Towards a Predictive Capability for Local Helicity Injection Startup

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on behalf of the PEGASUS Team

University of Wisconsin-Madison

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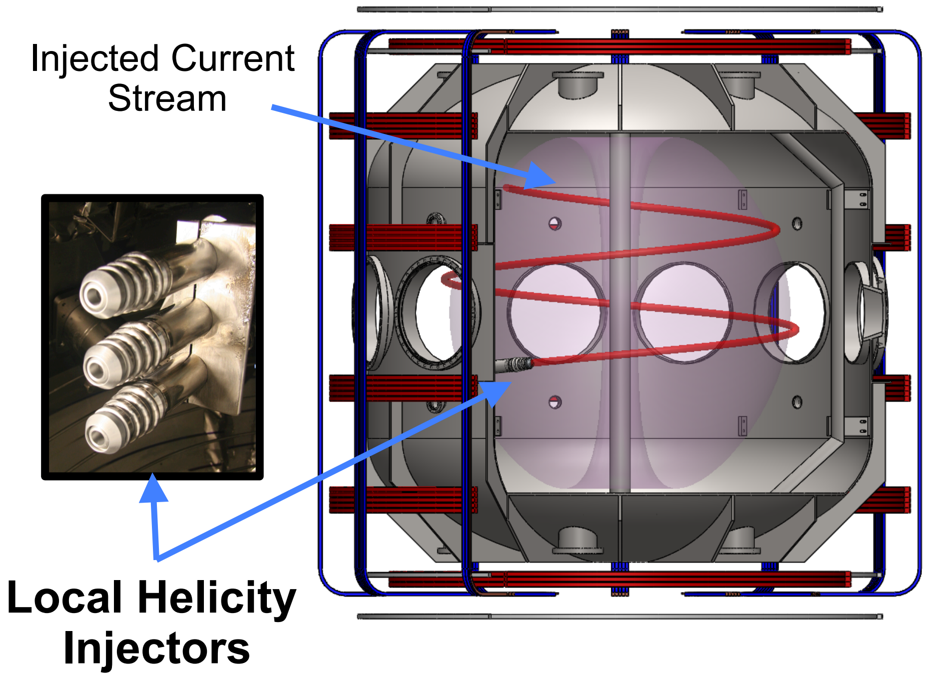
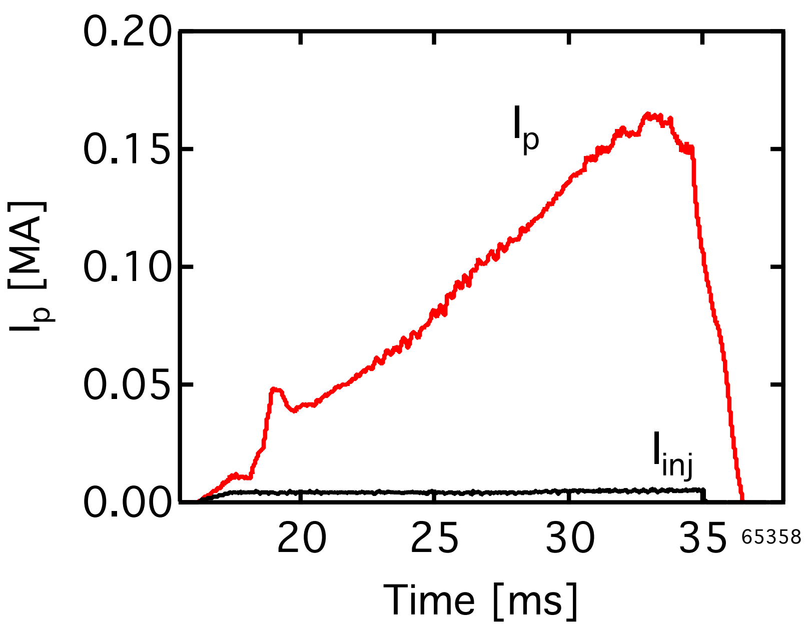
PEGASUS Toroidal Experiment

Local Helicity Injection (LHI) is a Promising Non-Solenoidal Startup Technique

Current injected on outboard side



$I_p \leq 0.18$ MA (I_{inj} = 5 kA)



- Unstable current streams form tokamak-like state via Taylor relaxation
- Appears scalable to MA-class startup

J.L. Barr, APS DPP 2014

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A Hierarchy of Predictive Models Being Developed for LHI Startup

1. Maximum I_p limits*

Taylor Relaxation

$$I_p \leq I_{TL} \sim \sqrt{\frac{I_{TF} I_{inj}}{w}}$$

Helicity Conservation

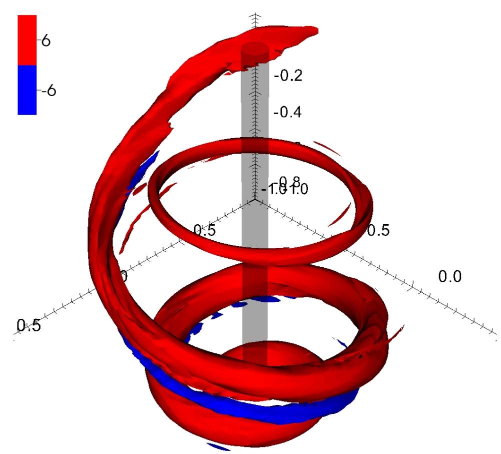
$$V_{LHI} \approx \frac{A_{inj} B_{\varphi,inj}}{\Psi} V_{inj}$$

2. 0-D power-balance $I_p(t)$

$$I_p \left[V_{LHI} + V_{IR} + V_{IND} \right] = 0; \;\; I_p \leq I_{TL}$$

3. 3D Resistive MHD (NIMROD)**
   – See Sovinec, GP8.00047

Simulated current stream reconnection in NIMROD



*D.J. Battaglia, et al. Nucl. Fusion **51** (2011) 073029.
*N.W. Eidietis, Ph.D. Thesis, UW-Madison, 2007.
**J. O'Bryan, Ph.D. Thesis, UW-Madison, 2014.
J. O'Bryan, C.R. Sovinec, Plasma Phys. Control. Fusion **56 064005 (2014)

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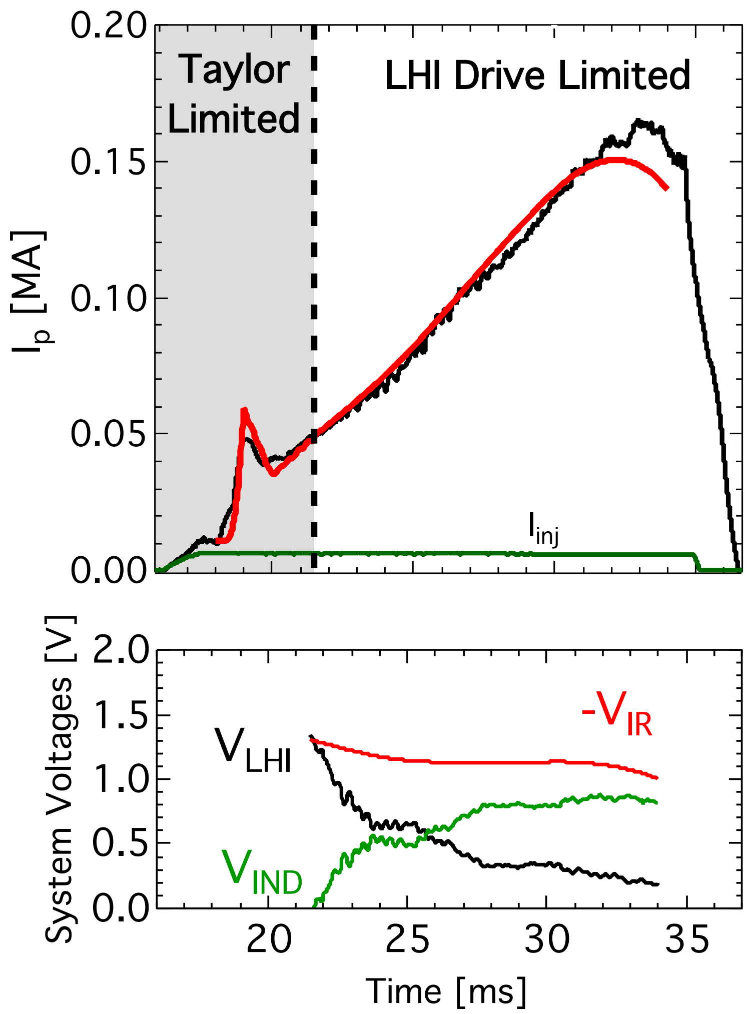
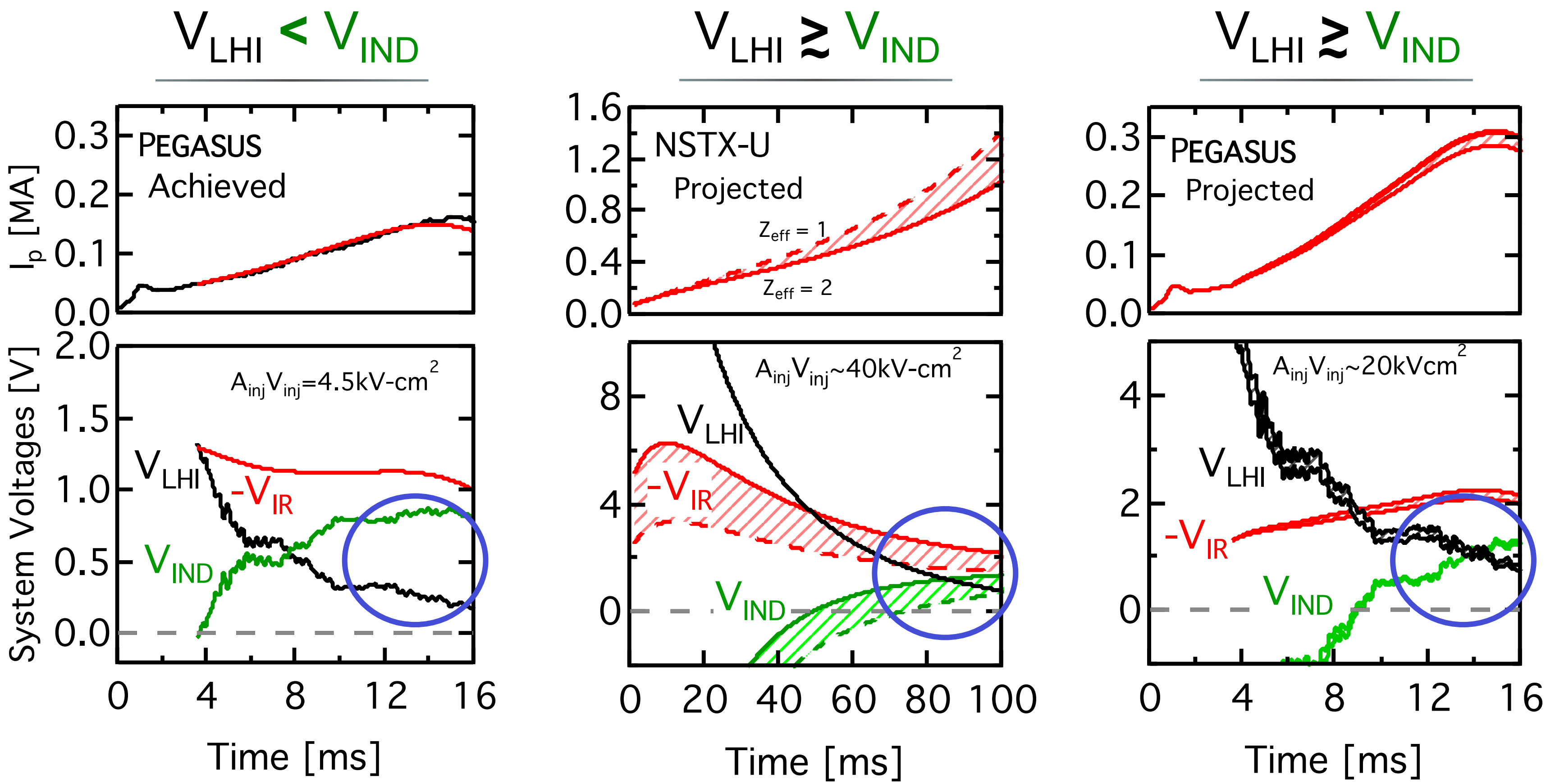
2. 0-D Power-Balance: Lumped-Parameter Model for Predictive $I_p(t)$

