

Theory verification and numerical benchmarking on neoclassical toroidal viscosity torque

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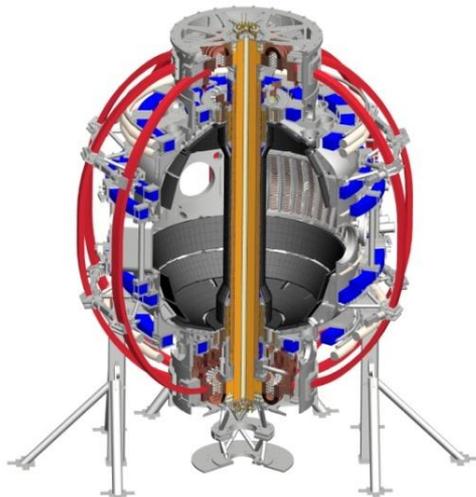
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Motivation

NTV torque → externally optimize plasma rotation and its profile → affect plasma instabilities and performance

To increase the predictability for NTV torque and the controllability for plasma rotation in tokamaks, it is important to understand the different approaches of NTV theory with proper cross-benchmark.

Besides the particle simulation of NTV torque ([K. Kim et al, NO6.00001](#)), various semi-analytic methods have been developed in recent years.

Outline

- Introduction of three different approaches with corresponding codes
- Benchmark of NTV torque (precession and bounce-harmonic resonance of trapped ions)
- Importance of bounce-harmonic resonance in NTV calculation
- Summary

The Semi-Analytic Methods of NTV Torque

- In general, the semi-analytic methods find the NTV torque by solving the bounce averaged drift kinetic equation.
- Mainly three approaches with corresponding codes are established.

Connected NTV formula	Combined NTV theory	Drift kinetic energy theory
<ul style="list-style-type: none"> • Analytic NTV formulation is derived in the collisionality regimes, SBP, $\nu - \sqrt{\nu}$ and $1/\nu$ separately. • Pade approximation to smoothly connect the formulations in different collisionality regimes. 	<ul style="list-style-type: none"> • NTV torque due to different particle motions are combined by a generalized equation. • No need to separate the collisionality regimes 	<ul style="list-style-type: none"> • Drift kinetic energy is studied to determine ideal MHD instabilities e.g. Resistive Wall Modes. • The torque is calculated based on the equivalence between NTV torque and drift kinetic energy. $T_\phi = 2in \delta W_k$
Precession resonance	Precession resonance + Bounce-harmonic resonance	
Pitch angle scattering collisional operator (accurate in the $\nu - \sqrt{\nu}$ regime)	Krook collisional operator (allows to derive one generic equation of NTV torque and drift kinetic energy)	
Geometric simplification	Full toroidal geometry	
MARS-Q	IPEC-PENT	MARS-K
Liu and Sun, PoP 20 (2013) 022505 Sun et al, NF 51 (2011) 053015 Shaing et al, NF 50 (2010) 025022	Park et al, PRL 102 (2009) 065002 Logan et at, submitted to PoP	Park, PoP 18 (2011) 110702 Liu et al, PoP 15(2008) 112503

N. Logan et al, NO6.00003 (Detailed formulation)

Perturbed Equilibrium

NTV torque is induced when the equilibrium is perturbed by non-axisymmetric magnetic perturbation.

