NSTX-U

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M3D-K Simulations of Wave-Particle Interaction with Realistic Fast Ion Distribution

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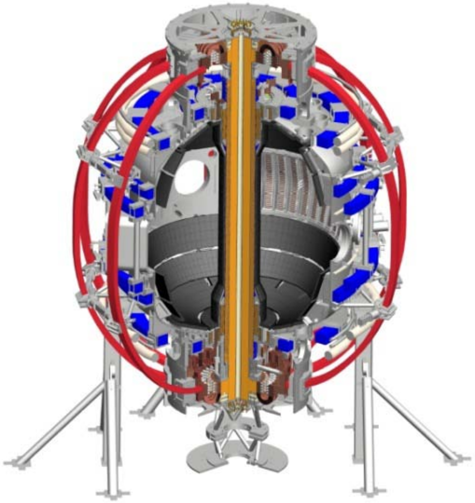

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Initialization with Realistic Fast Ion Distributions could Improve Accuracy of Hybrid simulations

- Fluid/kinetic hybrid simulation codes such as M3D-K typically use a simple analytic form to specify the initial fast ion distribution.
  →This could limit the simulation accuracy when the analytic distribution is a poor fit to the actual distribution.

- An interface between the output of tokamak fast ion Monte Carlo code TRANSP/NUBEAM and M3D-K has been developed.
  → More realistic fast ion distributions from NUBEAM can be directly used as M3D-K inputs to improve simulation accuracy.

- M3D-K linear simulations of non-resonant kink mode and TAE mode on NSTX have been performed with analytic and realistic fast ion distributions to validate the new method and check the possible benefits.

M3D-K: Global Kinetic/MHD Hybrid Code

$$\rho\frac{d\mathbf{v}}{dt}+\rho\left(\mathbf{v}_i^*\cdot\nabla\right)\mathbf{v}_\perp=-\nabla\cdot P-\nabla\cdot\mathbf{P}_h+\mathbf{J}\times\mathbf{B}-\mathbf{bb}\cdot\nabla\cdot\prod_i$$

$$\mathbf{P}_h=P_\perp\mathbf{I}+\left(P_\parallel-P_\perp\right)\mathbf{bb}$$

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\left(\rho\mathbf{v}_i\right)=0$$

$$\rho\left[\frac{\partial\mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}+\left(\mathbf{v}_i^*\cdot\nabla\right)\mathbf{v}_\perp\right]=-\nabla\mathbf{p}+\mathbf{J}\times\mathbf{B}+\mu\nabla^2\mathbf{v}$$

$$\mathbf{E}+\mathbf{v}\times\mathbf{B}=\eta\mathbf{J}-\frac{\nabla_\parallel p_e}{ne}$$

$$\frac{\partial\mathbf{B}}{\partial t}=-\nabla\times\mathbf{E}$$

$$\mathbf{J}=\nabla\times\mathbf{B}$$

$$\frac{\partial p}{\partial t}+\mathbf{v}\cdot\nabla p=-\gamma p\nabla\cdot\mathbf{v}+\nabla\cdot n\chi_\perp\nabla\left(\frac{p}{\rho}\right)-\mathbf{v}_i^*\cdot\nabla p-\gamma p\nabla\cdot\mathbf{v}_i^*+\frac{\mathbf{J}\cdot\nabla p_e}{ne}+\gamma p_e\mathbf{J}\cdot\nabla\left(\frac{1}{ne}\right)$$

$$\frac{\partial p_e}{\partial t}+\mathbf{v}\cdot\nabla p_e=-\gamma p_e\nabla\cdot\mathbf{v}+\nabla\cdot n\chi_{\perp e}\nabla\left(\frac{p_e}{\rho}\right)+\frac{\mathbf{J}_\parallel\cdot\nabla p_e}{ne}-\gamma p_e\nabla\cdot\left(\mathbf{v}_e^*-\frac{\mathbf{J}_\parallel}{ne}\right)$$

where $\mathbf{v}_e^*\equiv-\frac{\mathbf{B}\times\nabla p_e}{neB^2}$, $\mathbf{v}\equiv\mathbf{v}_i-\mathbf{v}_i^*=\mathbf{v}_e-\mathbf{v}_e^*+\frac{\mathbf{J}_\parallel}{ne}$ $\mathbf{v}_i^*\equiv\mathbf{v}_e^*+\frac{\mathbf{J}_\perp}{ne}$,

- ➢**Extended MHD equations (Single fluid, Two-fluid (but not used in this work))**
- ➢**Hybrid (Kinetic fast ions)**
- ➢**Kinetic fast ions couple back to fluid model through pressure tensor, which is calculated by gyrokinetic or drift-kinetic equation**

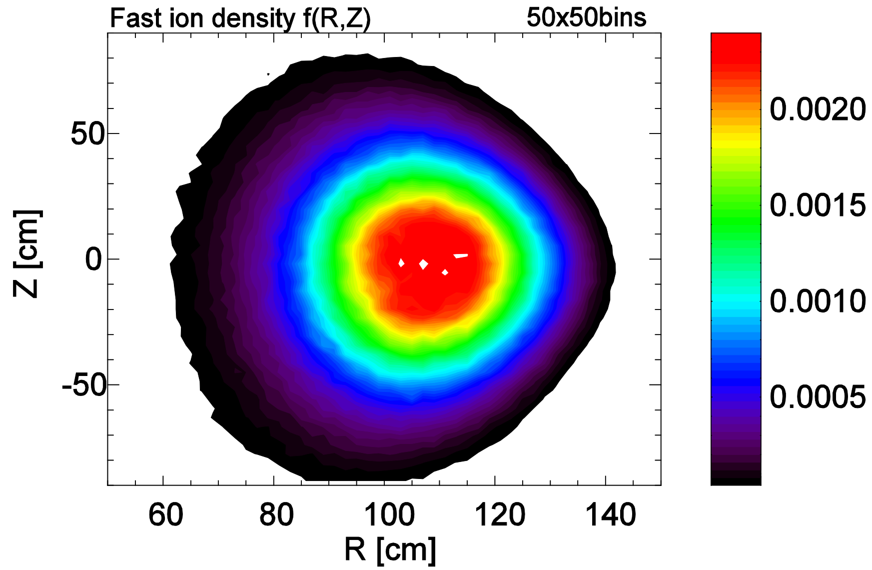
**Fu et al. Phys. Plasmas 2006*

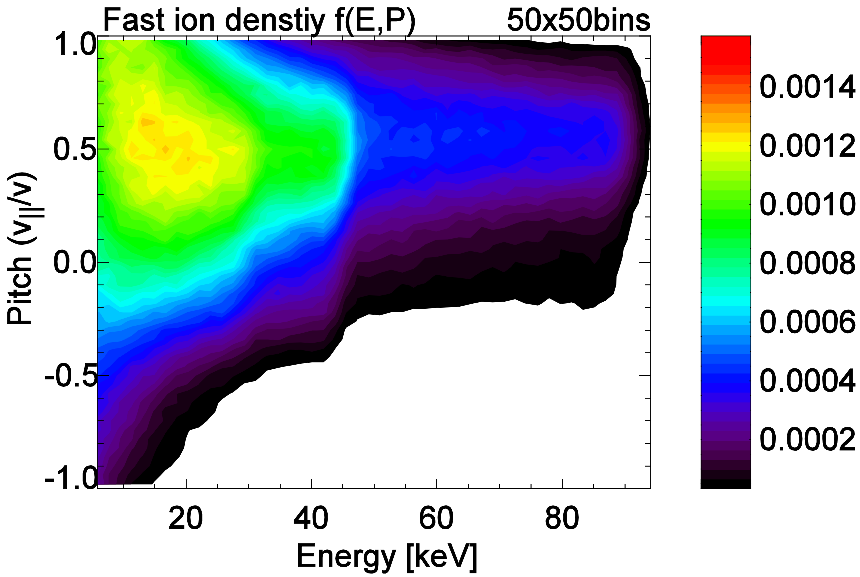
Main Features of M3D-K

- Use realistic geometry
- Use experimental parameters and profiles
- Global
- Linear and nonlinear
- Non-perturbative fast ion effects (i.e., can model EPM)
- Numerical equilibrium calculation with plasma rotation and anisotropic fast ion pressure
- Interface with TRANSP/NUBEAM fast ion distribution is developed and tested.

