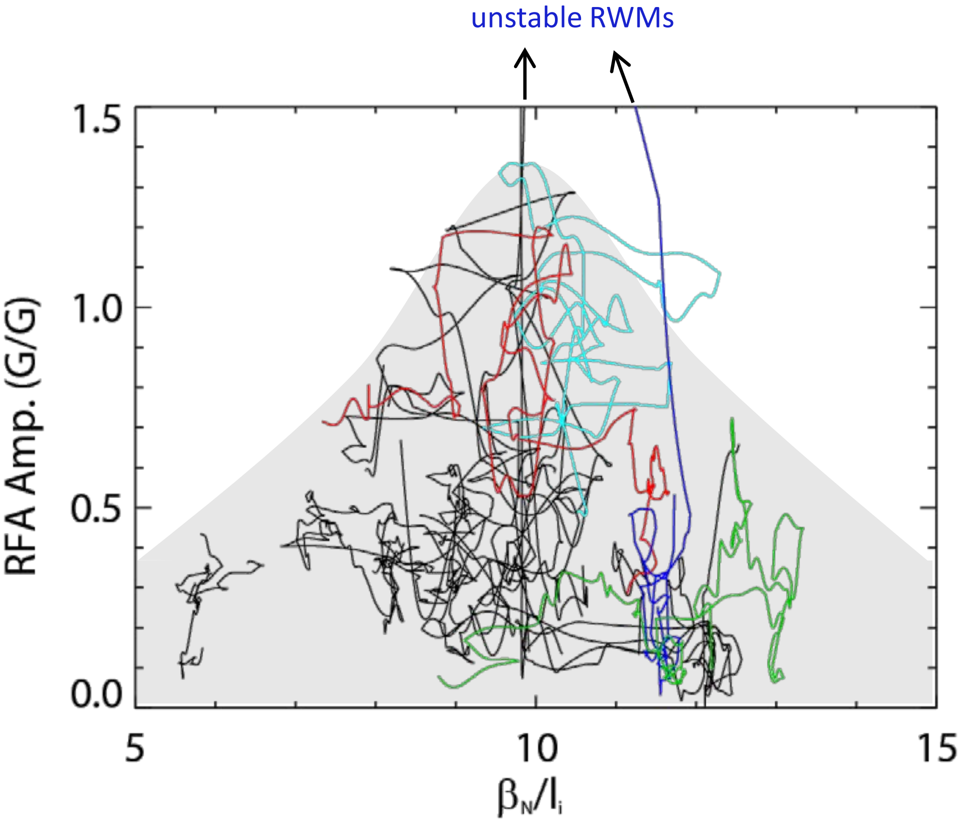


Calculation (MISK)