- Model elements:
  - Inputs: $R_0(t)$, shape(t), $V_{LHI}(t)$, $\eta(t)$, $\ell_i(t)$
  - Low-A inductance, force-balance
- Reasonable agreement between calculated $I_p(t)$ and measurement
- High-I_p: current drive dominated by PF induction, geometry evolution

0-D model predictions vs data



Physics Test for MA Startup on NSTX-U Requires Increased Helicity Injection Drive



- Confinement when $V_{LHI} \gtrsim V_{IND}$ is a critical issue
  - At $I_p \sim$ 0.2-0.3 MA in PEGASUS

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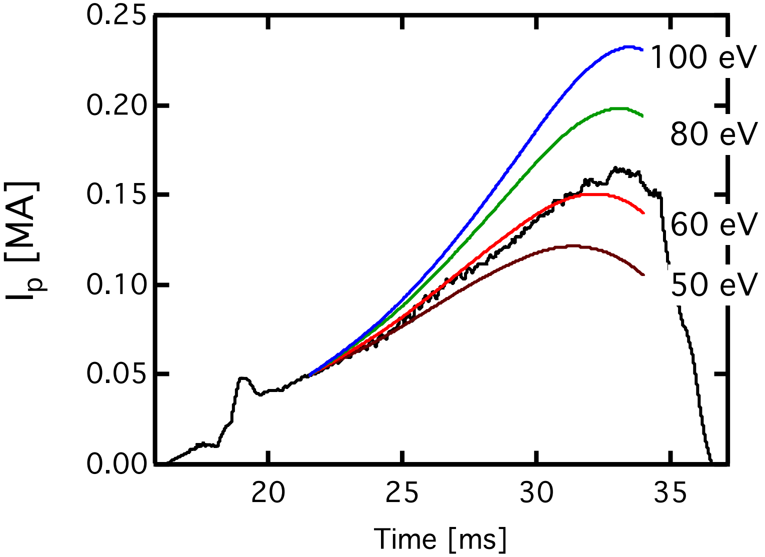
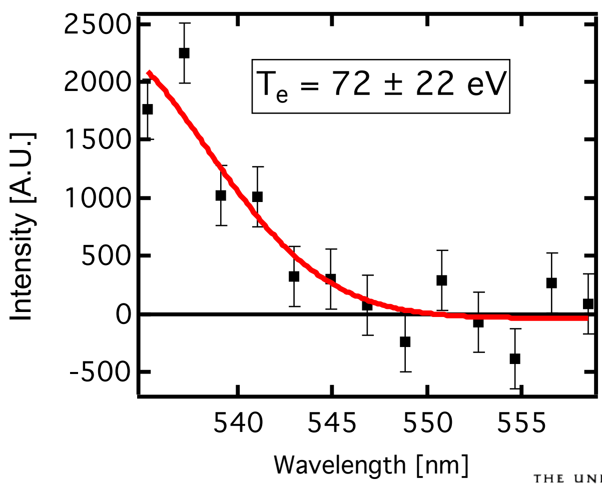
Knowledge of Confinement an Important Question for Predictability

- $I_p(t)$ model depends critically on η
  - Initial Thomson scattering: $T_e(0)$~70 eV



- Dual confinement regimes?

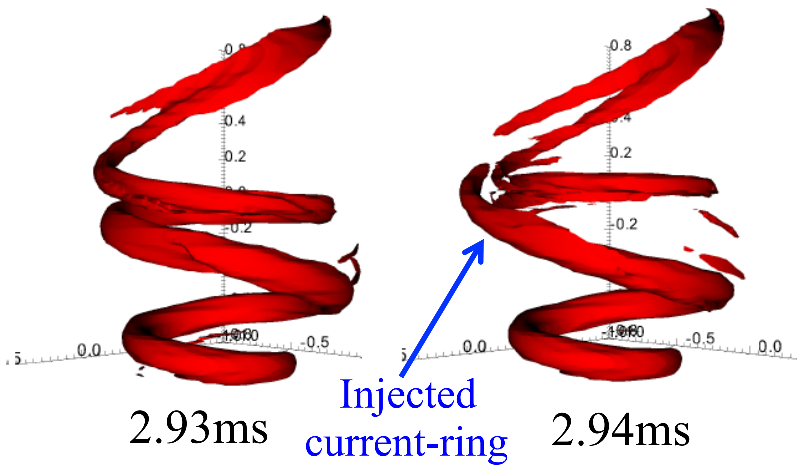
Warm Core	Cool Edge
OH-like	Stochastic
Inductive drive	Reconnection
Low $\tilde{B}/B$	Large $\tilde{B}/B$

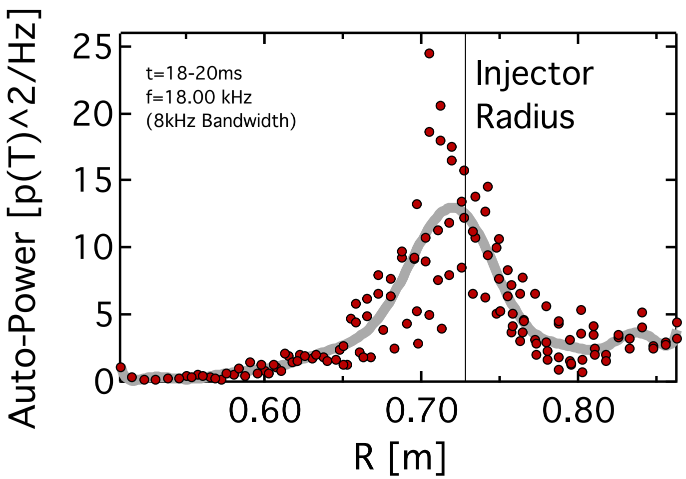


3. NIMROD Simulations Show I_p Growth Resulting from Reconnection in Edge

- Resistive MHD modeling (NIMROD)
- Divertor injection
  - Coherent current streams reconnect
  - Inject current rings
- Qualitative agreement to experiment:
  - Injector-localized MHD
  - Intermittent MHD bursts
  - $\Delta I_p \sim I_{inj}$ jumps
  - Reconnection driven anomalous ion heating observed (M.G. Burke, PP8.00095)

Calculated current rings in NIMROD**





***J. O'Bryan, PhD. Thesis, UW Madison 2014*

Understanding Injector Physics Enables High V_{inj} Operation

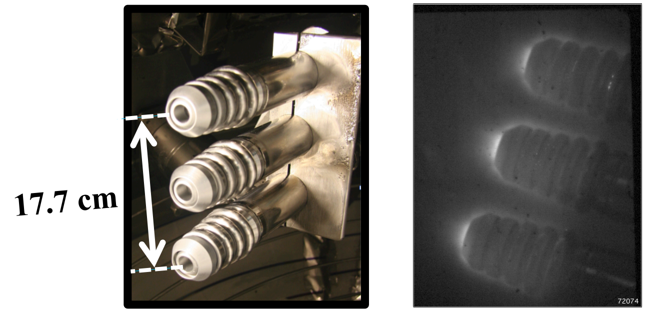
- High-power, low-PMI while immersed in scrape-off plasma
  - Cathode shaping mitigates cathode spots
  - Shield rings, local limiter prevent arc-back

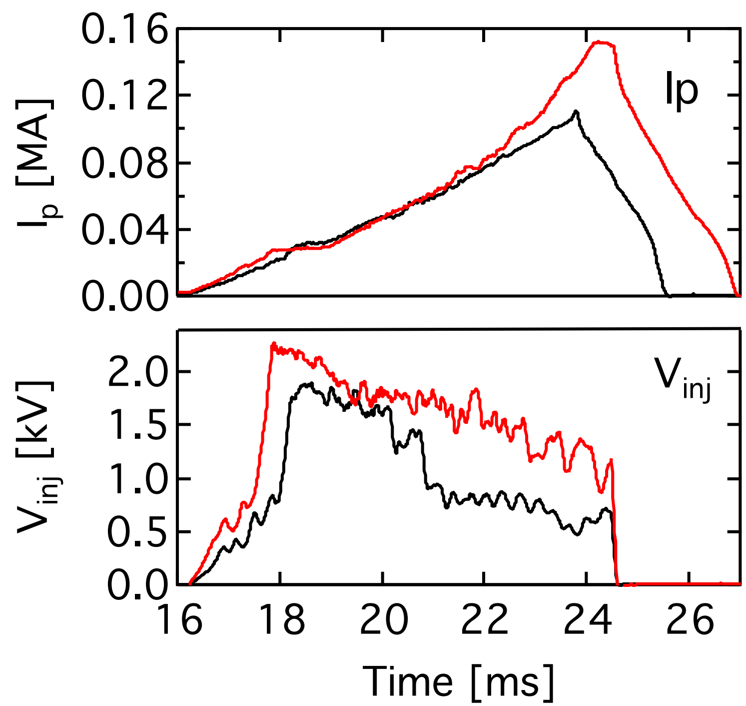
- I_p increases with V_{inj}

- J_{inj}, V_{inj} depend on tokamak scrape-off density
  - Space-charge neutralization of streams

$$J_{INJ} = n_{edge} e \sqrt{\frac{2e}{m_e}} \sqrt{V_{INJ}}$$

Advanced injectors with quiescent operation





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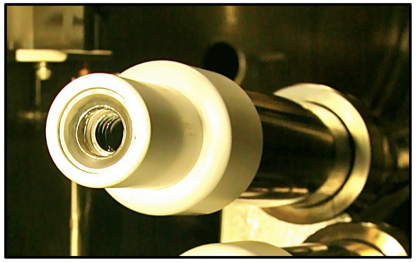
Technology & Science Challenges for NSTX-U & Beyond

