NSTX-U

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M3D-K Simulations of Toroidicity-induced Alfvén Eigenmodes on NSTX

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D. Liu[1], G. Y. Fu[2], J. A. Breslau[2], E. D. Fredrickson[2], M. Podestà[2] , N. A. Crocker[3] and S. Kubota[3]

[1] *University of California, Irvine*
[2] *Princeton Plasma Physics Laboratory*
[3] *University of California, Los Angeles*

55th Annual Meeting of APS Division of Plasma Physics
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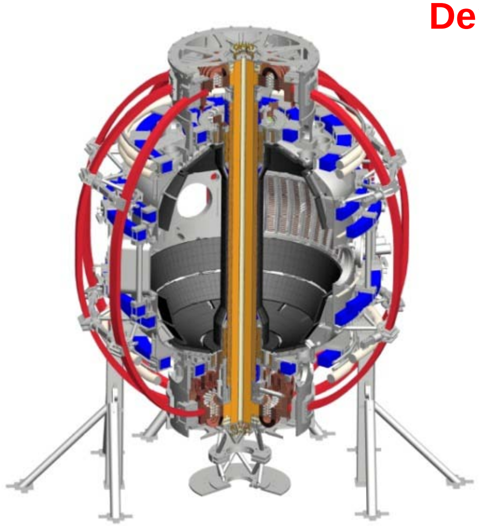
Denver, Colorado



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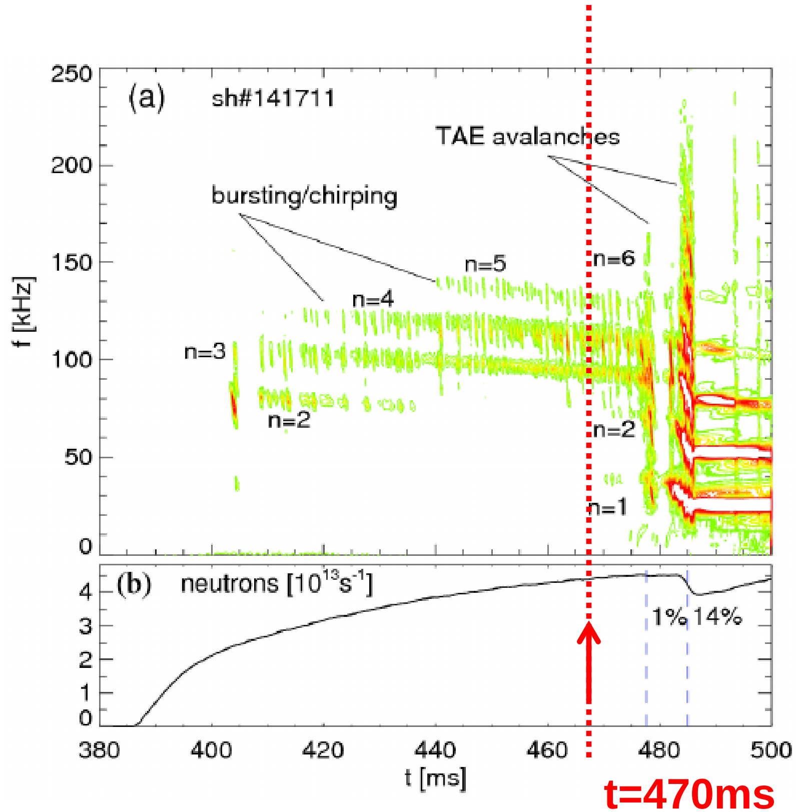
NSTX Provides an Excellent Test-Bed for Fast Ion Study and Code Validation

**Podesta et al. Nucl. Fusion 2012*

- Low field (0.3-0.55T), high density($<10\times10^{19}m^{-3}$)
  NB energy 60-100 keV → $v_{fi}/v_{alfven} \approx 1-5$
- TAEs: routinely observed in NSTX; cause fast ion redistribution/loss.
- In this work,
  - Carried out linear simulations of beam-ion-driven TAE and compared with measurements
  - Explored the effects of rotation, q profile change
  - Developed an interface to use realistic fast ion distribution from NUBEAM output



Start with linear simulation at t=470ms for code validation

M3D-K: Global Kinetic/MHD Hybrid Code

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla P - \nabla \cdot P_f + \mathbf{J} \times \mathbf{B}$$

**Fu et al. Phys. Plasmas 2006*

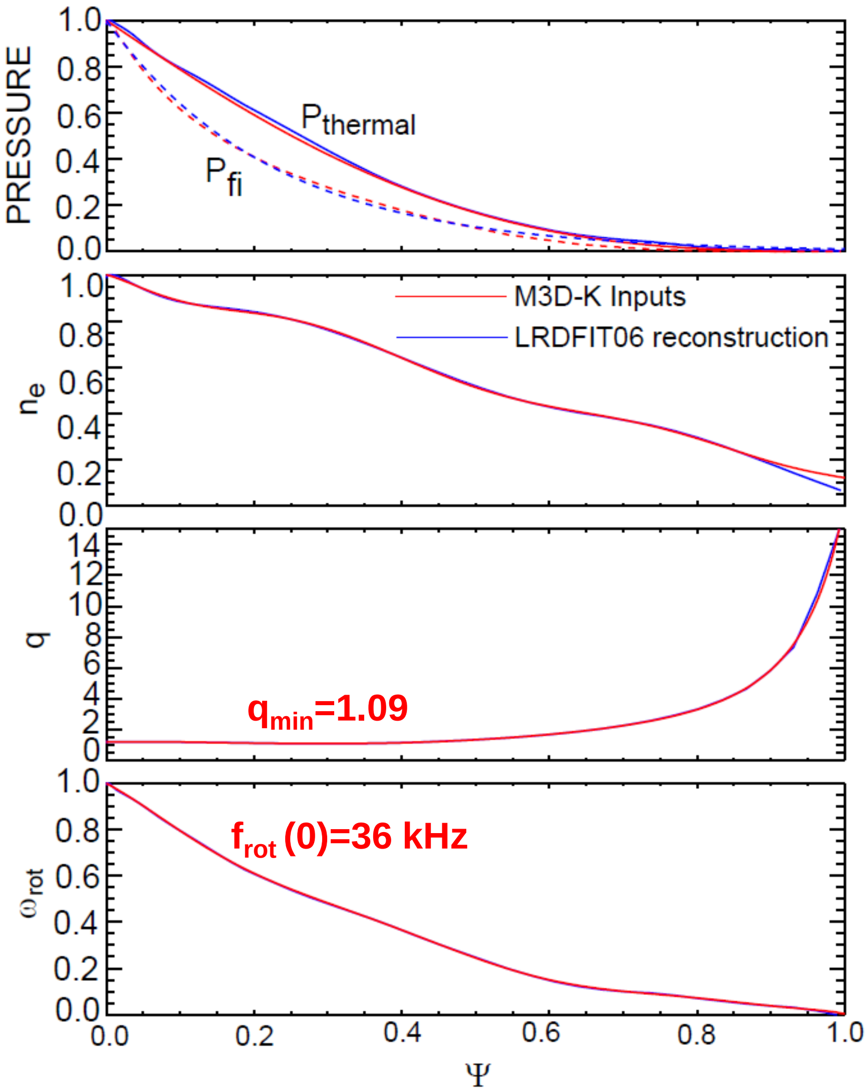
- Global
- Single fluid MHD equations
- Hybrid (kinetic fast ions)
- Non-perturbative fast ions effects

In this work, we use

- Realistic geometry, experimental parameters and profiles
- Plasma rotation & anisotropic fast ion pressure included
- Analytical slowing-down fast ion distribution

$$f = \frac{cH(v_0 - v)}{v^3 + v_c^3} \exp(-\psi / \Delta\psi) \exp[-(\Lambda - \Lambda_0)^2 / \Delta\Lambda^2], \Lambda = \frac{\mu B}{E}$$

Experimental Plasma Parameters and Profiles are Used for Inputs of TAE Simulation



➢ NSTX parameters and profiles

- B_0 =0.55T, R=0.85m, a=0.67m
- $n_e(0)$=4.4x 10^{13} cm^{-3}
- $T_e(0)$=1.4 keV, $T_i(0)$=1.3 keV

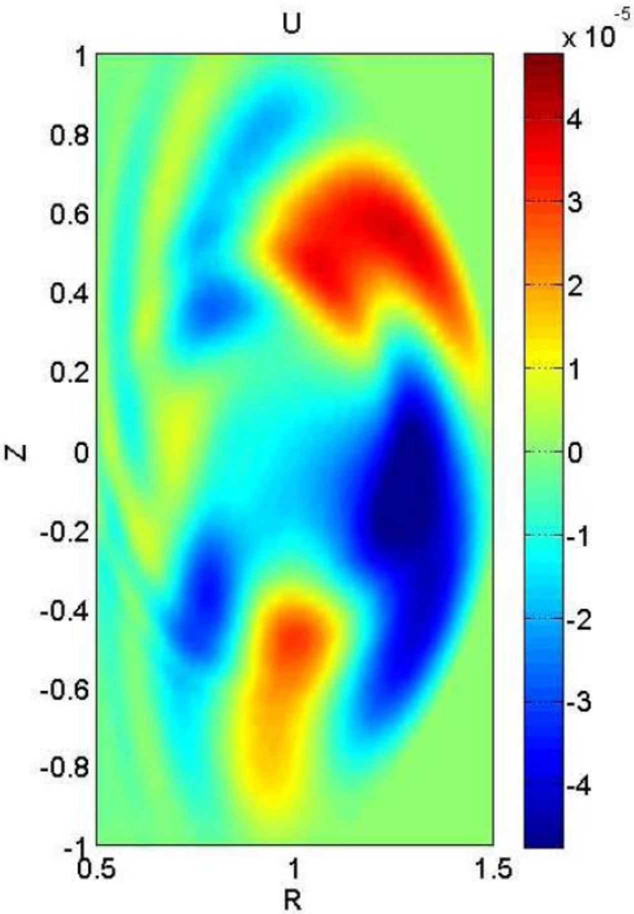
- v_{fi} / v_{alfven}=2.5
- $\beta_{tot}(0)$=18.4%, $\beta_{fi}(0)$=6.5%

•Analytic slowing down distribution

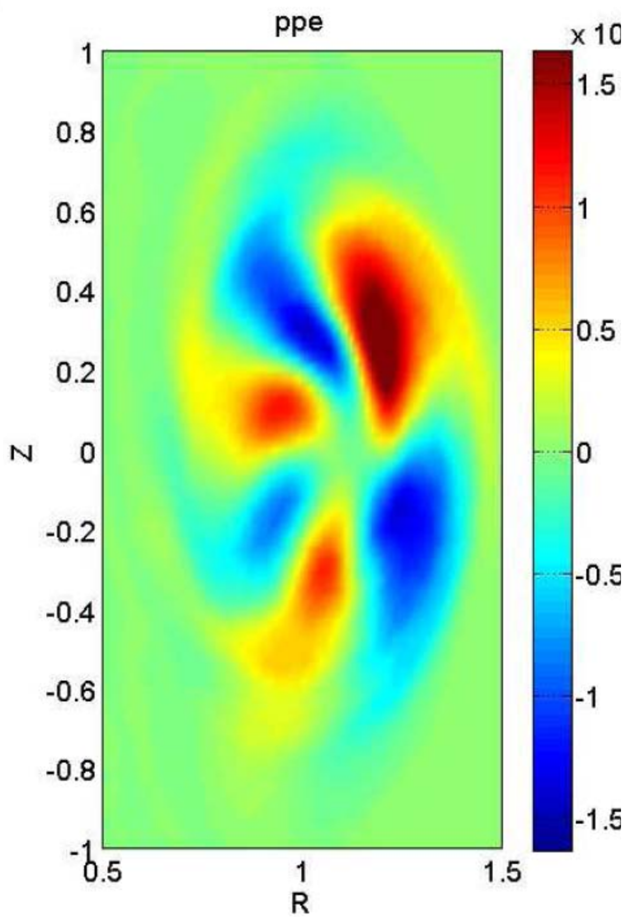
➢ Self-consistent equilibrium with rotation and anisotropic fast ion pressure

