The 3D Structure of Flux Tubes That Admits Flute Instability in the Scrape-Off-Layer (SOL) of Tokamaks

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ABSTRACT

The 3D Structure of Flux Tubes That Admits Flute Instability in the Scrape-Off-Layer (SOL) of Tokamaks, HIRONORI ('HIRO') TAKAHASHI, PPPL – A severe reduction in size down to an ion gyro-radius scale, commonly known as "squeezing," in a lateral dimension of the cross section of a flux tube is traditionally thought to inhibit the occurrence of the flute instability in the Scrape-off-Layer of a diverted tokamak by isolating the main volume of the flux tube from its ends at electrically conducting target plates. A study reported here in the 3D flux tube structure reveals the absence of squeezing for a flux tube that is sufficiently large in its toroidal extent (small toroidal harmonic number n) and located in a layer of low field-line shear around the "sweet spot" (about mid-way between the primary and secondary separatrices). The low-shear layer does not hence inhibit the flute instability through the squeezing mechanism, and may thus restore the flute instability, among the most virulent in the magnetized plasma, to the ranks of candidate electrostatic instabilities thought to underlie the turbulence in the SOL in tokamaks. Variations along the flux tube of geometrical characteristics including the cross section will be calculated to develop criteria for the absence of squeezing.

Highlights

Does 3D Flux Tube Geometry 'Squeeze' out Flute Instability as a Turbulence-generating Mechanism in Tokamak SOL?

This work revisits the long-held and widely accepted notion that a sever reduction in the flux tube cross section known as "squeezing" inhibits flute instability, among the most virulent of electrostatic instabilities, removing it from among mechanisms thought to underlie turbulence in the tokamak SOL.

Flux Tube Geometry as Instability Inhibitor

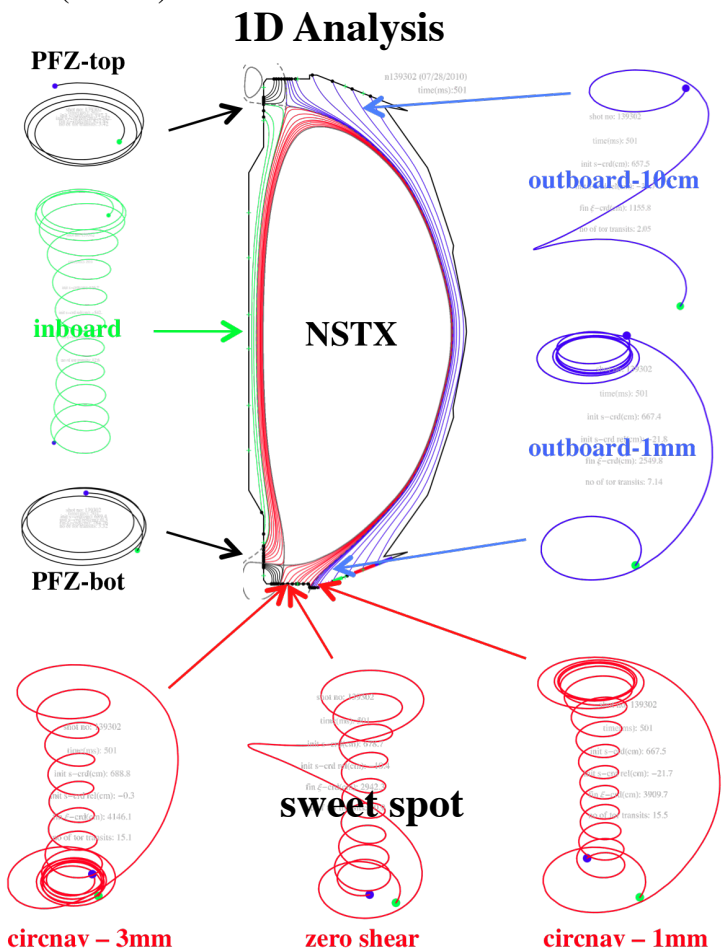
A volume in the SOL in an outboard mid-plane region, harboring conditions conducive to driving flute mode unstable, may prescribe a 3D flux tube extending to limiter tiles. But the stability of flute mode depends critically on the conditions at the flux tube's ends, where field-aligned currents of an opposite polarity come together. For example, Berk, Ryutov, and Tsidulko (BRT)(PoP '91) described temperature-gradient instability induced by conducting end walls that may be relevant to flux tubes in the tokamak SOL.

However, Farina, Pozzoli, and Ryutov (FPR)(NF '93) concluded that the flux tube geometry in the tokamak SOL inhibits flute instability because a severe reduction due to field line shear in a flux tube cross-sectional dimension down to an ion gyro-radius scale effectively isolates the main flux tube volume from its end walls. This view has profoundly influenced the development of the study of electrostatic turbulence in the tokamak SOL over the following two decades. The present work revisits the subject studied in this impactful article.

STRUCTURE
OF
FIELD LINES (1D),
FIELD RIBBONS (2D),
AND
FLUX TUBES (3D)

Primary and Secondary Separatrices Demarcate SOL

#139302 (501 ms)



A field line executes a large number of toroidal transits, if it passes near an x-point – primary or secondary.

Primary and secondary separatrices demarcate the SOL volume into five distinct non-communicating zones:

(i) Poloidally Circumnavigating Zone
(ii) Outboard Zone
(iii) Inboard Zone
(iv) Private Flux Zone - Top
(v) Private Flux Zone - Bottom.

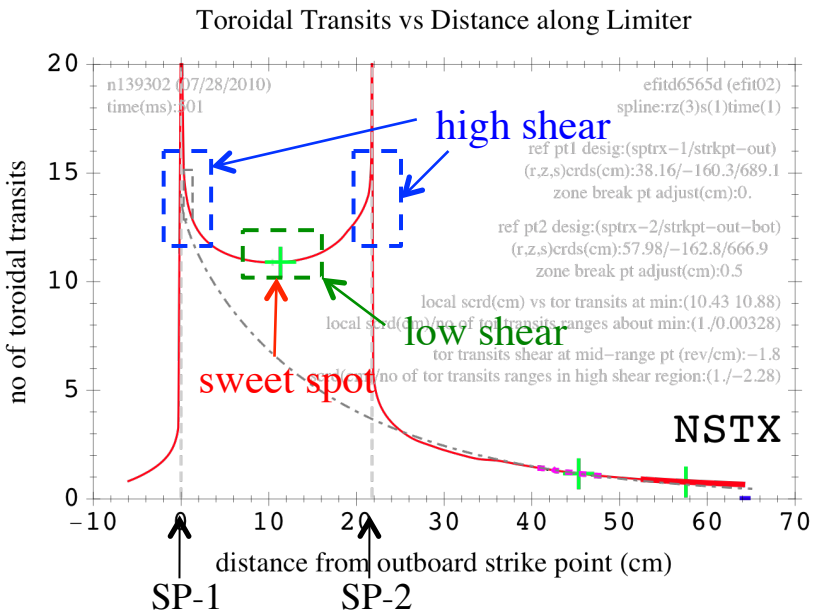
Field line geometry differs qualitatively in these zones. A flux tube that straddles a zone boundary has a complex geometry, splitting into multiple parts that end in different divertors and non-contiguous toroidal areas – an issue usually skirted around in the literature by considering part of flux tube only near the outboard mid-plane.

