On the conditions for the onset of nonlinear chirping structures in NSTX

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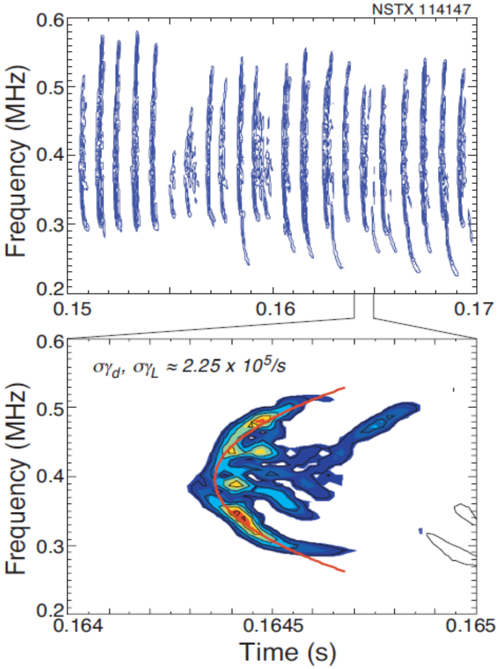
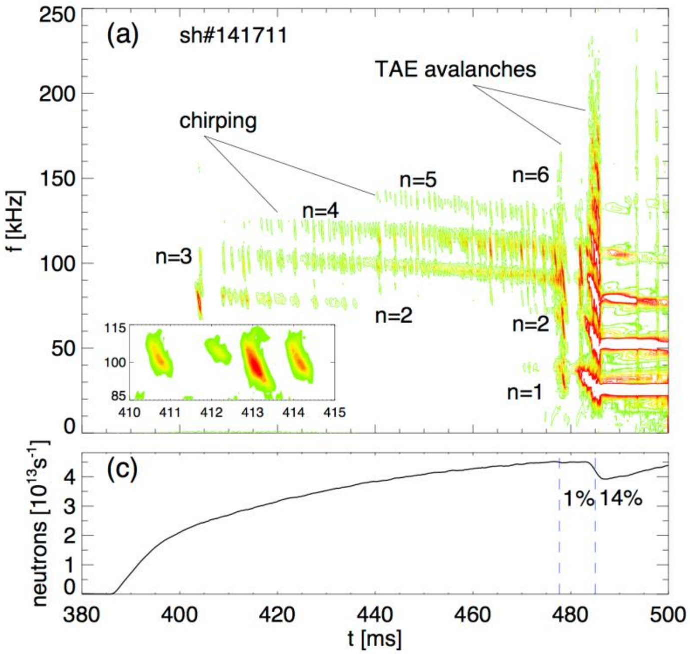
APS Meeting, Savannah, GA, Nov 18, 2015

Chirping modes in NSTX can degrade the confinement of energetic particles

GAE/CAE
Fredrickson et al, PoP 2006.



TAE Podestà et al, NF 2012



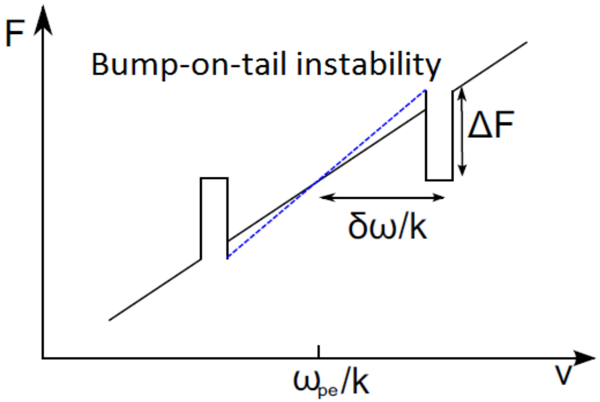
Up to 40% of injected beam is observed to be lost in DIII-D and NSTX

Chirping is ubiquitous in NSTX but rare in DIII-D. Why??