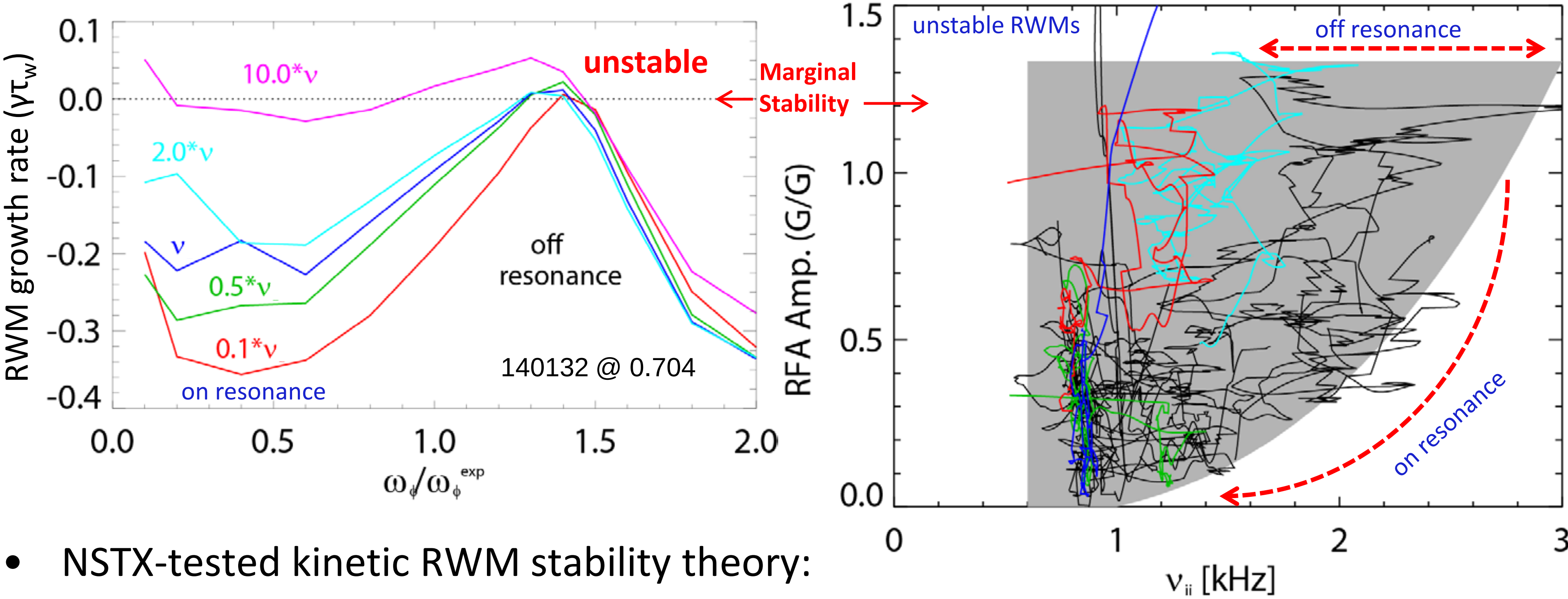
RFA measurements confirm previous NSTX result that the highest β_N/l_i is not the least stable





- NSTX can reach high β, low l_i range where next-step STs aim to operate
  - Active control experiments reduced disruption probability from 48% to 14%, but mostly in high β_N/l_i
- RFA amplitude from 20-shot database also peaks at intermediate β_N/l_i
  - Increased stability at high β_N/l_i due to kinetic stabilization from resonances

Theory: Reduced ν is stabilizing near kinetic resonances
Experimental Confirmation: Reduced ν -> reduced low RFA