Four flux tubes will be discussed, a pair of them with a large and small toroidal span in the outboard and circumnavigating zones.

Adapted from APS '11

High-Shear, Low-Shear, and "Sweet Spot"

Toroidal Transits Executed by a Field Line



Number of toroidal transits that a field line executes, as it travels poloidally from a start point on a tile surface to an end point on another tile surface, is plotted as a function of the start-point distance measured along the limiter surface from the bottom outboard strike point of the primary separatrix. The plot covers three zones, bottom PFZ (d < 0), circumnavigating (0 < d < ~ 22 cm), and outboard (~ 22 < d cm) zones (see previous slide).

The rate of change in the number of toroidal transits for the varying location of field-line starting point is the (field-line-averaged) "shear" in analogy to the (flux-surface-averaged) shear in the safety factor for field lines inside the main plasma.

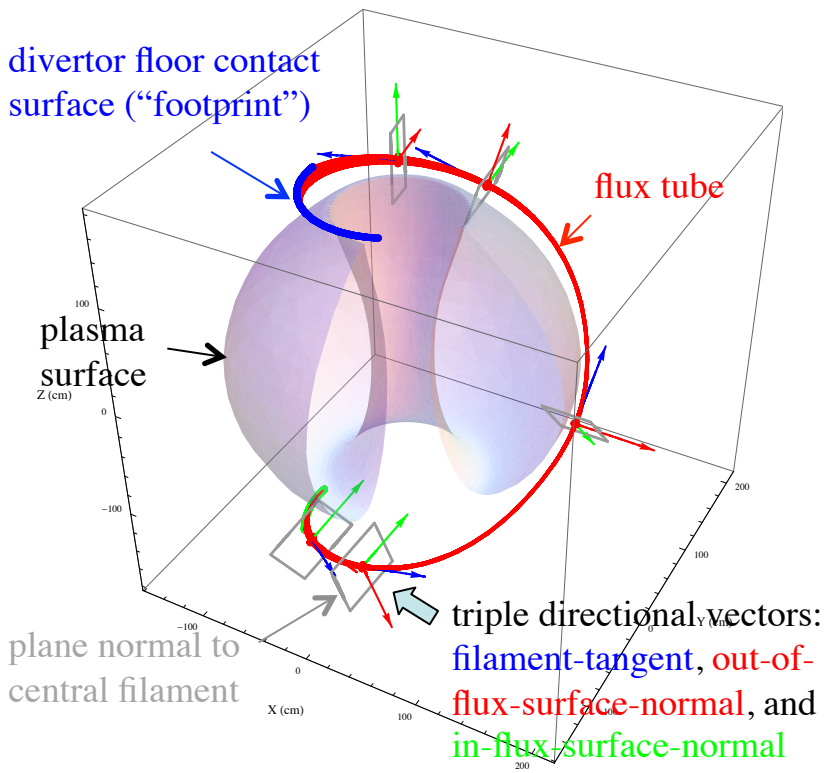
The number of transits does not decrease monotonically (as dotted exponential line) because of the presence of a *secondary separatrix*.

Instead, the number of transits possesses two regions of high shear (rapid variations with respect to the starting-point distance) near the primary (SP-1) and secondary (SP-2) strike points and a region of low shear about a zero-shear "sweet spot."

Adapted from APS '11

3D Flux Tubes in Tokamak SOL

3D Flux Tube (*)



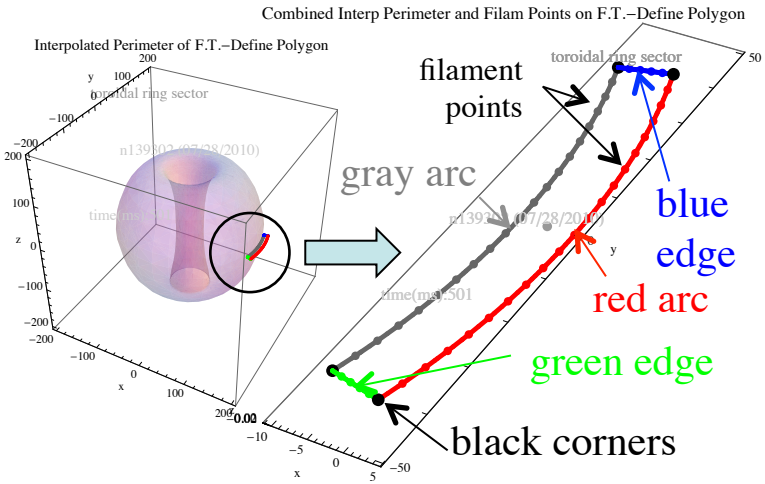
(*) The flux tube in this explanatory drawing has a circular normal cross section at the magnetic axis plane when viewed head-on along its central filament, and is different from the flux tube discussed below with a more rectangular cross section.

Adapted from APS '13

A flux tube in the SOL is a 3D volume bounded by filaments on its side surfaces and structural components on its end surfaces. A flux tube is meaningfully identifiable as such, only if some properties of the volume, such as particle, heat, and current densities, differ from those of the surrounding regions.

Because of the lack of symmetry-generating rapid transport along irrational field lines operative within the main plasma, asymmetries in the SOL form 3D flux tubes. Asymmetries may arise from *operational* procedures: for example, the plasma heating and fueling are usually not symmetric. Asymmetries may arise spontaneously as instability in the SOL plasma. Of particular interest are current-carrying flux tubes that may generate error field affecting the stability of the main plasma and those that go unstable themselves.

Field Ribbon and Flux Tube Specifications



Toroidal Ring Sector

Location, toroidal arc span, and "radial" width of a toroidal ring sector are varied in this study. Field lines form an "in-flux-surface" field ribbon through the red or gray arc, and an "out-of-flux surface" ribbon through the blue or green edge.

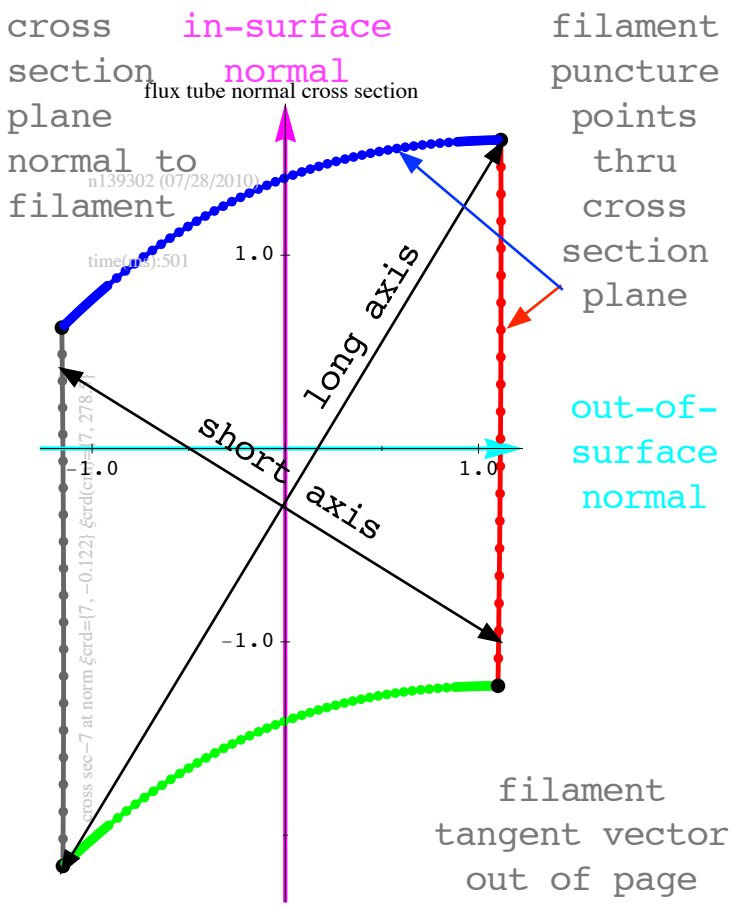
Any closed 3D space curve in the SOL linking magnetic flux defines a flux tube consisting of side walls formed by field lines and end caps on the limiter surface.