n=3 Linear Simulation Exhibits Ballooning Feature

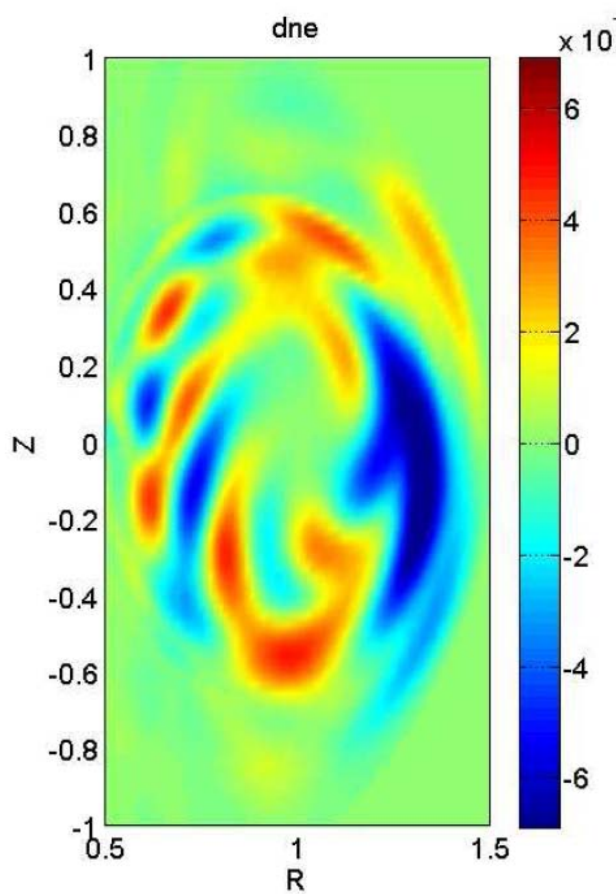
Streaming Function U



Pressure Perturbation $P_{//}$



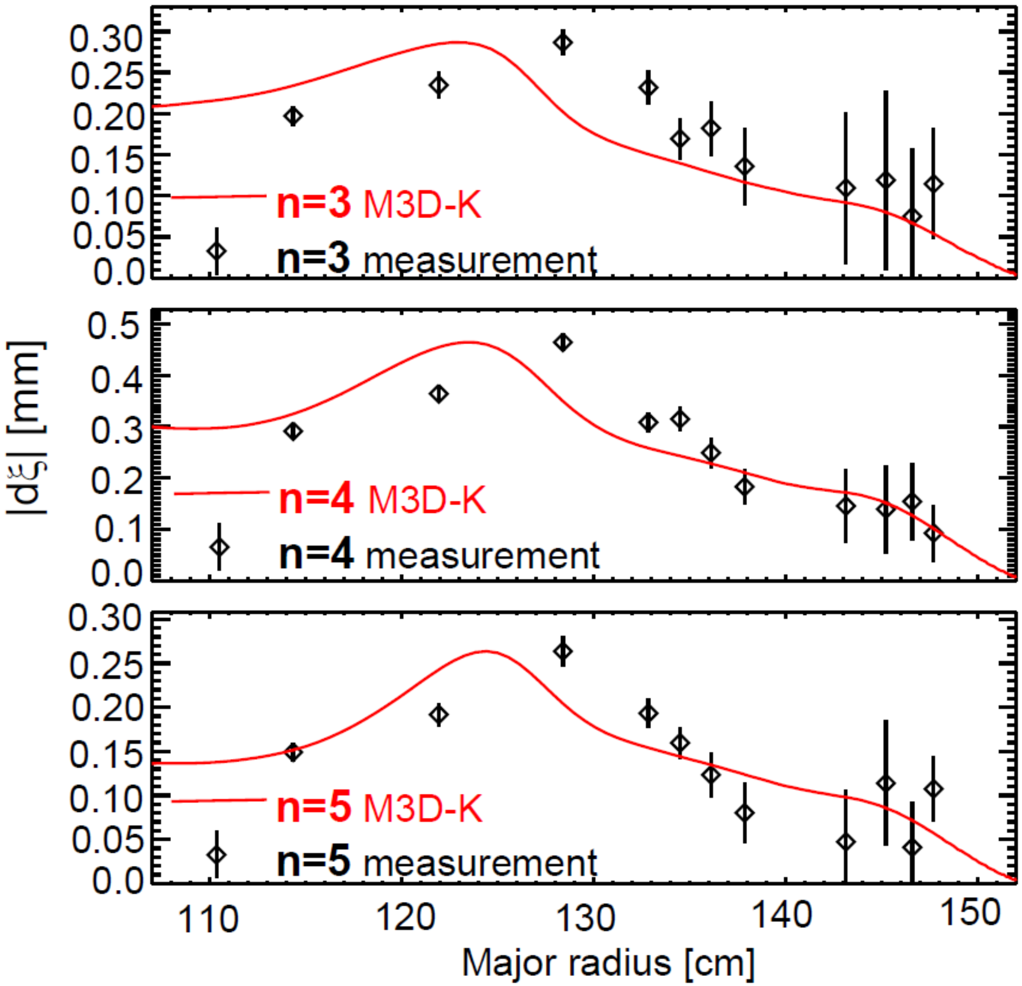
Density Perturbation δn



Structure + Frequency → TAE

- ➢n=4 and n=5 linear simulations also show TAE-like ballooning structure

Mode Structure and Mode Frequency of Simulated n=3,4,5 TAE are Similar to Experimental Measurements



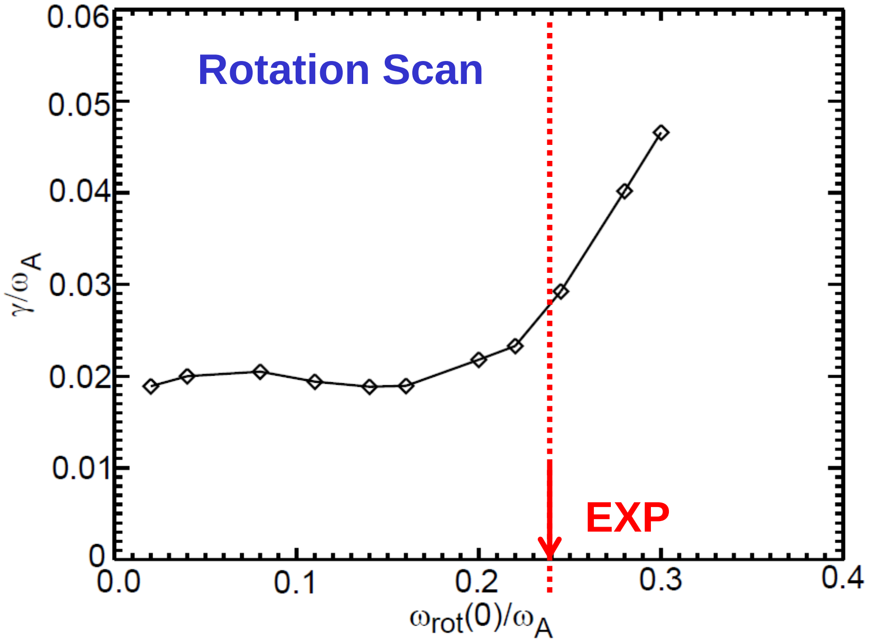
Black: reflectometer measurements

Red: M3D-K synthetic reflectometer signal

	n=3	n=4	n=5
f_{exp} (kHz)	**100**	**120**	**140**
f_{M3D-K} (kHz)	**106**	**130**	**149**

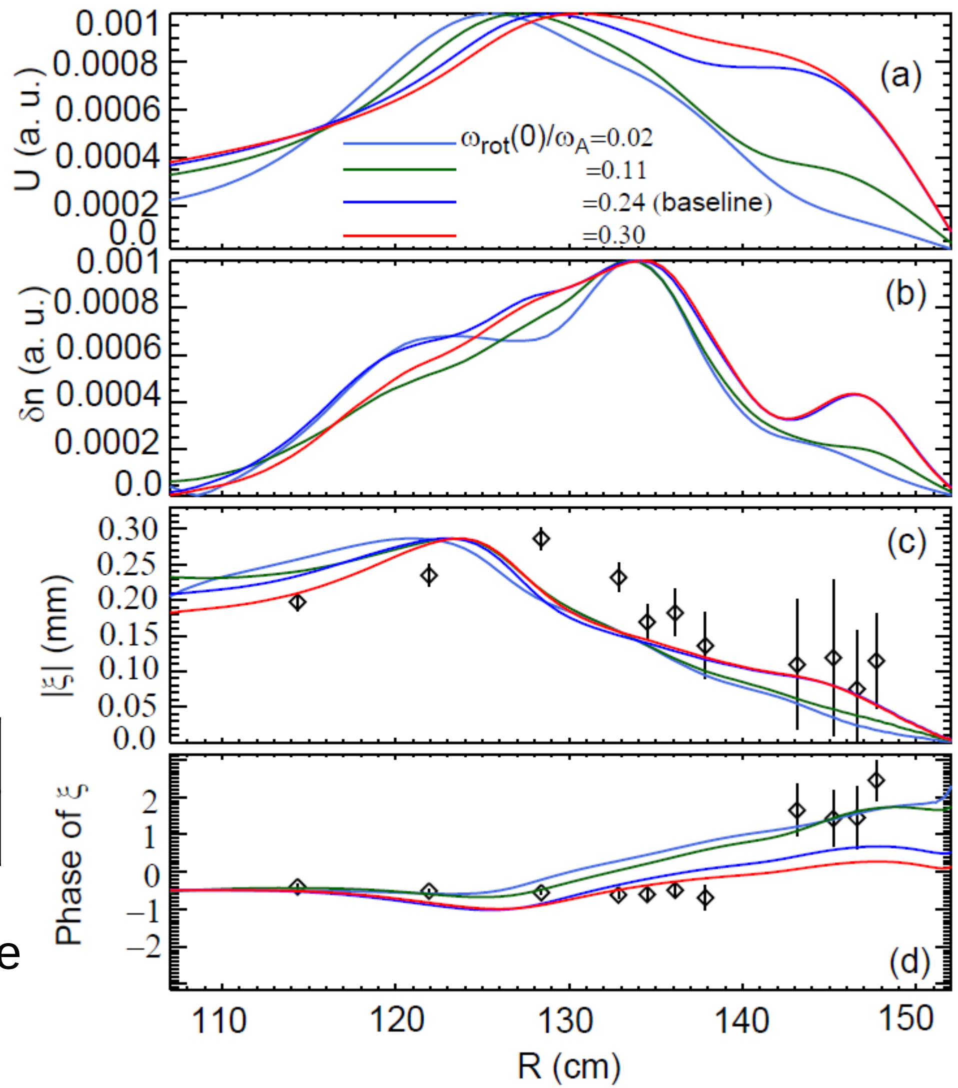
- Reflectometer response (ξ) modeled for M3D-K δn with WKB approximation

Strong Rotation Affects Mode Stability



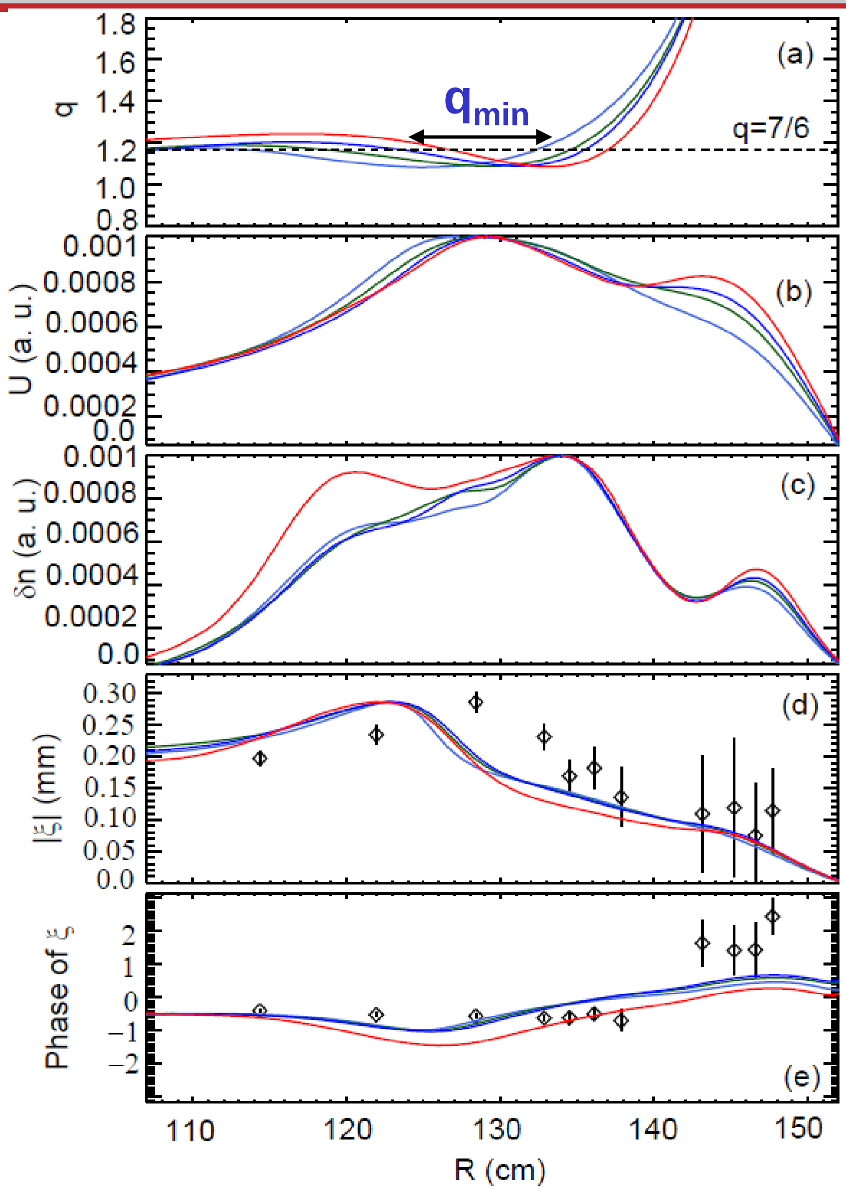
f_{exp} (kHz)=100kHZ

Rotation	0.02	0.11	**0.24**	0.30
$f_{M3D\text{-}K}$(kHz)	61	80	**106**	117



- Strong rotation can destabilize the mode
- Rotation affects the phase variation
- Reasons under investigation

Mode Growth Rate is Sensitive to q Profile, but Mode Structure and Frequency are Less Affected



	γ/ω_A (%)	Frequency (kHz)
Ψ at q_{min}=0.19	6.1	106
Ψ at q_{min}=0.26	4.7	106
Ψ at q_{min}=0.30 (baseline)	**2.9**	**106**
Ψ at q_{min}=0.35	1.4	106