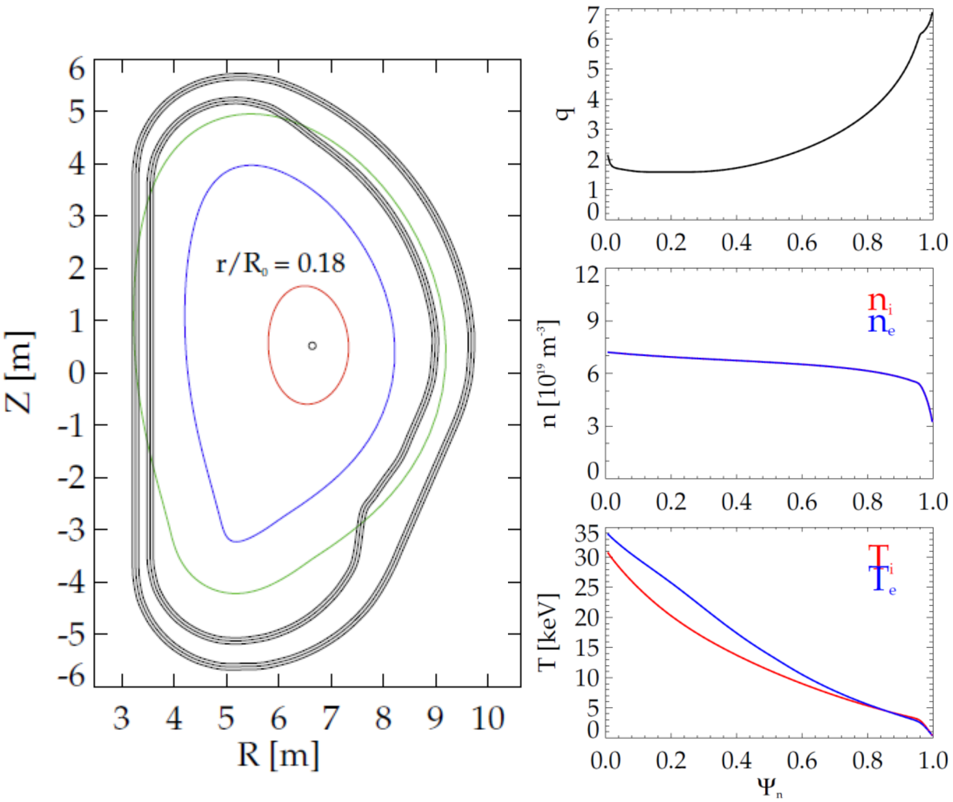


- NSTX-tested kinetic RWM stability theory:
  - Stabilizing collisional dissipation reduced (expected from early theory)
  - Stabilizing resonant kinetic effects enhanced (contrasts early RWM theory)
- RFA amplitude reduced at lower ν for low RFA (stable) plasmas, little effect on higher RFA (marginal) plasmas
- Expectations in NSTX-U, tokamaks at lower ν (ITER)
  - Stronger stabilization near ω_ϕ resonances; almost no effect off-resonance

[J. Berkery *et al.*, Phys. Rev. Lett. **106**, 075004 (2011)]

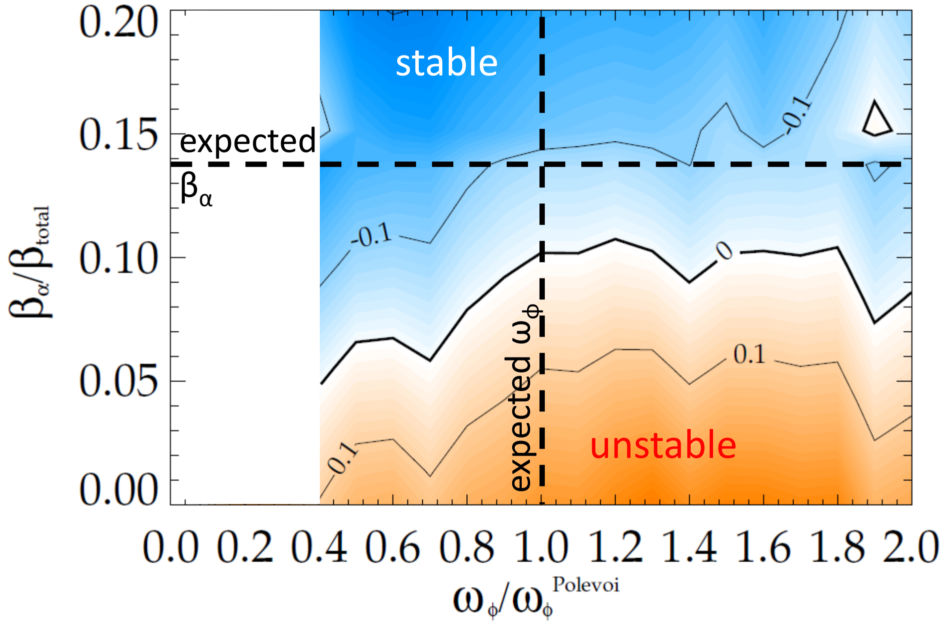
ITER advanced scenario requires alpha particles for RWM stability across all rotation values

In a previously analyzed case [J.W. Berkery *et al.*, Phys. Plasmas 17, 082504 (2010)], αs were required for stability across all ω_ϕ.



