Field line reconstruction in edge transport modeling of non-axisymmetric tokamaks configurations

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Motivation: modeling of non-axisymmetric divertor head loads

- Control of divertor **heat loads** (both steady state and transient) remains one of the key challenges for a fusion based reactor and for a compact **fusion nuclear science development facility (FSNF)**
- **Resonant magnetic perturbations (RMPs)** are a promising method for ELM control $\rightarrow$ breaking of axisymmetry
- **Advanced divertors** (specialized divertor geometry): Snowflake, X-divertor) for steady state heat flux reduction
- Likely, both concepts will have to work together, and NSTX-U is a well suited device to study this
- **Reliable plasma transport simulations require an accurate representation of the magnetic field configuration**

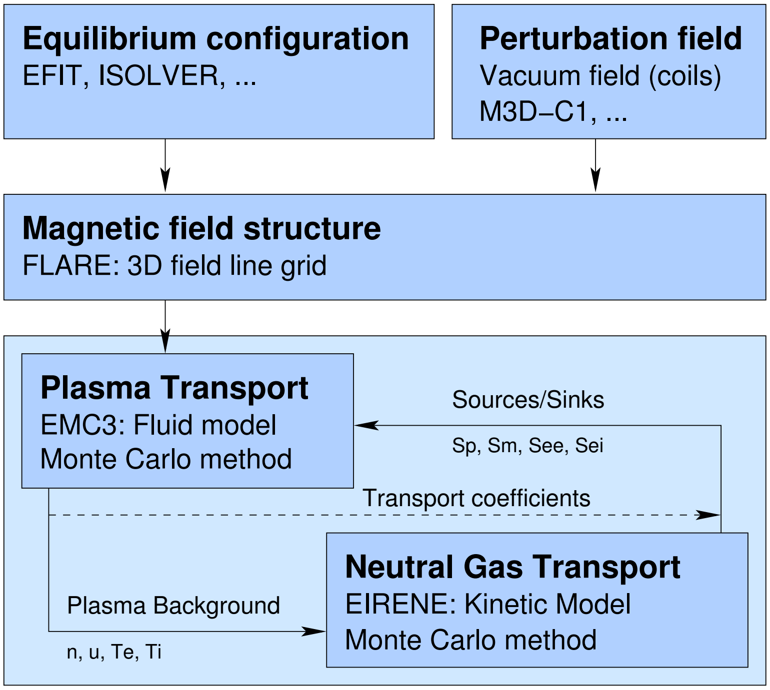
1. Introduction to 3D edge modeling (for NSTX-U)
   - The NSTX-U setup
   - Magnetic perturbation effects
   - The 'snowflake' divertor configuration'
2. Finite resolution equilibrium representation
   - Impact of the spline order
   - Impact of the mesh resolution
3. The field line representation in transport simulations
   - Field line deviation analysis
   - Field line loss analysis
   - Divertor heat load analysis

Introduction to 3D edge modeling

- Transport modeling is based on a given magnetic field configuration (as in 2D edge modeling)
- Finite resolution effects need to be understood to quantify uncertainties in simulation results:

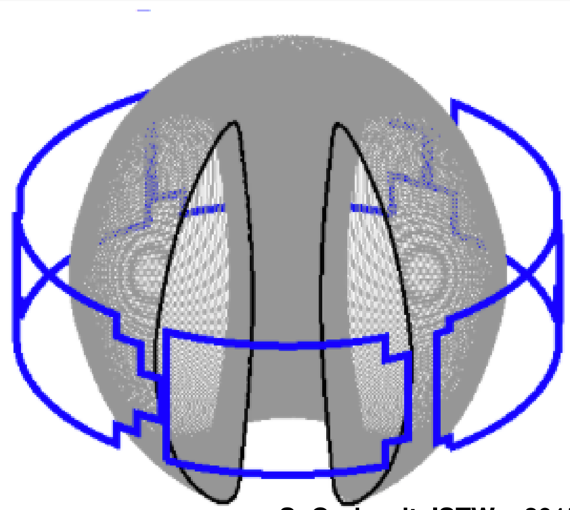
  1. for the underlying equilibrium
  2. for the field line representation in the transport code

- The FLARE code provides tools to characterize the magnetic field configuration, and to generate and analyze grids for transport simulations



The NSTX-U setup

- NSTX(-U) has a set of 6 midplane coils that can be used to apply resonant magnetic perturbations $\rightarrow$ we are looking at configurations with toroidal mode number $n = 3$