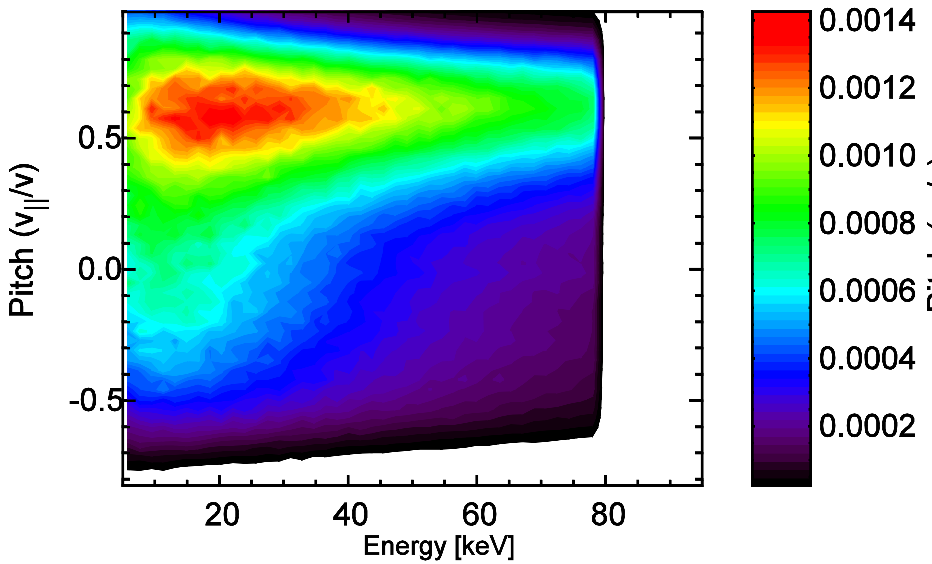
f_{exp} (kHz)=100kHZ

- Although U shifts as q=7/6 surface moves, but synthetic reflectometer response is less sensitive

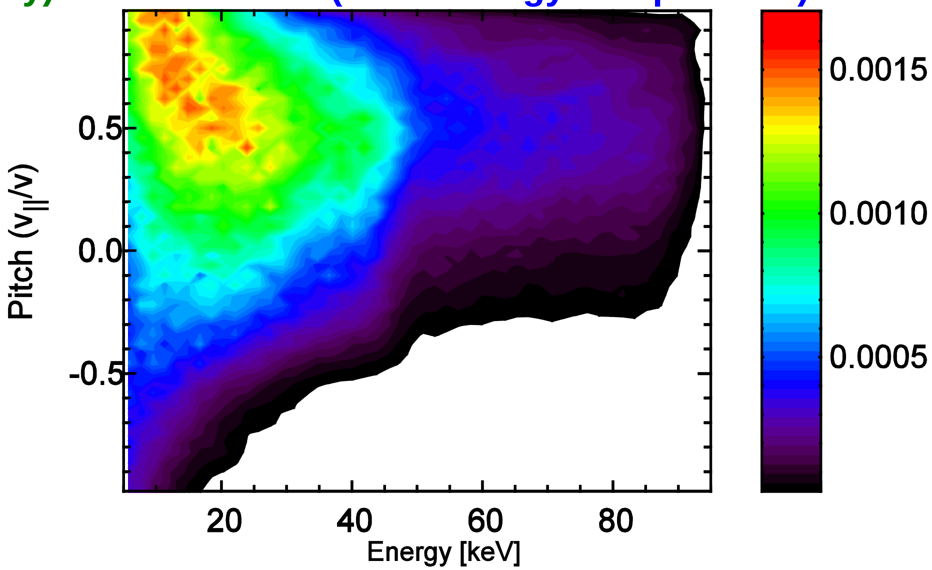
Initialization with Realistic Fast Ion Distributions is needed for Accurate simulations

➢Analytic form for fast ion distribution may not be a good fit to the actual distribution.

Analytic anisotropic (full energy component only)



NUBEAM (three energy components)



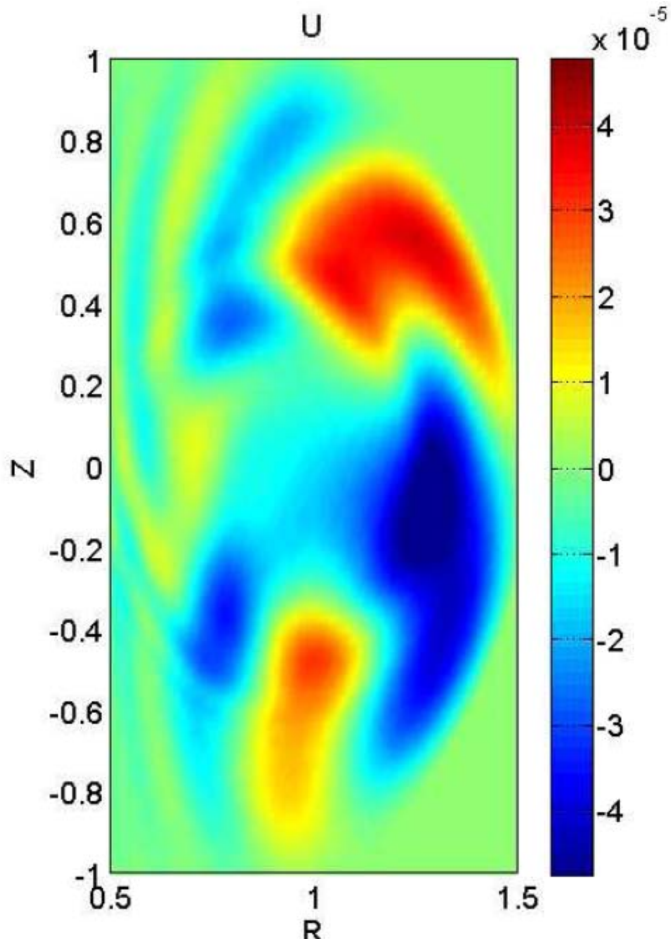
➢NUBEAM gives more realistic and classical fast ion distribution $F(R,Z, \lambda=v_{||}/v, E)$

➢Hybrid simulation codes need fast ion distribution in (P_φ, μ, E) and the function must be smooth enough to allow derivatives to be taken with E and P_φ

➢Convert fast ion distribution from $F(R,Z, \lambda=v_{||}/v, E)$ to $f(P_\varphi, \mu, E)$

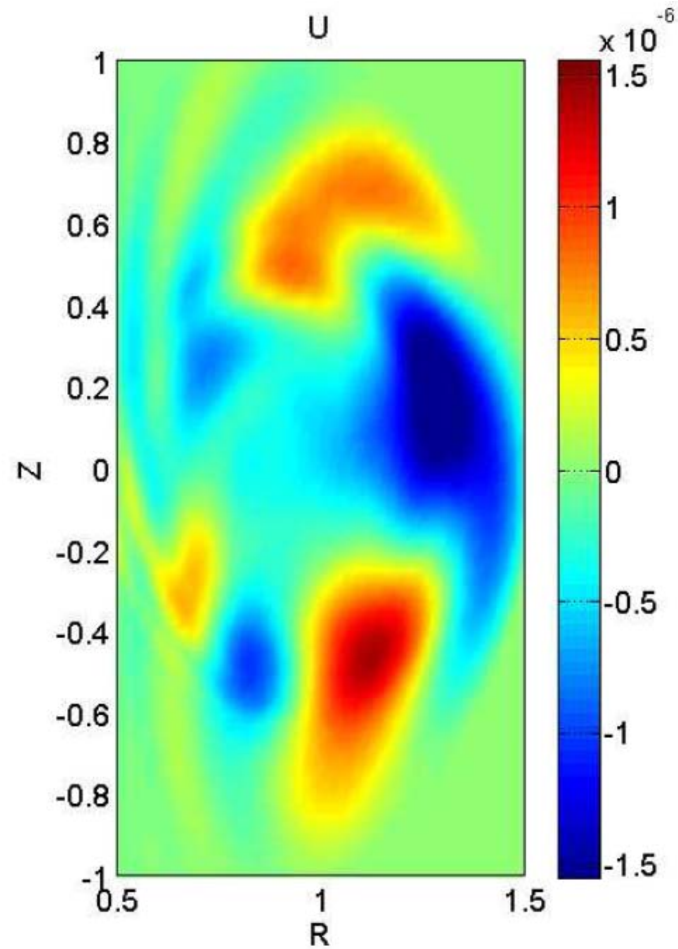
$$\int F(R,Z,\lambda,E)R\sqrt{E}\,dRdZd\lambda dE = \sum_{v_{sign}}\int f_{v_{sign}}(P_\phi,\mu,E)\Im_{v_{sign}}(P_\phi,\mu,E)dP_\phi d\mu dE$$

Mode Structure are Similar for the Runs with Analytic and NUBEAM Fast Ion Distributions



$$\Lambda_0 = \frac{\mu B}{E} = 0.6, \Delta\Lambda = 0.30$$

Analytic anisotropic f_0
γ/ω_A =3.1%, f=106kHz



NUBEAM f_0
γ/ω_A =3.2%, f=99kHz

Summary

- Both the mode structure and frequency of n=3,4,5 TAEs from linear M3D-K simulations are consistent with experimental measurements.

- A sensitivity study shows that mode structure and frequency are relatively insensitive to q profile variation (within experimental error), but mode growth depends strongly on rotation and q profile.

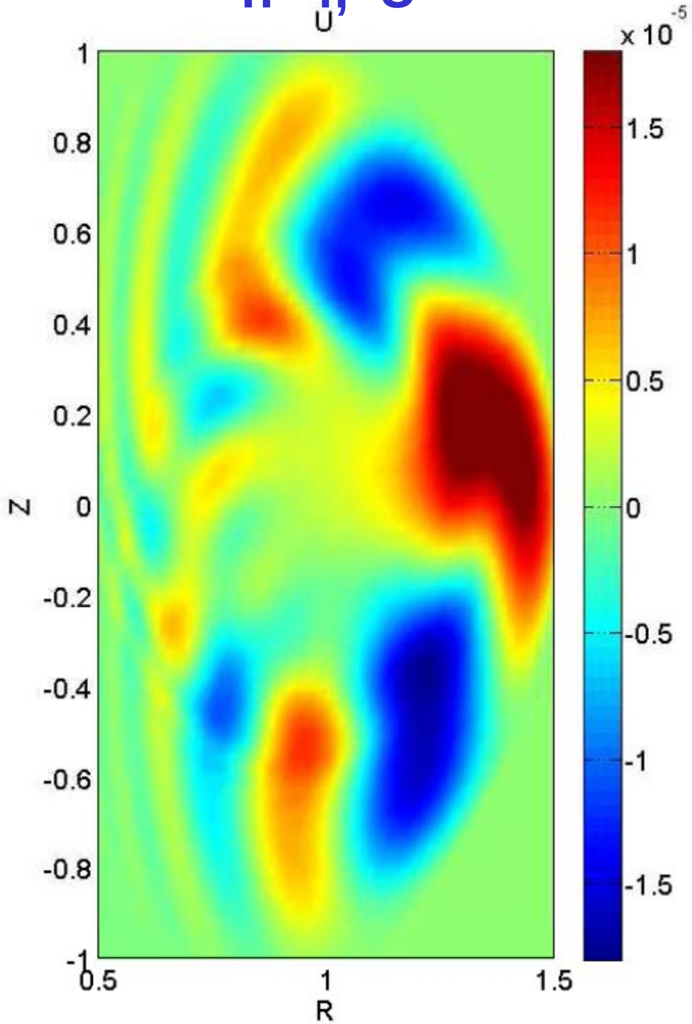
- Realistic NUBEAM fast ion distribution now can be used as inputs of M3D-K or other kinetic simulation codes.