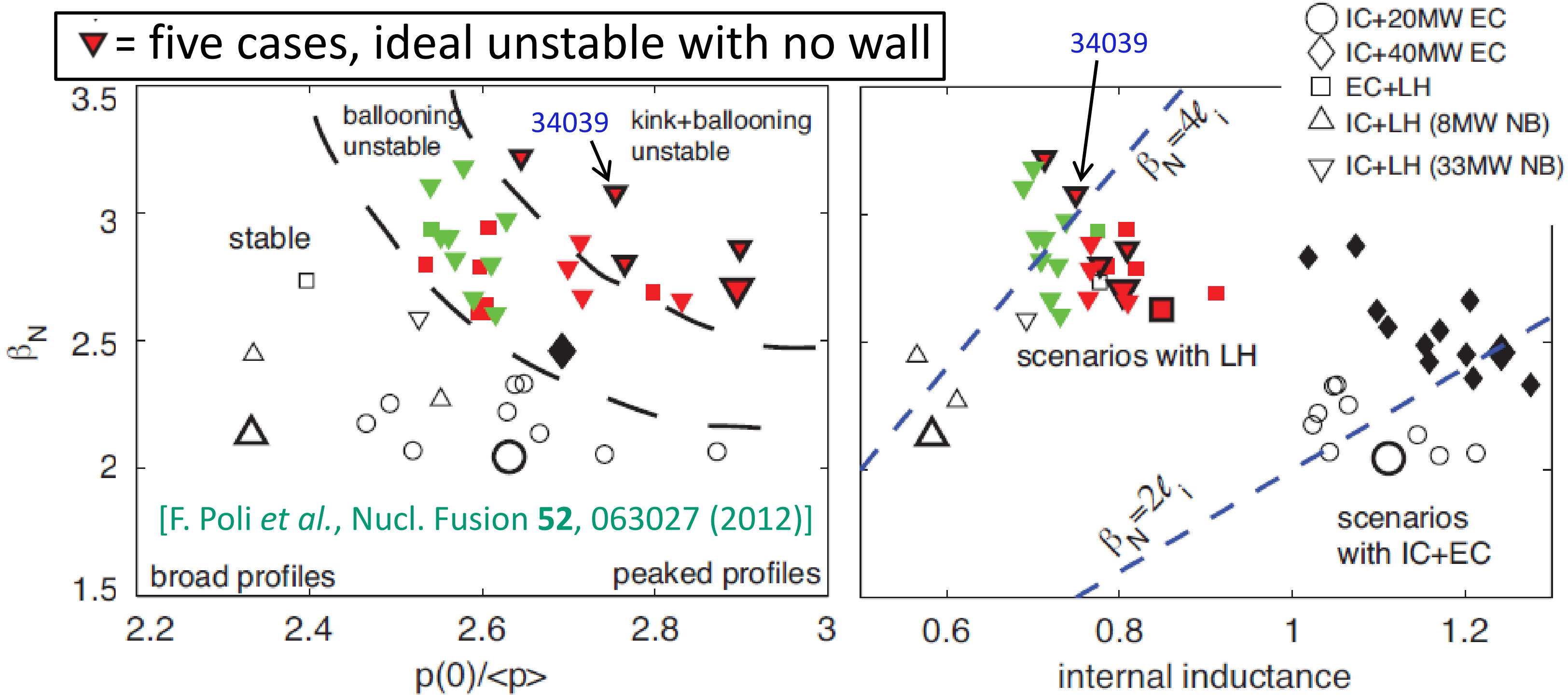
- ITPA MHD WG7 equilibrium
    - I_p = 9 MA, β_N = 2.9 (7% above no-wall limit)

$\gamma\tau_W$ contours vs. β_α and ω_ϕ



- Calculation revisited with physics improvements (inc. correction to ω_D in MISK)
    - Makes calculation somewhat more stable, but generally consistent. Doesn't affect the conclusions.

Kinetic RWM stability analysis started with MISK for a wider database of ITER advanced scenario equilibria



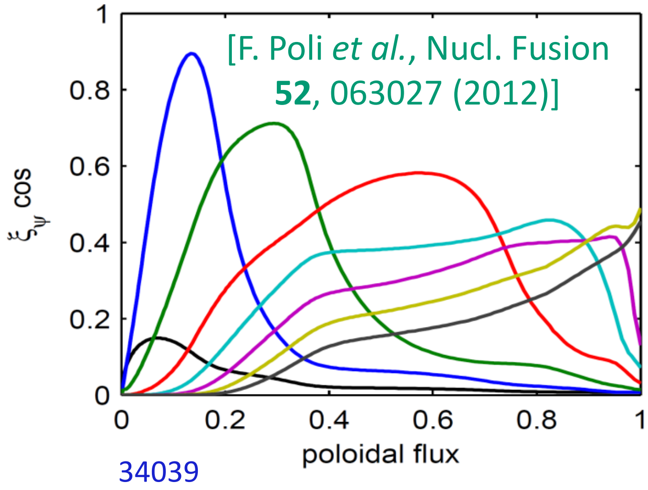
▼ = five cases, ideal unstable with no wall