IPEC, MARS-K/Q solves the perturbed equilibrium

$$\vec{j} \times \vec{B} + \vec{J} \times \vec{b} - \nabla p = 0$$

$$p = -\vec{\xi} \cdot \nabla P - \Gamma P \nabla \cdot \vec{\xi}$$

$$\vec{b} = \nabla \times (\vec{\xi} \times \vec{B})$$

$$\vec{j} = \nabla \times \vec{b}$$

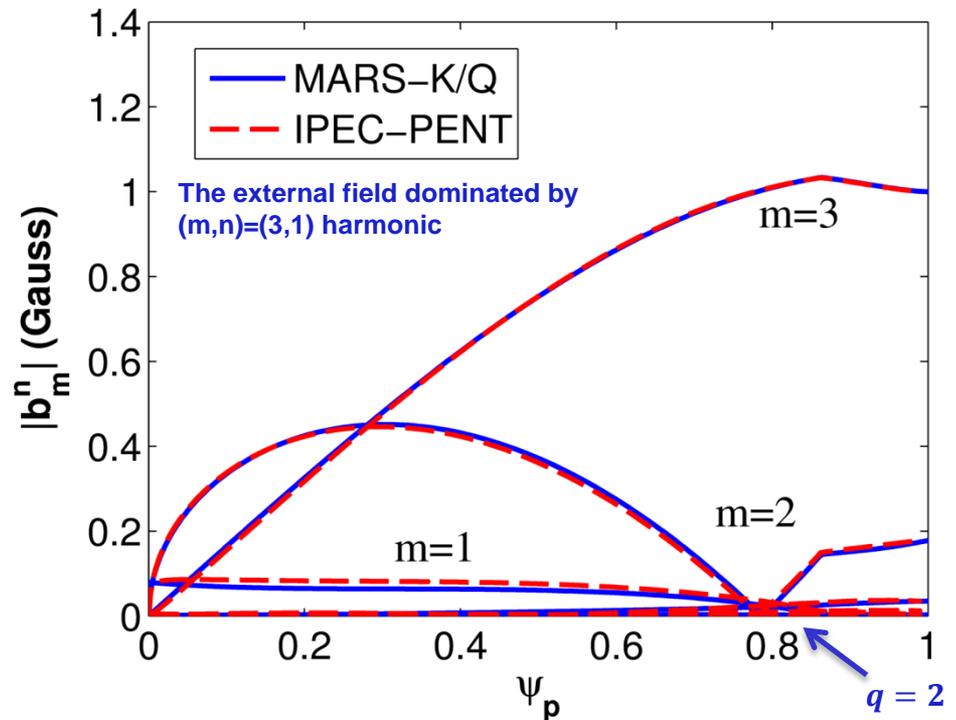
A perturbed tokamak equilibrium is used:

- Circular cross-section
- Large aspect ratio, $R/a=10$
- Stable to ideal kink
- $q_0 = 1.1, q_a = 2.52$

The continuous coil is located close to plasma, $b_c=1.1a$.

$$\nabla \times \vec{b} = \vec{j}_{coil}, \quad \nabla \cdot \vec{j}_{coil} = 0.$$

The external magnetic perturbation is generated by the coil with $(m,n)=(3,1)$ helical current.

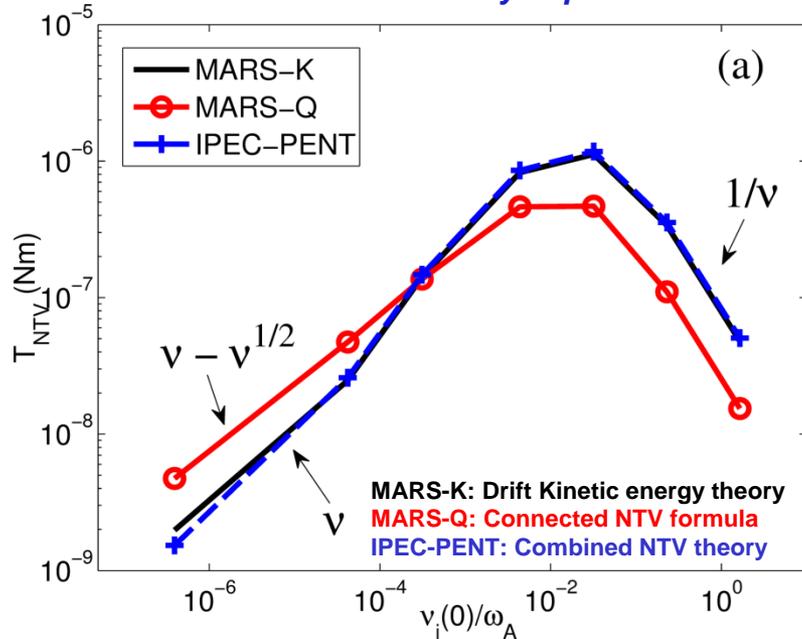


A good agreement of perturbed equilibrium among MARS-K/Q and IPEC-PENT.