- Future work
  - Nonlinear simulations of multiple TAEs and mode avalanches
  - Consider kinetic effects of thermal plasmas

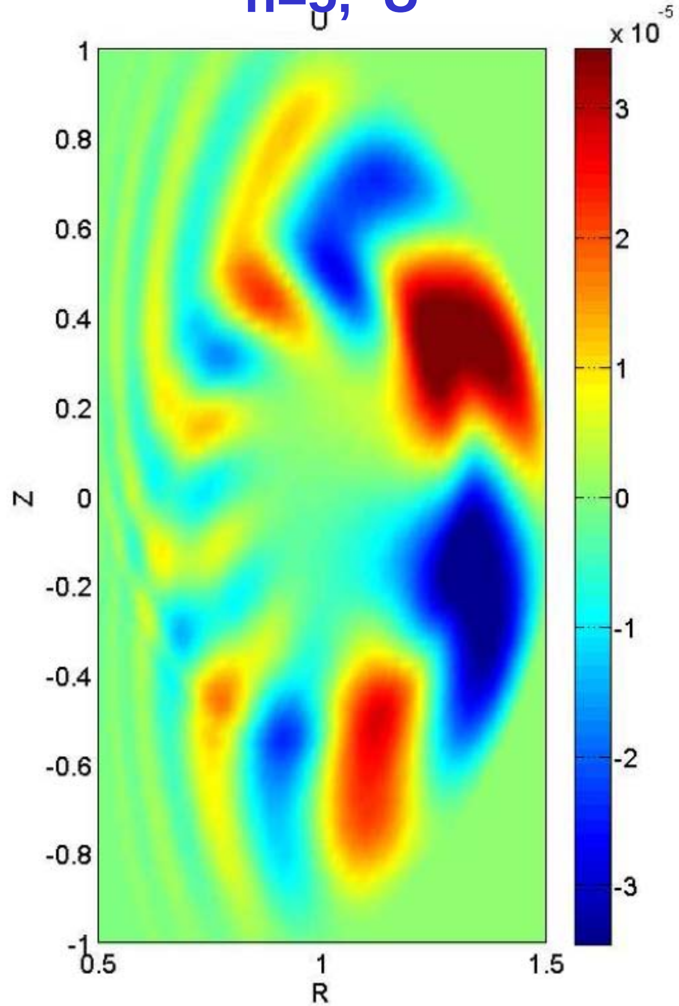
Backup Slides

n=4,5 Simulations also Exhibit TAE-like Mode Structure

n=4, U

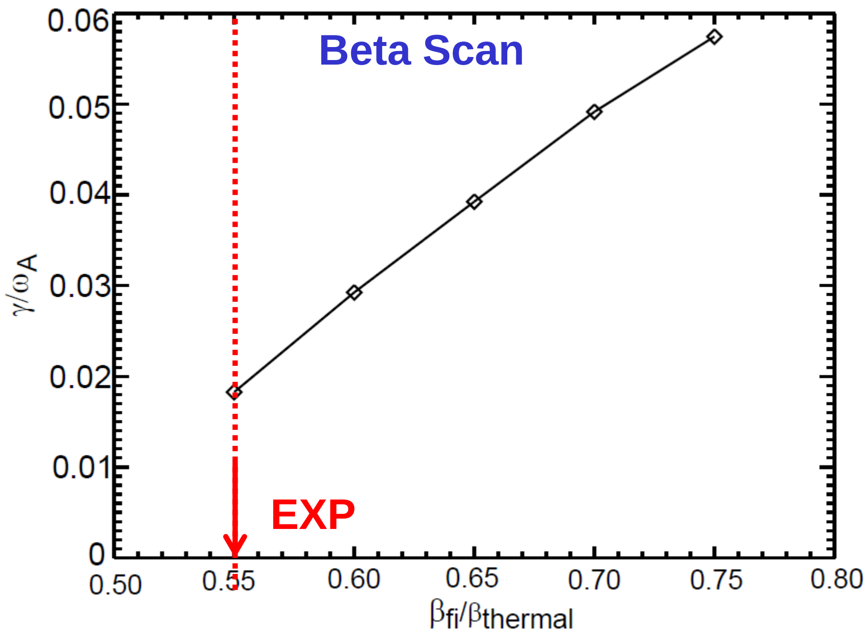


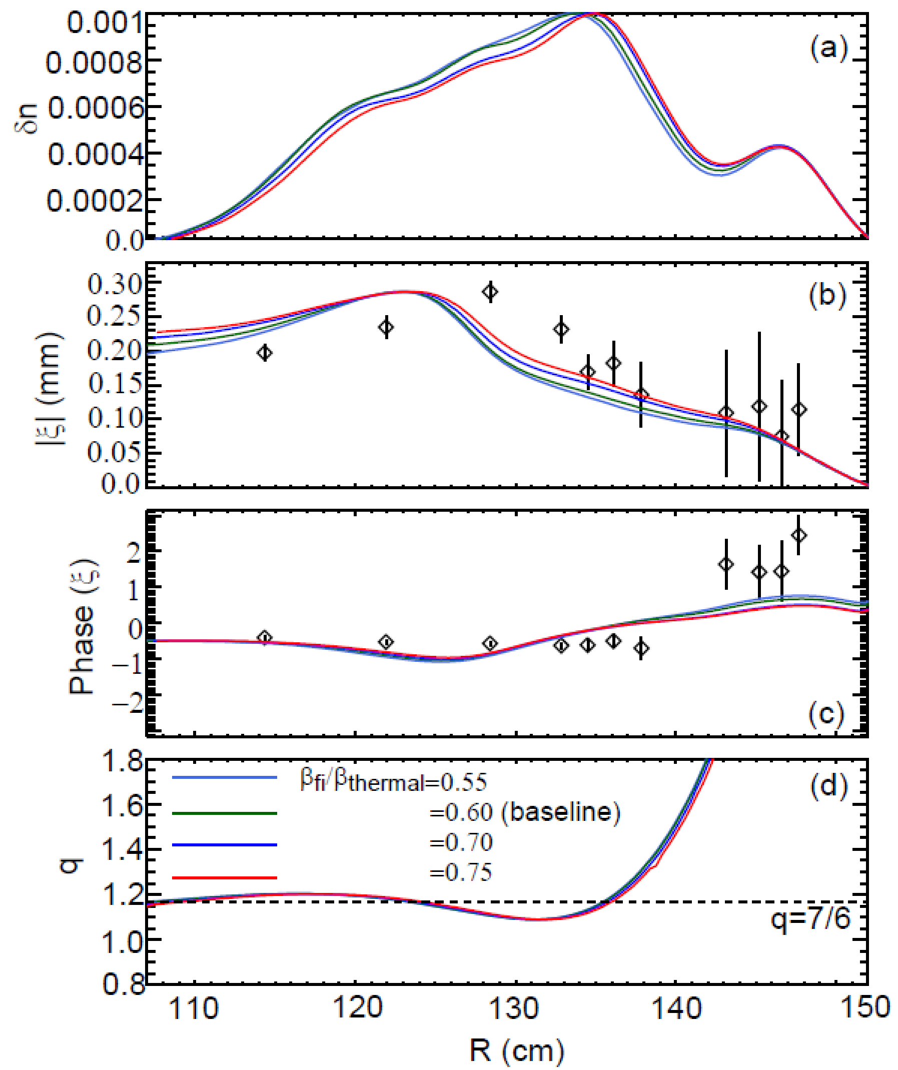
n=5, U



* Single mode linear simulation

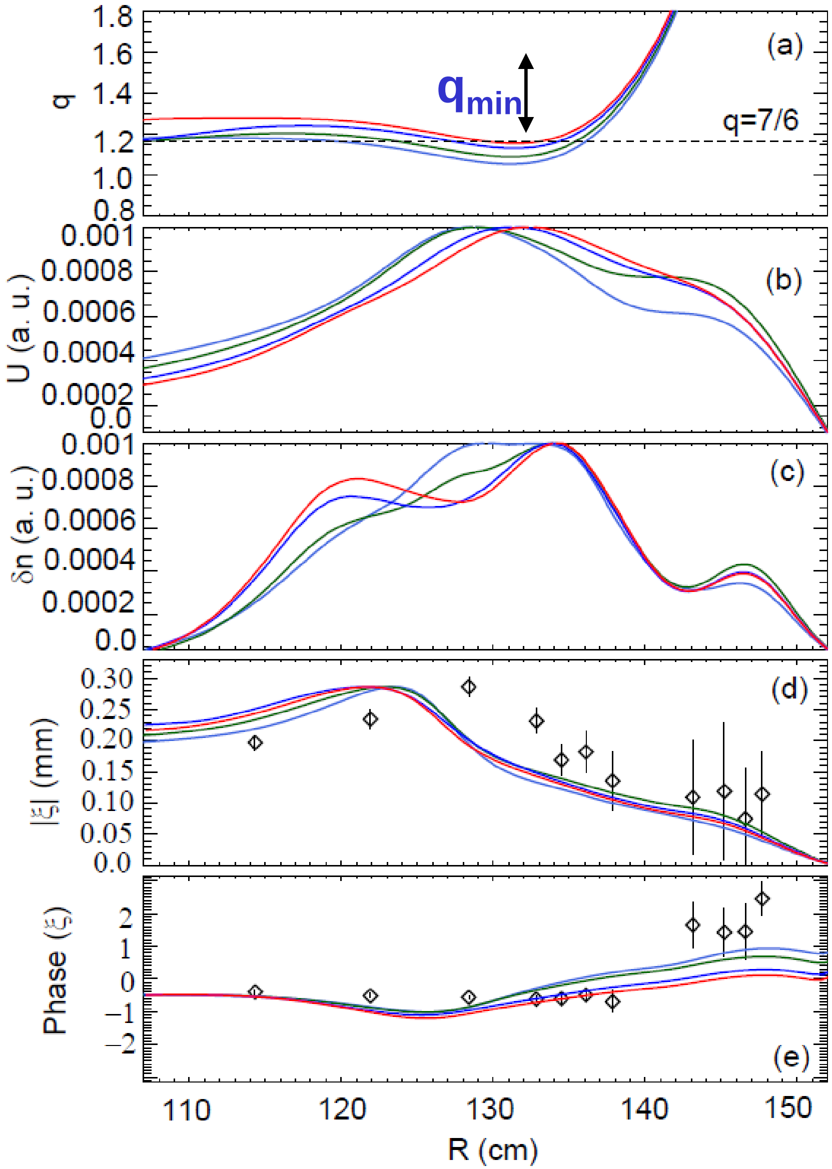
Fast Ion Beta has Weak Effect on Mode Structure





- $\beta_{thermal}$=11.9% fixed
- Mode growth rate increases linearly with driving term, as expected

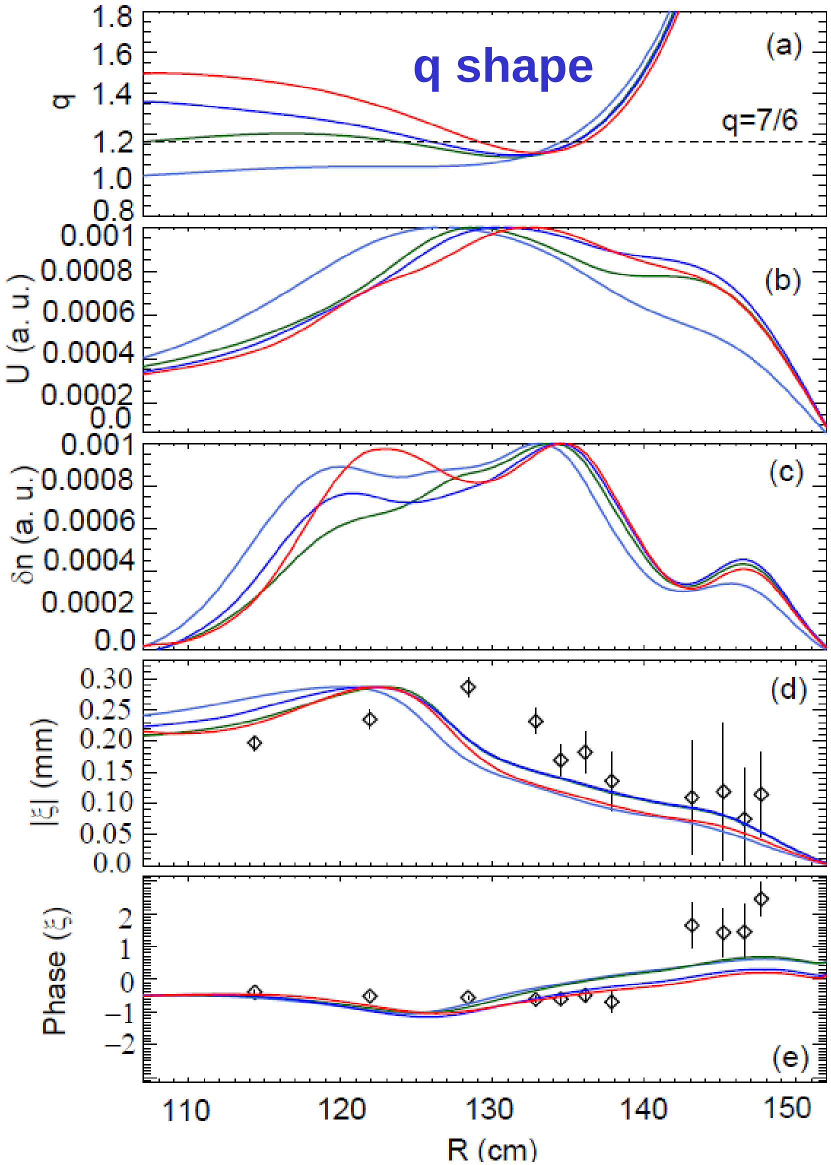
q_{min} Value Scan: Mode Growth Rate, Mode Structure and Frequency are Weakly Affected