The choice of a curve embodies the physics under study. The perimeter of a piece of hardware, e.g., a limiter tile, biased electrode, or Langmuir probe, is a clear-cut case of flux-tube prescribing curves. An SOL volume prone to instability or a cloud of gas puff may represent a more nebulous case.

For want of a specific guidance from instability analysis, this study uses a toroidal ring sector of a conical surface (*). The curve consists of four segments: two toroidal circular arcs and two straight edges. These segments, also called "field ribbons," are color-coded to allow separate tracking of cross-sectional deformation attributable to contraction/expansion of each of them.

(*) While a circular cross section common in the literature has an intuitive visual appeal, it lacks the flexibility of independent toroidal and radial size variations. Limited radial space in the SOL might have precluded an investigation of toroidally wide flux tubes because of insistence on a circular cross section.

Flux Tube Cross Section Analysis



Short Axis

A single numerical value is desirable for characterizing the flux tube cross-sectional size to compare with the ion gyro-radius.

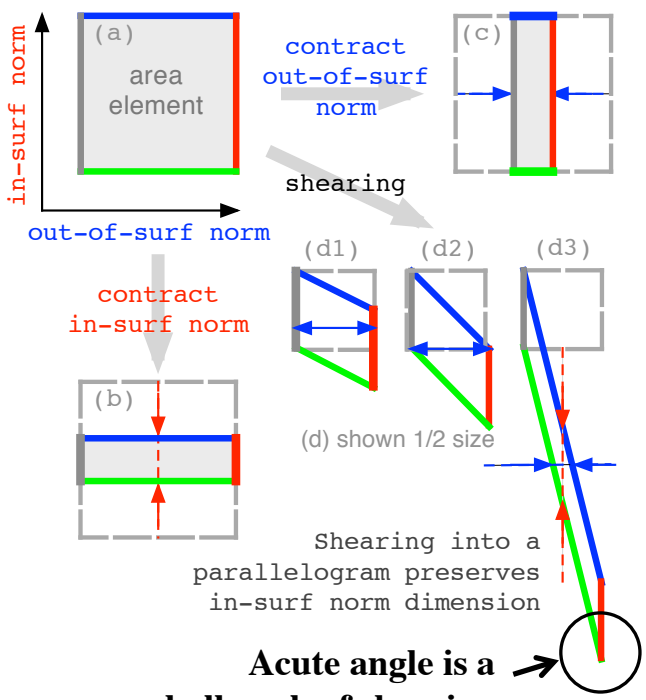
The longest straight line segment that a cross section can accommodate will be called the long axis. The distance between the intersect points of the cross section boundary and a line segment drawn normal to the long axis at its half-way point will be called the short axis. The short axis will be measured as multiples of 50 eV deuteron gyro-radius (~ 2.8 mm at the flux tube prescribing reference point in the cases studied below(*)).

(*) The deuteron gyro-radius, not much smaller than a flux surface variation scale length, arises from low field in NSTX, and may pose its own issues with or without squeezing, which however are beyond the scope of this study.

Some Analysis Tools

Contraction and Shearing

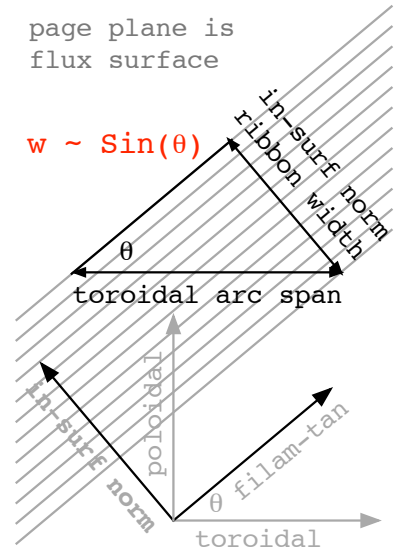
page face is cross-sectional plane



Acute angle is a hallmark of shearing

Toroidal Arc Ribbon

In-Surface Normal Ribbon Width



The constancy along the length of a flux tube of its toroidal arc span (when expressed in units of angle) leads to a universal curve that can approximately(*) describe the variation of the width of a toroidal arc ribbon when expressed in ion gyro-radius (ρ_i) multiples:

$$w(\xi)/\rho_i(\xi)/w(\xi_{ref})/\rho_i(\xi_{ref}) = \mathrm{Sin}(\theta(\xi))/\mathrm{Sin}(\theta(\xi_{ref})) \quad \text{Eq. (1)}$$

where ξ is the distance coordinate along central field line, $\theta(\xi)$ is the angle that the field line makes with respect to toroidal circle (somewhat akin to but not same as the pitch angle), and subscript "ref" is the flux tube prescribing reference point.

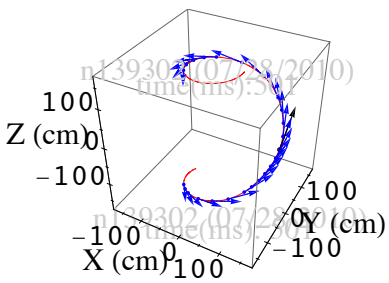
(*) approximate in having substituted a curved arc length for a straight line joining its ends: arc length $R(\xi)*\phi$, where $R(\xi)$ is major radius and ϕ is arc angle, in-surface linear ribbon width $R(\xi)*\phi*\mathrm{Sin}(\theta(\xi))$, but $R(\xi) \sim \rho_i(\xi)$. The constancy of ϕ leads to the above expression when normalized by its value at the reference point.

CROSS-SECTIONAL STRUCTURE OF FLUX TUBES IN OUTBOARD AND CIRCUMNAVIGATING ZONES

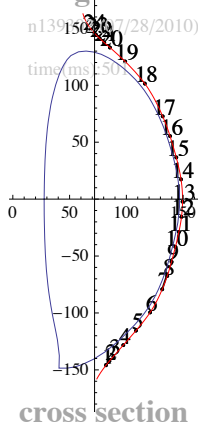
Wide Flux Tubes Have Enough Margin

Outboard Zone Flux Tube

toroidal ring sector 2 cm outside secondary separatrix, 2 cm wide by 30 deg long.