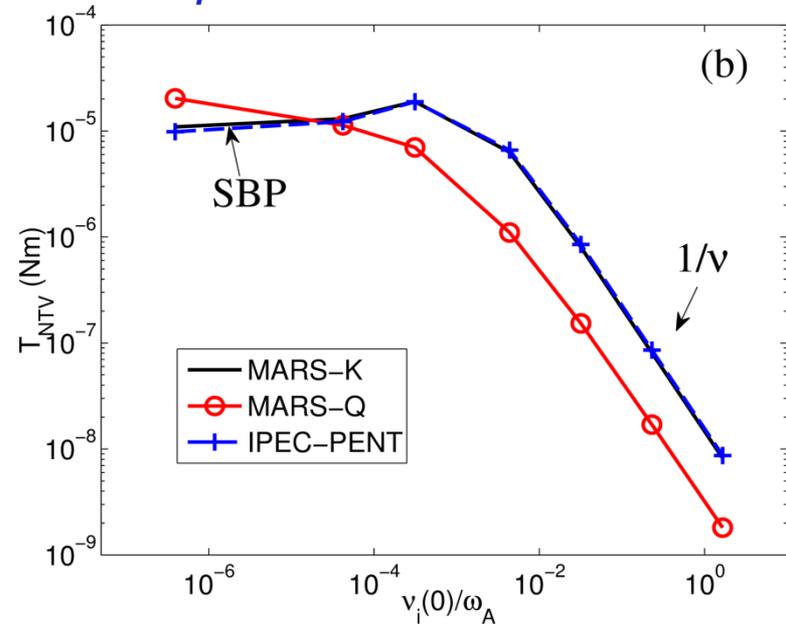
NTV Torque Due to Precession Resonance of Trapped Ions

NTV torque contributed solely by the precession motion of trapped ions is compared.

Collisionality dependence of NTV torque due to the precession resonance



High rotation case: $\omega_E = 0.1\omega_A$



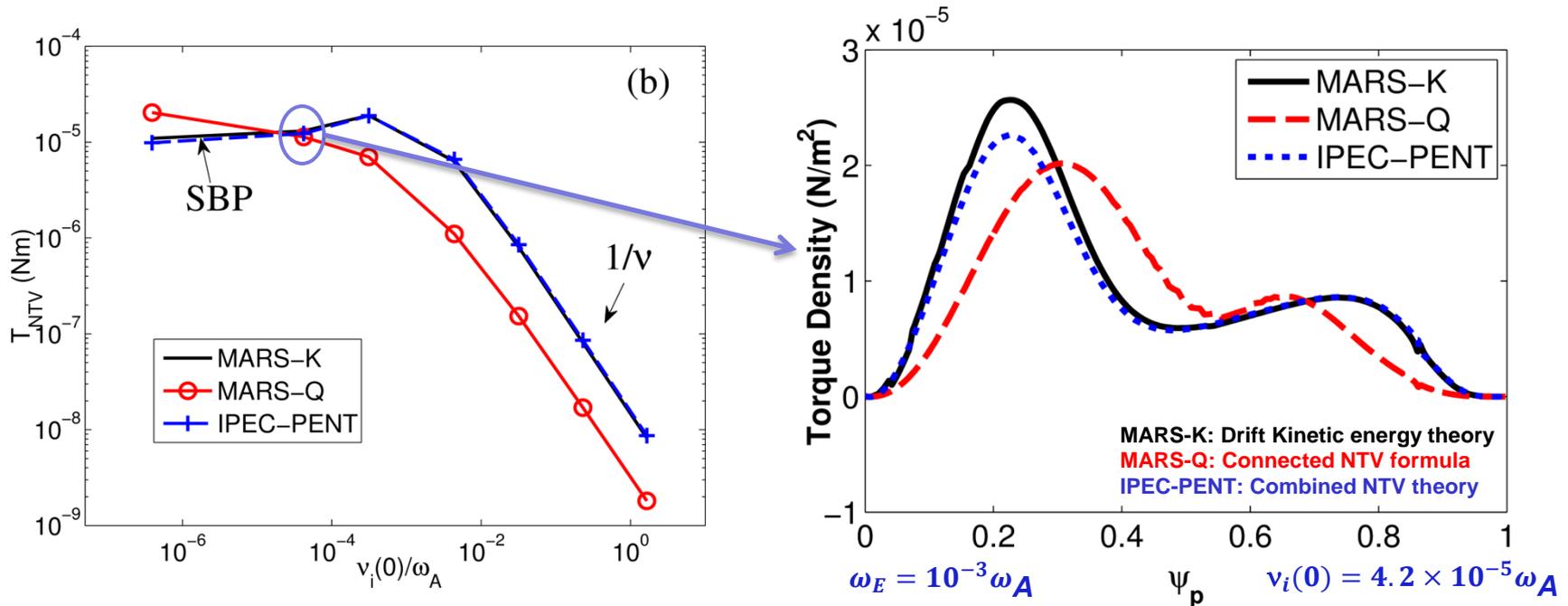
Low rotation case: $\omega_E = 10^{-3}\omega_A$

The three codes agree with the predicated behavior of torque for all collisionality regimes.