	γ/ω_A (%)	Frequency (kHz)
q_{min}=1.02 at Ψ=0.30	3.5	100
q_{min}=1.09 at Ψ=0.30 (baseline)	**2.9**	**106**
q_{min}=1.13 at Ψ=0.30	3.4	107
q_{min}=1.15 at Ψ=0.30	3.8	106

f_{exp} (kHz)=100kHZ

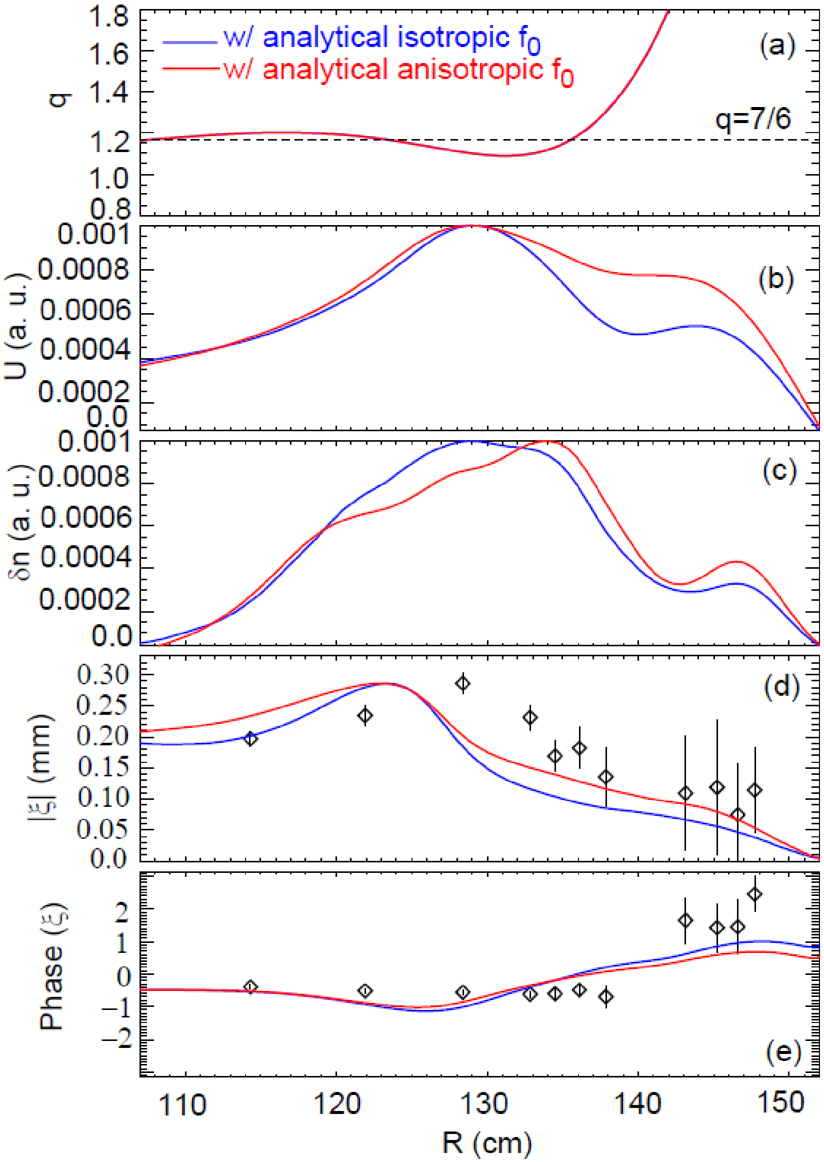
q Shape Scan: Mode Growth Rate is Sensitive to q Profile, but Mode Structure and Frequency are Less Affected



	γ/ω_A (%)	Frequency (kHz)
q_0=1.02	6.9	99
$q_{0=}$=1.09 (baseline)	**2.9**	**106**
q_0=1.32	3.1	108
q_0=1.50	2.4	109

f_{exp} (kHz)=100kHZ

Initial Fast Ion Distribution Weakly Affects Mode Structure and Mode Frequency



	γ/ω_A (%)	Frequency (kHz)
Isotropic f_0	1.9	101
Anisotropic f_0	3.1	106

f_{exp} (kHz)=100kHZ

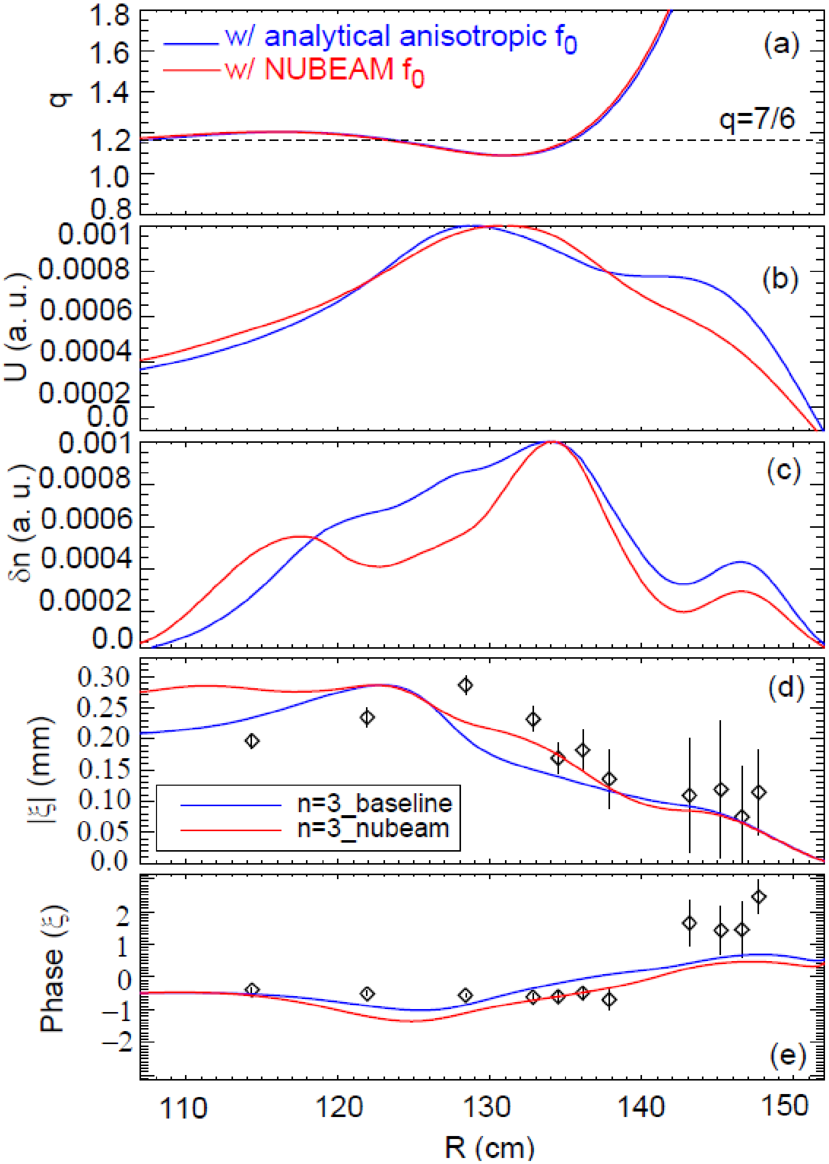
- Analytical slowing-down distribution

$$f = \frac{cH(v_0 - v)}{v^3 + v_c^3} \exp(-\psi / \Delta\psi) \exp[-(\Lambda - \Lambda_0)^2 / \Delta\Lambda^2], \Lambda = \frac{\mu B}{E}$$

Isotropic $\Delta\Lambda = 10$

Anisotropic $\Lambda_0 = \frac{\mu B}{E} = 0.6, \Delta\Lambda = 0.30$

Mode Structure and Mode Frequency are Similar for the Runs with Analytic and NUBEAM Fast Ion Distributions



	γ/ω_A (%)	Frequency (kHz)
Anisotropic f_0 (baseline)	3.1	106
NUBEAM f_0	3.2	99

f_{exp} (kHz)=100kHZ

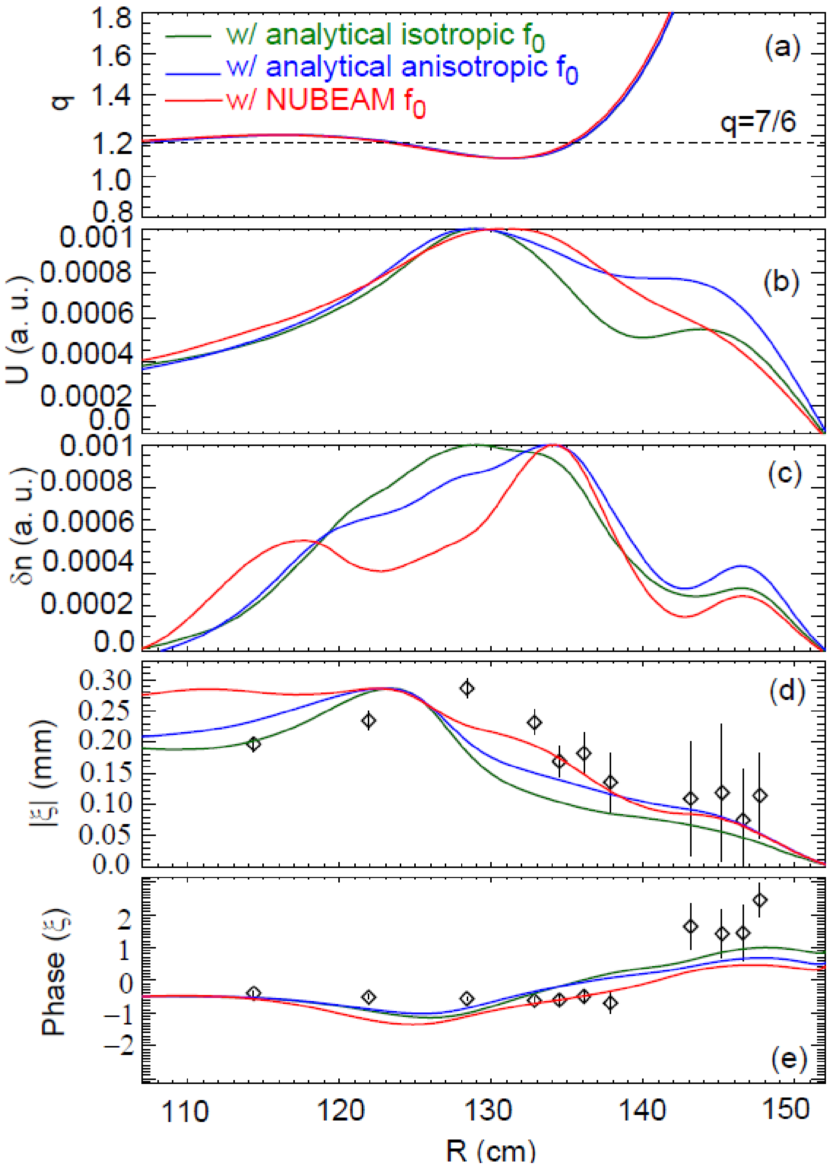
➢Analytical slowing-down f_0

$$f = \frac{cH(v_0 - v)}{v^3 + v_c^3} \exp(-\psi / \Delta\psi) \exp[-(\Lambda - \Lambda_0)^2 / \Delta\Lambda^2], \Lambda = \frac{\mu B}{E}$$

$$\Lambda_0 = \frac{\mu B}{E} = 0.6, \Delta\Lambda = 0.30$$

➢NUBEAM f_0 :
TRANSP run with experimental plasma profiles

Initial Fast Ion Distribution Affects Mode Structure and Mode Stability



	γ/ω_A (%)	Frequency (kHz)
Isotropic f_0	1.9	101
Anisotropic f_0 (baseline)	**3.1**	**106**
NUBEAM f_0	3.2	99

f_{exp} (kHz)=100-90kHZ

➢Analytical slowing-down fast ion distribution

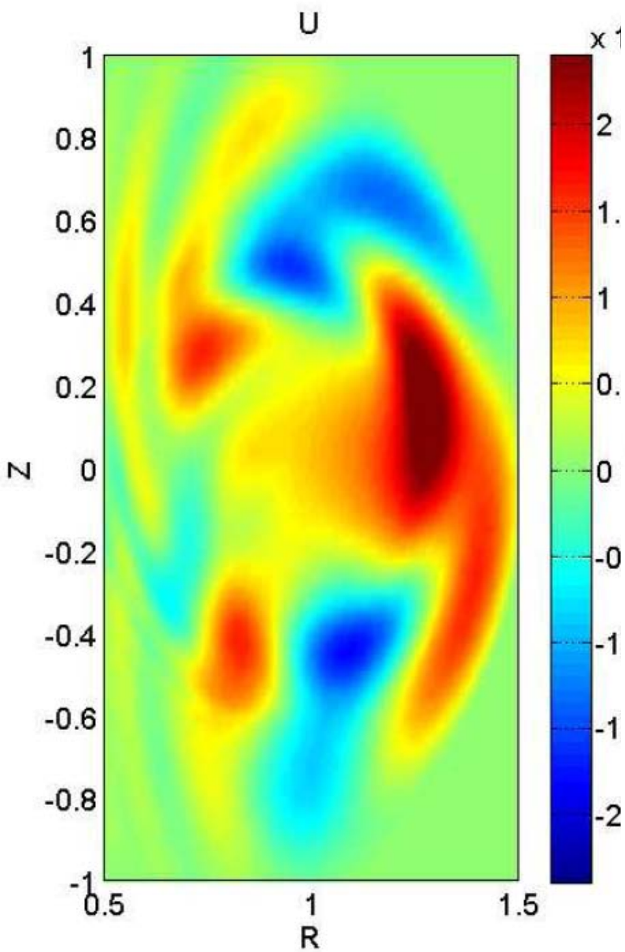
$$f = \frac{cH(v_0 - v)}{v^3 + v_c^3} \exp(-\psi / \Delta\psi) \exp[-(\Lambda - \Lambda_0)^2 / \Delta\Lambda^2], \Lambda = \frac{\mu B}{E}$$