[F. Poli *et al.*, Nucl. Fusion **52**, 063027 (2012)]

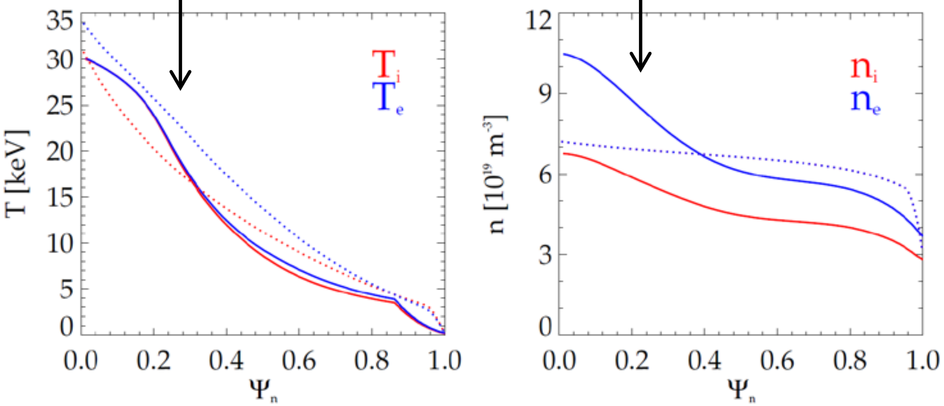
- Five discharges selected; self-consistent variation in parameter space
  - — Full discharge evolutions – created by combination of TSC and TRANSP codes
  - — Range of β_N = 2.65 – 3.25; ideal n=1 no-wall unstable
  - — Have internal transport barriers
- Include EPs from: 33MW N-NBI (D), 20 MW IC, 40 MW LH
  - — Next steps: include anisotropic EP dist.: slowing-down for beams, bi-Max for RF.

Low rotation, ITBs in ITER can cause stabilizing precession drift resonance in plasma core

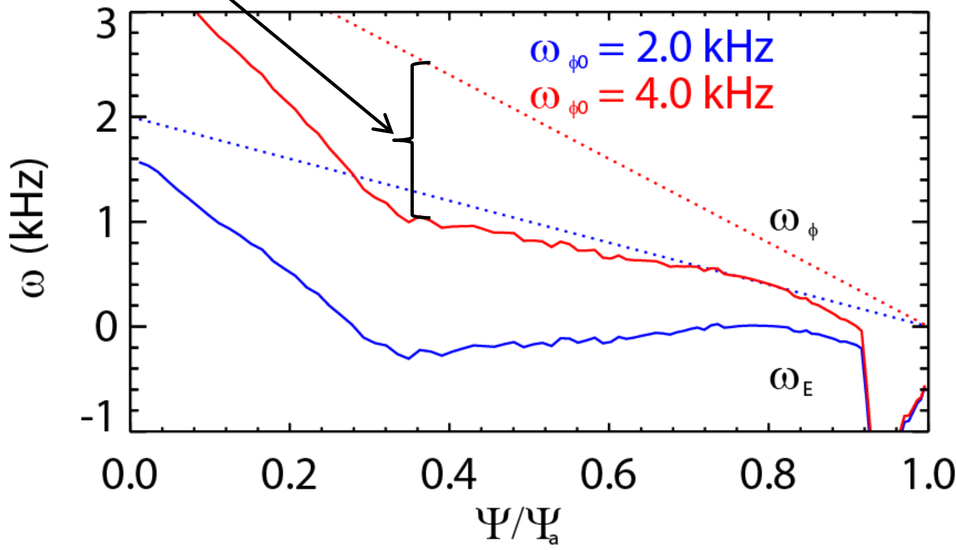
- “Infernal” type eigenfunction peaks near core.
- Strong internal gradients create large ω_*
  - — Cause difference between ω_ϕ and ω_{EXB}.
  - — Enables resonance with precession drift of trapped thermal ions if ω_ϕ is low.