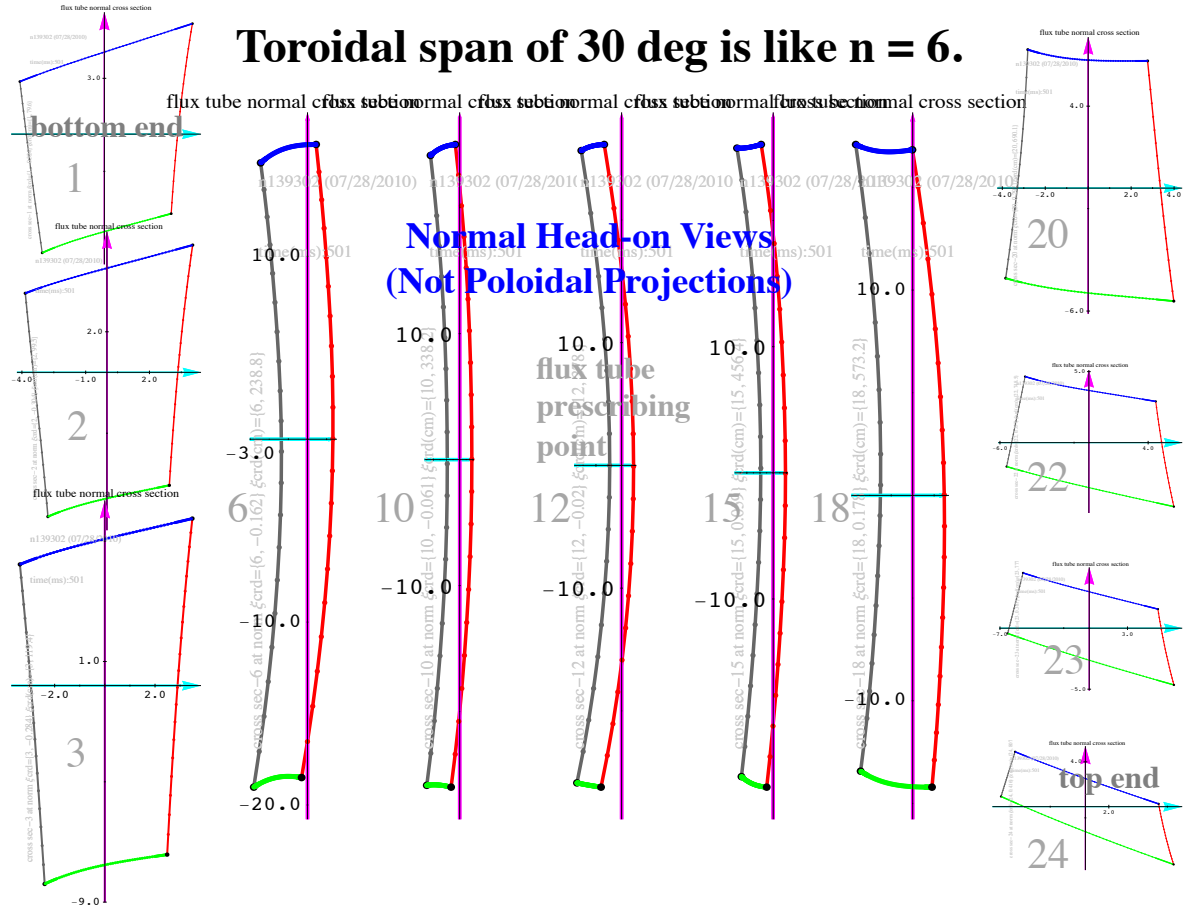


central filament and tangent vectors



cross section plane locations

Toroidal span of 30 deg is like n = 6.



Away from the flux-tube-prescribing reference point (12) toward either end (1 or 24), red filaments become ever more crowded into a rapidly diminishing toroidal arc span, which however remains sizable even at an end. Little shearing is evident (no acutely angled corners).

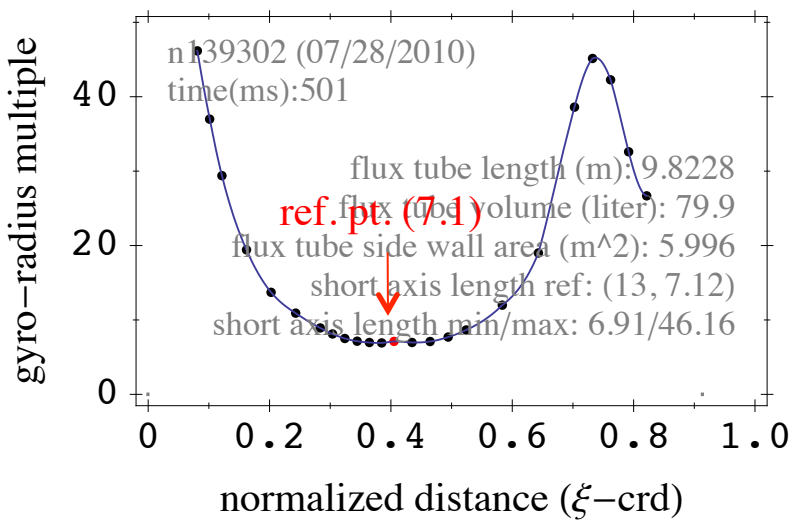
Wide Flux Tube Can Expand, Not Contract

Outboard Zone Flux Tube

toroidal ring sector 2 cm outside secondary separatrix, 2 cm wide by 30 deg long.

Normalized Short Axis Length

Cross Section Short Axis Length along Flux Tube



(*) The analysis presently does not extend over the entire central filament length because the cross section becomes clipped by the limiter surface near the flux tube ends.

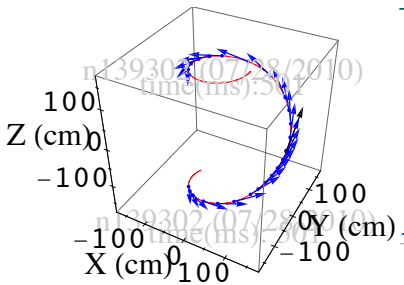
The "radial" width (2 cm) of this flux-tube-defining toroidal ring sector, much smaller than the toroidal arc length (78.3 cm), is the main determiner of the short axis around the reference point, which is nearly aligned with the out-of-surface normal. Away from the reference point and toward an end, the short axis initially grows (instead of being "squeezed") from 7.12 to 46.2 (for upper end) in gyro-radius multiples. This increase is a reflection of the well-known "flux surface expansion" effect near an x-point.

But the rapidly shrinking toroidal arc length takes over as the determiner of the short axis, which is now nearly aligned with the in-surface normal, and reaches 26.7 in gyro-radius multiples at the last point (*) calculated near the upper divertor end.

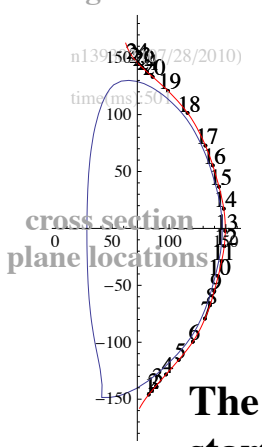
Same Mechanism Contracts Narrow Flux Tubes