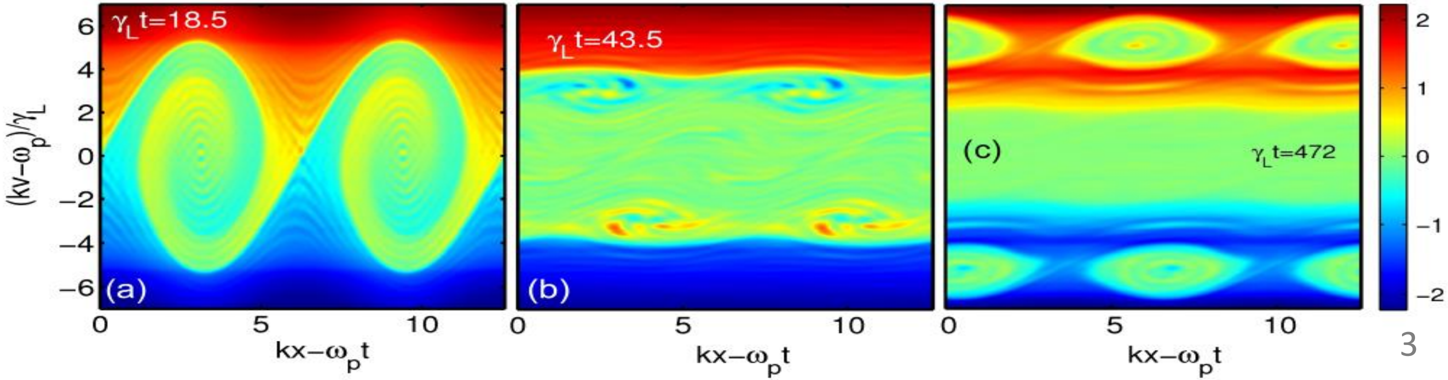
This presentation focuses on the conditions for chirping onset rather than their long-term evolution

Phase space holes and clumps in kinetically driven, dissipative systems

- Nonlinear Landau damping perspective: incomplete phase mixing leads to small sideband oscillations that may tap free energy at the edges of the plateau
- Chirping in frequency may allow for a continuous interplay between the free energy from the distribution function and the wave dissipation
- Collisions eventually degrade the resonant island plateau, and the process restarts



Vlasov simulations by Lilley and Nyqvist, PRL 2014



Nonlinear dynamics of driven kinetic systems close to threshold

Assumptions:

- Perturbative procedure for $\omega_b \ll \gamma$
- Truncation at third order due to closeness to marginal stability
- Bump-on-tail modal problem, uniform mode structure

Cubic equation: lowest-order nonlinear correction to the evolution of mode amplitude A:

$$\frac{dA}{dt} = A - \int_0^{t/2} d\tau \tau^2 A(t-\tau) \int_0^{t-2\tau} d\tau_1 e^{-\nu_{scatt}^3 \tau^2 (2\tau/3+\tau_1) + i\nu_{drag}^2 \tau(\tau+\tau_1)} A(t-\tau-\tau_1) A^*(t-2\tau-\tau_1)$$

Berk, Breizman and Pekker, PRL 1996

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stabilizing

destabilizing (makes integral sign flip)