Numerical Fast Ion Distribution is Calculated by Monte Carlo Modeling NUBEAM

- NUBEAM module in TRANSP: a Monte Carlo code for 4D (R,Z, $\lambda = v_{||}/v$, E) time dependent simulation of fast ion transport in tokamak devices.
- Assume fast ions behave classically; could add anomalous radial diffusion operator based on user provided diffusivities
- Includes guiding center drift orbiting, collisional, and atomic physics effects





Fast Ion Distribution Needed to be Transformed from (R,Z,λ,E) to Constants-of-Motion Space

- M3D-K code needs fast ion distribution f(R,Z,λ,E) expressed in terms of constants of the motion (P_φ, μ ,E) and the function must be smooth enough to allow derivatives to be taken with E and toroidal angular momentum P_φ
- In addition to P_φ, μ ,E, the sign of $v_{||}$ is also needed to resolve orbit type.
- NUBEAM calculated fast ion distribution f(R,Z,λ,E) needs to be transformed to f(P_φ, μ ,E) coordinates.

$$\int f(x,y,z,vx,vy,vz)d^3\mathbf{x}d^3\mathbf{v}$$
$$=\int f(R,Z,\lambda,E)R\sqrt{E}dRdZd\lambda dE$$
$$=\sum_{v_{sign}}\int f_{v_{sign}}(P_\varphi,\mu,E)\Im_{v_{sign}}(P_\varphi,\mu,E)dP_\varphi d\mu dE$$

where

$$P_\varphi=(eA_\varphi+mv_\varphi)R\approx e\psi(R,Z)+mv_{||}R\frac{B_\varphi(R,Z)}{B(R,Z)}$$

$$\mu=\frac{\frac{1}{2}mv_\perp^2}{B}=\frac{[1-(v_{||}/v)^2]E}{B(R,Z)} \qquad \lambda=v_{||}/v$$

- Use TRANSP "Plasma State" utility to get ψ and B at each particle location (R,Z), and then calculate P_φ, μ

Jacobian of the Transform Strongly Depends on Particle Orbit Topology

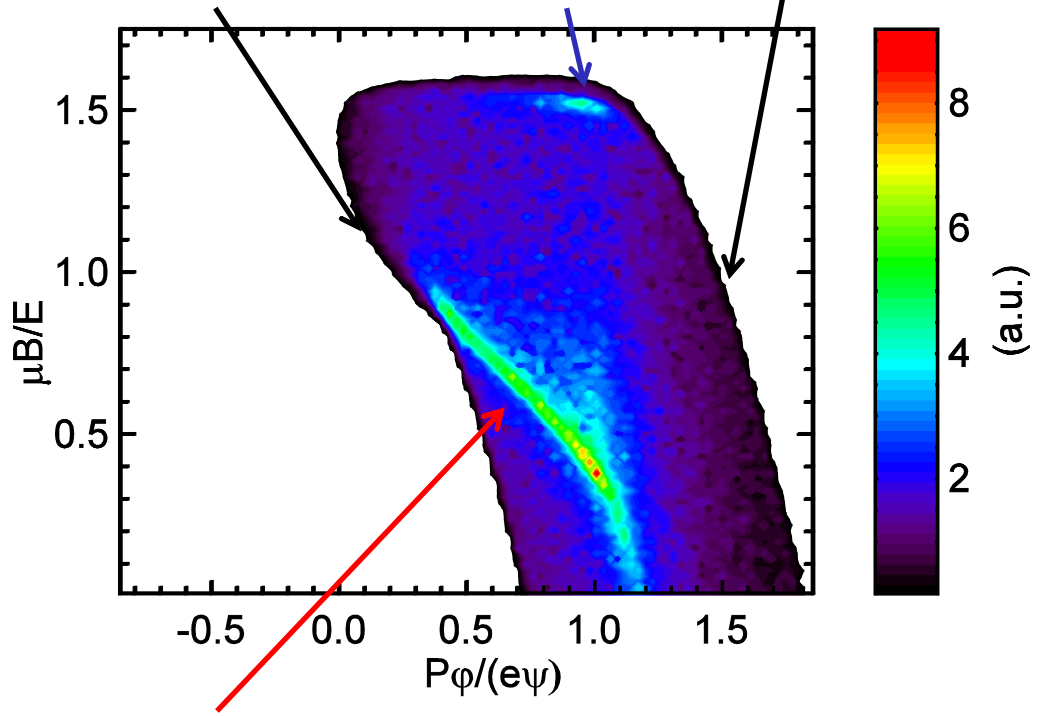
counter-current Jacobian of the transform
from $(R, Z, v_{\perp}^{2}, v_{\parallel})$ **to** (P_{φ}, μ, E)

E=85keV

Magnetic axis

Stagnation orbits

Plasma boundary



Trapped/passing boundary

- ➢Sharp change of Jacobian appears in the boundary of different particle orbit topology because

$$\Im(P_{\varphi}, \mu, E) = \frac{4\pi^{2}\tau_{bounce}}{qm^{2}}$$

**J. Egedal Nucl. Fusion 2005*

Jacobian of the Transform is Calculated by Monte Carlo Method

$$\int f(x,y,z,vx,vy,vz)d^3\mathbf{x}d^3\mathbf{v} = \int f(R,Z,\lambda,E)R\sqrt{E}\,dRdZd\lambda dE = \sum_{v_{sign}}\int f_{v_{sign}}(P_\varphi,\mu,E)\Im_{v_{sign}}(P_\varphi,\mu,E)dP_\varphi d\mu dE$$

- An exact analytic $\Im_{v_{sign}}(P_\varphi,\mu,E)$ cannot be calculated because it requires integration over phase space orbits.
- A Monte Carlo approach is used to calculate Jacobian

- Procedure:

(1)Generate a large population of ~50 million particles uniformly in phase space (R,Z,λ,E), within the same range as in the NUBEAM distribution

(2)Transform coordinates to constants-of-motion space

(3)Bin the particles in (P_φ, μ ,E) space for each $v_{||}$ sign.

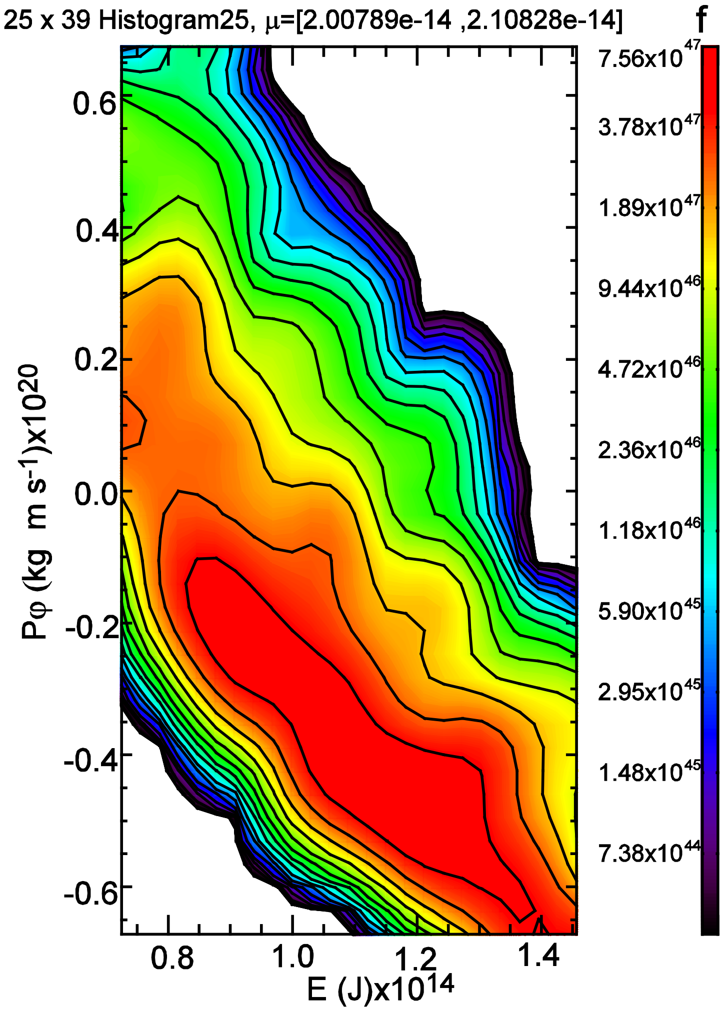
(4)The particle number in each (P_φ, μ ,E) bin proportional to $\Im_{v_{sign}}(P_\varphi,\mu,E)$