- IPEC-PENT vs. MARS-K: A very good quantitative agreement.
- The torque computed by MARS-Q also qualitatively agrees with IPEC-PENT and MARS-K's result.

NTV Torque Due to Precession Resonance of Trapped Ions

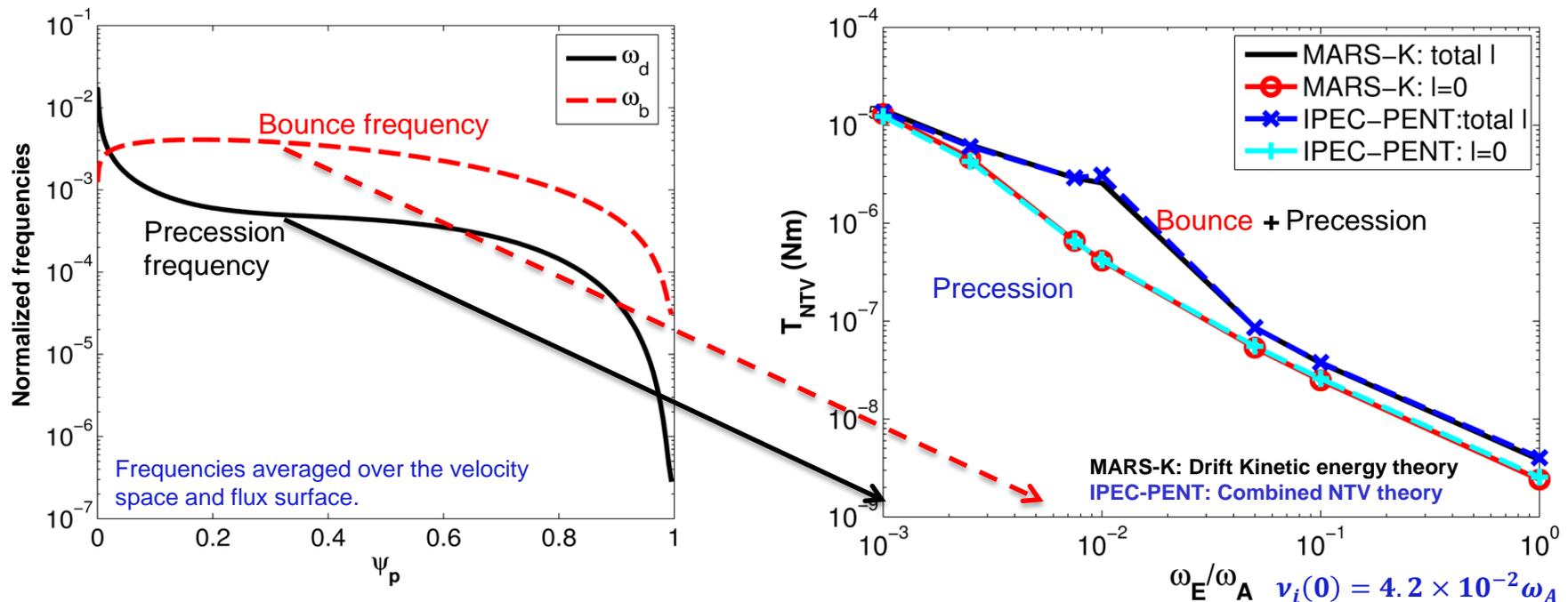
The comparison of torque density profile among IPEC-PENT and MARS-K/Q.



- The torque computed by MARS-K is very close to IPEC-PENT's result.
- MARS-Q also presents a similar torque density profile.