Berk, Breizman and Pekker, PRL 1996 Lilley, Breizman and Sharapov, PRL 2009

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stabilizing

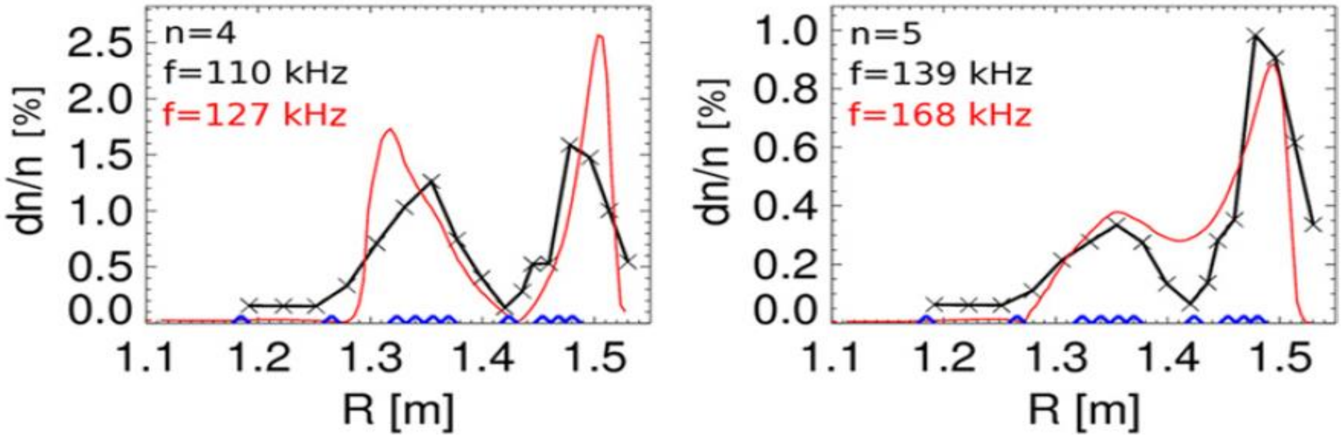
destabilizing (makes integral sign flip)

- If nonlinearity is weak: linear stability, solution saturates at a low level and f merely flattens (system not allowed to further evolve nonlinearly).
- If solution of cubic equation explodes: system enters a strong nonlinear phase with large mode amplitude and can be driven unstable (precursor of chirping modes).

Berk, Breizman and Pekker, PRL 1996 Lilley, Breizman and Sharapov, PRL 2009

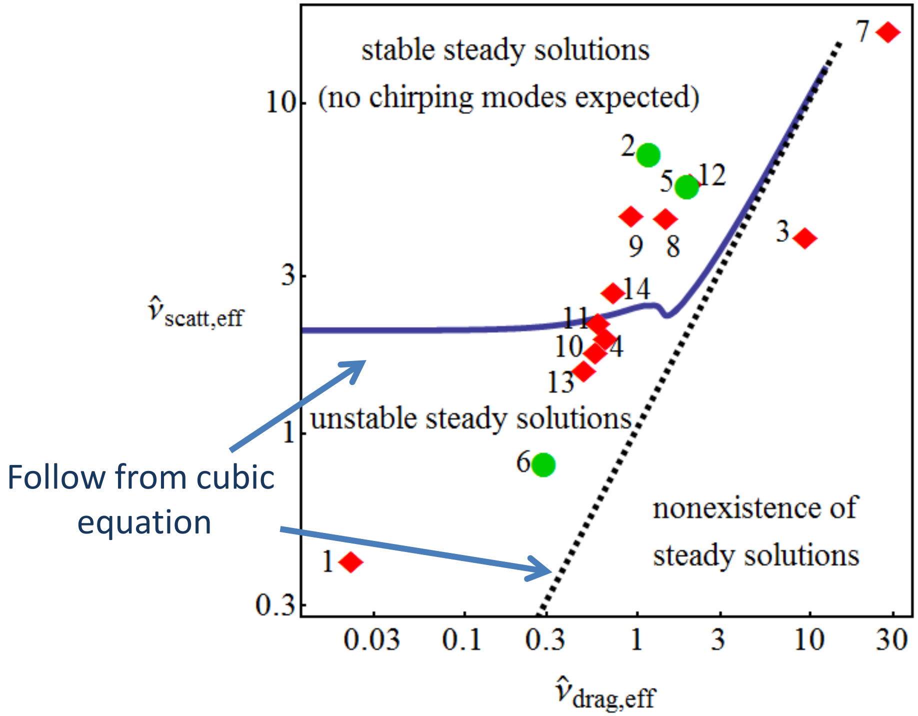
Mode structure identification

- NOVA code: finds linear, ideal mode structures
- Its kinetic postprocessor NOVA-K computes resonance surfaces and provides damping and linear growth rates. Phase sapace and bounce averages are necessary calculate effective collisional coefficients
- NOVA's mode structures are compared with NSTX reflectometer measurements (fluid displacement times the local density gradient is equivalent to the density fluctuation)



Chirping in terms of effective collisional coefficients for realistic resonances and mode structures

Experimental observations:
Red diamonds: chirping was obseved
Green dots: no chirping observed