Constructing Fast Ion Density Function in Constants-of-Motion Space

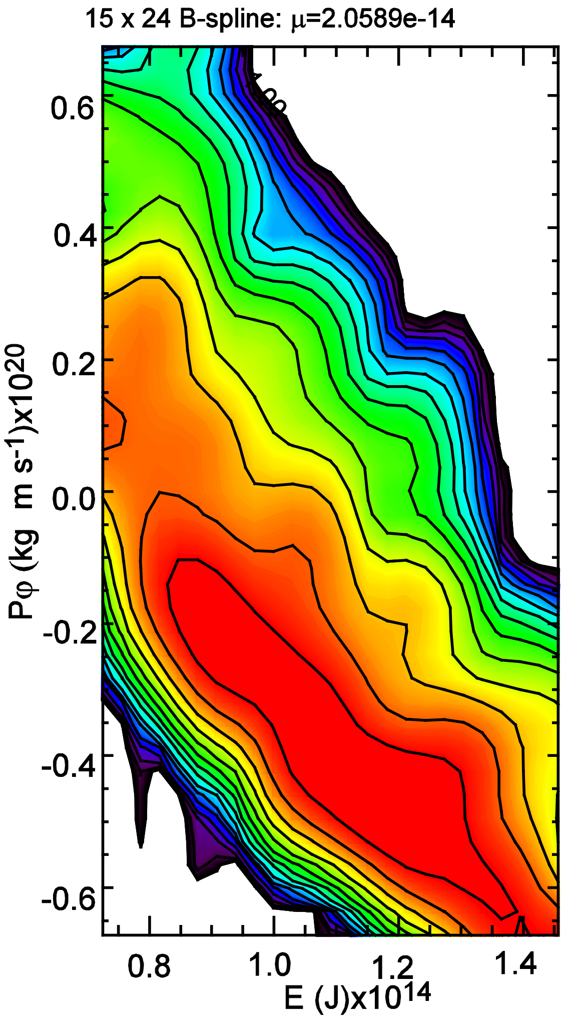
- Divide NUBEAM output particle data into two subsets based on $v_{||}$ sign
- For each subset, sort particles by μ and divide them into several subpopulations of equal width in μ.
- Divide each subpopulation into a number of bins in the P_φ and E directions. The bin width in P_φ and E direction are the same for each subpopulation.
- Multiply by the numerically calculated Jacobian.
- Apply Gaussian smoothing in both P_φ and E directions.
- Fit 2D cubit B-spline to the smoothed data using GSL routines, with uniform knots and a number of coefficient s in each direction approximately 5/8 the number of bins.
- Store the spline coefficients in a file, which can be used to construct fast ion density function in constants-of-motion space and perform quick spline and derivative interpolations at arbitrary location.
- **Improved from the work of Breslau et al. Sherwood Meeting 2011*

Good Agreement between Raw Fast Ion Density Function and Spline Fit in Constants-of-motion Space

Raw data from NUBEAM



Reconstructed from cubic B-spline fit

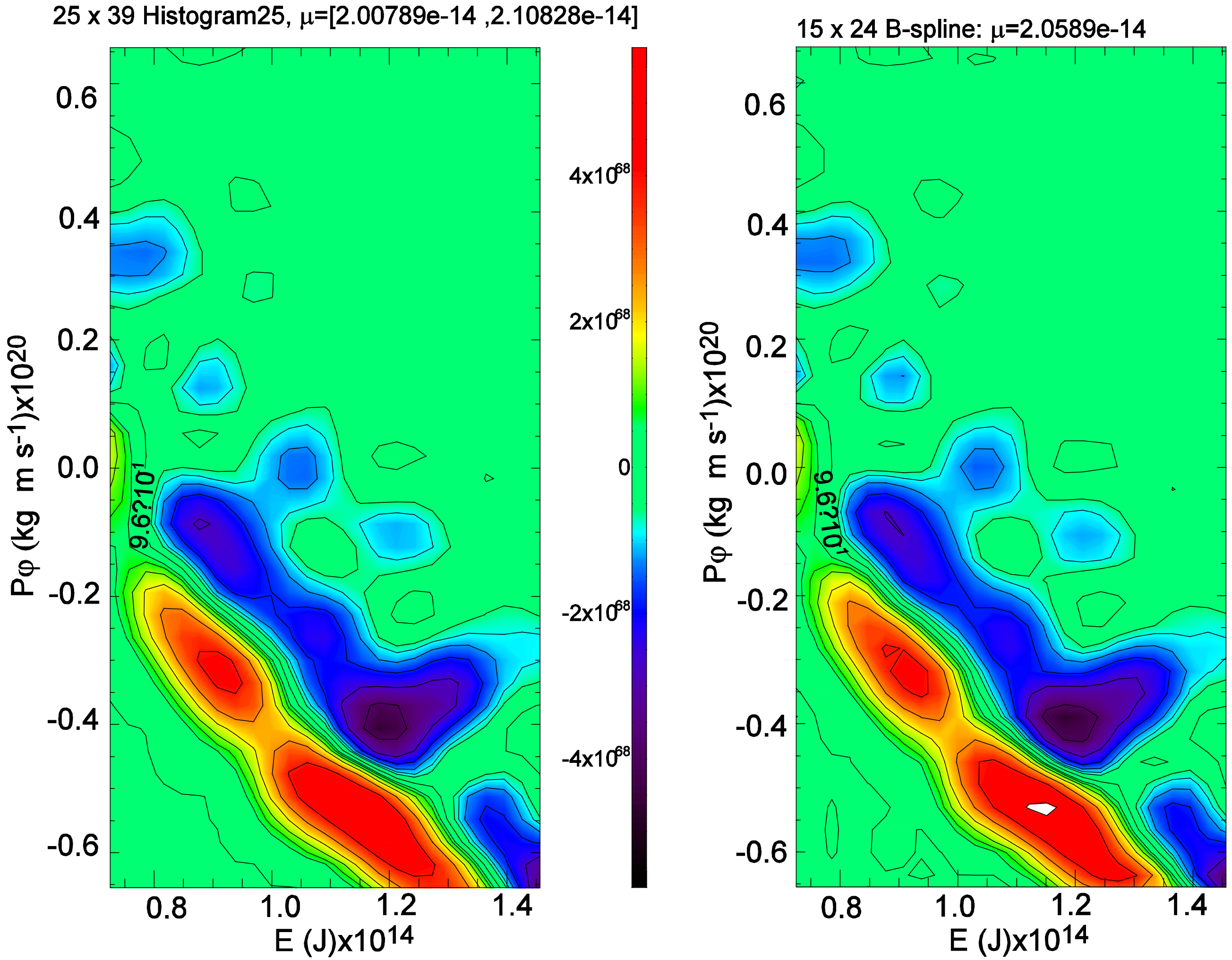


$$f_{co-current}(P_\varphi, E, \mu)|_{\mu=2.058e-14}$$

- **40 bins in μ for $v_{||}$>=0**
  38 bins in μ for $v_{||}$<0
- **B-spline fits for high-occupancy μ subset are good**
- **B-spline fits for low-occupancy μ subsets are adequate**

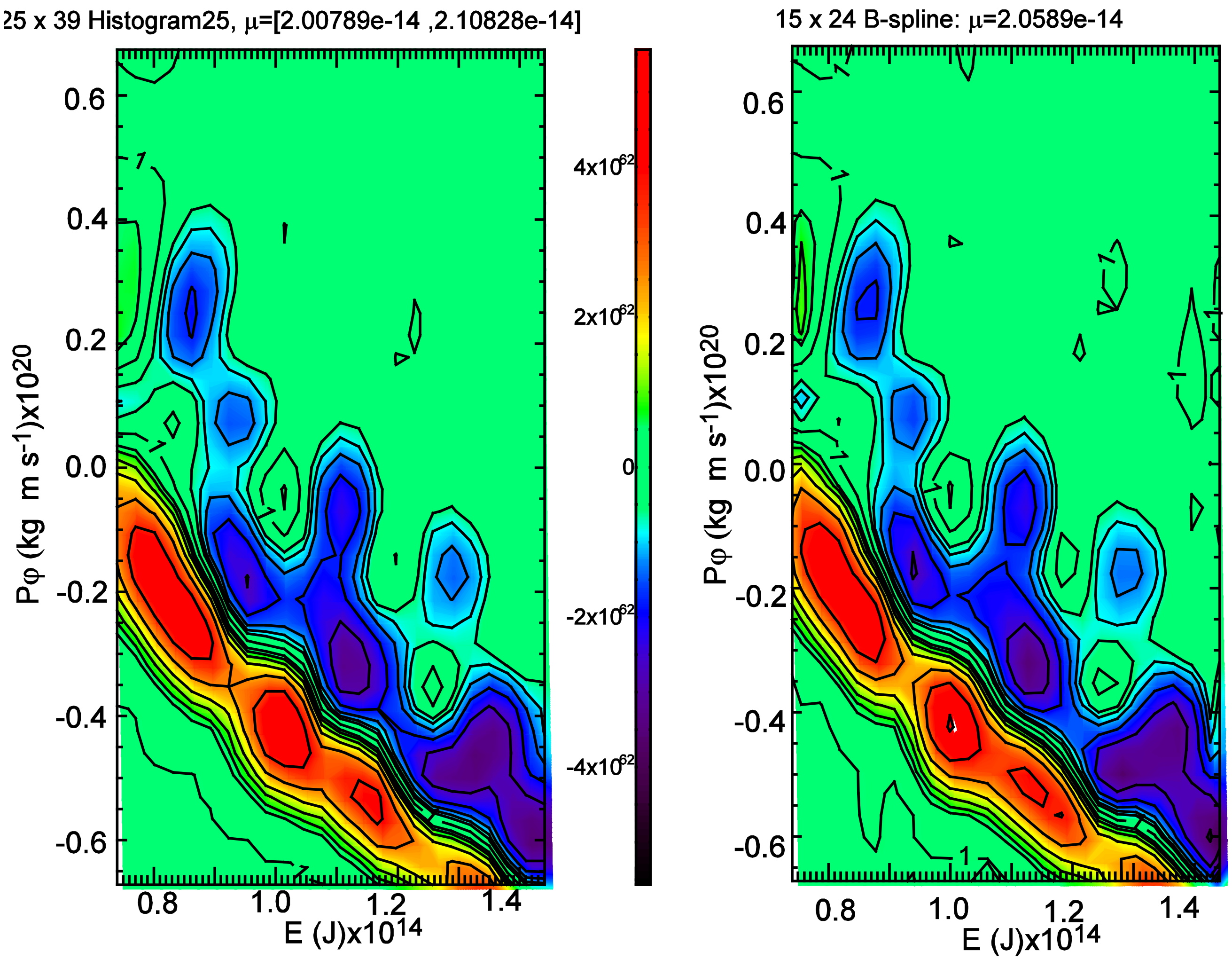
Gradients are Smooth, Match well with NUBEAM Raw Data