NTV Torque Due to Bounce-Harmonic Resonance of Trapped Ions

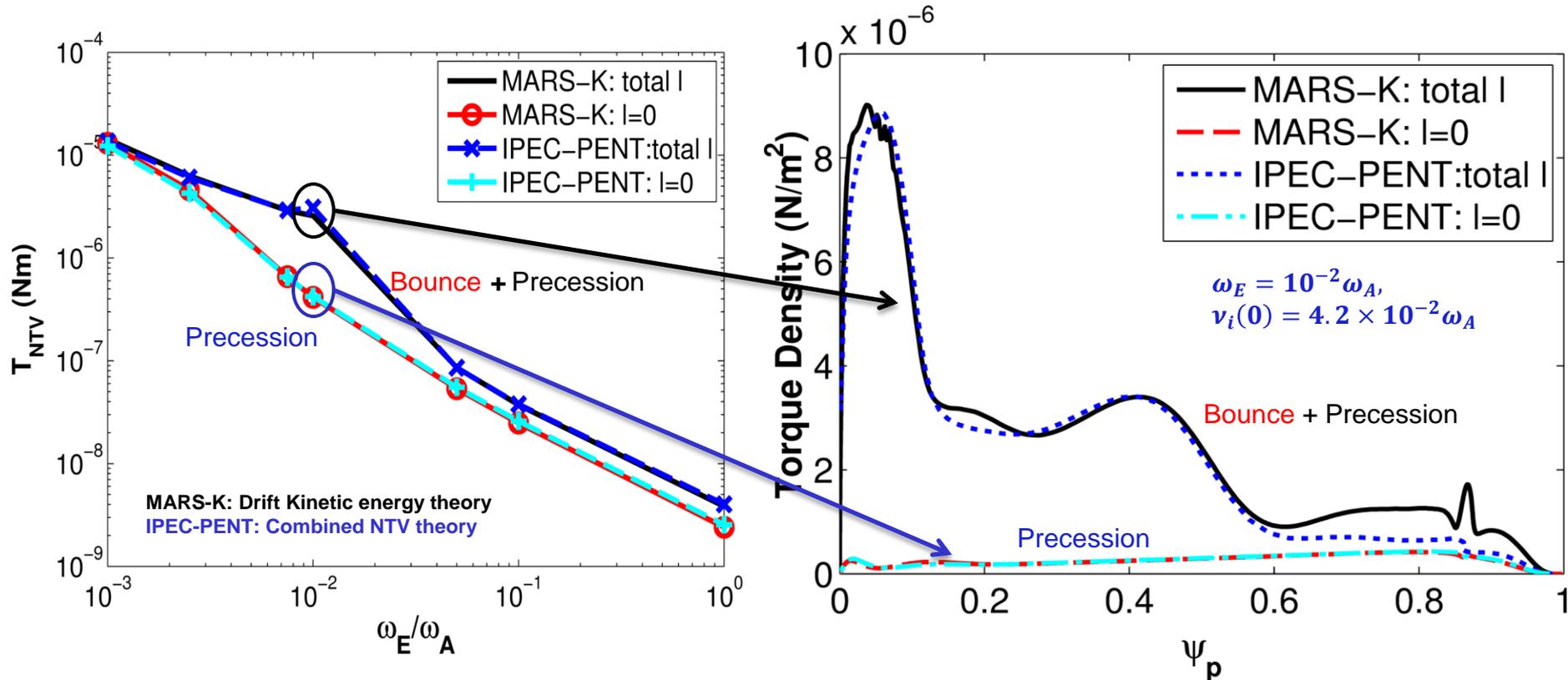
The comparison of NTV torque due to bounce-harmonic and precession resonance by scanning $E \times B$ drift frequency.



- The bounce-harmonic resonance starts to play a major role when ω_E is comparable with the averaged ion bounce frequency $\langle \omega_b \rangle$.
- When $\omega_E \sim \langle \omega_d \rangle < \langle \omega_b \rangle$ or $\omega_E \gg \langle \omega_b \rangle$, the contribution of torque is mainly from the precession resonance.

NTV Torque Due to Bounce-Harmonic Resonance of Trapped Ions

A quantitative agreement of torque profiles computed by IPEC-NTV and MARS-K



- Comparing with the precession resonance, the bounce-harmonic resonance has a significant contribution to NTV torque.
- The bounce-harmonic resonance is dominant everywhere along the radial direction.

When is Bounce-Harmonic Resonance Important?