Pitch-angle scattering: leads to loss of correlation (loss of phase information from one bounce to another)

Drag (slowing down): coherently moves structures down in velocity

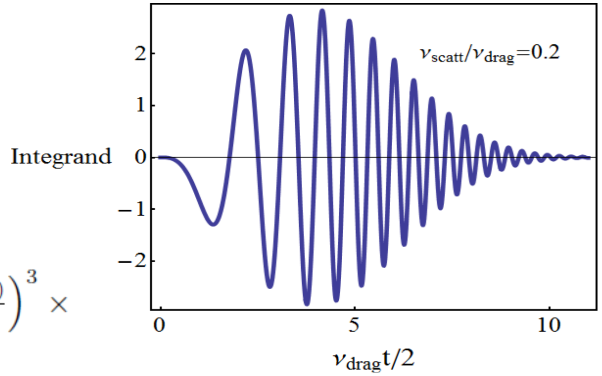
Bump-on-tail modeling is not enough to resolve the regions in collisions space that allows for chirping modes

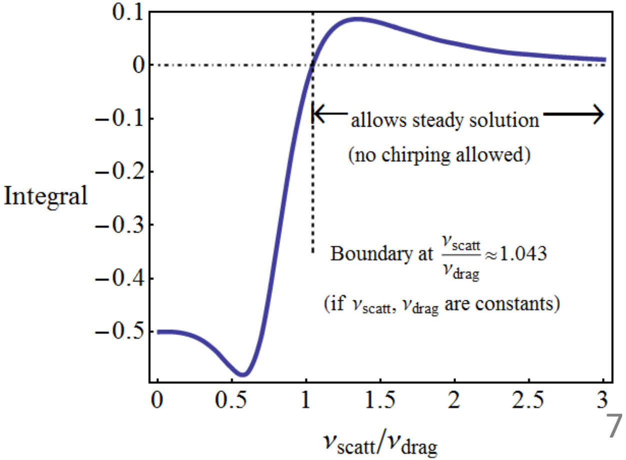
Missing physics in the simplified theoretical prediction: mode structure, (multiple) resonance surfaces and phase-space and bounce averages

Generalization: cubic equation with collisional coefficients varying along resonances and particle orbits

Action-angle formalism for the general problem, with a similar perturbative approach employed before, leads to the **generalized criterion for existence of steady-state solutions** (no chirping):

$$\begin{aligned}
&\lim_{t\to\infty} Re\Bigg\{ e^{i\varphi}\sum_{\sigma_\|}\int dP_\varphi \int d\mu\, \tau_b\left(P_\varphi,\mathcal{E}'+\omega_0 P_\varphi/n,\mu\right)\left|V\left(P_\varphi,\mathcal{E}'+\omega_0 P_\varphi/n,\mu\right)\right|^4 \left(\frac{\partial\Omega(P_\varphi,\mathcal{E}'+\omega_0 P_\varphi/n,\mu)}{\partial I}\right)^3 \times \\
&\times \frac{\partial F(P_\varphi,\mathcal{E}'+\omega_0 P_\varphi/n,\mu)}{\partial\Omega}\frac{1}{\nu_{drag}^4(P_\varphi,\mathcal{E}'+\omega_0 P_\varphi/n,\mu)}\int_0^{\nu_{drag}t/2} dz\, z\, exp\left[-2\frac{\nu_{diff}^3(P_\varphi,\mathcal{E}'+\omega_0 P_\varphi/n,\mu)}{\nu_{drag}^3(P_\varphi,\mathcal{E}'+\omega_0 P_\varphi/n,\mu)}z^3/3+iz^2\right] \\
&\times \frac{-1+exp\left[\left(-\frac{\nu_{diff}^3(P_\varphi,\mathcal{E}'+\omega_0 P_\varphi/n,\mu)}{\nu_{drag}^3(P_\varphi,\mathcal{E}'+\omega_0 P_\varphi/n,\mu)}z^2+iz\right)\left(\nu_{drag}(P_\varphi,\mathcal{E}'+\omega_0 P_\varphi/n,\mu)t-2z\right)\right]}{-\frac{\nu_{diff}^3(P_\varphi,\mathcal{E}'+\omega_0 P_\varphi/n,\mu)}{\nu_{drag}^3(P_\varphi,\mathcal{E}'+\omega_0 P_\varphi/n,\mu)}z+i}\Bigg\} > 0
\end{aligned}$$