➢NUBEAM fast ion distribution

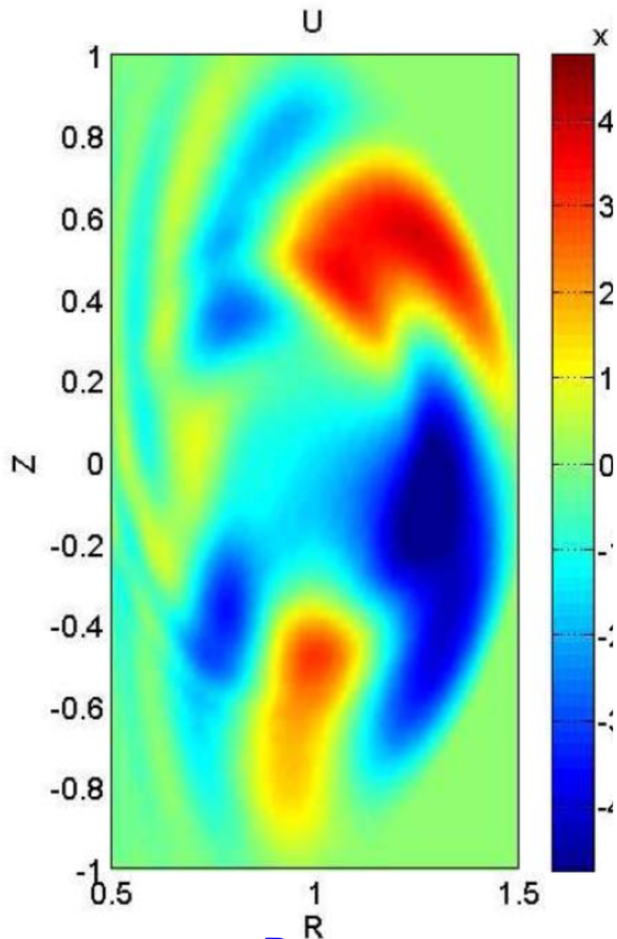
NUBEAM: a Monte Carlo module in TRANSP code for 4D (R,Z, $\lambda = v_{||}/v$, E) time dependent simulation of fast ion transport

- Assume fast ions behave classically;
- Include guiding center drift orbiting, collisional, and atomic physics effects

Comparison of Mode Structure Obtained with Analytic and NUBEAM Fast Ion Distributions

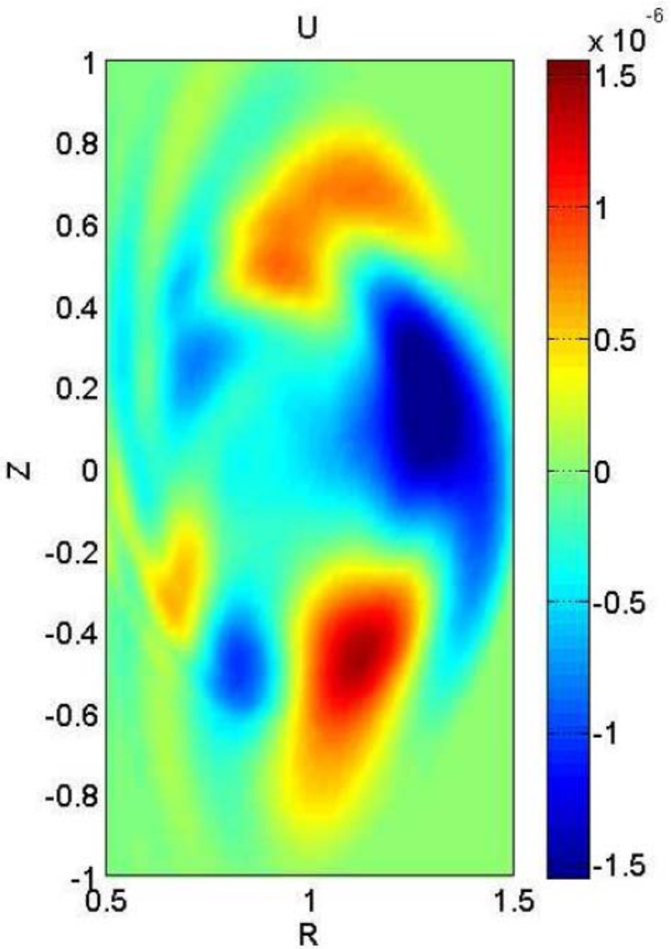


Analytic isotropic f_0
γ/ω_A =1.9%, f=101kHz



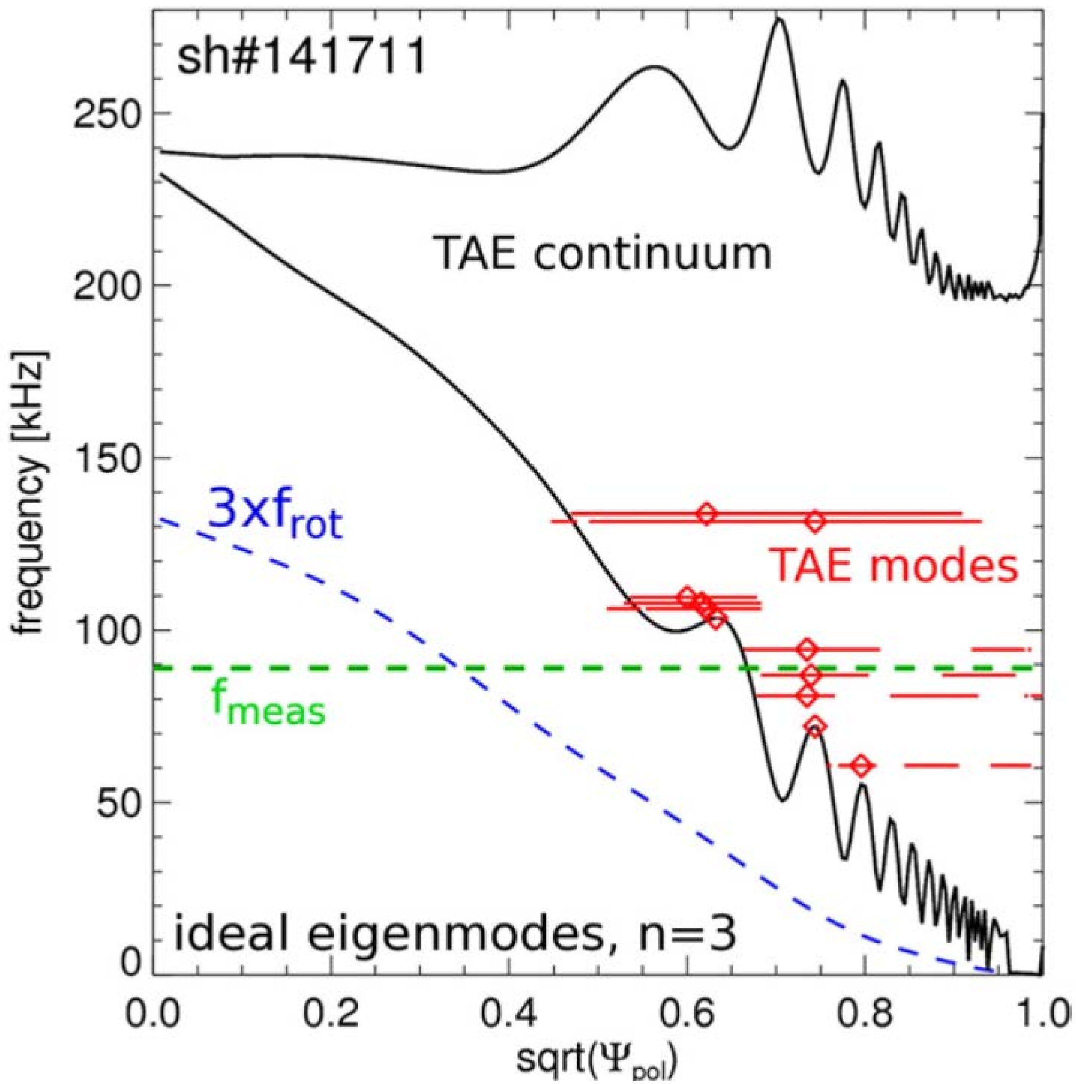
$$\Lambda_0 = \frac{\mu B}{E} = 0.6, \Delta\Lambda = 0.30$$

Analytic anisotropic f_0
γ/ω_A =3.1%, f=106kHz



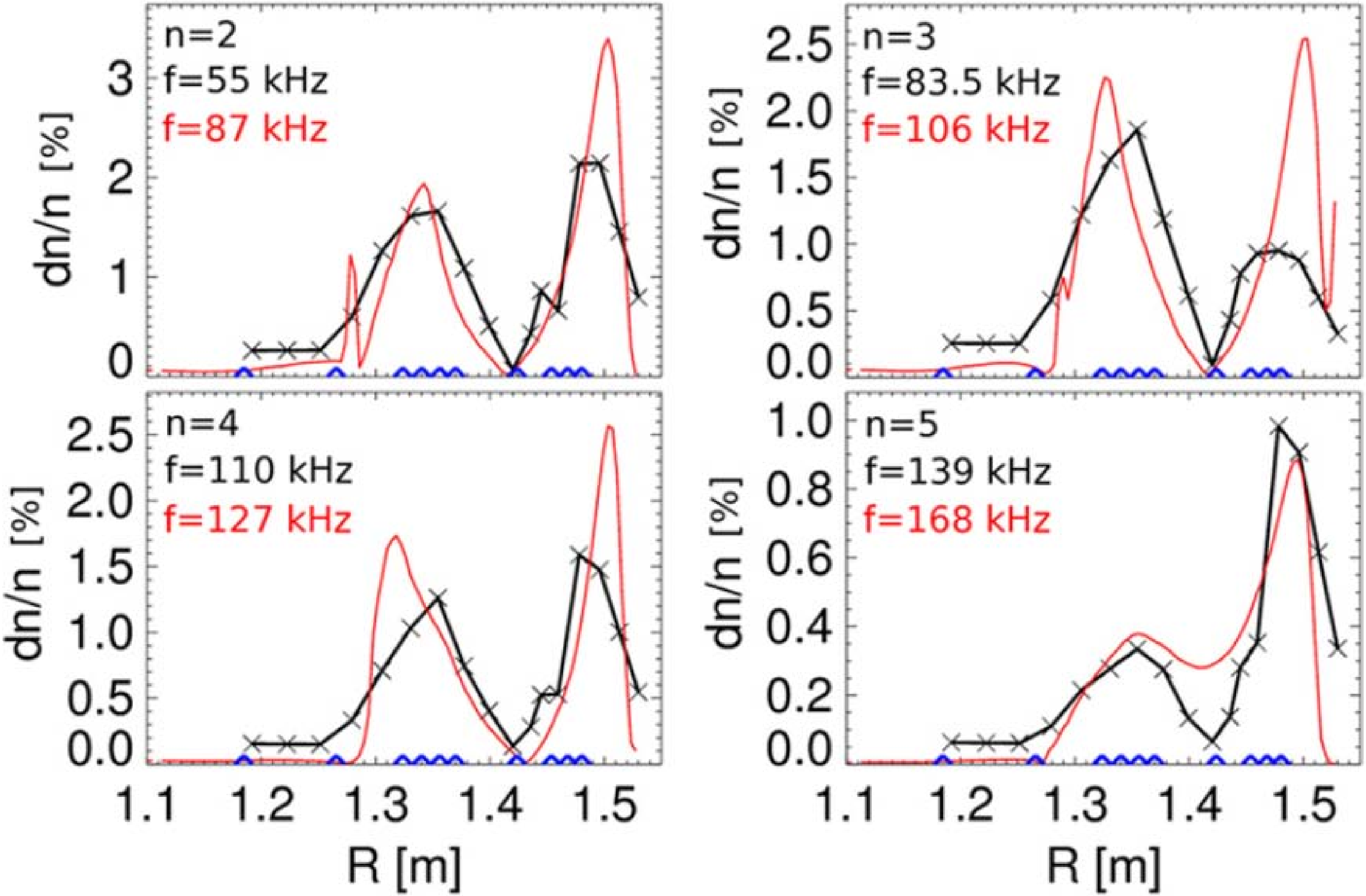
NUBEAM f_0
γ/ω_A =3.2%, f=99kHz

TAE Continuum and/RSAE Eigenmodes from NOVA



**M . Podesta et al. NF 2012*

Comparison of δn/n from Measurements and NOVA (t=484ms)



Jacobian of the Transformation from Velocity Space to Constants-of-Motion Space Strongly Depends on Particle Orbit Topology

➢Transformation from velocity space to constants-of-motion space.

$$\int G(x,y,z,v_x,v_y,v_z)d^3\mathbf{x}d^3\mathbf{v}$$
$$=\int F(R,Z,\lambda,E)R\sqrt{E}dRdZd\lambda dE$$
$$=\sum_{v_{sign}}\int f_{v_{sign}}(P_\phi,\mu,E)\Im_{v_{sign}}(P_\phi,\mu,E)dP_\phi d\mu dE$$