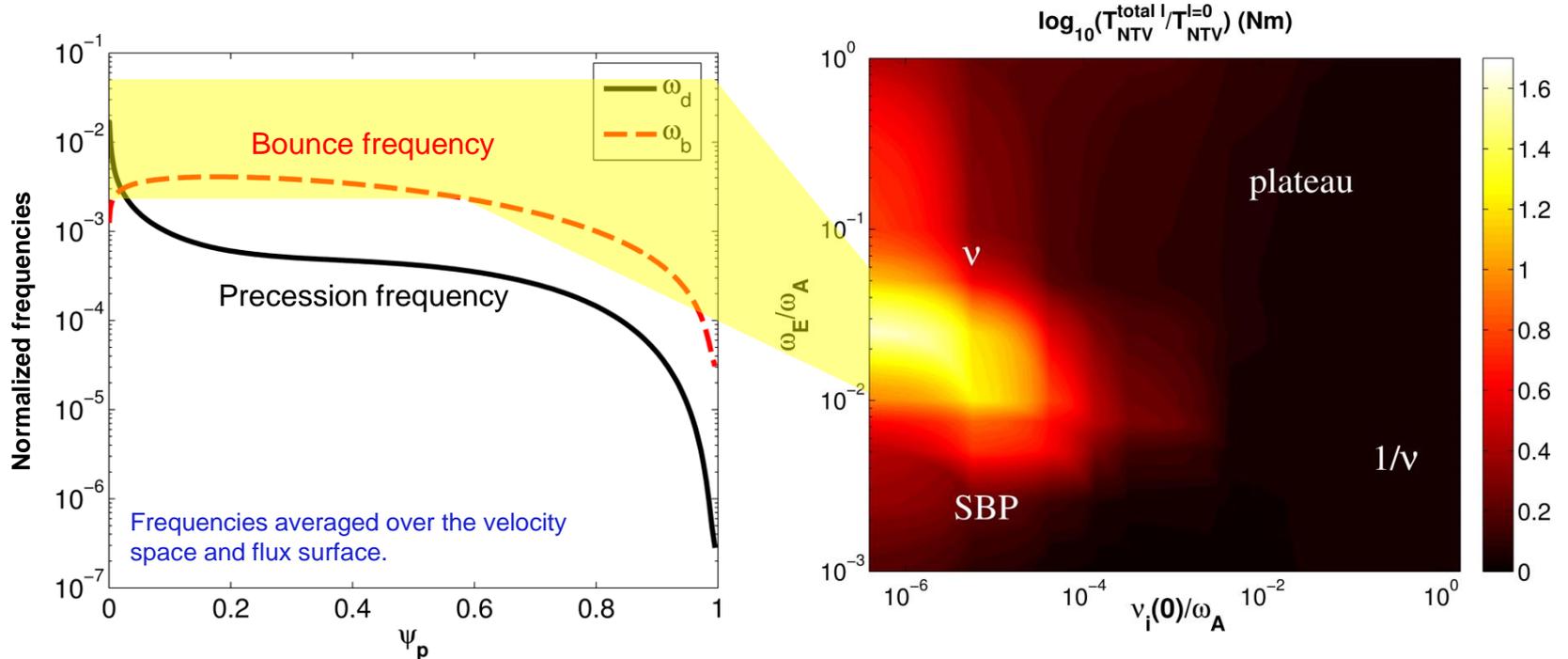
The maximum value of torque due to bounce-harmonic resonance is found in the ν regime when $\omega_E \sim 2 \times 10^{-2} \omega_A \sim \omega_b$.

$$\text{Max} \left(\frac{T_{\phi}(\text{precession+bounce})}{T_{\phi}(\text{precession})} \right) \sim 40$$

The NTV torque due to the bounce-harmonic resonance is observed in the particle simulation (Kim et al, PRL 110 (2013) 185004) and

the recent KSTAR experiment (Park et al, PRL 111 (2013) 095002).

Collisionality and ω_E dependence of the ratio $T_{\phi}(\text{precession} + \text{bounce})/T_{\phi}(\text{precession})$



MARS-K is used to perform the computation. IPEC-PENT agrees with these results.

Summary

- A successful numerical benchmarking among three different approaches of NTV theory has been carried out.