- Long-pulse startup (0.1 s)
  - Injector heat load
  - Edge density control
  - Plasma control
- High B_{TF}
  - Confinement scaling
  - Effective injector size
  - Initial tokamak formation
  - Reconnection with high guide field
- Plasma size scaling
- Close fitting wall
  - Potentially complicates relaxation

Injector technology evolving to meet physics challenges

Concave cathode



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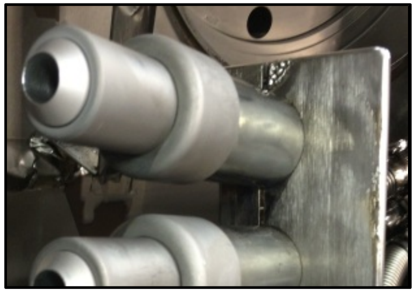
Frustum cathode, local limiter



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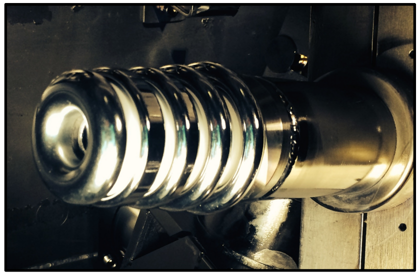
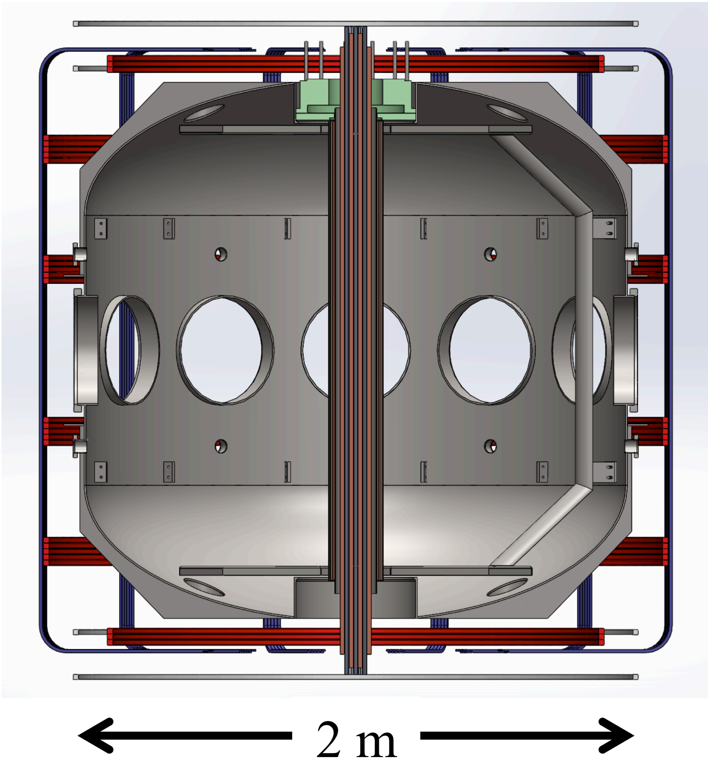
High heat-flux cathode, shield rings



PEGASUS-U to Address Physics, Technology for Scalable LHI Startup

- Long pulse startup (0.1 s)
- New central column (A~1.2)
  - Increased B_{TF} (5x)
  - New OH solenoid, PPPL (6x V-s)
- Enhanced divertor coils
- Upgraded High-V_{LHI} Injectors
  - Remotely insertable
- Core diagnostics:
  - Multipoint TS
  - CHERS via DNB
- Supporting the 5 year plan for NSTX-U

PEGASUS-U



Moving Towards Predictable, MA-Class, Non-Solenoidal Startup

- A hierarchy of models is being developed for LHI startup:
  - Max I_p: helicity conservation, Taylor relaxation
  - $I_p(t)$: 0-D power-balance (future: TSC)
  - Detailed dynamics: resistive MHD (NIMROD)

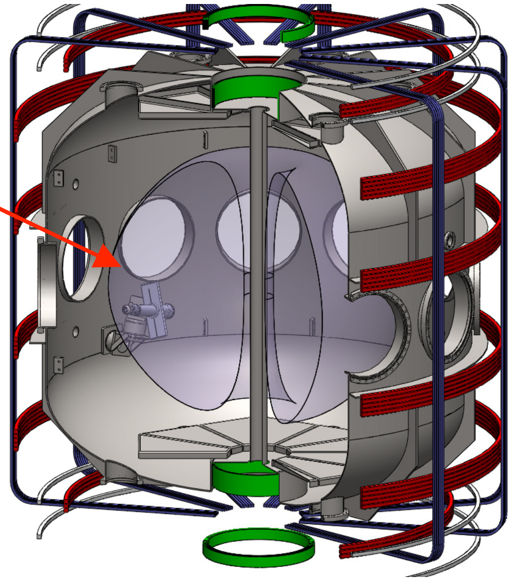
- Outstanding issues:
  - Scaling to high toroidal field, longer-pulse, large size
  - Confinement, stability in LHI drive dominant regime
  - Edge density, J_{inj} control strategies
  - Advanced injector development

- PEGASUS-U will address critical LHI physics issues for NSTX-U

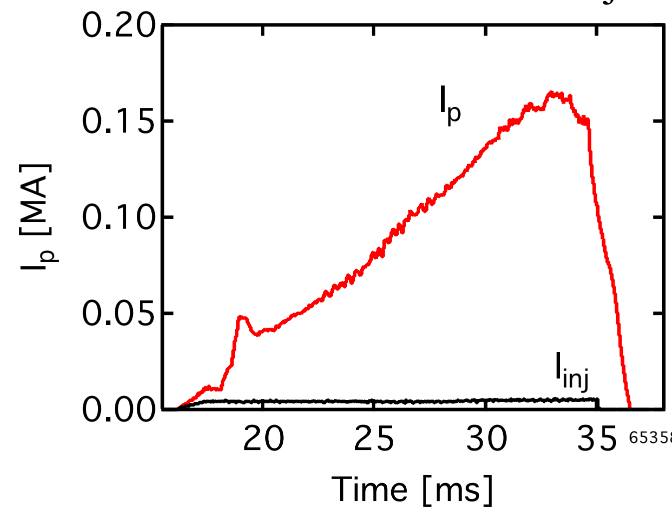
K.E. Thome talk: next this session, GO3
PEGASUS group posters: session PP8

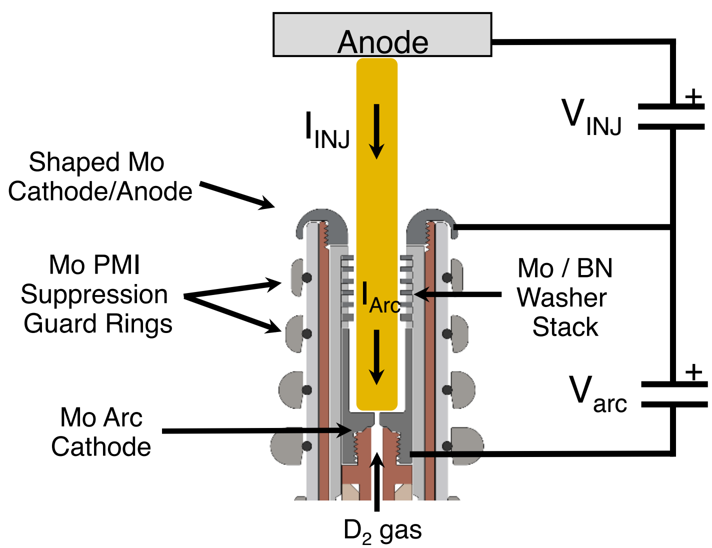
Outboard LHI Provides Robust Startup on the PEGASUS ST

Helicity Injectors



$I_p \leq 0.18$ MA via LHI ($I_{inj} = 5$ kA)



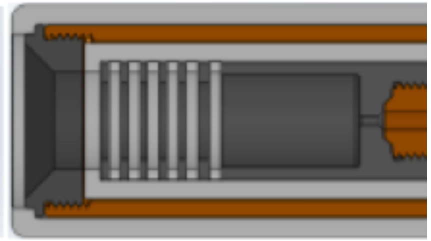


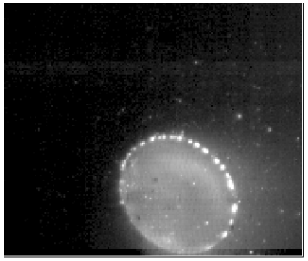
Plasma Parameters		Injector Parameters	
I_p	≤ 0.23 MA	$\sum I_{inj}$	≤ 14 kA
τ_{shot}	≤ 0.025 s	I_{inj}	≤ 2 kA
B_T	0.15 T	V_{inj}	≤ 2.5 kV
A	1.15–1.3	N_{inj}	≤ 4
R	0.2–0.45 m	A_{inj}	$= 2$ cm^2
a	≤ 0.4 m	I_{arc}	≤ 2 kA
κ	1.4–3.7	V_{arc}	≤ 0.5 kV

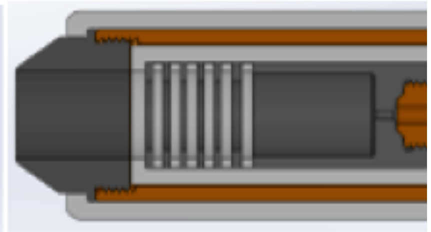
Controlling Plasma-Material Interaction is Enabling High V_{INJ} Operation

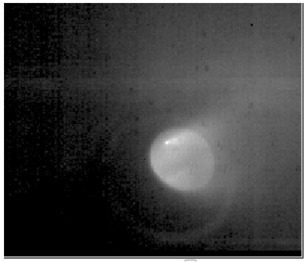
- Injector requirements include
  - Large A_{inj}, J_{inj}
  - $V_{inj} > 1$ kV
  - $\Delta t_{pulse} \sim 10\text{-}100$ ms
  - Minimize PMI

…all adjacent to tokamak LCFS



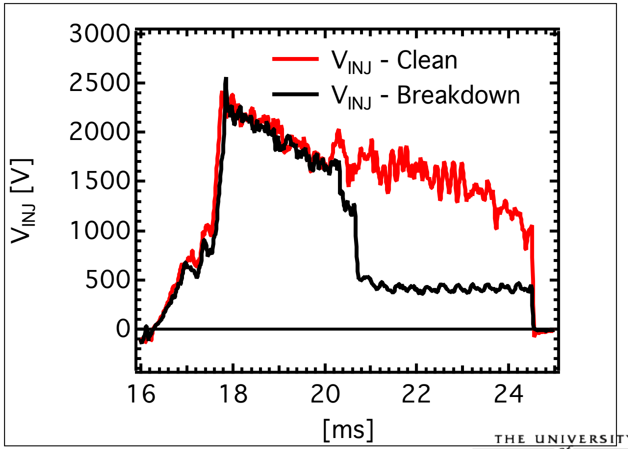






- Advanced injector design enables high V_{inj}
  - Cathode shaping to mitigate cathode spots
  - Shielding of cathode, insulators to prevent injector breakdown
  - 3x improvement in V_{inj}, Δt_{pulse}