Raw data from NUBEAM **Reconstructed from cubic B-spline fit**



$$\frac{\partial f_{co-current}(P_{\varphi}, E, \mu)}{\partial P_{\varphi}}$$

Gradients are Smooth, Match well with NUBEAM Raw Data

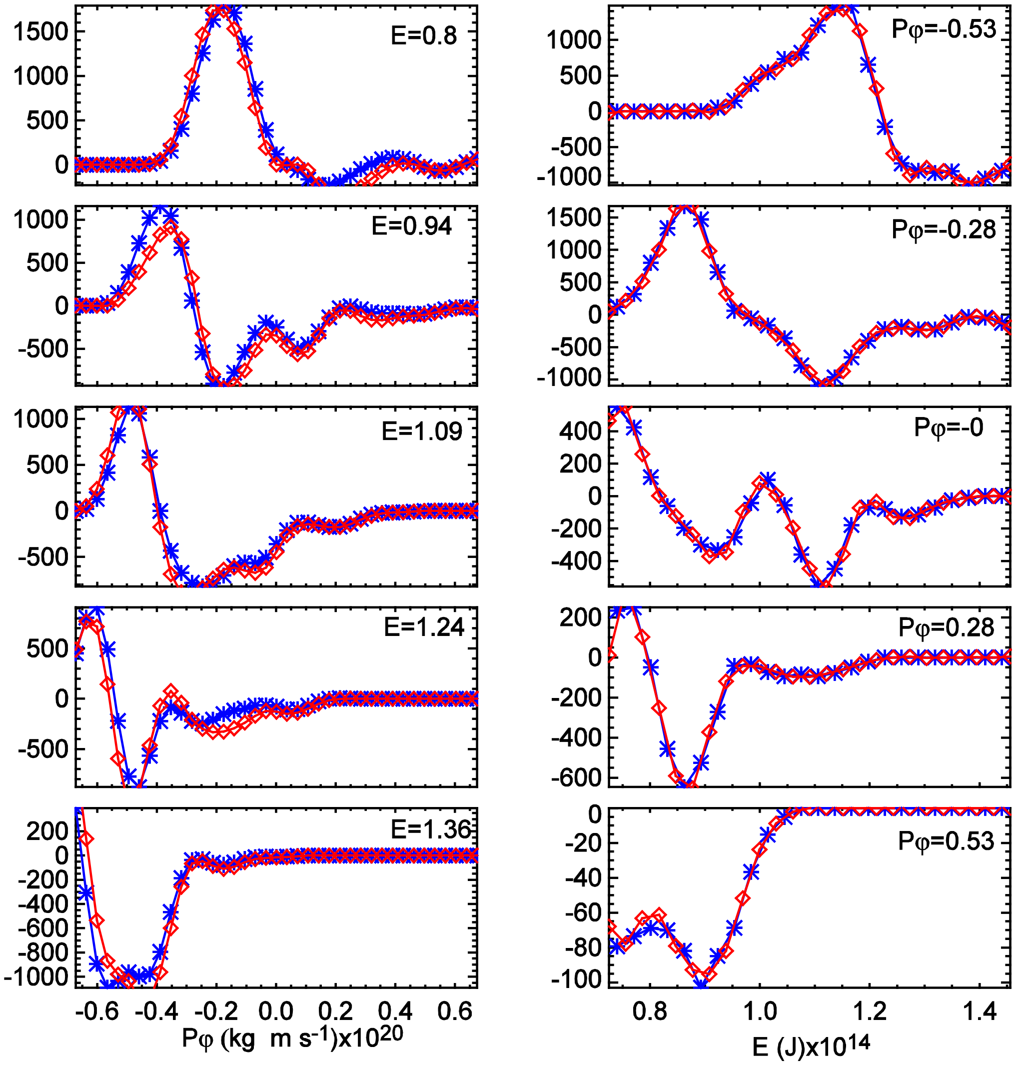


Raw data from NUBEAM

Reconstructed from cubic B-spline fit

$$\frac{\partial f_{co-current}(P_{\varphi}, E, \mu)}{\partial E}$$

Gradients are Smooth, Match well with Raw Data (cont'd)

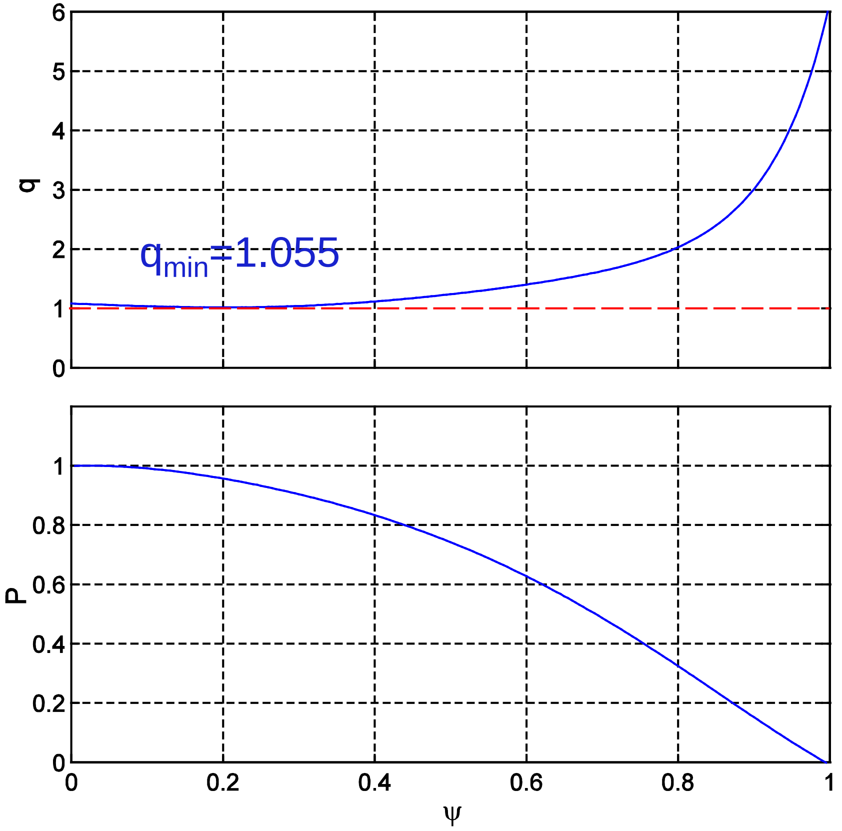


Blue: raw data from NUBEAM

Red: reconstructed from cubic B-spline fit

$$\frac{\partial f_{co-current}(P_\varphi, E, \mu)}{\partial E}$$

M3D-K Linear Simulations of Non-Resonant Kink Mode on NSTX



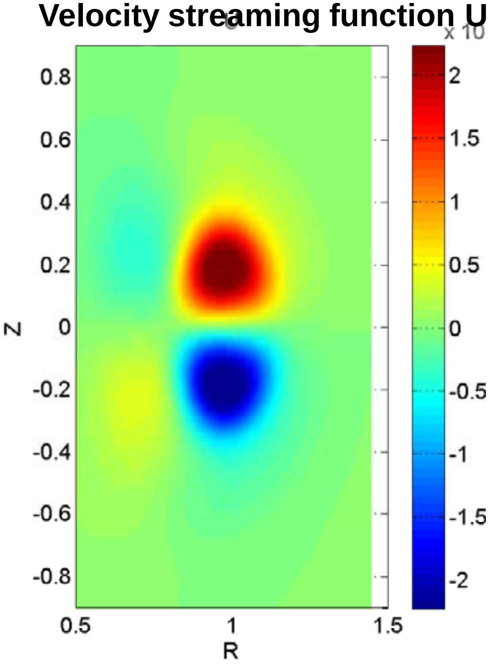
- **Both experiments and simulations show that (1,1) non-resonant kink mode is unstable in NSTX when the shear is low and central safety factor is close but greater than one.**
- **The mode is a pressure-driven mode. It can saturate nonlinearly and persist, and can develop m=2 magnetic islands.**
- **The mode could severely impact performance: flatten rotation, redistribute fast ions, trigger neoclassical tearing modes.**