Combined NTV (IPEC-PENT), Connected NTV formula (MARS-Q) and Drift kinetic energy (MARS-K)

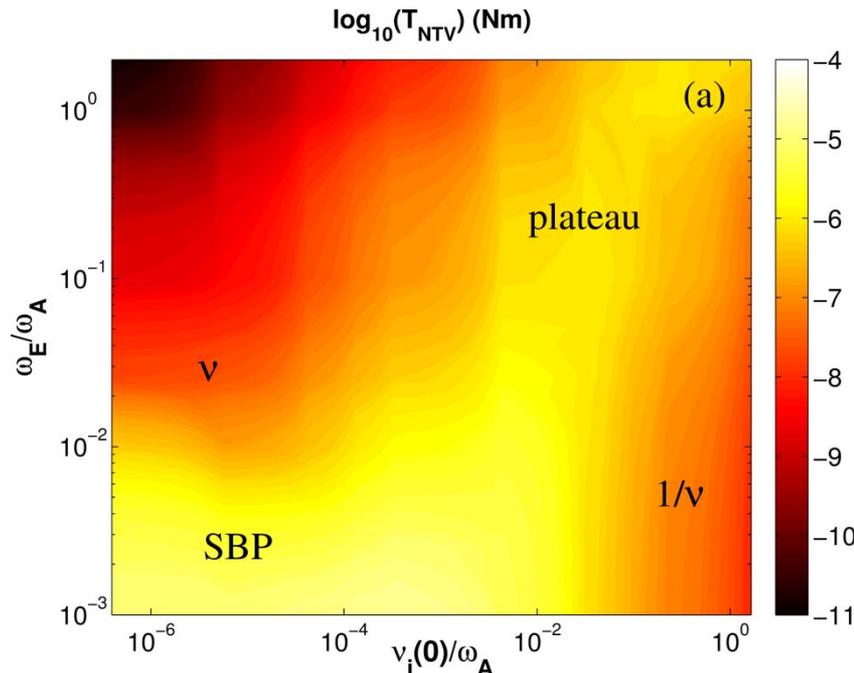
- The excellent agreement between IPEC-PENT and MARS-K affirms the equivalence between combined NTV torque and drift kinetic energy.
- The rotation and collisionality scan shows the importance of including the bounce-harmonic resonance in NTV torque.

The low collisionality ($\sim \nu$ regime), $E \times B$ drift frequency matches the bounce frequency

- The detailed modeling of combined NTV theory and drift kinetic energy will be presented by **N. Logan et al, NO6.00003**

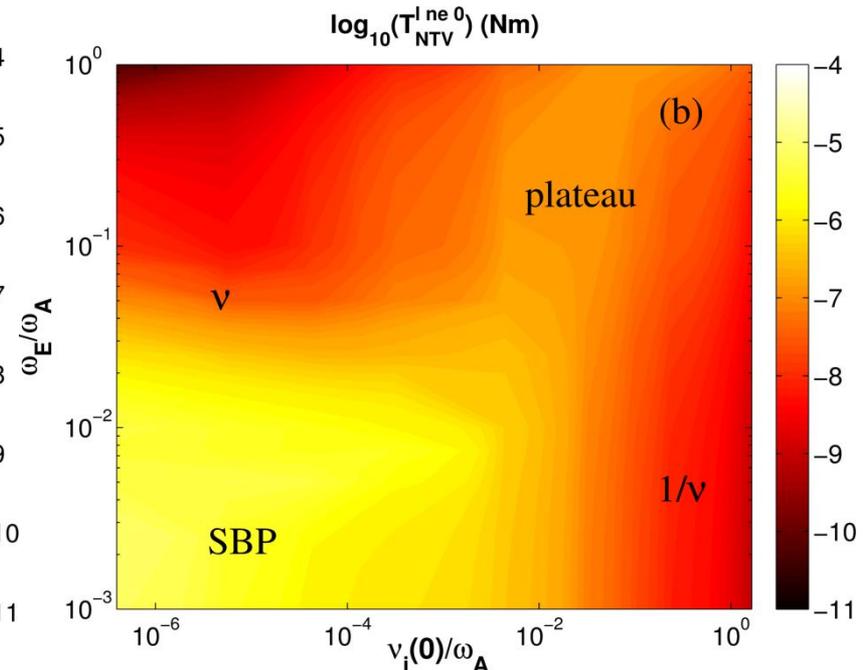
Collisonality and ω_E dependence of NTV torque

Precession resonance



Collisionality regime defined with respect to precession resonance

Bounce-harmonic resonance



MARS-K is used to perform the computation. IPEC-PENT agrees with these results.

- Precession resonance:**
- A large torque due to mainly occurs in the SBP regime.
 - The plateau between the ν regime and the $1/\nu$ regime is observed.

- BH resonance:**
- A wider ω_E range has a significant contribution to torque.
 - In the $1/\nu$ and plateau regimes, the torque decreases more quickly since bounce motion can be strongly affected by the collisionality.