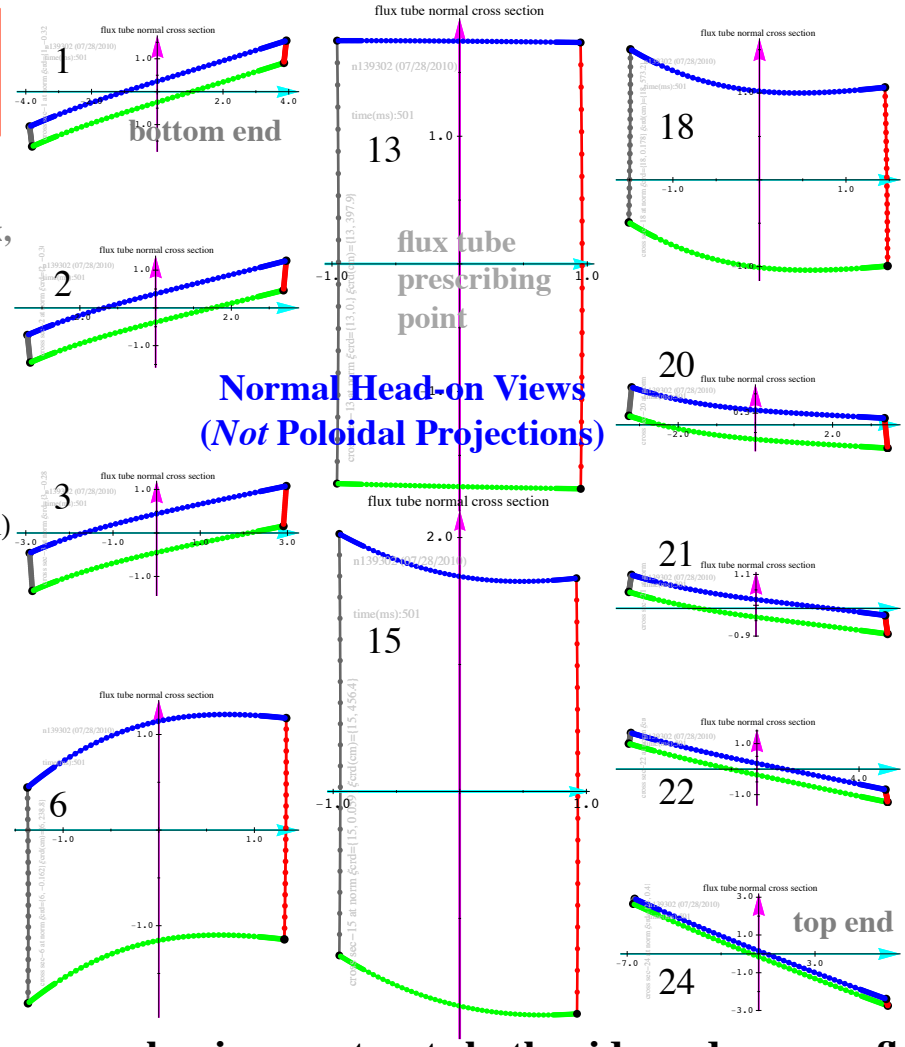
Outboard Zone Flux Tube

toroidal ring sector 2 cm outside secondary separatrix, 2 cm wide by 2 deg long.



central filament and tangent vectors





Normal Head-on Views (*Not* Poloidal Projections)

Away from the ref. point (13) toward either end (1 or 24), the cross-sectional deformation, largely due to contraction in the toroidal arc span (red or gray), can be significant (by ~ 4.6 here), reaching a minimum at a flux tube end (not near an x-point).

Little shearing is evident (no acutely angled corners).

The same mechanism contracts both wide and narrow flux tubes, but a narrow one starts narrow and gets even narrower.

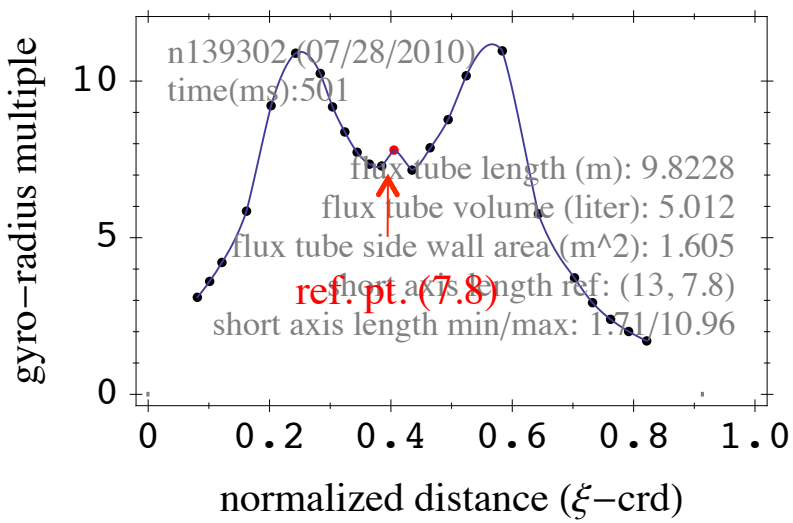
Narrow Flux Tubes Have Little Margin

Outboard Zone Flux Tube

toroidal ring sector 2 cm outside secondary separatrix, 2 cm wide by 2 deg long.

Normalized Short Axis Length

Cross Section Short Axis Length along Flux Tube



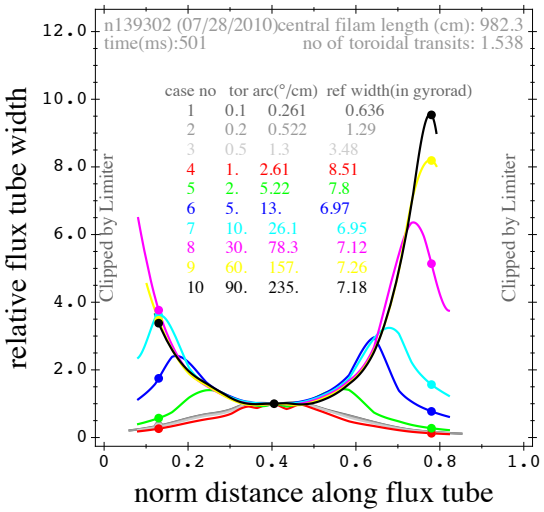
It is contraction of the toroidal arc span that dooms a narrow flux tube. So, it helps to start with a big enough capital.

The "radial" width (2 cm) of this flux-tube-defining toroidal ring sector, much smaller than the toroidal arc width (5.22 cm), is the main determiner of the short axis around the reference point, which is nearly aligned with the out-of-surface normal. Away from the reference point and toward an end, the short axis initially grows (instead of being "squeezed") from 7.8 to 11 (for upper end) in gyro-radius multiples. This increase is a reflection of the well-known "flux surface expansion" effect near an x-point.

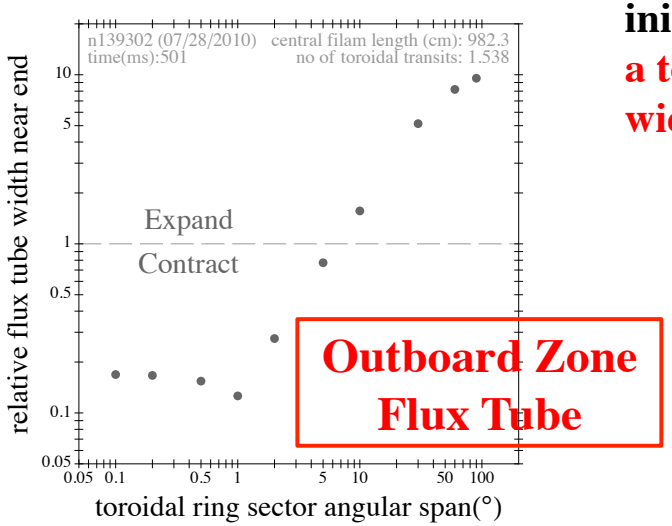
But the rapidly shrinking toroidal arc length takes over as the determiner of the short axis, which is now nearly aligned with the in-surface normal, and reaches 1.71 in gyro-radius multiples at the last point calculated near the upper divertor end.