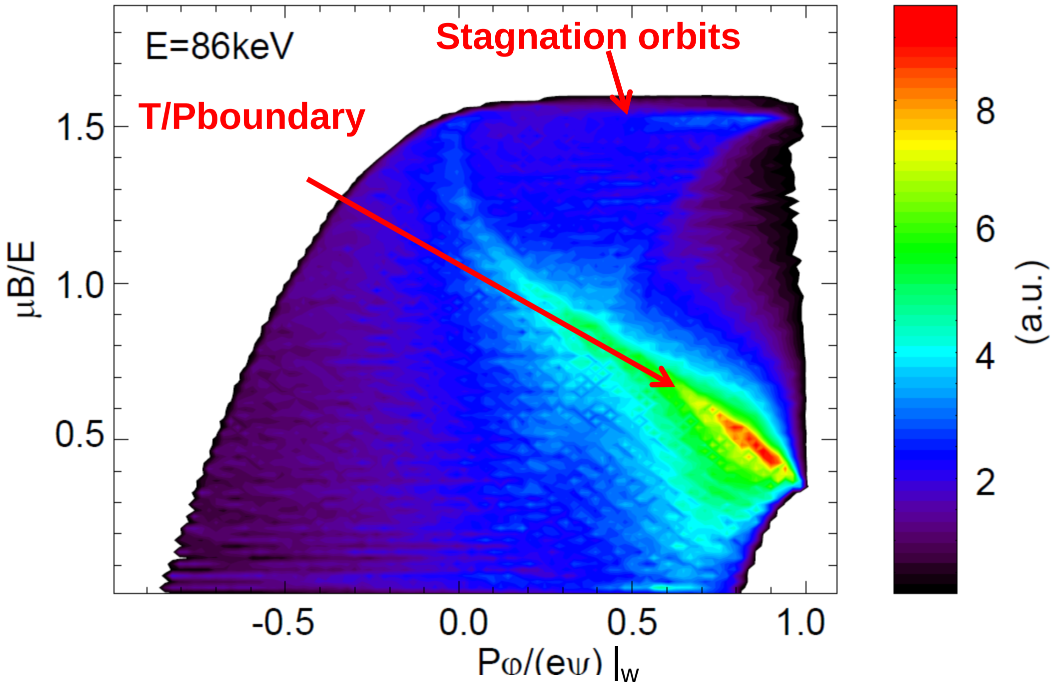
$$P_\varphi=(eA_\varphi+mv_\varphi)R\approx e\psi(R,Z)+mv_\parallel R\frac{B_\varphi(R,Z)}{B(R,Z)}$$

$$\mu=\frac{\frac{1}{2}mv_\perp^2}{B}=\frac{[1-(v_\parallel/v)^2]E}{B(R,Z)} \qquad \lambda=v_\parallel/v$$

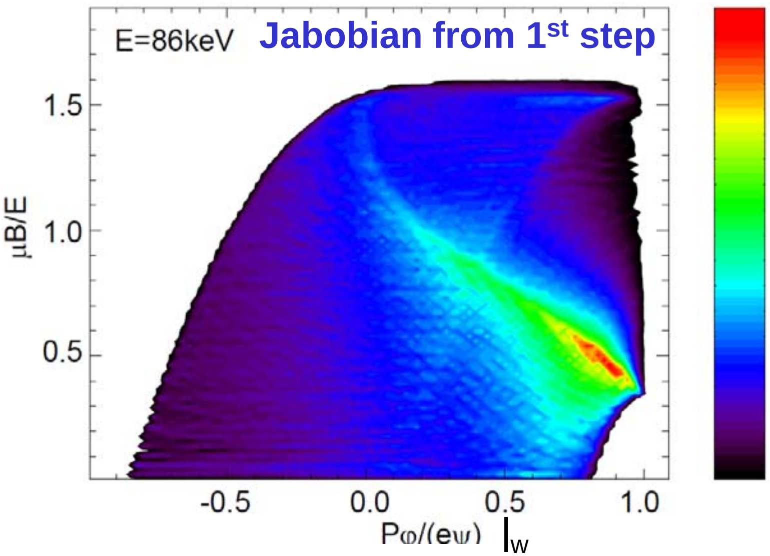
➢An exact analytic $\Im_{v_{sign}}(P_\varphi,\mu,E)$ cannot be easily calculated because it requires integration over phase space orbits.

$$\Im(P_\varphi,\mu,E)=\frac{4\pi^2\tau_{bounce}}{qm^2}$$

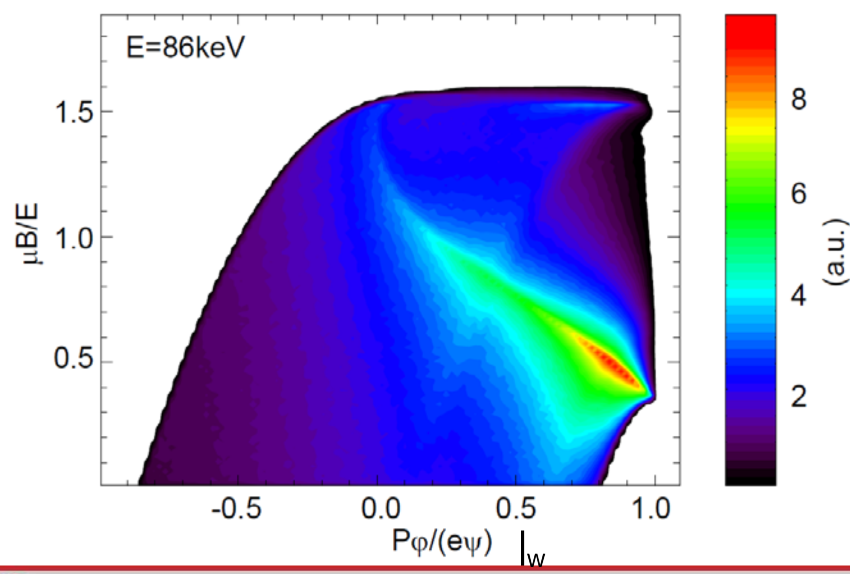
*J. Egedal Nucl. Fusion 2005



Jocabian Can be Calculated Accurately with Iterative Monte Carlo Method



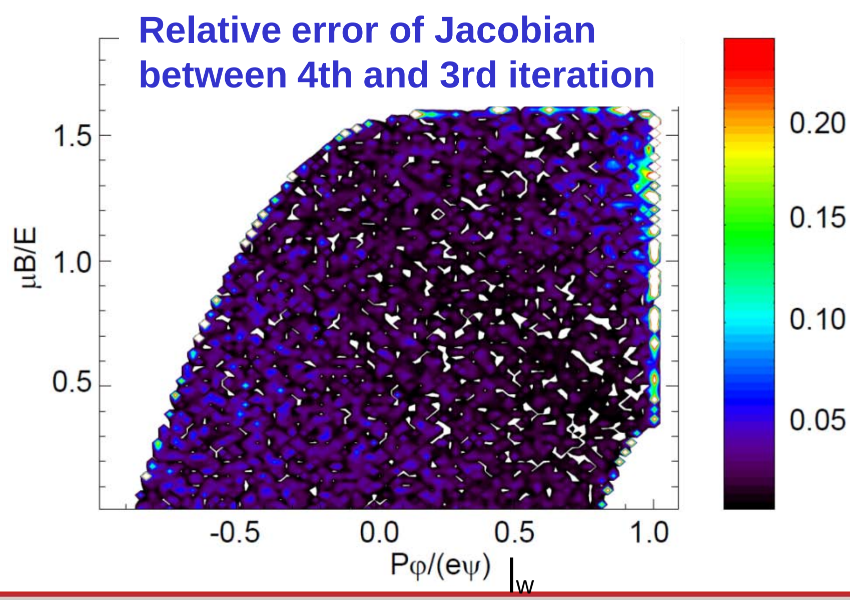
Jabobian from 4th iteration



- ➢Jacobian can be considered as the ratio of infinitesimal volume in (P_φ, μ ,E) space to the infinitesimal volume in (R, Z, λ, E) space

$$\int F(R,Z,\lambda,E)R\sqrt{E}dRdZd\lambda dE = \sum_{v_{sign}}\int f_{v_{sign}}(P_\phi,\mu,E)\Im_{v_{sign}}(P_\phi,\mu,E)dP_\phi d\mu dE$$

- ➢Iterative MC method (*Improved from Breslau's work)
- ▪Keep f uniformly in (R, Z, λ, E) space
- ▪Launch more particles in the region where the Jocabian has large relative error, adjust their weight



Jocabian Can be Calculated Accurately with Iterative Monte Carlo Method

- Jacobian can be considered as the ratio of infinitesimal volume in (Pφ, μ ,E) space to the infinitesimal volume in (R,Z,λ,E) space
- Step 1: Launch 10M random particles uniformly in (R,Z,λ,E) grids, Jacobian $\Im(P_\phi, \mu, E)$ is inversely proportional to the particle number/weight in each (P_φ, μ ,E) grid

The accuracy of $\Im(P_\phi, \mu, E)$ is generally not good near T/P or C/L boundaries.
To improve the accuracy of Jacobian, set i_level=1 for all (R,Z,λ,E) grids

- 2nd iteration: Double the particle number (20M) and reduce the particle weight by half; re-launch particles uniformly in (R,Z,λ,E) grids; recalculate the Jacobian; find the grids in (R,Z,λ,E) space whose corresponding Jacobian has relative larger error and change their iteration marker i_level to i_level+1

…

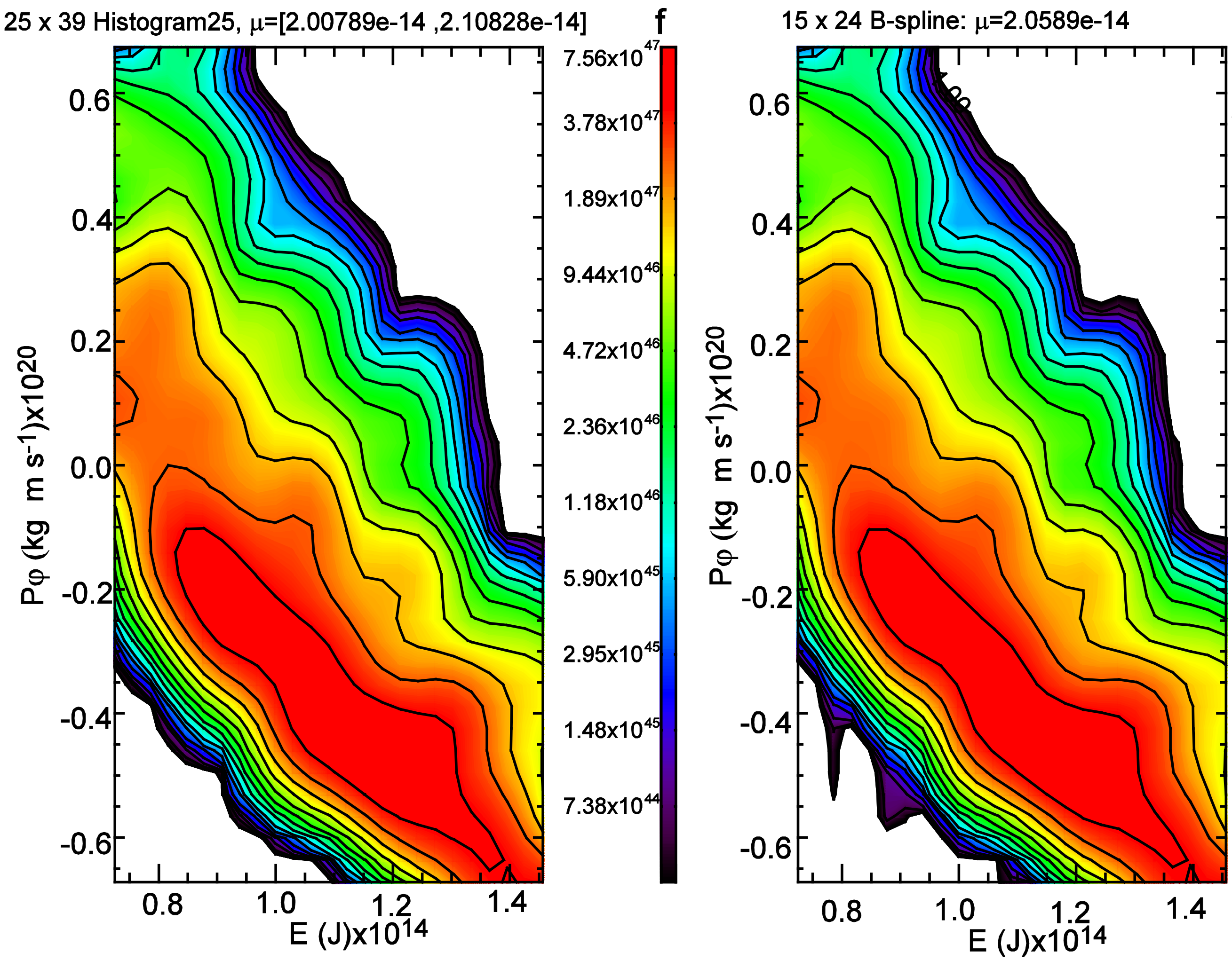
- n-th Iteration: Launch ~20M particles uniformly in (R,Z,λ,E) space except the grids whose Jacobian has relative larger error. For those grids, increase the particle number by 2^(i_level), and decrease their weight by (½)^i_level. ; recalculate the Jacobian; find the grids in (R,Z,λ,E) space whose corresponding Jacobian still has relative larger error and set their iteration marker i_level to i_level+1
- Repeat n-th Iteration until relative error is acceptable.