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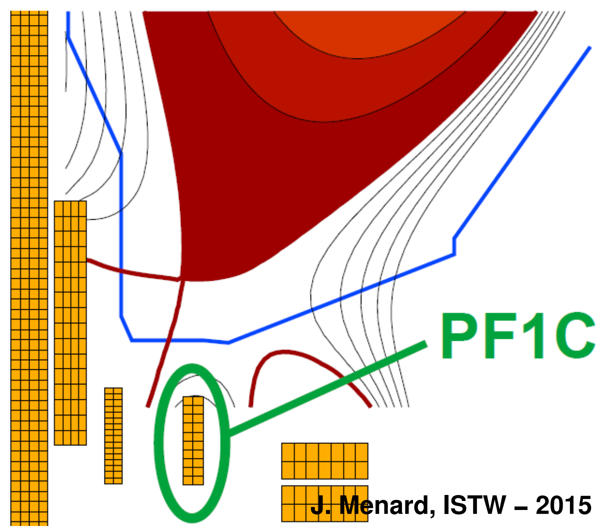
- NSTX-U has additional poloidal field coils for snowflake / X-divertor configurations



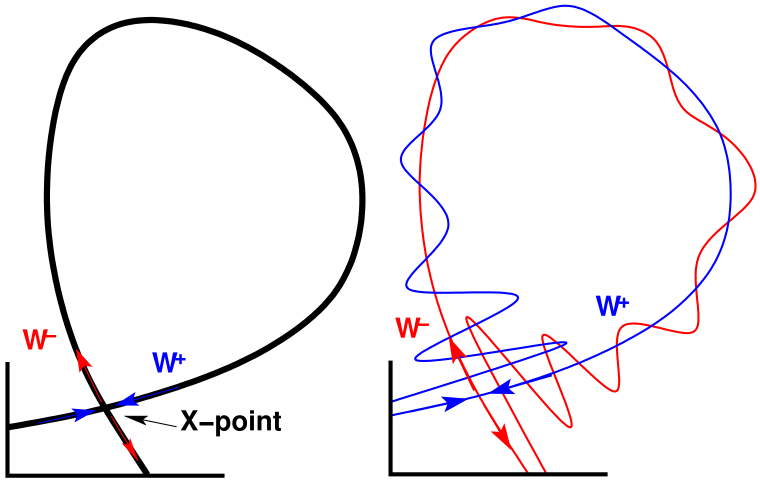
Perturbation of the magnetic separatrix $\rightarrow$ lobes

- The separatrix associated with **X** has 2 branches (set of field line trajectories, invariant manifolds):

$$W^+ = \left\{ \mathbf{p} \middle| \lim_{l\to\infty} F_{\mathbf{p}}(l) \rightarrow \mathbf{X} \right\}$$

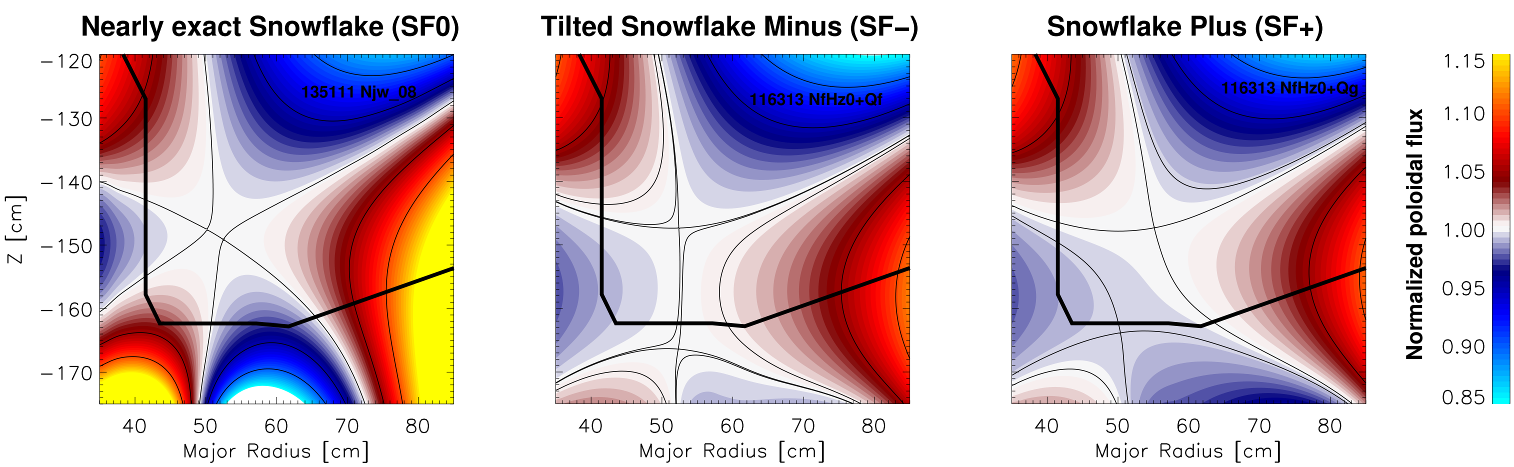
$$W^- = \left\{ \mathbf{p} \middle| \lim_{l\to-\infty} F_{\mathbf{p}}(l) \rightarrow \mathbf{X} \right\}$$