**S. Gerhardt Nucl. Fusion 2011; J. Breslau Nucl. Fusion 2011; F. Wang Sherwood Meeting 2012*

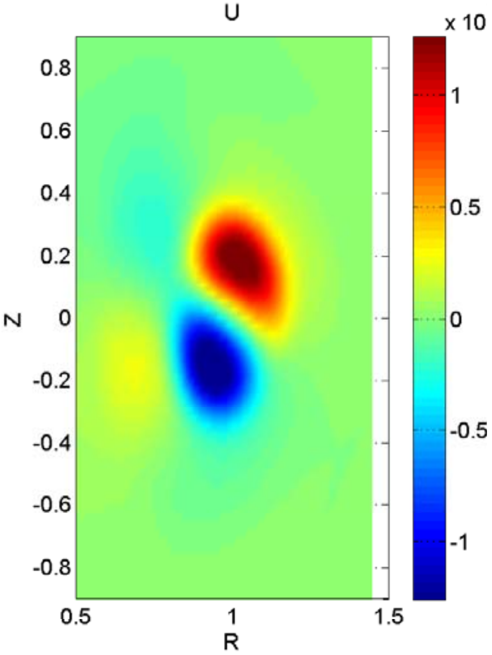
- **NSTX #124379 at t=0.635s equilibrium profiles**
- **R=0.86m, a=0.6m, $B_t(0)$=0.44T, $n_e(0)$=9.3x$10^{13}$$cm^{-3}$**
- **q_{min}=1.055, $\beta_{thermal}(0)$=24%, $\beta_{fi}(0)$=5%**

Reasonable M3D-K Simulations Results Obtained with both Analytic and NUBEAM Fast Ion Distributions

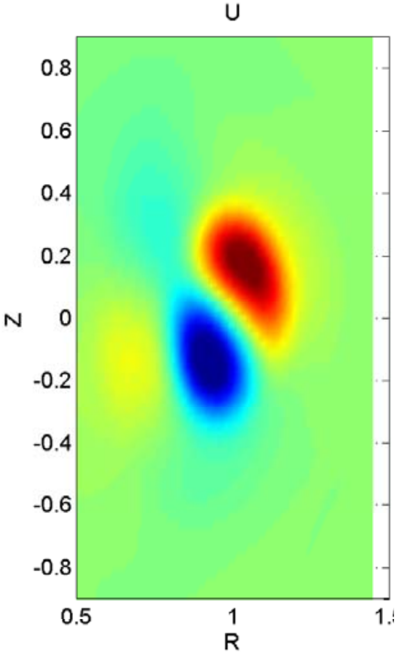
Velocity streaming function U



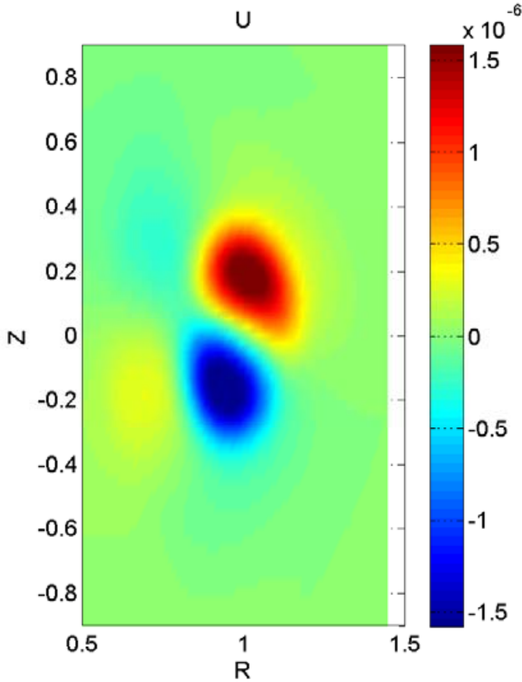
Without fast ions
γ/ω_A=3.04%



Analytic isotropic fast ions
γ/ω_A=2.56%



Analytic anisotropic fast ions
γ/ω_A =2.80%

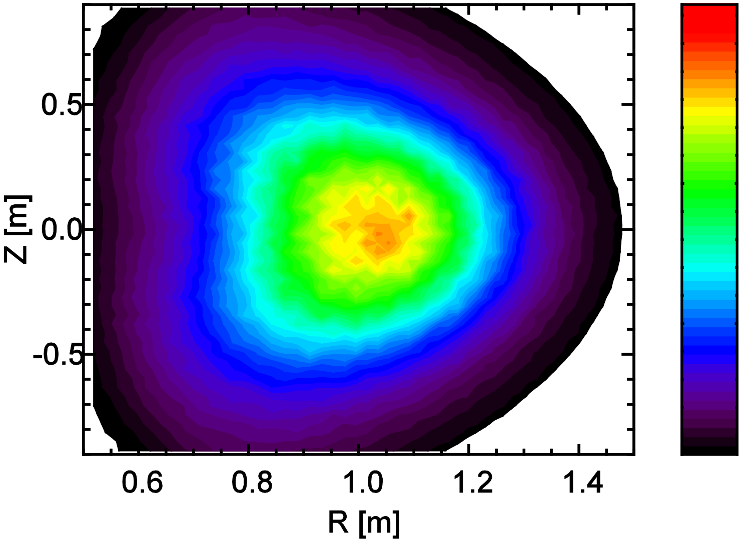


NUBEAM fast ions
γ/ω_A =2.70%

$$f = \frac{cH(v_0 - v)}{v^3 + v_c^3} \exp(-\psi / \Delta\psi) \exp[-(\Lambda - \Lambda_0)^2 / \Delta\Lambda^2], \Lambda = \frac{\mu B}{E}$$

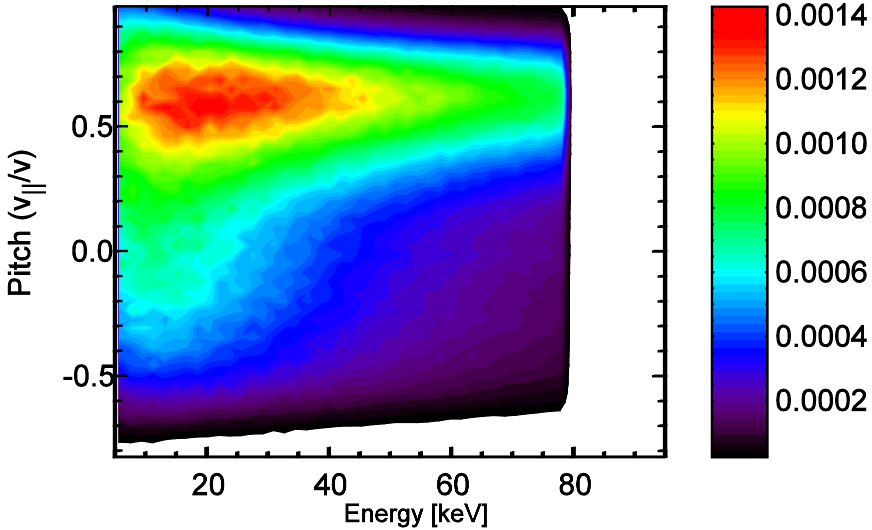
- **$\beta_{thermal}(0)$=24% fixed in all cases, $\beta_{fi}(0)/\ \beta_{tot}(0)$= =5% for the cases with fast ions;**
- **Fast ions weakly stabilize non-resonant kink mode, and induce mode rotation;**
- **Different fast ion distributions can change mode growth rate and mode frequency.**

Initial Analytic & NUBEAM Fast Ion Distributions are Different, and they remain Unchanged during Linear Simulation