[F. Poli *et al.*, Nucl. Fusion **52**, 063027 (2012)]

Strong gradients



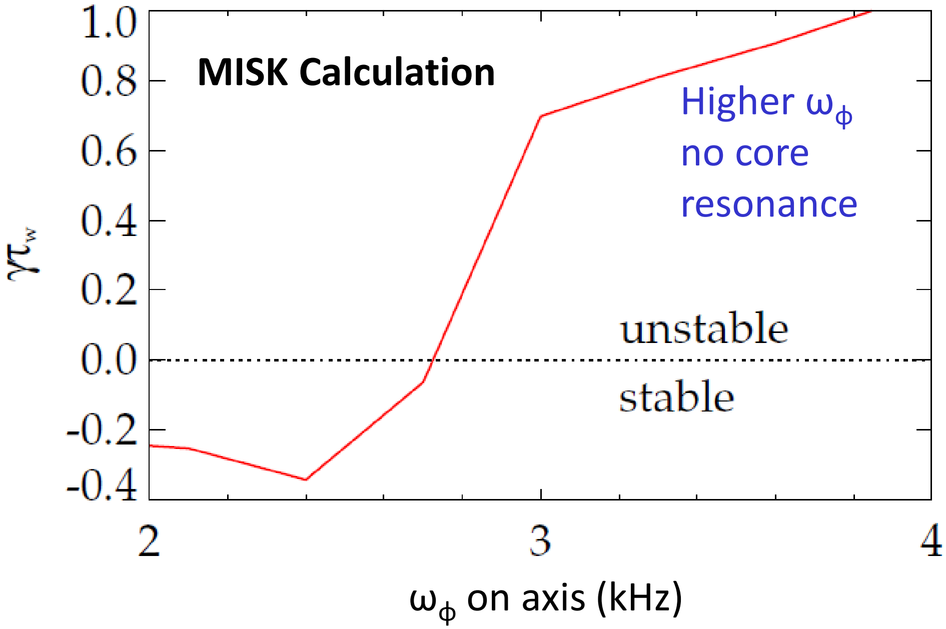
dashed lines = “standard” ITER advanced scenario



$$\omega_\phi \approx \omega_E + \omega_{*i}$$

Internal transport barriers may be beneficial to RWM stability by lowering the E×B frequency

- Stable region found with no alphas
  - At low ω_ϕ, due to precession resonance of trapped thermal ions, coupled with infernal eigenfunction
  - Unstable region at higher ω_ϕ similar to previous results



- Caveat: ITBs can be transient
  - This may result in RWM instability if profile dynamics move the plasma off-resonance
  - Active RWM control would then be needed during the period when the plasma profiles are away from stabilizing kinetic resonances