Flux Tube May Either Expand or Contract

Relative Flux Tube Width



Flux Tube Contract/Expand



The flux tube cross sectional size variation is studied with a toroidal ring sector of the same specifications (reference point 2 cm outside secondary separatrix in the outboard zone with a 2 cm "radial" width) except toroidal angular span varying over 3 orders of magnitude from 0.1 to 90 deg. The short axis length relative to its value at the reference point is shown in the upper plot.

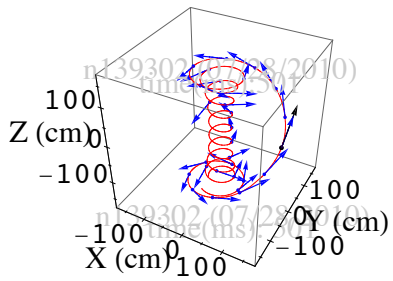
Flux tubes, with a toroidal arc span small in relation to the "radial" width, have a cross-sectional size that more or less monotonically falls toward either end. Large toroidal arc span flux tubes in contrast have a cross-sectional width that initially rises before falling toward either end. Flux tubes with a toroidal arc span sufficiently wide have a cross-sectional width larger everywhere than that at the reference point.

The lower plot (log-log scales) shows that the flux tube cross-sectional size is nearly linear with the toroidal arc span, except at really small values, and whether the flux tubes in the above set expanded or contracted when examined at a point near the inboard divertor end (right-hand points on the curves in the upper plot).

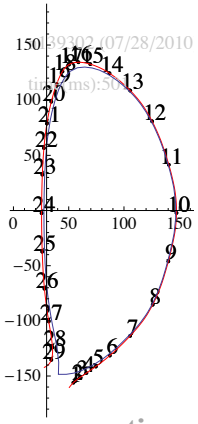
Flux Tubes Sheared Severely on Inboard Side

Circumnavigating Zone Flux Tube

toroidal ring sector at sweet spot, 30% wide by 30 deg long.

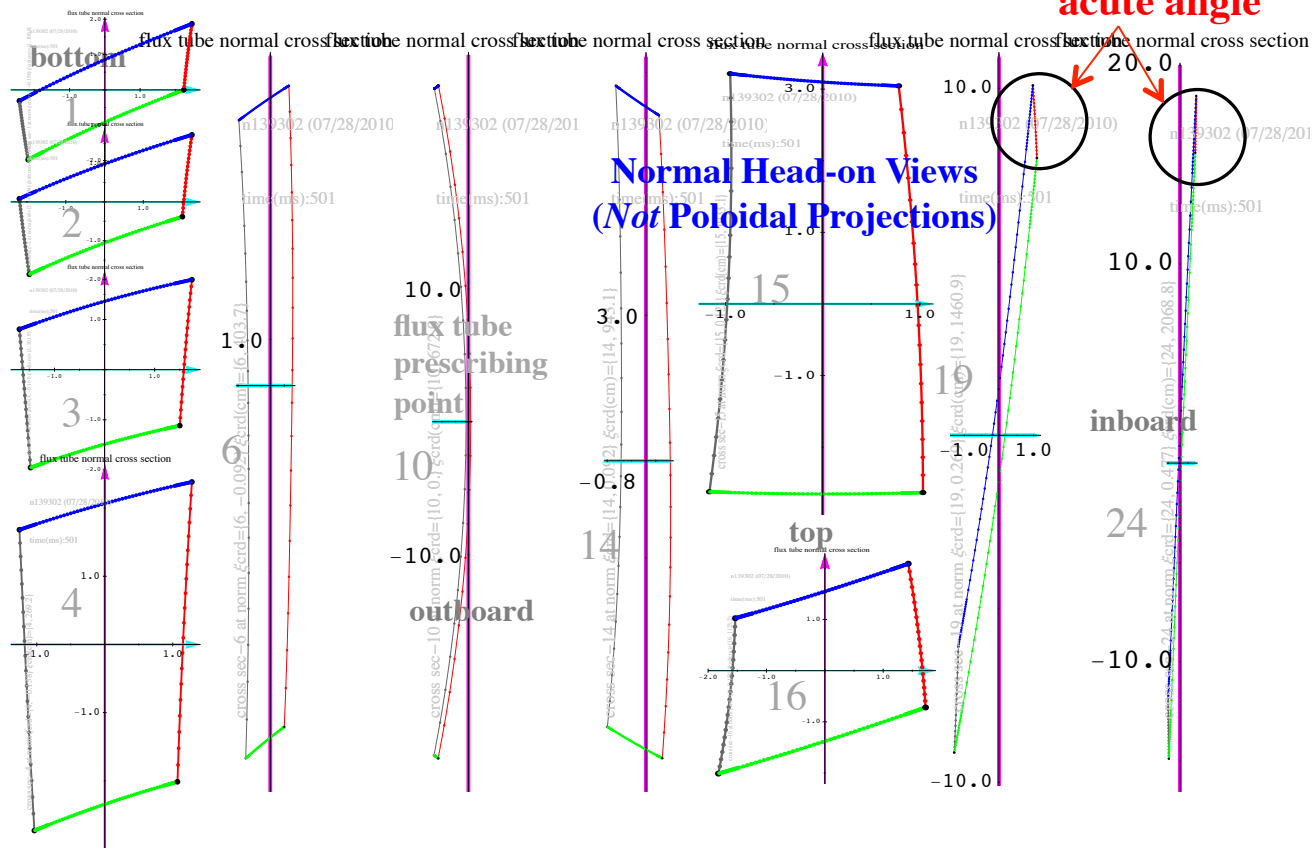


central filament and tangent vectors



cross section plane locations

Toroidal span of 30 deg is like n = 6.



Shearing is not evident on the outboard side but severe on the inboard side, presenting an interesting situation of the flux tube main body isolated from one of its ends but not the other. Can flute mode go unstable here?

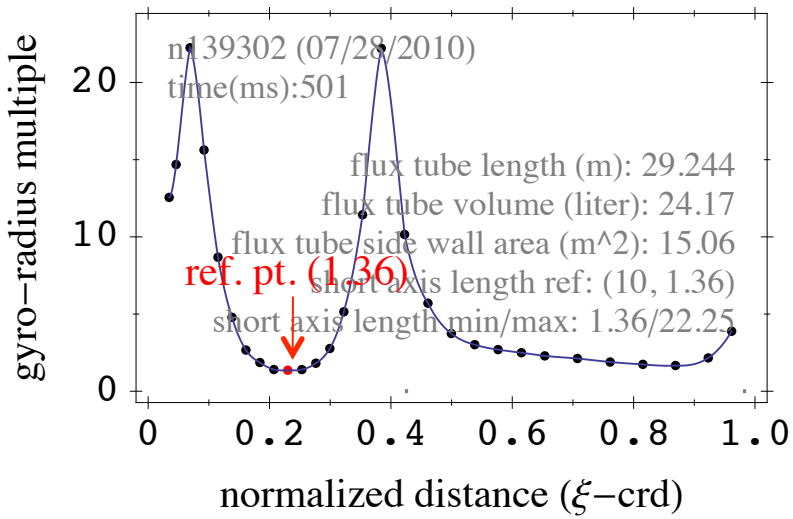
Flux Tube May Either Expand or Contract

Circumnavigating Zone Flux Tube

toroidal ring sector at sweet spot, 30% wide by 30 deg long.