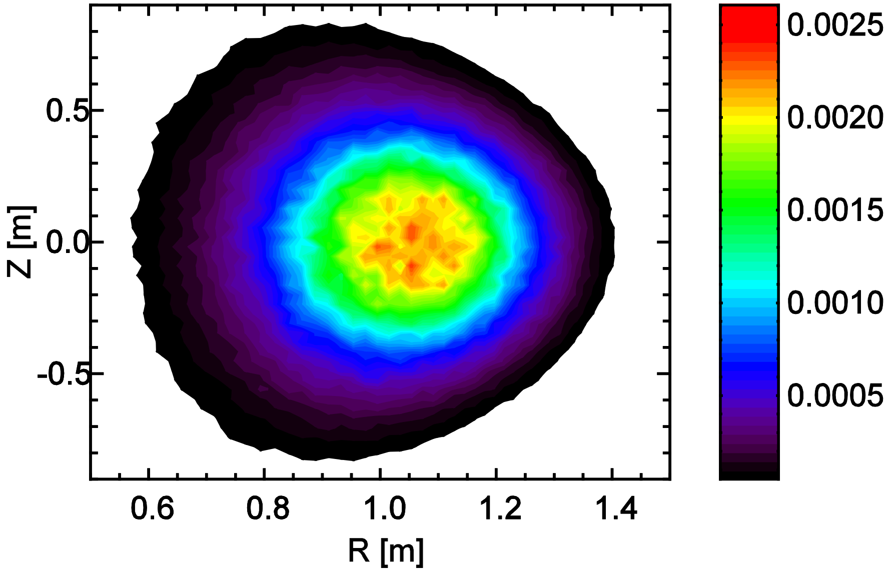


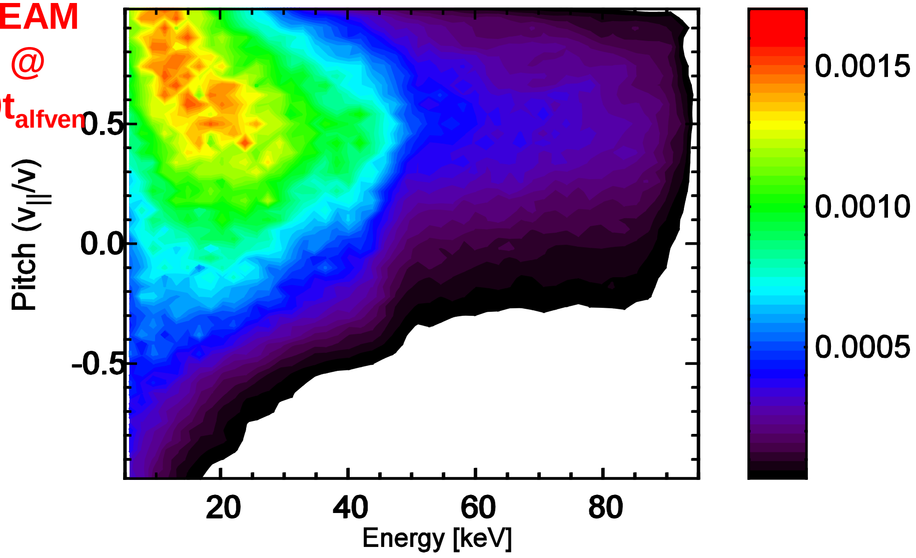
Analytic case @ t=150t$_{alfven}$

Full Energy component, only



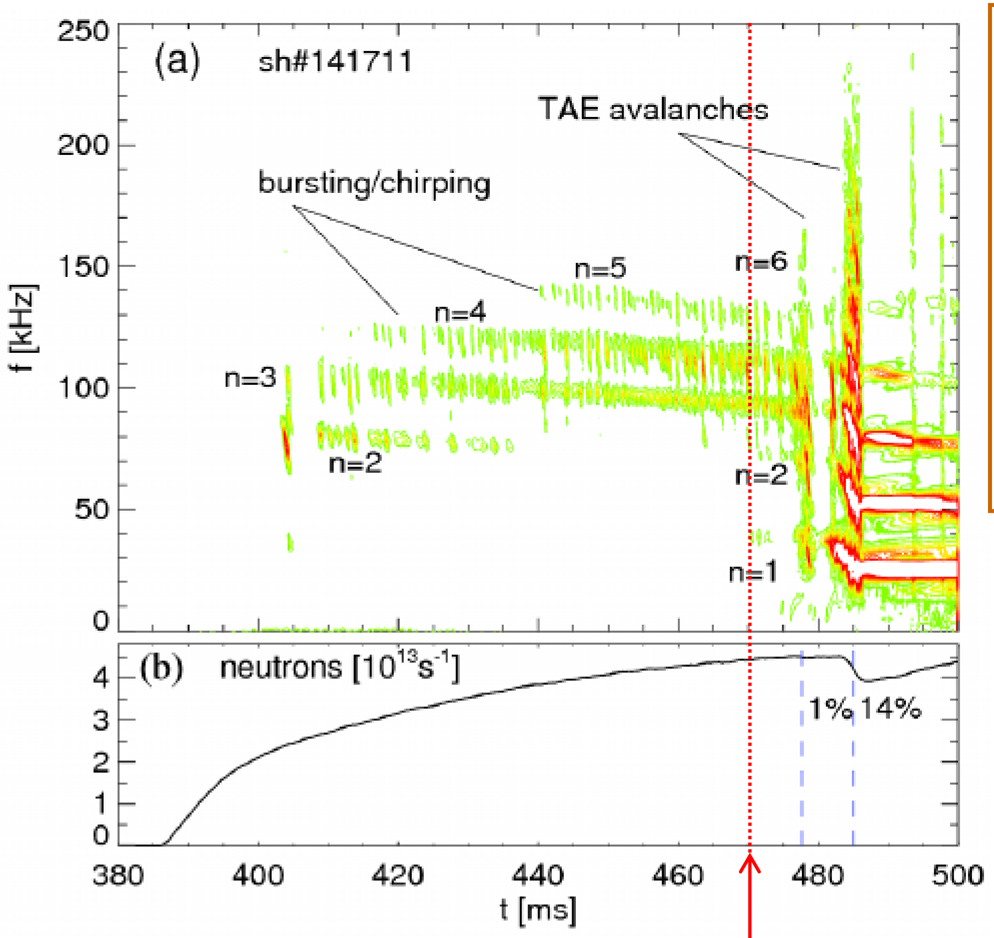
NUBEAM case @ t =80t$_{alfven}$





Similar to original NUBEAM distribution in slide 5

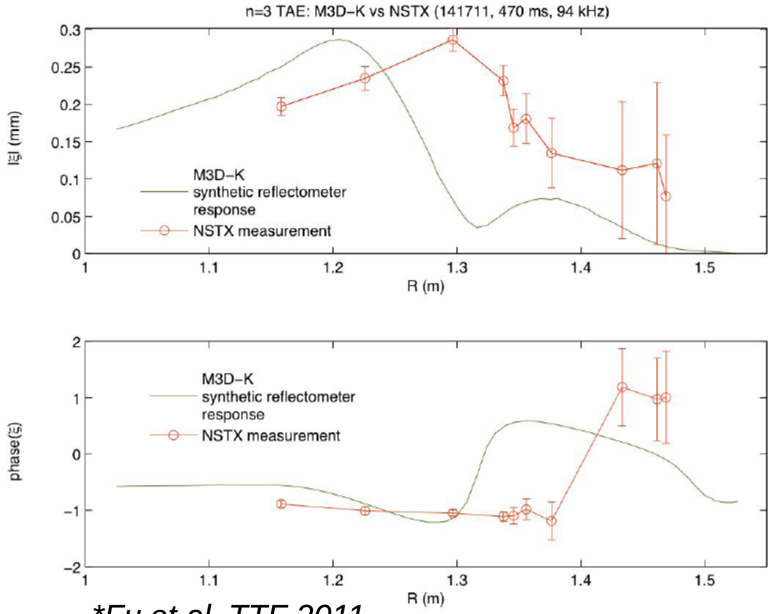
Previous M3D-K Linear Simulations Qualitativity Agree with Experimental Measurements