Voltage of quiescently operating injectors (red) and voltage after breakdown of overdriven injectors (black)



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Technology & Science Challenges for NSTX-U & Beyond

- Long-pulse startup (100ms)
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  - Edge density control
  - Plasma control
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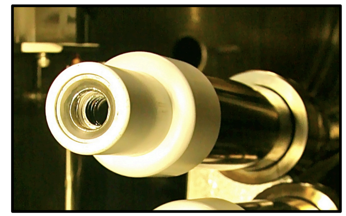
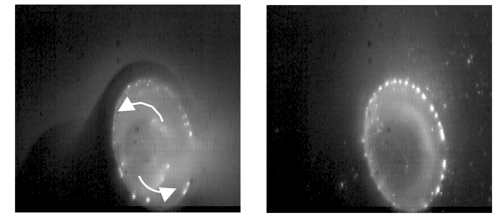
Injector technology evolving to meet physics challenges

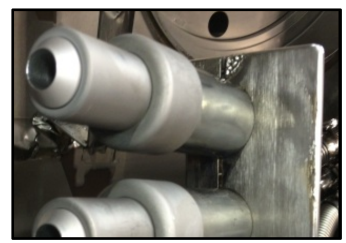
Concave cathode:



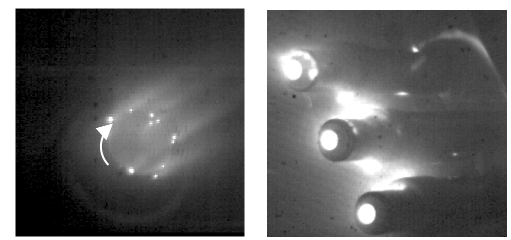
Cathode spots near BN → PMI



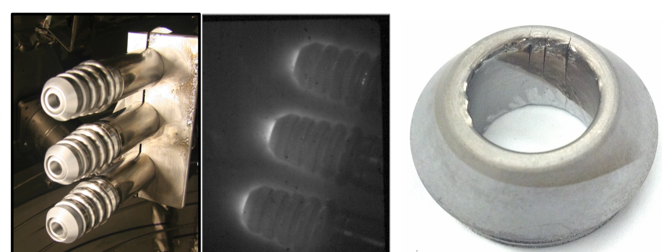
Frustum cathode, local limiter:



Cathode spot control, but arc-back



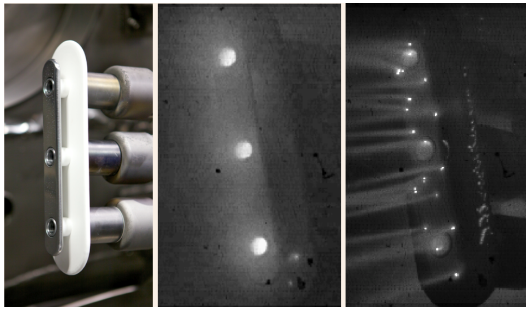
Stepped shielding: Mitigate spots & arc-back, but heat-load cracking



Spore-node, shield rings

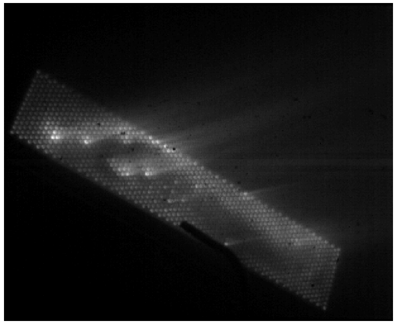
Tests of Various Injector Concepts Favor Smaller A_{INJ}, High V_{INJ}

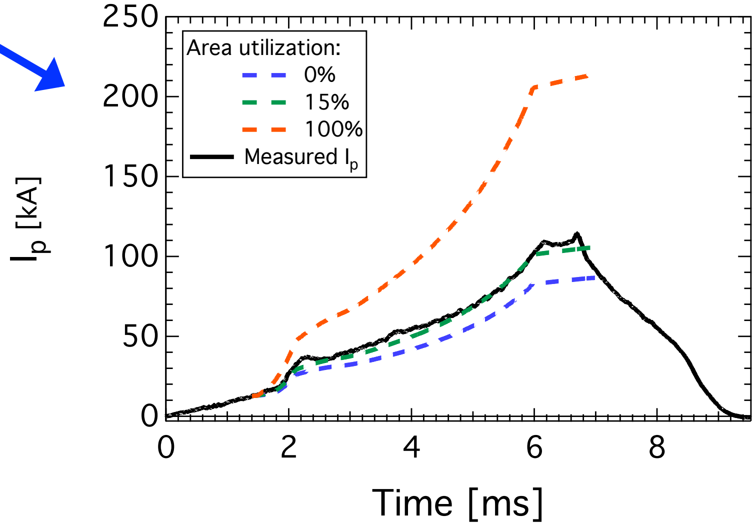
- Tests of large-area passive electrodes show reduced effective A_{INJ}
  - Integrated metallic electrode surface
    - Cathode spot emission



Above: integrated electrode injector assembly

  - Large area gas-effused Mo Plate
    - Hollow cathode emission
    - Non-uniform J_{INJ}, effective A_{INJ} = 0.15 $A_{GEOMETRIC}$

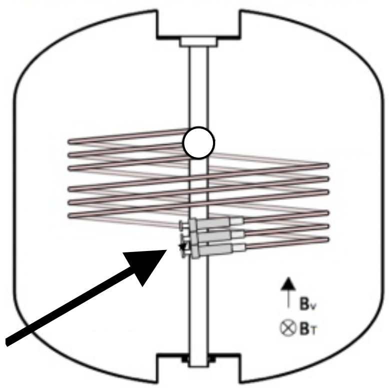




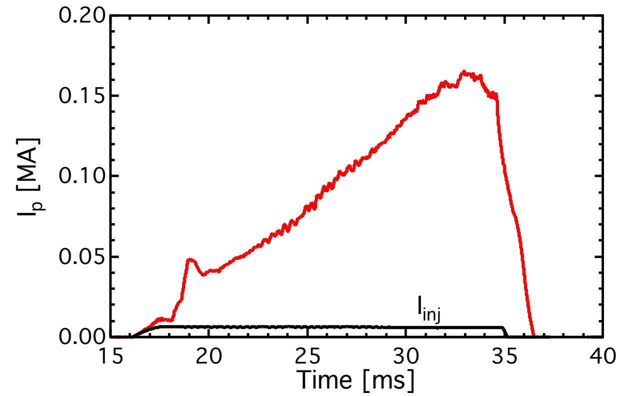
- Conclusion: Arc-based injector system most effective path to maximize HI rate

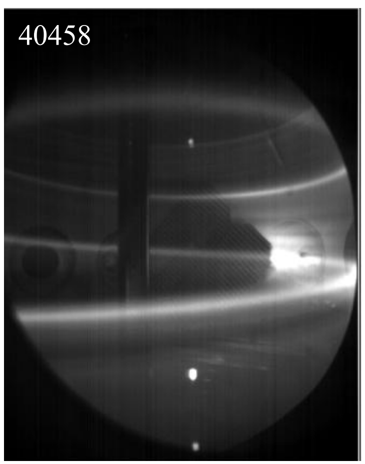
J.L. Barr, APS DPP 2014

LHI Injects Current Streams that Relax, Form Tokamak-Like Plasma



Local Helicity Injectors

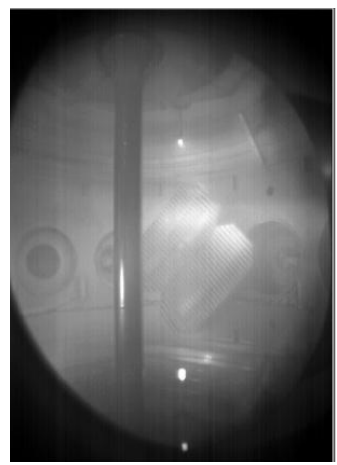




Force-free current streams

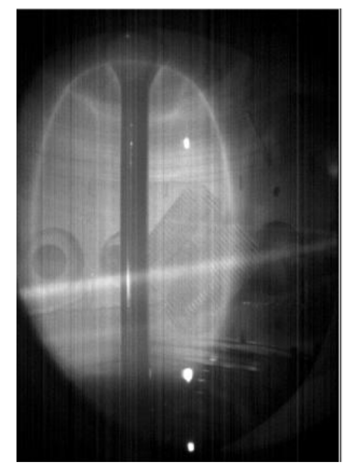
Null Formation

Relaxation



Tokamak-like state

Injector Shutoff



Tokamak forms

Battaglia et al., Nucl. Fusion ***51****, 073029 (2011)*

Helicity Balance, Taylor Relaxation Criteria Set Max Achievable I_p from LHI

Helicity balance in a Tokamak geometry:

$$\frac{dK}{dt} = -2\int_V \eta \mathbf{J}\cdot\mathbf{B}\, d^3x - 2\frac{\partial\psi}{\partial t}\Psi - 2\int_A \Phi\mathbf{B}\cdot d\mathbf{s} \;\Rightarrow\; I_p \le \frac{A_p}{2\pi R_0\langle\eta\rangle}\left(V_{ind} + V_{eff}\right)$$

- Helicity injection provides an effective loop voltage*,**:

$$V_{eff} \approx \frac{A_{inj}B_{\phi,inj}}{\Psi_T}V_{inj}$$

Taylor relaxation of a force-free equilibrium:

$$\nabla\times B = \mu_0 J = \lambda B$$

$$\langle\lambda_p\rangle \le \langle\lambda_{edge}\rangle$$

$$\Rightarrow\; \frac{\mu_0 I_p}{\Psi_T} \le \frac{\mu_0 I_{inj}}{2\pi R_{inj} w B_{\theta+V,inj}} \;\overset{**}{\Rightarrow}\; I_p \le \sqrt{\frac{1}{B_{\theta+V,inj}/I_p}\frac{\Psi_T I_{inj}}{2\pi R_{inj} w_{inj}}}$$

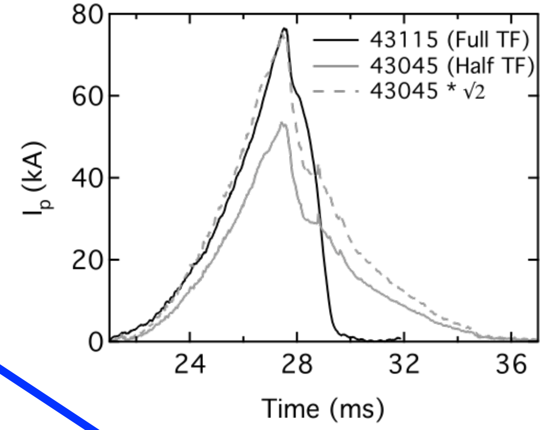
A_p	Plasma area	Ψ_T	Plasma toroidal flux
w_{inj}	Edge current channel width	$B_{\theta+V,inj}$	Poloidal field in injection region

*D.S. Darrow, *et al. Physics of Fluids B: Plasma Physics*, 2(6): 1415, 1990.