Constructing Fast Ion Density Function in Constants-of-Motion Space

- Divide NUBEAM output particle data into two subsets based on $v_{||}$ sign
- For each subset, sort particles by μ and divide them into several subpopulations of equal width in μ.
- Divide each subpopulation into a number of bins in the P_φ and E directions. The bin width in P_φ and E direction are the same for each subpopulation.
- Multiply by the numerically calculated Jacobian.
- Apply Gaussian smoothing in both P_φ and E directions.
- Fit 2D cubit B-spline to the smoothed data using GSL routines, with uniform knots and a number of coefficient s in each direction approximately 5/8 the number of bins.
- Store the spline coefficients in a file, which can be used to construct fast ion density function in constants-of-motion space and perform quick spline and derivative interpolations at arbitrary location.
- **Improved from the work of Breslau et al. Sherwood Meeting 2011*

Good Agreement between Raw Fast Ion Density Function and Spline Fit in Constants-of-Motion Space

Raw data from NUBEAM **Reconstructed from cubic B-spline fit**

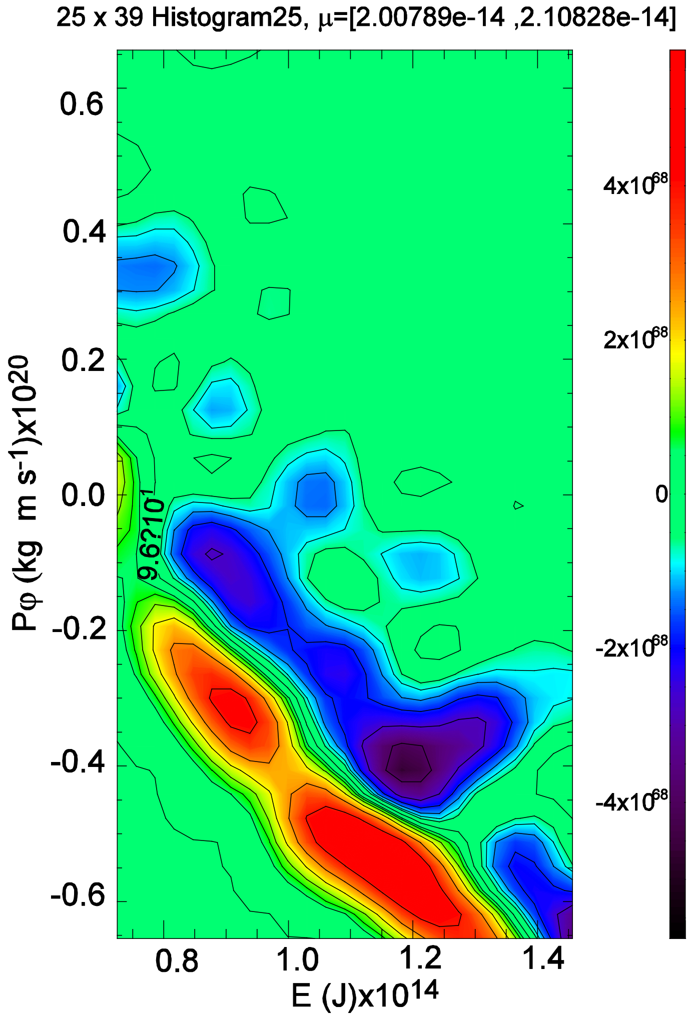


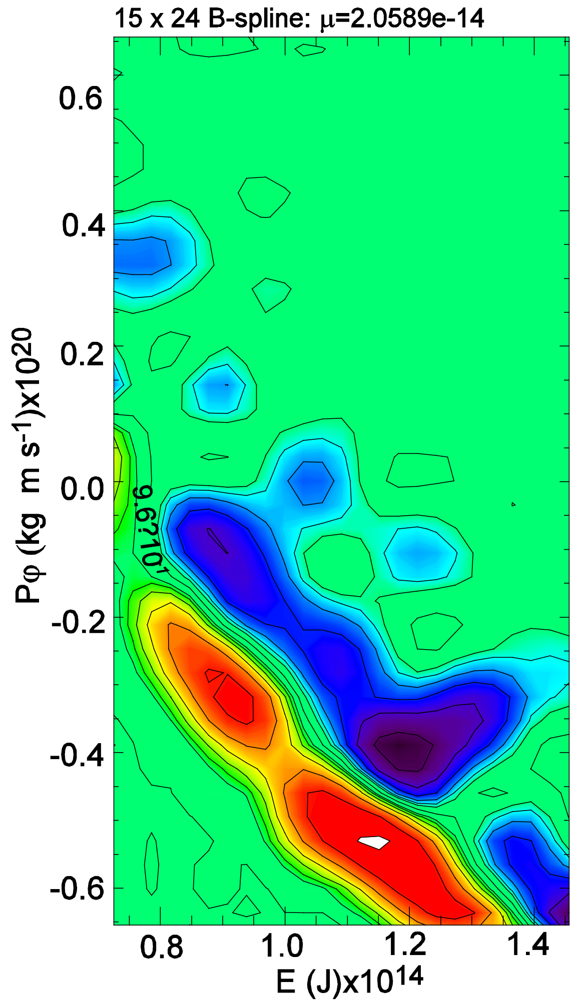
$$f_{co-current}(P_\varphi, E, \mu)\,|_{\mu=2.058e-14}$$

- ➢**40 bins in μ for $v_{||}$>=0**
  38 bins in μ for $v_{||}$<0
- ➢**B-spline fits for high-occupancy μ subset are good**
- ➢**B-spline fits for low-occupancy μ subsets are adequate**

Gradients are Smooth, Match well with NUBEAM Raw Data

Raw data from NUBEAM **Reconstructed from cubic B-spline fit**





$$\frac{\partial f_{co-current}(P_\varphi, E, \mu)}{\partial P_\varphi}$$

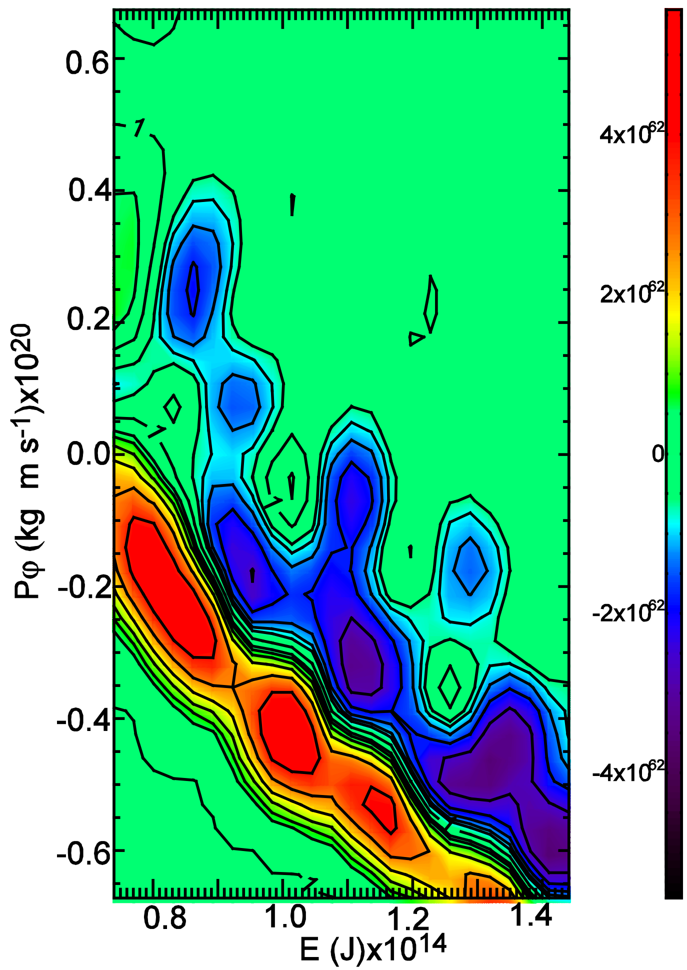
Derivative relative to P_φ

Gradients are Smooth, Match well with NUBEAM Raw Data

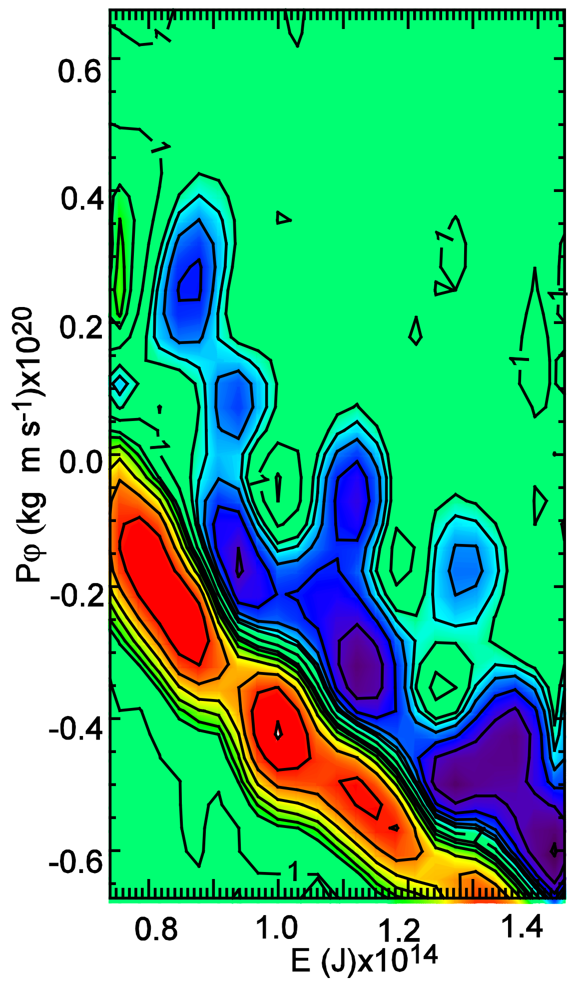
Raw data from NUBEAM

Reconstructed from cubic B-spline fit

25 x 39 Histogram25, μ=[2.00789e-14 ,2.10828e-14]



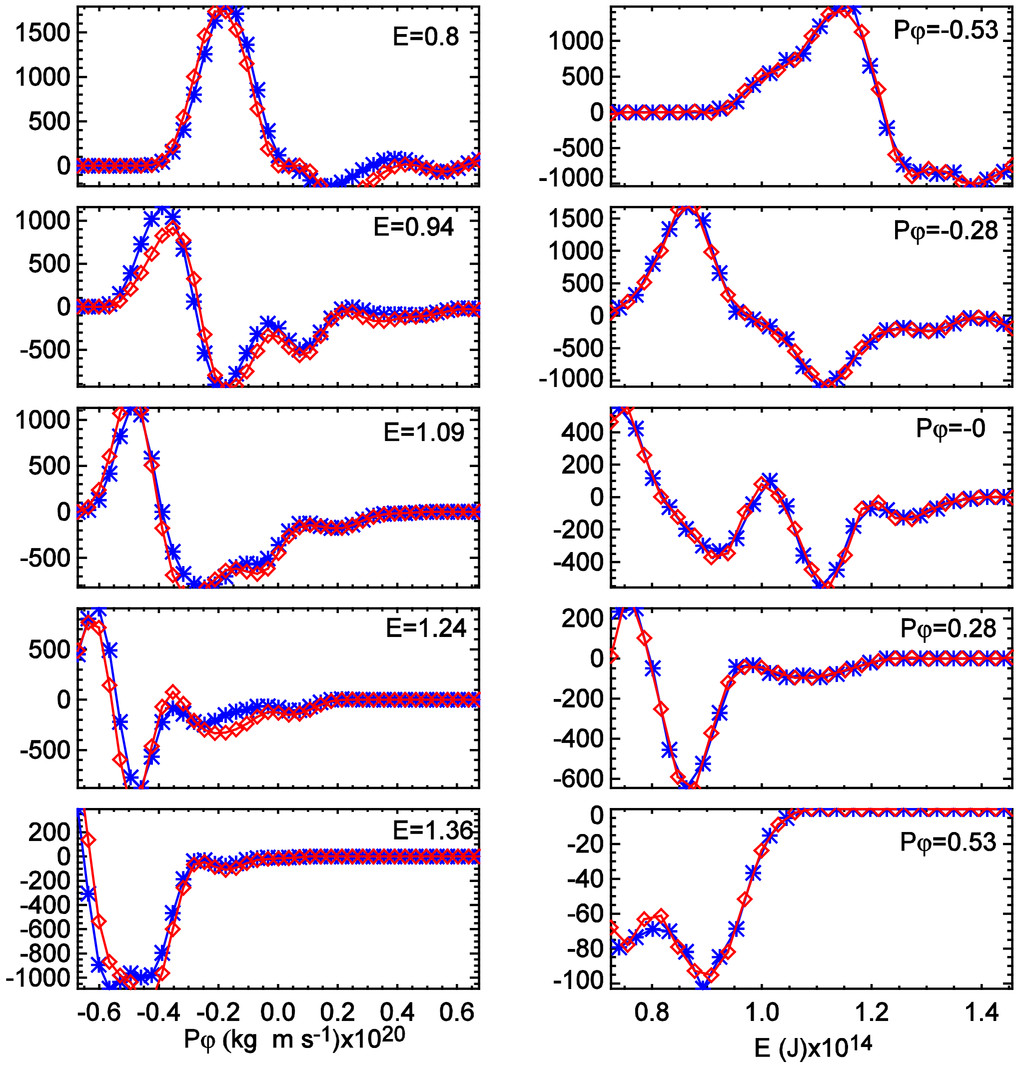
15 x 24 B-spline: μ=2.0589e-14



$$\frac{\partial f_{co-current}(P_\varphi, E, \mu)}{\partial E}$$

Derivative relative to E

Gradients are Smooth, Match well with Raw Data



Blue: raw data from NUBEAM

Red: reconstructed from cubic B-spline fit

$$\frac{\partial f_{co-current}(P_\varphi, E, \mu)}{\partial E}$$