$F_{\mathbf{p}}(l)$: field line through $\mathbf{p}$



- Both branches overlap in the unperturbed configuration, but split when magnetic perturbations are applied
- This opens up a **connection between the plasma interior and the divertor targets**

The 'snowflake' configuration allows for a variety of magnetic topologies at NSTX-U



- 'Nearly exact snowflake': approximation to 'classical' snowflake (2nd order magnetic null, $B_{\mathrm{pol}} \sim r^2$ scaling)
- Here SF0 is actually 'snowflake minus', which is topologically equivalent to 'connected double null' (CDN)
- The secondary X-points in the (tilted) SF- and SF+ configurations here are outside the divertor targets $\rightarrow$ topology is 'lower single null' (LSN)

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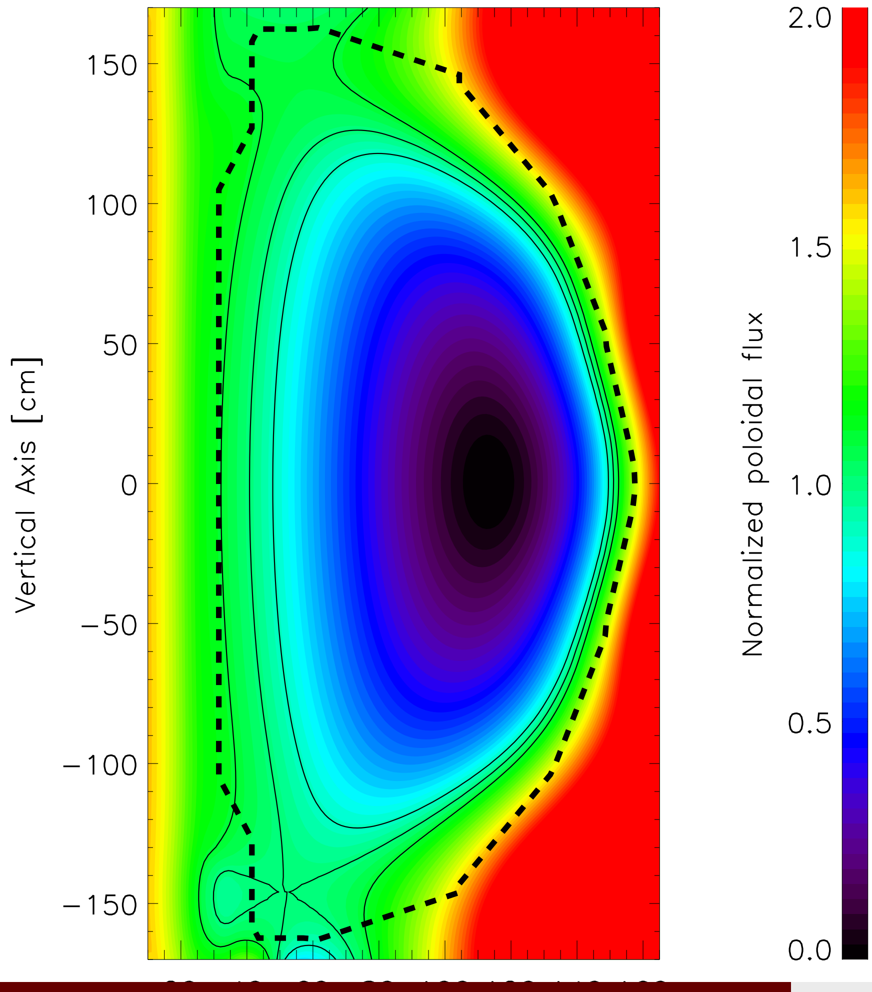
The FLARE code can take an equilibrium from a 'g-file' as input