Normalized Short Axis Length

Cross Section Short Axis Length along Flux Tube



(*) For a circumnavigating zone flux tube, the "radial" width may be specified in terms of poloidal flux between the two flux surfaces bounding the toroidal ring sector expressed as percentage of flux between two separatrices.

The "radial" width (30% or 0.323 cm)(*) of this toroidal ring sector, much smaller than the toroidal arc width (77.2 cm), determines the short axis around the reference point. Away from the reference point and toward an end, the short axis initially grows from 1.36 to 22.2 (for upper end) in gyro-radius multiples.

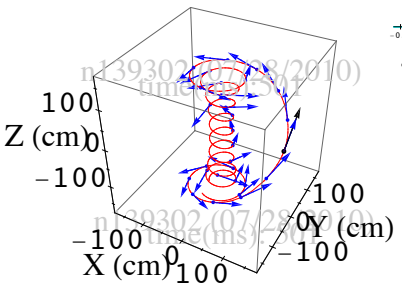
But the rapidly shrinking toroidal arc width takes over as the determiner of the short axis, which is now nearly aligned with the in-surface normal, and reaches 3.89 in gyro-radius multiples at the last point (*) calculated in the inboard bottom divertor.

The short axis length on the inboard side remains at about the same level or greater as the reference point in spite of severe shearing there.

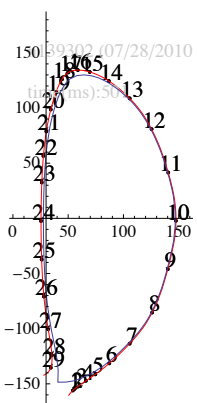
Different Mechanisms on Out/Inboard Sides

Circumnavigating Zone Flux Tube

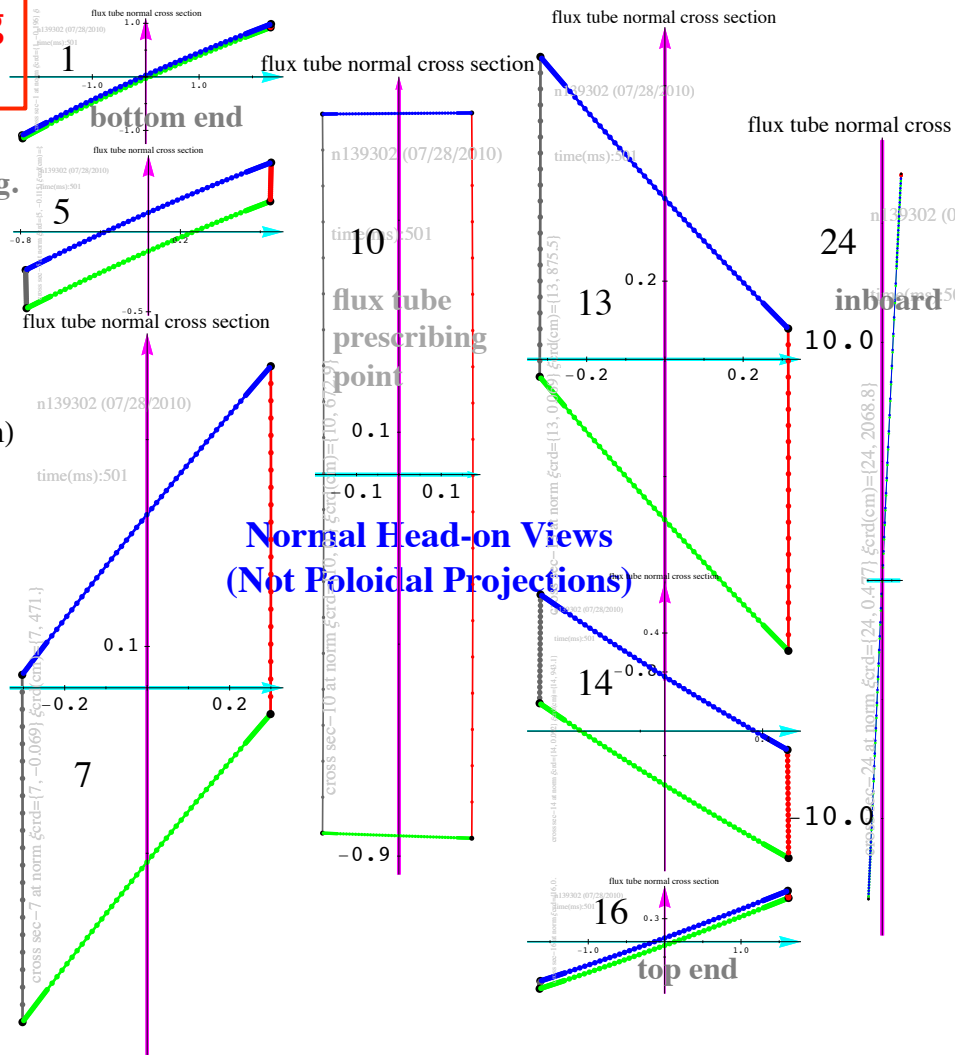
toroidal ring sector at sweet spot, 30% wide by 1 deg long.



central filament and tangent vectors



cross section plane locations



The cross section changes through contraction of the toroidal arc span (red or gray) on the outboard side, which is significant (factor ~ 4.6 reached at bottom end), but through severe field line shearing on the inboard side.

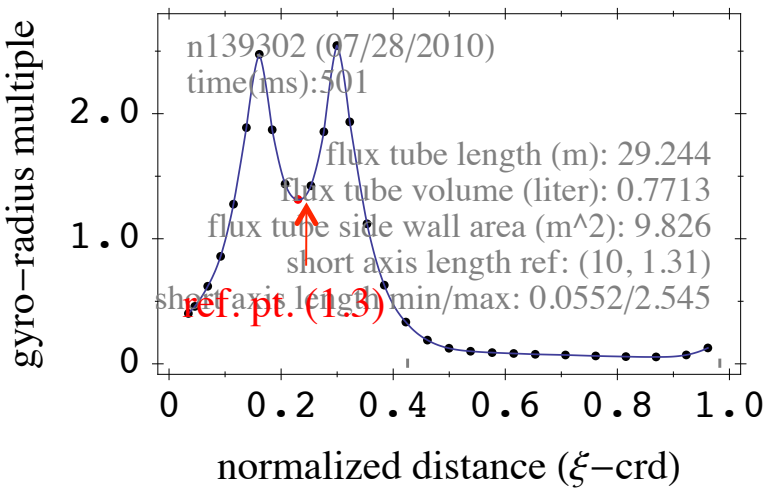
Severe Shearing Occurs on Inboard Side

Circumnavigating Zone Flux Tube

toroidal ring sector at sweet spot, 30% wide by 1 deg long.