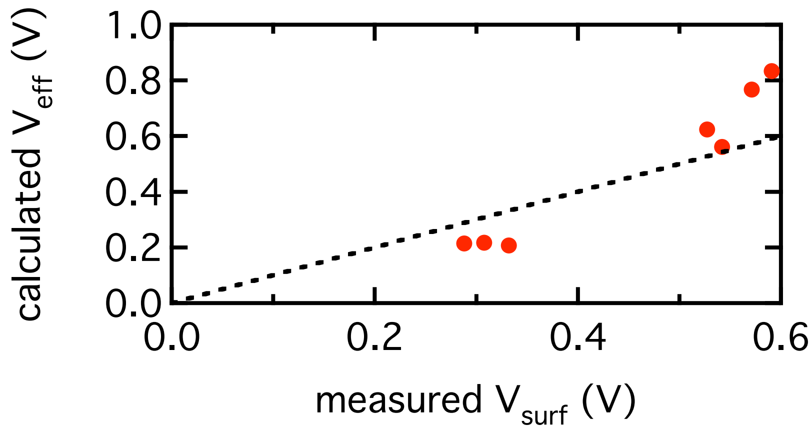
**D.J. Battaglia. *Non-solenoidal tokamak startup using localized current sources near the outboard midplane*. PhD thesis, UW-Madison, 2009.

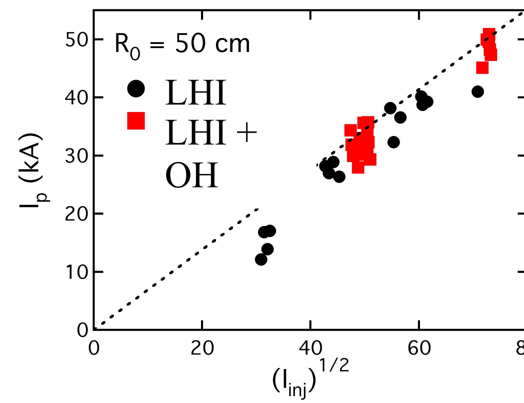
Global Helicity Balance, Taylor Relaxation Limits Confirmed

- At Taylor Limit: $I_p^{max} \propto \sqrt{I_{INJ}}, \sqrt{I_{TF}}, w^{-1/2}$



- V_{eff} agreement over limited data set

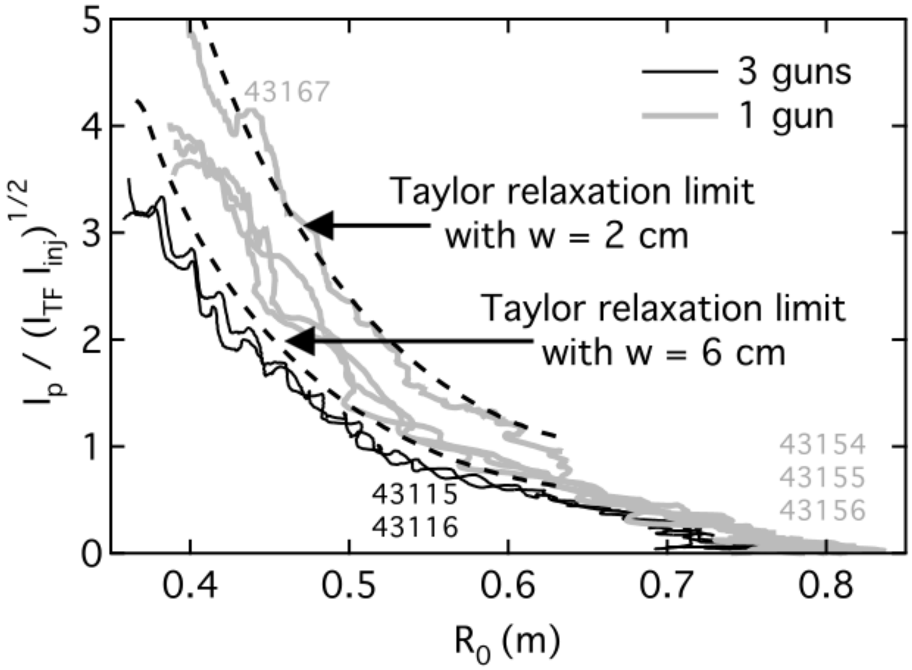


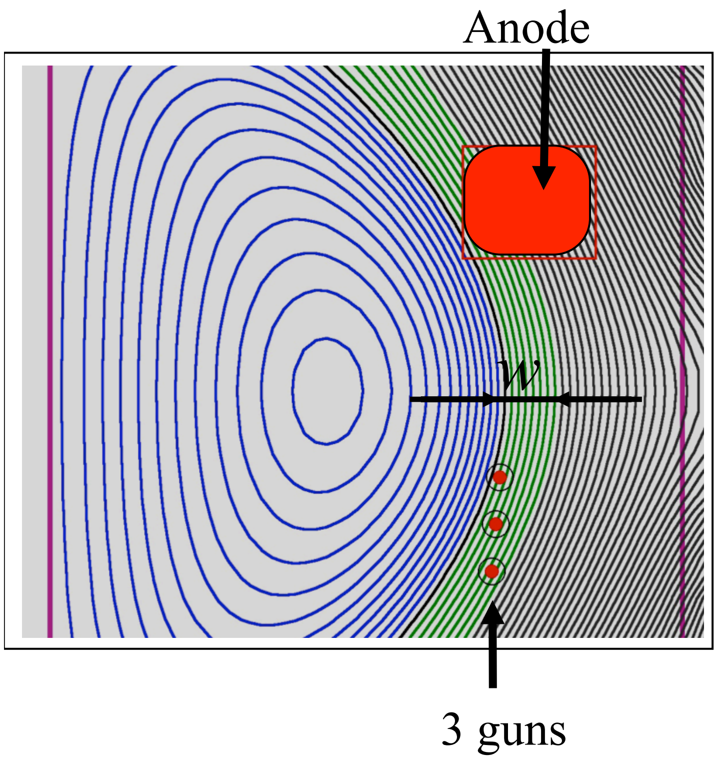


D.J. Battaglia, et al. *Nucl. Fusion* **51** (2011) 073029.
N.W. Eidietis. *Non-inductive startup of the Pegasus Spherical Torus using localized washer-gun current source,* Ph.D. Thesis, UW-Madison, 2007.

Experiments demonstrate dependence on the width of the driven current layer

- Relaxation current limit scales as $w^{-1/2}$
- One-gun discharges had higher limits than corresponding three-gun cases, indicating the gun array was misaligned:

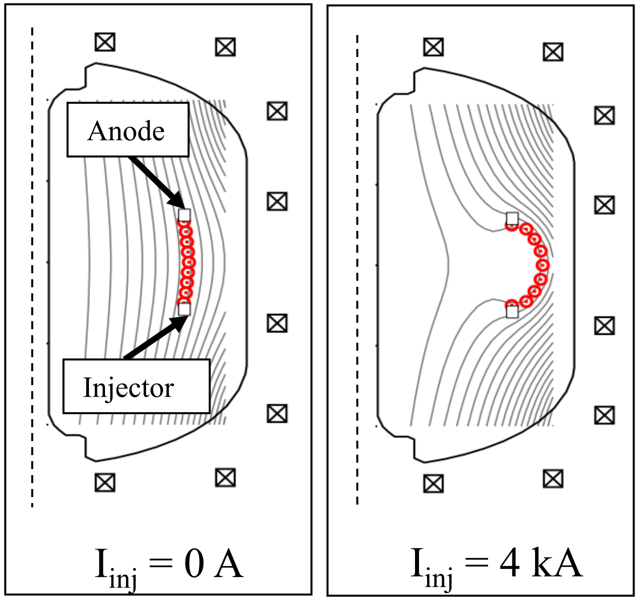




Internal Measurements Show Null Formation, J(R,t) Throughout LHI Discharge Evolution

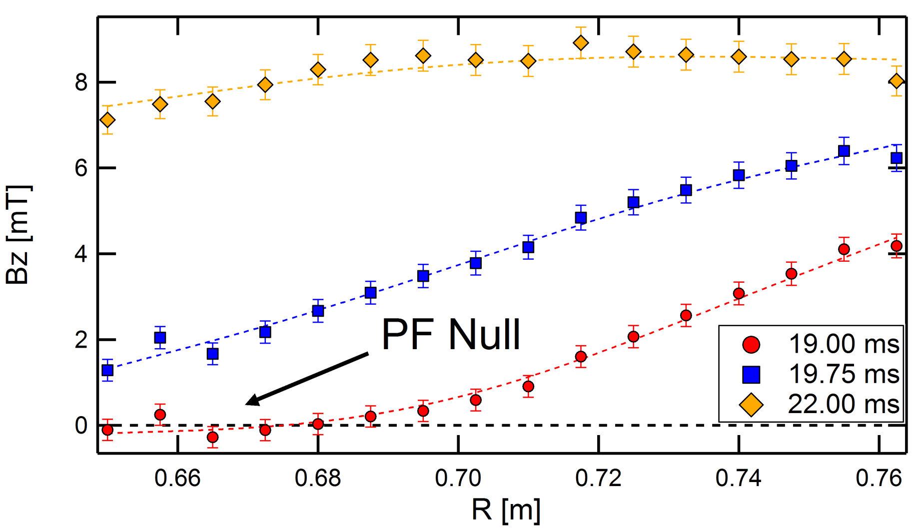
- Initial relaxation to tokamak-like topology coincident with inboard null formation
  - Injected current filaments perturb vacuum **B**
  - B_z must be sufficiently low and/or I_{inj} sufficiently high for null to form

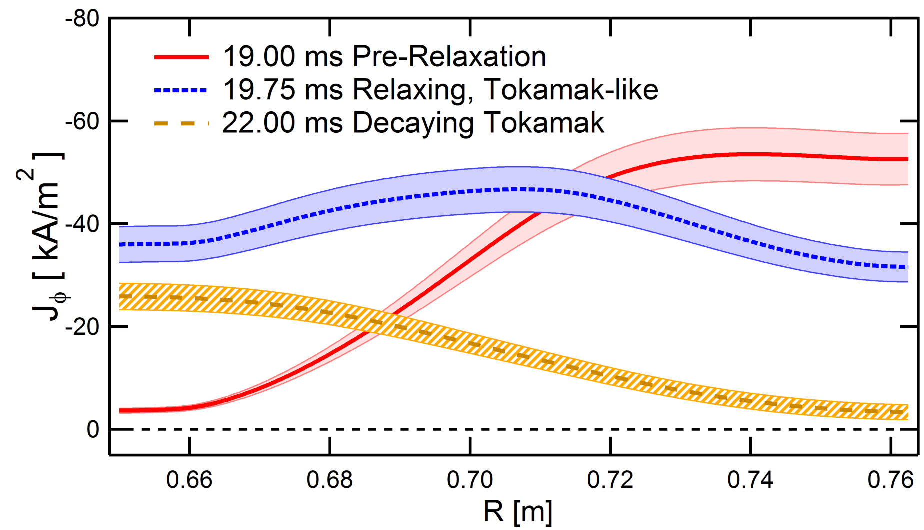
- Hall probe* $B_Z(R)$ provides $J_\phi(R)$ evolution
  - Predicted field null observed