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$$\times\frac{\partial F(P_\varphi,\mathcal{E}'+\omega_0P_\varphi/n,\mu)}{\partial\Omega}\frac{1}{\nu_{drag}^4(P_\varphi,\mathcal{E}'+\omega_0P_\varphi/n,\mu)}\int_0^{\nu_{drag}t/2} dz\, z\, exp\left[-2\frac{\nu_{diff}^3(P_\varphi,\mathcal{E}'+\omega_0P_\varphi/n,\mu)}{\nu_{drag}^3(P_\varphi,\mathcal{E}'+\omega_0P_\varphi/n,\mu)}z^3/3+iz^2\right]$$
$$\left.\times\frac{-1+exp\left[\left(-\frac{\nu_{diff}^3(P_\varphi,\mathcal{E}'+\omega_0P_\varphi/n,\mu)}{\nu_{drag}^3(P_\varphi,\mathcal{E}'+\omega_0P_\varphi/n,\mu)}z+iz\right)\left(\nu_{drag}(P_\varphi,\mathcal{E}'+\omega_0P_\varphi/n,\mu)t-2z\right)\right]}{-\frac{\nu_{diff}^3(P_\varphi,\mathcal{E}'+\omega_0P_\varphi/n,\mu)}{\nu_{drag}^3(P_\varphi,\mathcal{E}'+\omega_0P_\varphi/n,\mu)}z+i}\right\}>0$$

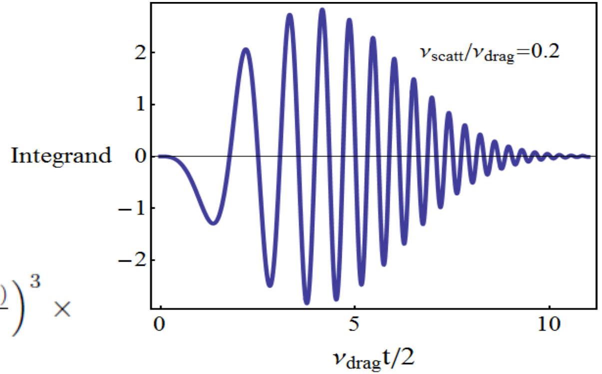
Phase space integration

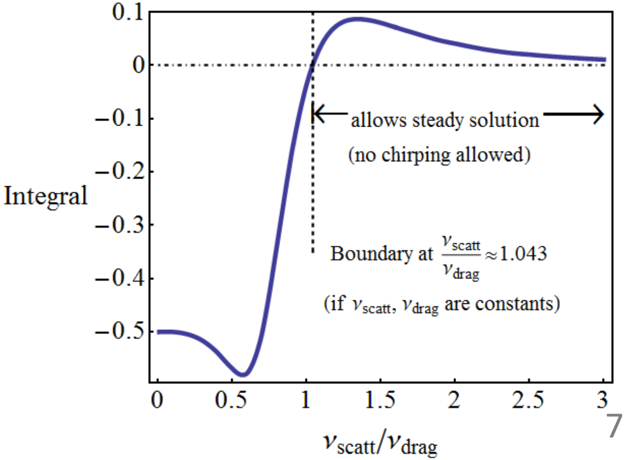
Eigenstructure information:

$$q\int dt\mathbf{v}_{dr}\cdot\delta\mathbf{E}e^{i\omega t}$$

Resonance surfaces:

$$\Omega\left(\mathcal{E},P_\varphi,\mu\right)=0$$





Future work

- Implementation of the generalized criterion using NOVA code
- Study of delay in chirping in terms of injection parameters and resonances
- Development of a line-broadened quasilinear diffusion solver coupled with NOVA and NOVA-K: chirping criterion is important for identification of parameter space for quasilinear validity

Thank you