**at Courtesy of M. Podesta et al*

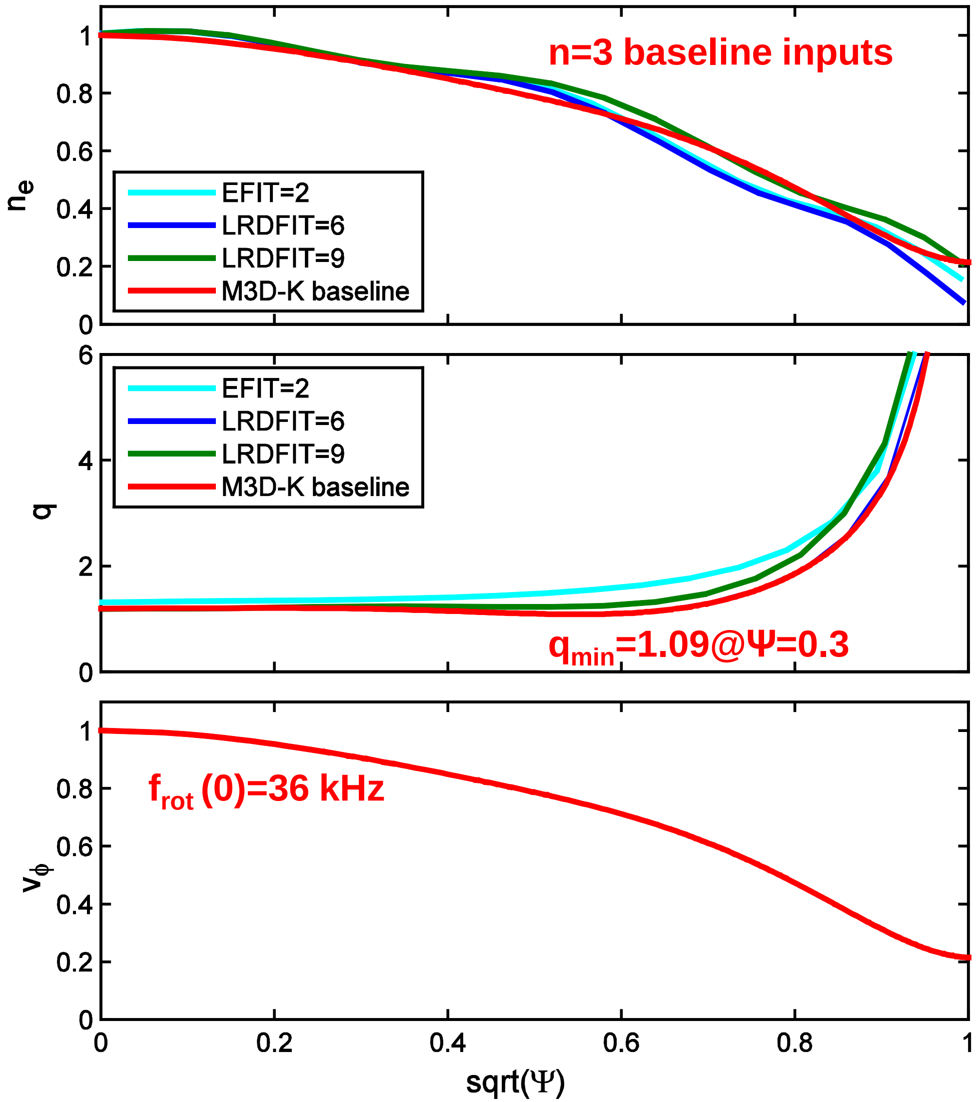
t=470ms

- ➢**Equilibrium and plasma profiles at t=470ms are used as M3D-K inputs.**
- ➢**Previous M3D-K linear simulation results (Fu TTF 2011) are similar to the measurements with respect to mode frequency, core-localized radial structure, and phase shift, but there is a significant radial shift between M3D-K simulations and measurements from multi-channel reflectometer.**



**Fu et al. TTF 2011*

Parameters and Profiles for M3D-K Simulation of n=3 TAE on NSTX



➢NSTX parameters (B=0.55T, R=0.85m, a=0.67m) and equilibrium profiles at 470 ms of shot 141711 are used as M3D –K inputs

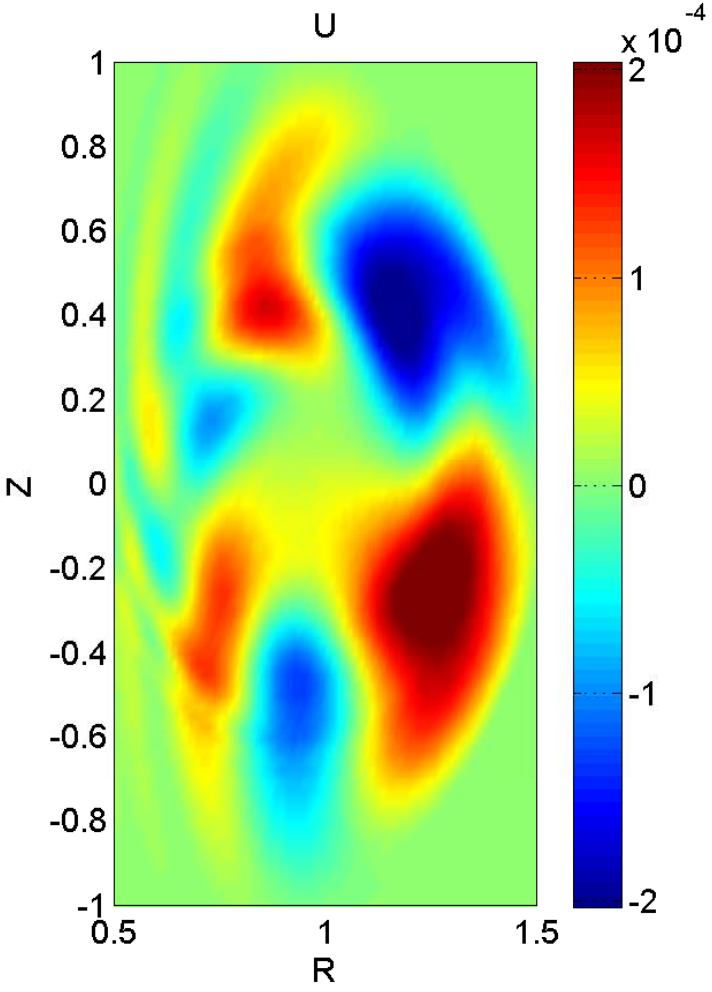
- $n_e(0)=4.4\times 10^{13}$ cm^{-3}
- $T_e(0)=1.4$ keV
- $T_i(0)=1.3$ keV

➢ Self-consistent equilibrium with plasma rotation and fast ion pressure included

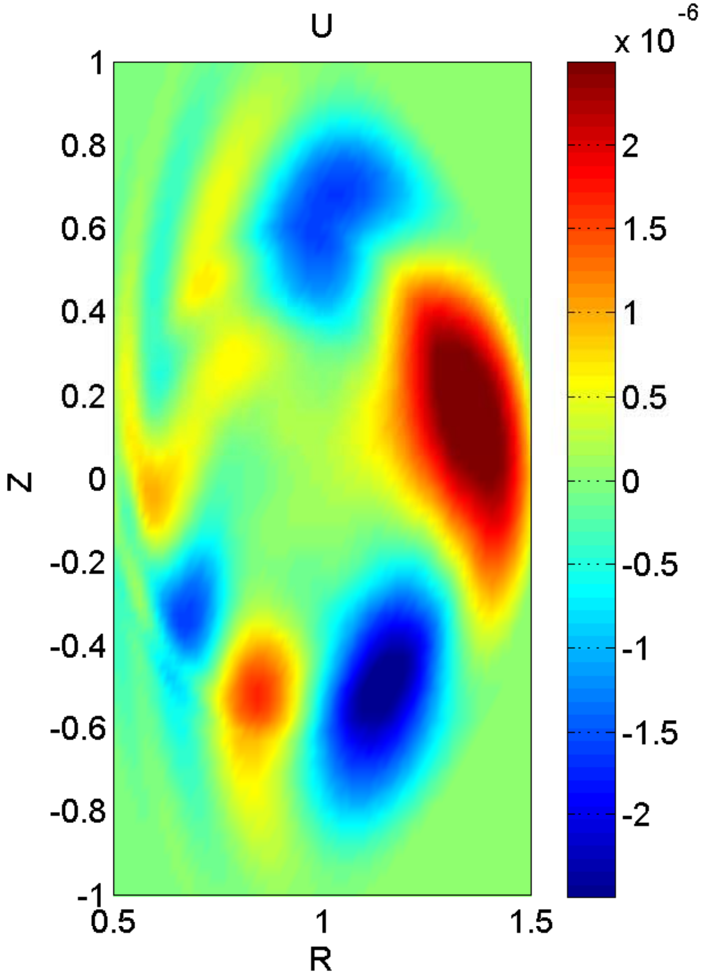
- $\beta_{tot}(0)=24\%$
- $\beta_{fi}(0)/\beta_{tot}(0)=0.67$