2-D force free current model

*: Bongard *et al.*, Rev. Sci. Instrum. **81**, 10E105 (2010)





2. When $I_p(t)$ < Taylor Limit 0-D Power Balance Model Predicts $I_p(t)$

- Lumped parameter model + helicity conservation:

$$I_p\left[V_{eff} + V_R + V_{PF} + V_{Lp}\right] = 0$$

  - V_{eff}: From helicity injection
  - V_R: Resistive dissipation from assumed flat Spitzer $T_e(R,t) = 70\text{eV}$
  - V_{PF}: Poloidal induction voltage
  - V_{Lp}: Voltage due to plasma self-inductance

- Inputs: $R_0(t)$, $a(t)$, $Ip(t_0)$, $\langle\eta_0\rangle$, $\kappa(t)$, ℓ_i (t)
  - Analytic low-A descriptions of L_p, B_z, plasma shape

- Differential equation in $I_p(t)$ solved when $I_p(t) < I_{Taylor}(t)$

Poynting's Theorem Applied at Plasma Boundary Defines Current Sources & Sinks

$$I_p V_s = \iiint \frac{\partial}{\partial t}\left(\frac{B_\theta^2}{2\mu_0}\right) dV + I_p^2 R_p - I_p V_{NICD}$$

Plasma surface voltage

– Inductive drive from OH, PF-ramping

– Self-inductance: contribution from *external* fields

$$V_s = -\frac{\partial}{\partial t}\left(\psi_{OH} + \sum_i \psi_{PF,i} + L_e I_p\right)$$

Internal magnetic energy

– Self-inductance: contribution from *internal* fields, static boundary

$$\frac{1}{I_p}\frac{\partial}{\partial t}\left(W_{B,p}\right) = \frac{1}{I_p}\frac{\partial}{\partial t}\left(\frac{1}{2} L_i I_p^2\right)$$

Resistive dissipation

– Uniform, constant Spitzer resistivity assumed

$$V_R = I_p R_p = I_p\left(\frac{\langle \eta_p \rangle 2\pi R_0}{A_p}\right)$$

Non-inductive current drive

– Local helicity injection

$$V_{NICD} = V_{eff} \approx \frac{A_{inj} B_{\phi,inj}}{\Psi_T} V_{inj}$$

*J.A. Romero and JET-EFDA Contributors 2010 Nucl. Fusion **50** 115002*
*S.P. Hirshman and G.H. Nielson 1986 Phys. Fluids **29** 790*
*S. Ejima et al 1982 Nucl. Fusion **22** 1313*

Plasma Self-Inductance Modeled with Analytic, Low-A Approximations

- Plasma self-inductance is partitioned into internal, external components:

$$L_p = L_i + L_e$$

- External component L_e is heavily aspect-ratio dependent:

$$L_e = \mu_0 R_0 \frac{a(\varepsilon)(1-\varepsilon)}{1-\varepsilon+\kappa b(\varepsilon)}$$

where $\varepsilon = \dfrac{1}{A}$

$$a(\varepsilon) = \left(1+1.81\sqrt{\varepsilon}+2.05\varepsilon\right)\ln\left(\frac{8}{\varepsilon}\right) - \left(2.0+9.25\sqrt{\varepsilon}+1.21\varepsilon\right)$$

$$b(\varepsilon) = 0.73\sqrt{\varepsilon}\left(1+2\varepsilon^4-6\varepsilon^5+3.7\varepsilon^6\right)$$

*S.P. Hirshman and G.H. Nielson 1986 *Phys. Fluids* **29** 790

- Internal component decreasing in time (typically ℓ_i = 0.5→0.2):

$$W_{B,p} = \iiint_{V_p} \frac{B_\theta^2}{2\mu_0} dV = \frac{1}{2} L_i I_p^2 = \frac{1}{2}\left(\frac{\mu_0 V_p}{C_p^2}\ell_i\right) I_p^2$$

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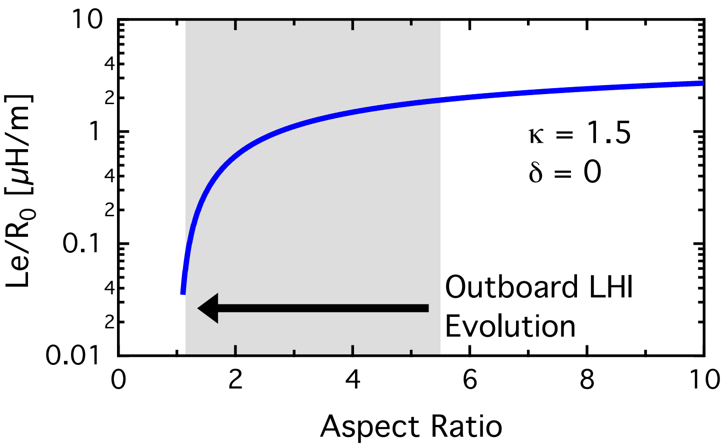

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J.L. Barr, APS DPP 2014

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Low-A, Analytic Models Approximate V_{PF}

- Applied vertical field provides force-balance and inductive loop-voltage
  - ψ_{PF} estimated with Hirshman & Nielson* mutual inductance-like M_v equation

$$V_{PF} = \frac{\partial}{\partial t}\left(\sum_i \psi_{PF,i}\right) \approx \frac{\partial}{\partial t}\left[M_V \pi R_0^2 \, B_V\big|_{R_0}\right]$$

where $$M_V(\varepsilon,\kappa) = \frac{(1-\varepsilon)^2}{(1-\varepsilon)^2 c(\varepsilon) + d(\varepsilon)\sqrt{\kappa}}$$

$$c(\varepsilon) = 1 + 0.98\varepsilon^2 + 0.49\varepsilon^4 + 1.47\varepsilon^6$$

$$d(\varepsilon) = 0.25\varepsilon\left(1 + 0.84\varepsilon - 1.44\varepsilon^2\right)$$

*S.P. Hirshman and G.H. Nielson 1986 *Phys. Fluids* **29** 790

- B_v required for force-balance is aspect-ratio and shape dependent
  - Uses Mitarai & Takase* formula for B_v for force-balance at low-A with κ:

$$B_V = \frac{\mu_0 I_p}{4\pi R_0}\left\{\frac{1}{\mu_0}\frac{\partial L_e}{\partial R} + \frac{\ell_i}{2} + \beta_p - \frac{1}{2}\right\}$$

*O. Mitarai and Y. Takase 2003 *Fusion Sci. Technol.*

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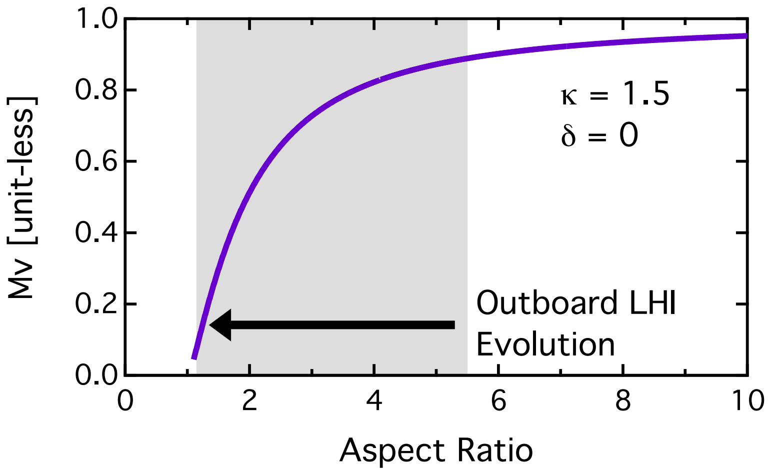

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*O. Mitarai and Y. Takase 2003 *Fusion Sci. Technol.*

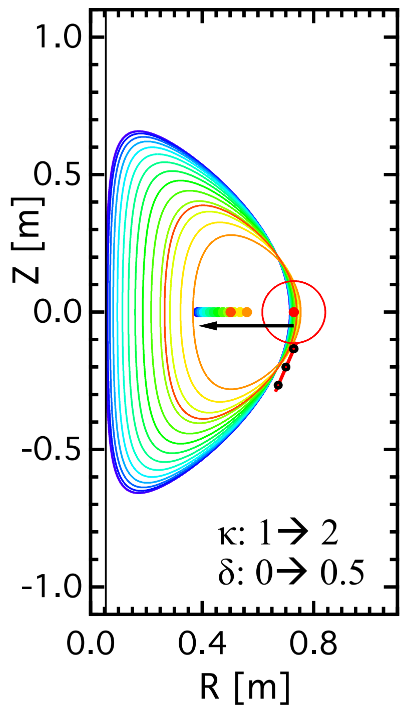
System Represented by ODE Initial Value Problem

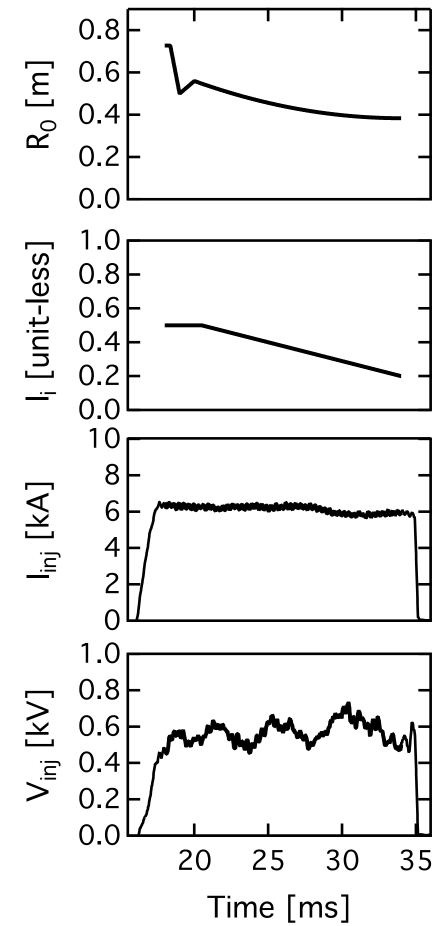
$$\frac{\partial I_p}{\partial t} = -\frac{\frac{1}{2}\frac{\partial L_i}{\partial t} + \frac{\partial L_e}{\partial t} + R_p + \frac{\partial}{\partial t}\left[M_V \pi R_0^2 \left(\frac{B_v}{I_p}\right)\right]}{L_p + M_V \pi R_0^2 \left(\frac{B_v}{I_p}\right)} I_p + \frac{V_{OH} + V_{eff}}{L_p + M_V \pi R_0^2 \left(\frac{B_v}{I_p}\right)}$$