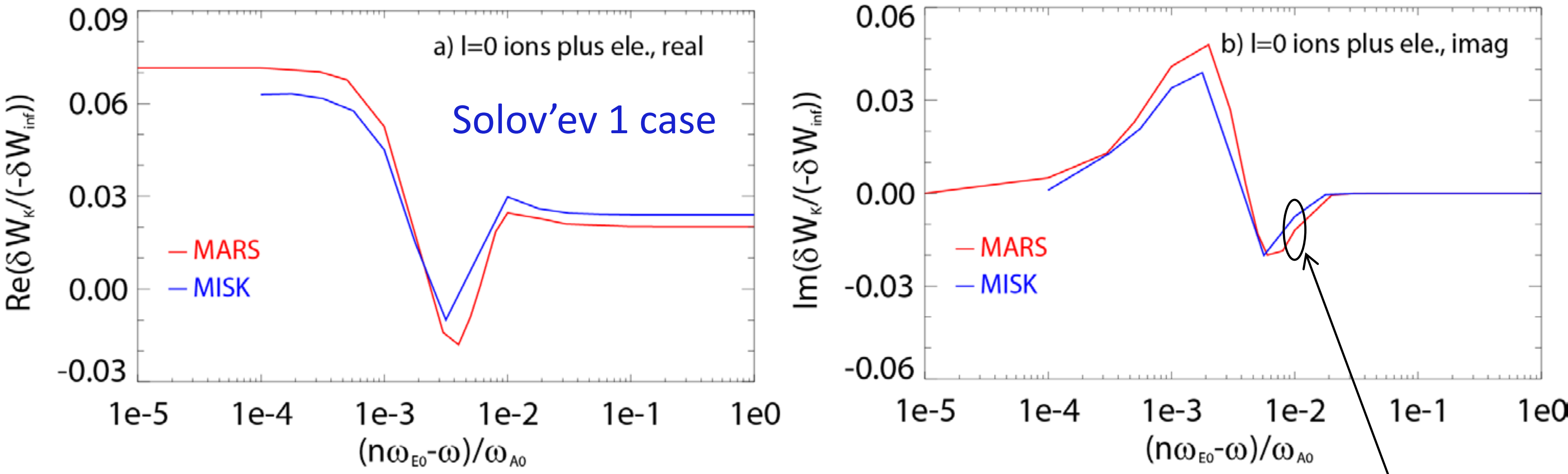
Agreement achieved between MISK and MARS-K under ITPA MHD Stability Group MDC-2 Benchmarking

Work in progress!

	r_{wall}/a	Ideal $\delta W/-\delta W_\infty$	$Re(\delta W_k)/\delta W_\infty$	$Im(\delta W_k)/(\delta W_\infty)$	$\gamma\tau_{wall}$	$\omega\tau_{wall}$	$\delta W_k/-\delta W_\infty$ ($\omega_E = \infty$)
Solov'ev 1 (MARS-K) (MISK)	1.15	1.187 1.122	0.0256 0.0271	-0.0121 -0.0077	0.804 0.847	-0.0180 -0.0124	0.157 0.153
Solov'ev 3 (MARS-K) (MISK)	1.10	1.830 2.337	0.0915 0.0284	-0.342 -0.0402	0.349 0.410	-0.226 -0.024	1.98 0.655
ITER (MARS-K) (MISK)	1.50	0.682 0.677	-4.51 0.653	-0.445 -0.746	-1.43 -0.041	-0.050 -0.538	229 8.46

- Calculations from MISK, and MARS-K (perturbative)
  - The relevant frequencies and eigenfunctions now match between codes for both analytical Solov'ev and ITER equilibria.
  - Numerical approach to the frequency resonance fraction energy integral taken in MISK is equivalent to analytical limits computed in MARS-K.

Agreement achieved between MISK and MARS-K under ITPA MHD Stability Group MDC-2 Benchmarking



- Comparing δW_K vs. ω_E scan (rather than single point (ω_E = 1e-2) in chart)
  - Good agreement for precession drift resonance (l=0 trapped particles) and circulating particles
  - "Light green" in chart can be deceiving: really OK agreement

$\mathrm{Im}(\delta W_k)/(\delta W_\infty)$

-0.0121
-0.0077

Global mode control and stabilization studies in high-β NSTX plasmas aid the goal of disruption avoidance in ITER

- Two active control techniques were used to avoid disruptions
  - Disruption probability was reduced from 48% to 14% in high β_N/l_i plasmas with dual field component (radial and poloidal) active control
  - A model-based state space controller sustained long-pulse, high-β_N discharges
- Dedicated resonant field amplification (RFA) experiments in NSTX revealed key dependencies of stability on plasma parameters
  - RFA measurements add additional support to the established theory of resistive wall mode (RWM) stability through kinetic mode-particle resonances
  - Stability is weakest at intermediate, not the highest, values of β_N/l_i, in agreement with other NSTX active control experiments
  - Relatively stable plasmas appear to benefit from reduced collisionality, in agreement with expectation from kinetic theory
- Application of the model to ITER plasmas indicates
  - Alpha particles may be needed for RWM stability
  - ITBs may be beneficial to RWM stability by lowering the E×B frequency

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