$P_{fi\_perpendicular} \neq P_{fi\_parallel}$

Similar Mode Structure Obtained with Analytic and NUBEAM Fast Ion Distributions

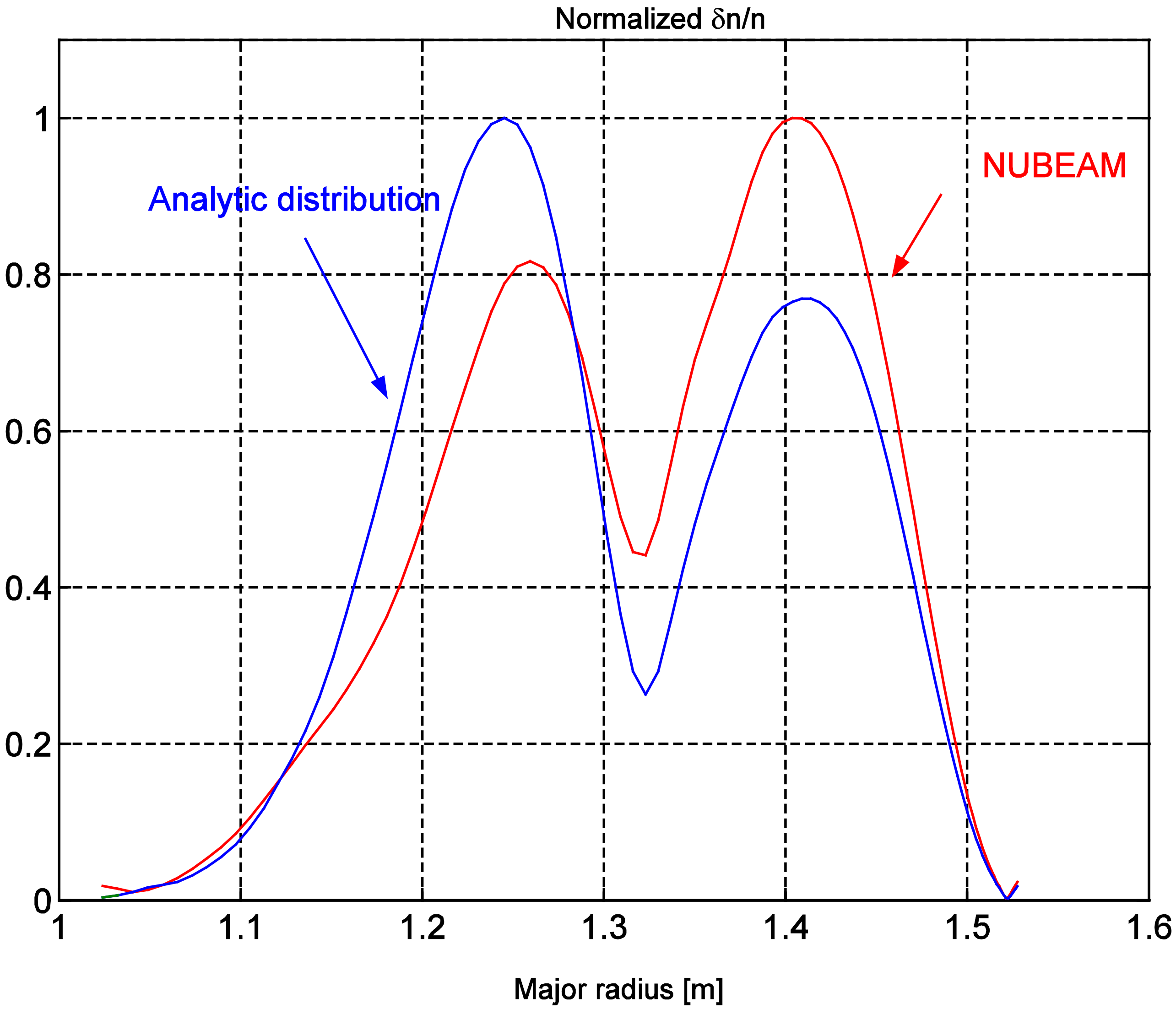


Analytic distribution, pitch(0)=0.64
γ/ω_A =6.60%, f=113kHz



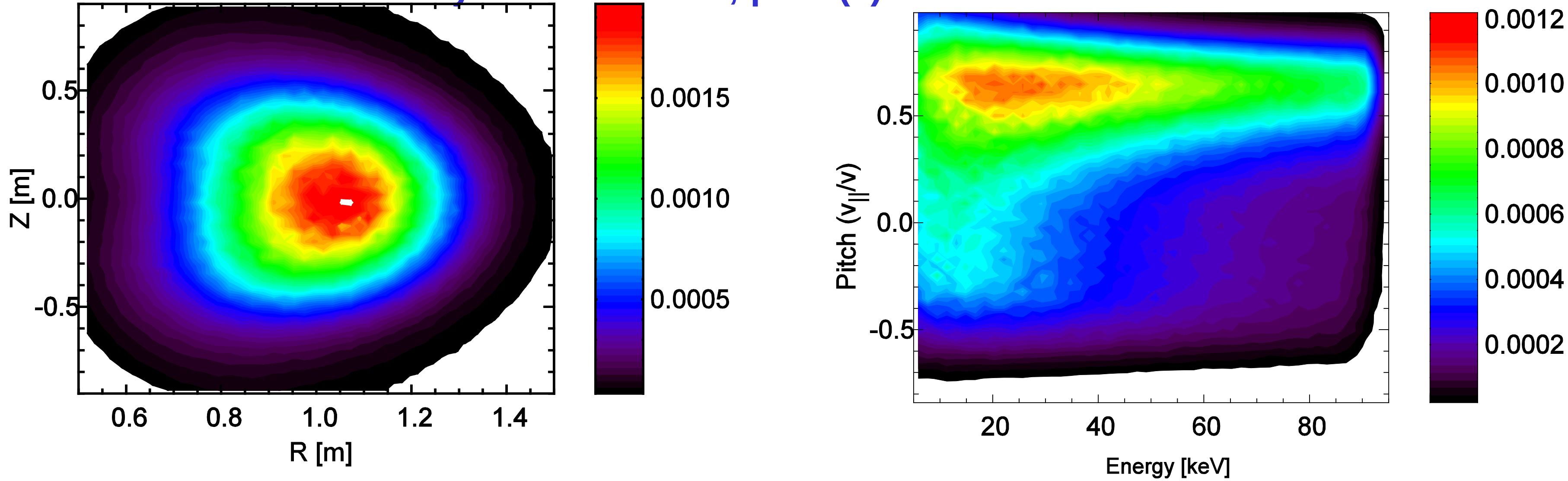
NUBEAM distribution
γ/ω_A =4.7%, f=110kHz

Detailed Mode Structure Affected by Initial Fast Ion Distribution

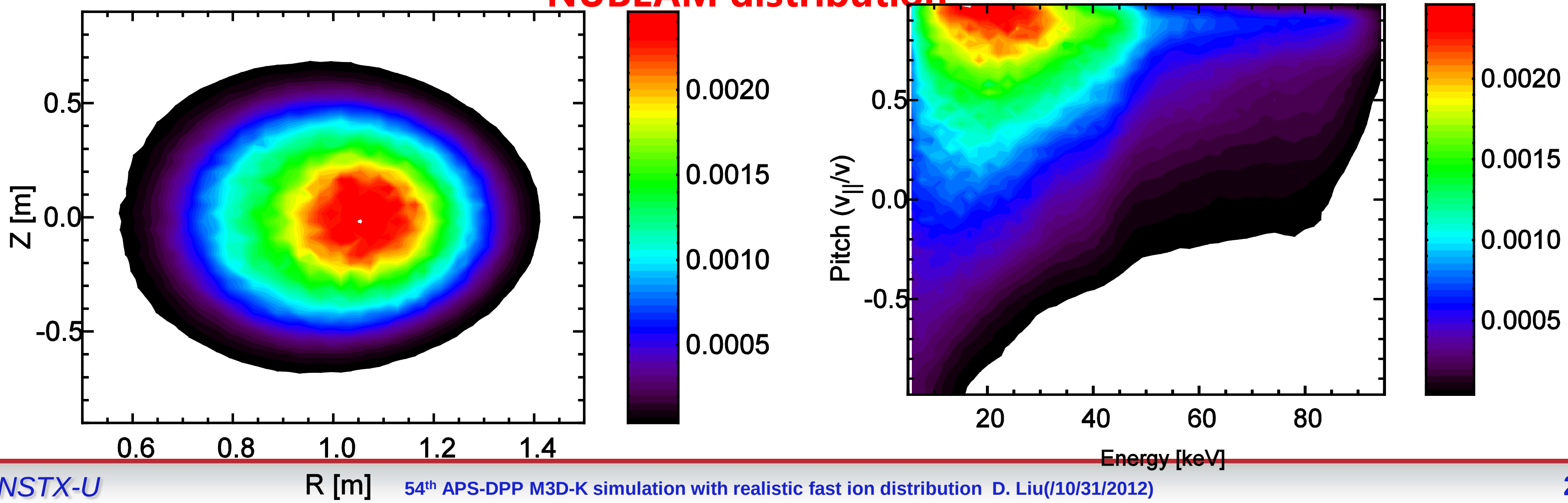


Comparison of Analytic and NUBEAM Fast Ion Distributions

Analytic distribution, pitch(0)=0.64



NUBEAM distribution



Conclusions and Future Plan

- **An interface between NUBEAM output and M3D-K has been developed and tested. More realistic NUBEAM fast ion distribution now can be used as M3D-K inputs to improve simulation accuracy.**

- **M3D-K linear simulation of non-resonant kink mode on NSTX show that fast ions can weakly stabilize the mode. Different fast ion distributions (analytic distribution and NUBEAM fast ion distribution) can affect mode growth rate and frequency.**

- **Preliminary simulation results of n=3 TAE mode with NUBEAM fast ion distribution are similar to previous results with analytic fast ion distribution. But detailed mode structure and frequency are affected by the input NUBEAM fast ion distribution.**

- **Future work**
- **Improve the accuracy of Jacobian calculation near the confined/lost boundary, trapped and passing bounday**
- **Convergence study and plasma parameter scan for n=3 TAE case**
- **Compare mode structure with reflectometer measurements and NOVA-K results**