- At initial relaxation to tokamak-like state:
  - Initial value from Taylor limit
- If $I_{p,\text{ Pow. Bal.}} > I_{p,\text{ Taylor}}$: follow Taylor limit
- Else: follow power-balance

Plasma Parameters Constitute Power-Balance Inputs

- $I_p(t_0)$
  – Initial condition to DE solver
- Shape
  – $R_0(t)$, $a(t)$, $\kappa(t)$, $\delta(t)$
- Plasma parameters
  – $\langle\eta\rangle$ assumed constant, Spitzer
  – $\beta_p = 0$
  – ℓ_i: 0.5 → 0.2
- LHI parameters
  – $A_{inj}(t)$, $V_{inj}(t)$





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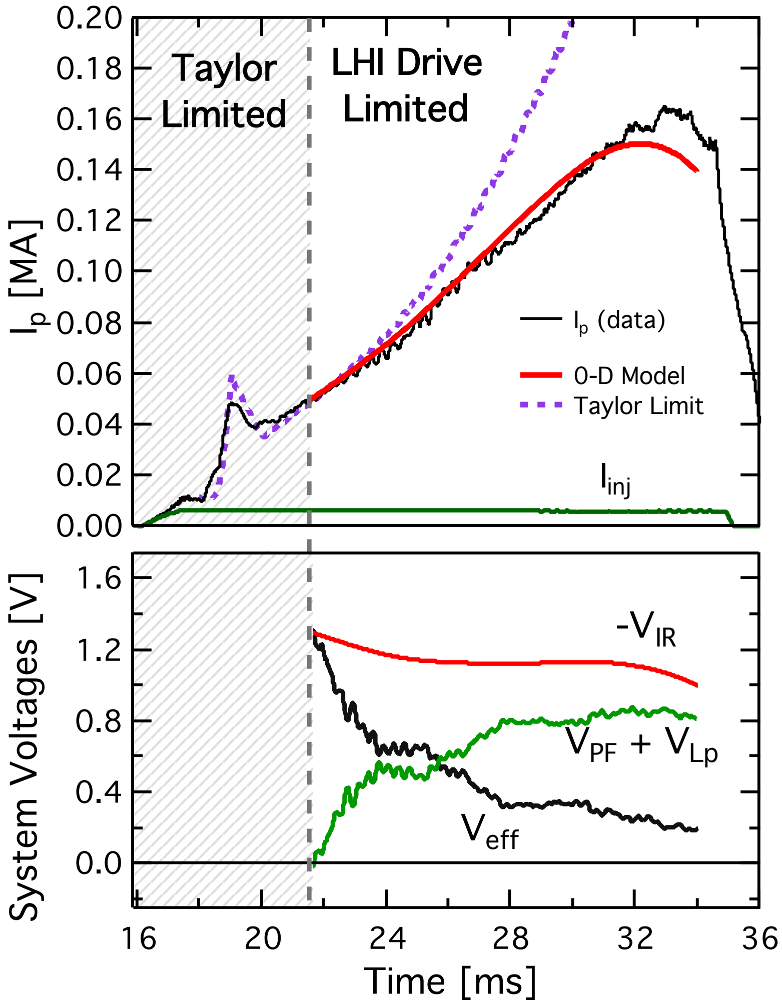
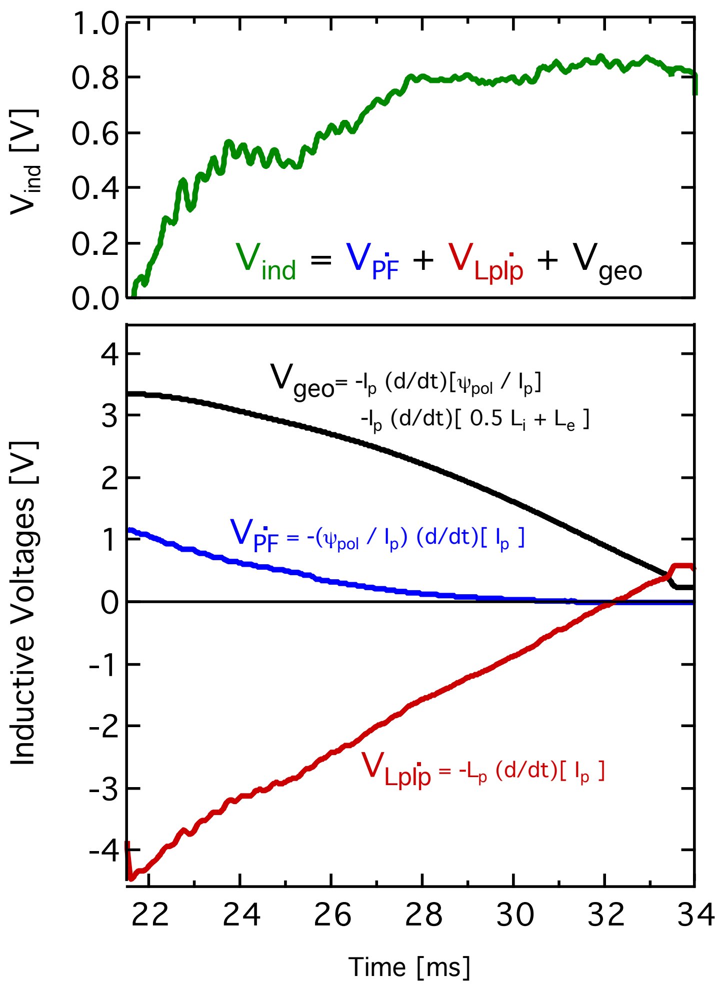
LHI Plasmas Undergo Two-Phase $I_p(t)$ Evolutions



- Low I_p: Taylor limited
  - Set by plasma geometry, I_{inj}
- Higher I_p: power-balance
  - Balance of LHI, inductive effects, resistive losses

Using 0-D Model as Interpretive Tool Gives Insight into LHI Current Dynamics



- Inductive voltages dominated by geometric evolution

- Geometry evolution ~ 3 V
- PF ramping ~ 1 V
- Inductive reactance ~ 4 V

Testing and Calibration of the Model Proceeding on Multiple Fronts

$$I_p V_s = \frac{\partial}{\partial t}\left(W_{B,p}\right) + I_p^2 R_p - I_p V_{NICD}$$

- $I_p(t)$ compared for various plasma evolutions

Plasma surface voltage

$$V_s = -\frac{\partial}{\partial t}\left(\psi_{OH} + \sum_i \psi_{PF,i} + L_e I_p\right)$$

Internal magnetic energy

$$\frac{1}{I_p}\frac{\partial}{\partial t}\left(W_{B,p}\right) = \frac{1}{I_p}\frac{\partial}{\partial t}\left(\frac{1}{2} L_i I_p^2\right)$$

- Approximations for $\psi_s(t)$, $W_m(t)$, $L_i(t)$ calibrated to experiment

Resistive Dissipation

$$V_R = I_p R_p = I_p\left(\frac{\eta_p 2\pi R_0}{A_p}\right)$$

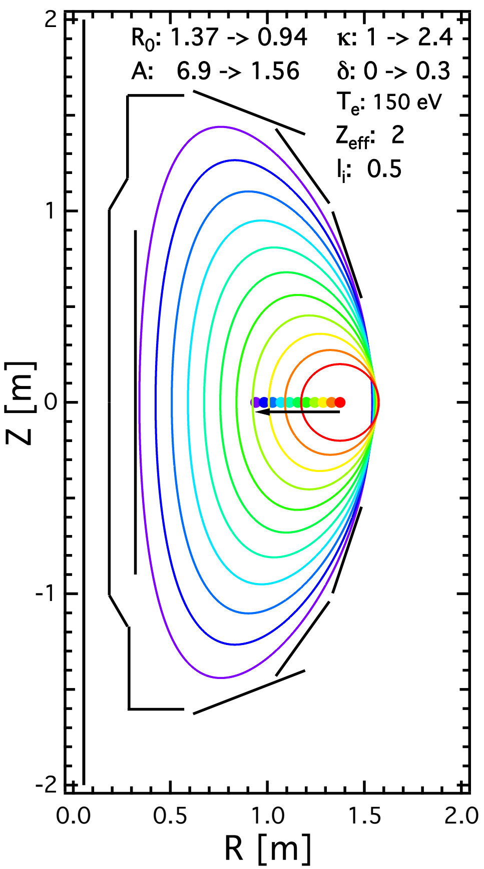
- Thomson Scattering
- TF variations

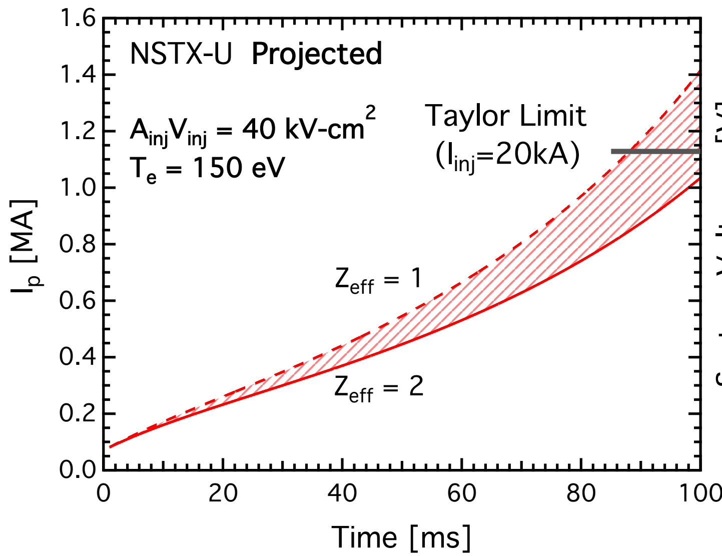
Non-inductive current drive

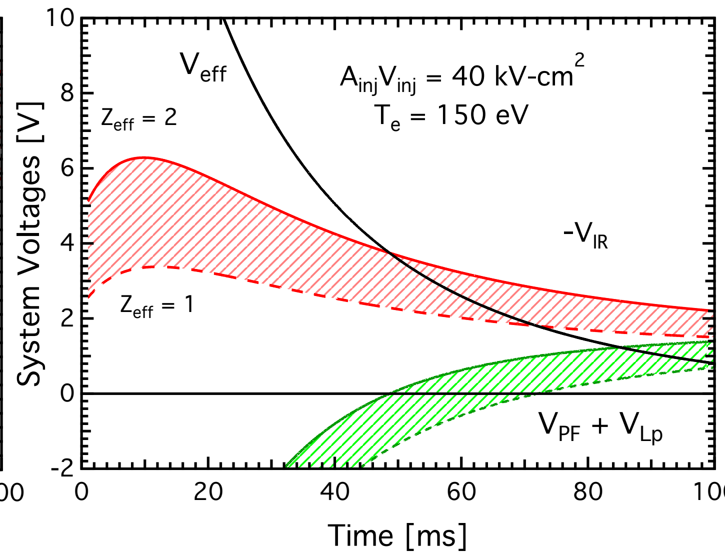
$$V_{NICD} = V_{eff}$$

- Upgraded injector systems to vary V_{eff}

Model Applied to NSTX-U Geometry for Initial I_p~1 MA Start-up Scenario Prediction