Normalized Short Axis Length

Cross Section Short Axis Length along Flux Tube



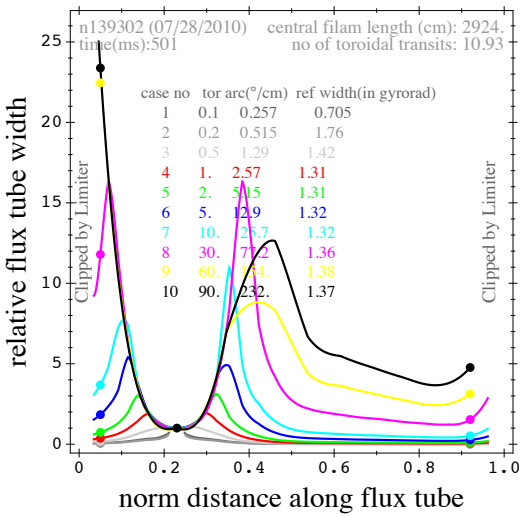
The "radial" width (30% or 3.23 mm) of this toroidal ring sector, much smaller than the toroidal arc length (2.57 cm), determines the short axis around the reference point. Away from the reference point and toward an end, the short axis initially grows from 1.31 to 2.54 (for upper end) in gyro-radius multiples.

But the rapidly shrinking toroidal arc length takes over as the determiner of the short axis, which is now nearly aligned with the in-surface normal, and reaches 0.127 in gyro-radius multiples at the last point calculated in the inboard bottom divertor.

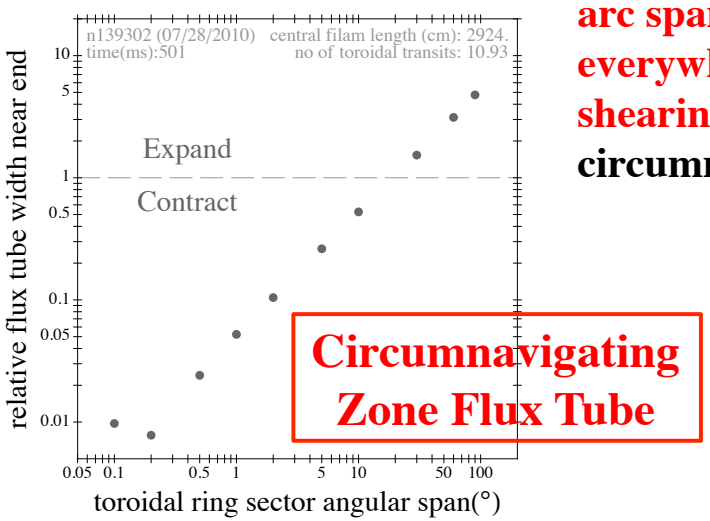
The short axis length is severely contracted by shearing on the inboard side.

Wide Flux Tubes Can Be Free from Contraction

Relative Flux Tube Width



Flux Tube Contract/Expand



The flux tube cross sectional size variation is studied with a toroidal ring sector of the same specifications (reference point at sweet spot in circumnavigating zone with a 30 % "radial" width) except toroidal arc span varying over 3 orders of magnitude from 0.1 to 90 deg. The short axis length relative to its value at the reference point is shown in the upper plot.

Flux tubes, with a toroidal arc span small in relation to the "radial" width, have a cross-sectional size that more or less monotonically falls toward either end. Large toroidal arc span flux tubes in contrast have a cross-sectional width that initially rises before falling toward either end. Flux tubes with a toroidal arc span sufficiently wide have a cross-sectional width larger everywhere than that at the reference point in spite of strong shearing on the inboard side. (Studies of flux tubes inside the circumnavigating zone have seldom, if ever, been reported).

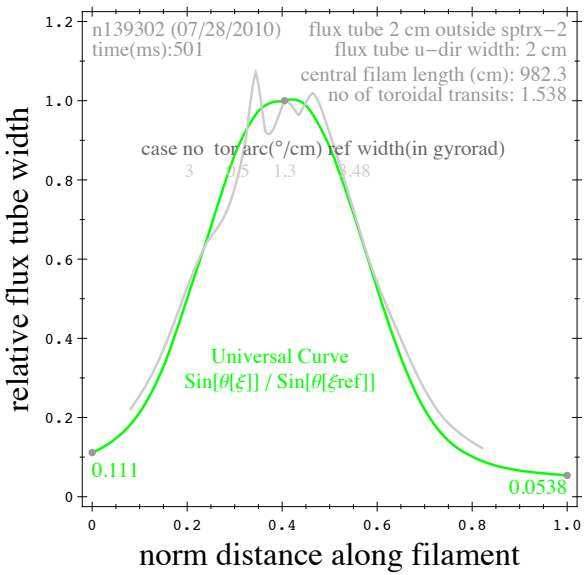
The lower plot (log-log scales) shows that the flux tube cross-sectional size is nearly linear with the toroidal arc span, and whether the flux tubes in the above set expanded or contracted when examined at a point near the inboard divertor end (right-hand points on the curves in the upper plot).

Universal Curve Describes Contraction Factor

Outboard Zone Flux Tube

toroidal ring sector 2 cm outside secondary separatrix, 2 cm wide by 0.5 deg long.

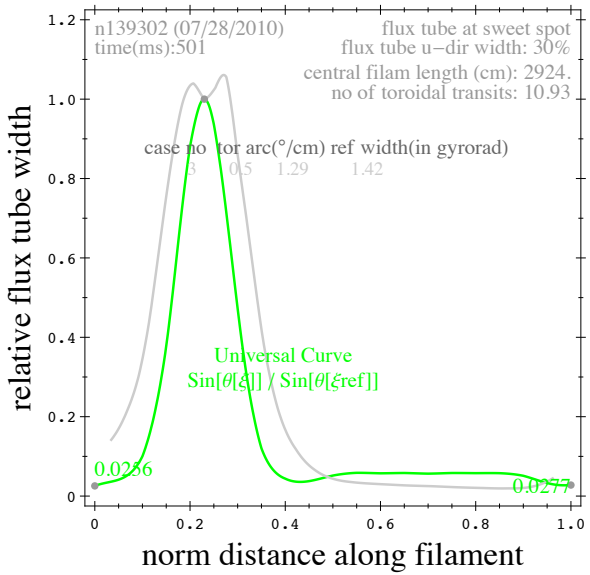
Universal F.T. Contraction Curve



Circumnavigating Zone Flux Tube

toroidal ring sector at sweet spot, 30% wide by 0.5 deg long.

Universal F.T. Contraction Curve



The universal curve (Eq. 1) represents well the cross-sectional contraction factor for cases with a small toroidal arc span when "flux surface expansion effect" is minimal. It is helpful in assessing the contraction factor at the ends where it is more involved to obtain it from cross-sectional analysis.

Conclusion

1. **Geometrical variations along the length of a flux tube may indeed be strong enough to alter the character of some phenomena in the tokamak SOL, as Farina, Pozzoli, and Ryutov (FPR) first reported in their impactful article (PoP '93) with respect to flute instability, but possibly for many other areas of the SOL physics.**

2. **Based on analysis made to date for a limited class of flux tubes prescribed on the plasma outboard side, severe contraction ("FPR Squeeze") of their cross sections down to an ion gyro-radius scale does not occur in flux tubes of a large enough toroidal extent; this finding possibly restores flute mode into the ranks of electrostatic instabilities thought responsible for the SOL turbulence.**

3. **Significant cross-sectional contraction occurs for flux tubes with a small toroidal extent in the outboard zone, not through the field line pitch angle shearing as proposed by FPR but through reduction in the in-surface normal dimension; Severe contraction ("FPR Squeeze") does occur for flux tubes with a small toroidal extent in the circumnavigating zone through shearing, not in the vicinity of an x-point as proposed by FPR but on the plasma inboard side (This is still under investigation whether the squeeze location is necessarily the inboard side, or simply far away or on the other side from the flux tube prescribing point).**