- Significant LHI drive required to achieve I_p ~ 1 MA
  - Predicted A_{inj} V_{inj} requirement estimates: ~ 40 cm^2-kV

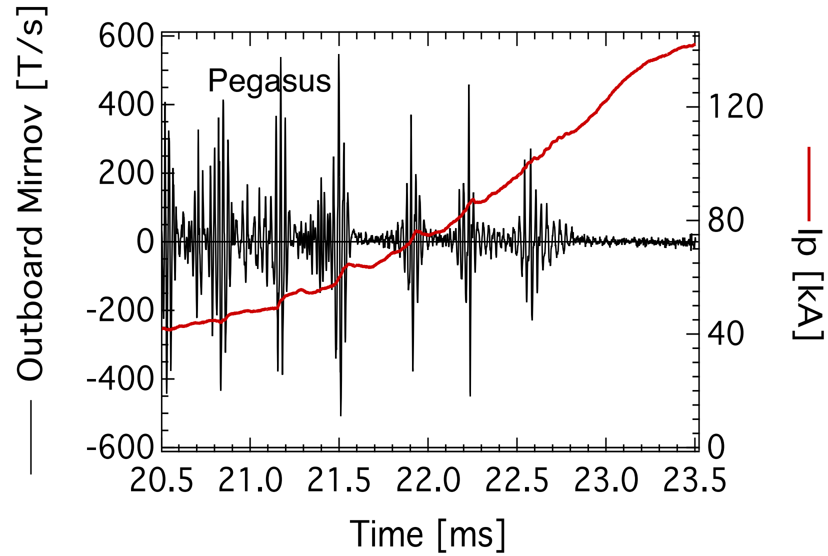
*C. Neumeyer *et all* (2009) *23rd IEEE/NPSS Symposium on Fusion Engineering*

**C. Neumeyer (2001) "NSTX Internal Hardware Dimensions"
http://nstx.pppl.gov/nstx/Engineering/NSTX_Eng_Site/Technical/Machine/NSTX_Engr_Machine_Dims_cm.html

Current Growth During LHI Correlated with Bursts of MHD Activity

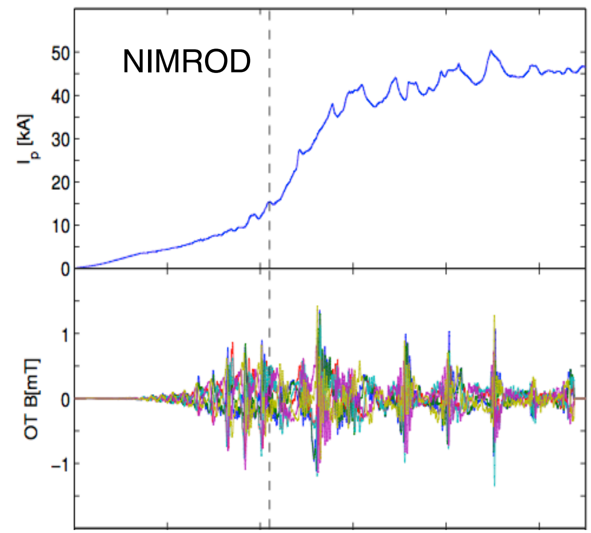
Measured burst properties include

- Two primary spectral components
  - n=1 ⇨ 10-20kHz @ R_{inj}, line-tied
  - n=0 ⇨ <5 kHz, plasma motion
- Typically correlates with sharp I_p increase



NIMROD simulations produce bursty MHD

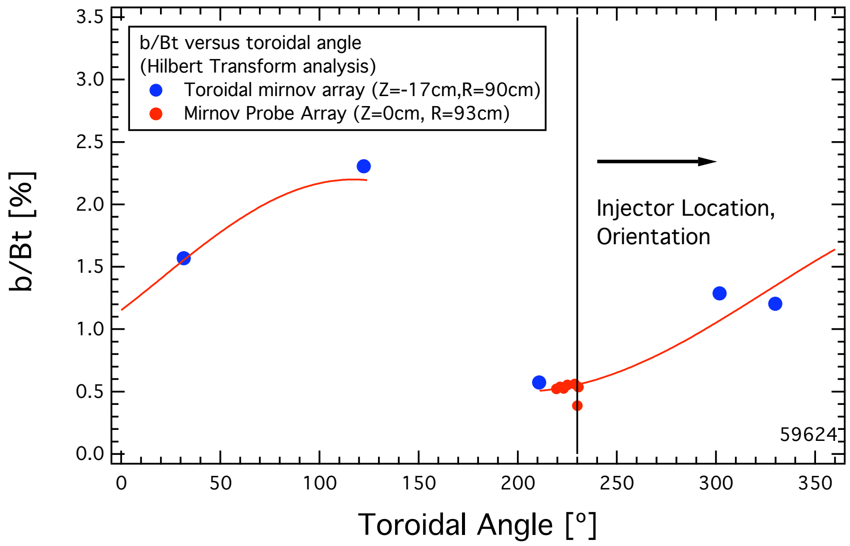
- Bursts from transient reconnection events
- Qualitative agreement with experiment

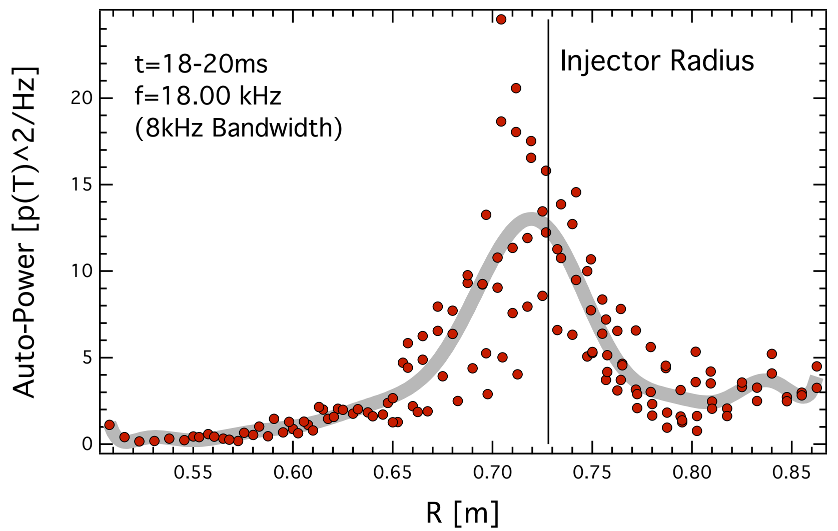


Battaglia *et al.*, Nucl. Fusion **51**, 073029 (2011)

Bursts show n=1

- Current multiplication, transport accompanied by MHD activity
- Two common spectral features
  - High-frequency 10–20 kHz n = 1
  - Low-frequency < 5 kHz n = 0
- n = 1 mode consistent with line tying
  - Activity localized near injector radius
  - Toroidal asymmetry in
- n = 0 localized to plasma interior
  - Inward radial motion



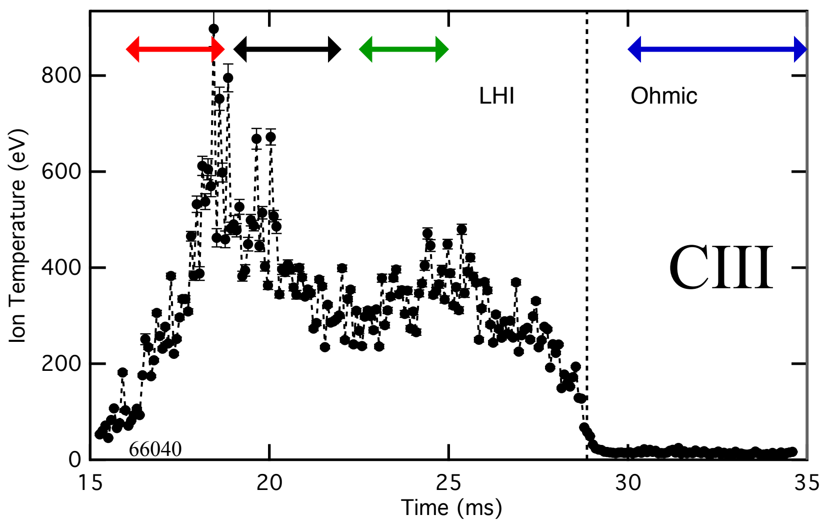


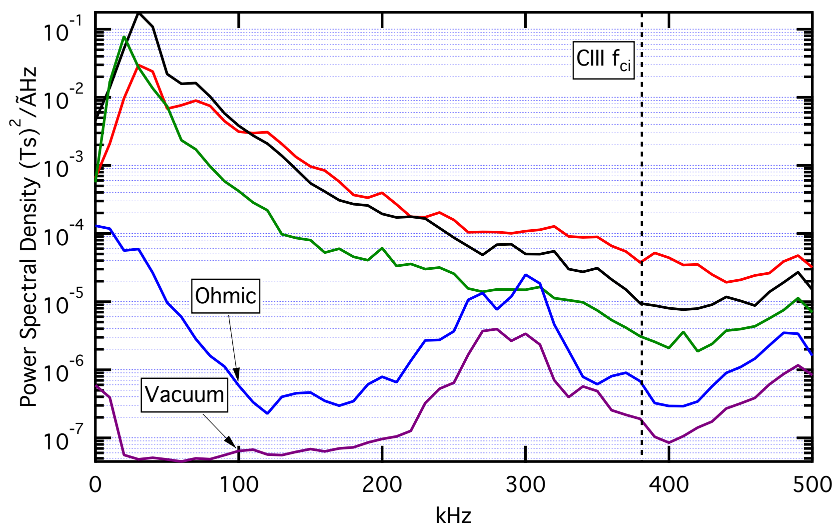
Barr *et al.*, APS-DPP 2012

Potential Reconnection-Induced Impurity Ion Heating Observed during LHI

- NIMROD shows magnetic reconnection during LHI
  - Ion heating commonly observed in reconnection experiments*

- Consistent with ion cyclotron heating mechanism**
  - Pegasus LHI MHD spectra show significant power in IC resonance region





*: Magee, *et al*. Phys. Rev. Lett. **107,** 065005 (2011)
**: Tangri

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Predictive Understanding of Plasma Impedance Required for Projecting to Higher I_p

- Determines feasible V_{INJ}, A_{INJ}, I_{INJ} and demands on power system

- Governed by plasma physics of arc source and tokamak edge
  - Low I_{INJ}: Double layer $J_{INJ} \sim V_{INJ}^{3/2}$
  - High I_{INJ}: Space charge neutralization of e-beam by edge plasma

$$J_{INJ} = n_{edge} e \sqrt{\frac{2e}{m_e}} \sqrt{V_{INJ}}$$

- n_{edge} dependence to be validated
  - Assuming $n_{edge} \sim$ fill pressure

Ramp-up I-V characteristics for Pegasus injectors agrees with 2-parameter model across wide range of fill pressures