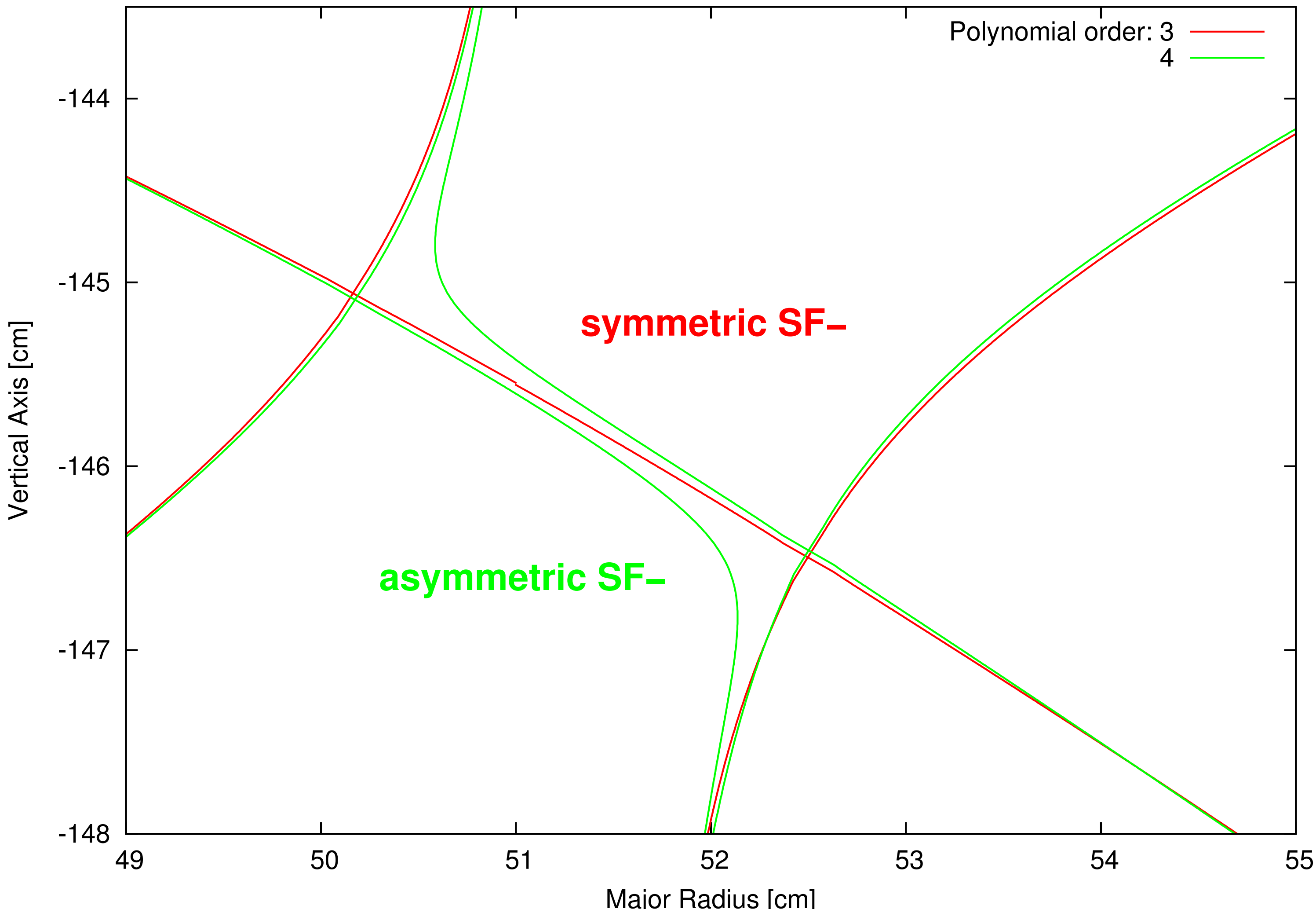


- The NSTX-U equilibrium with snowflake divertor was generated by ISOLVER: 81×81 equidistant data points are provided here
- A 2D B-spline interpolation allows to obtain a smooth representation of $\Psi(R, Z)$
- The numerical representation of the poloidal magnetic field

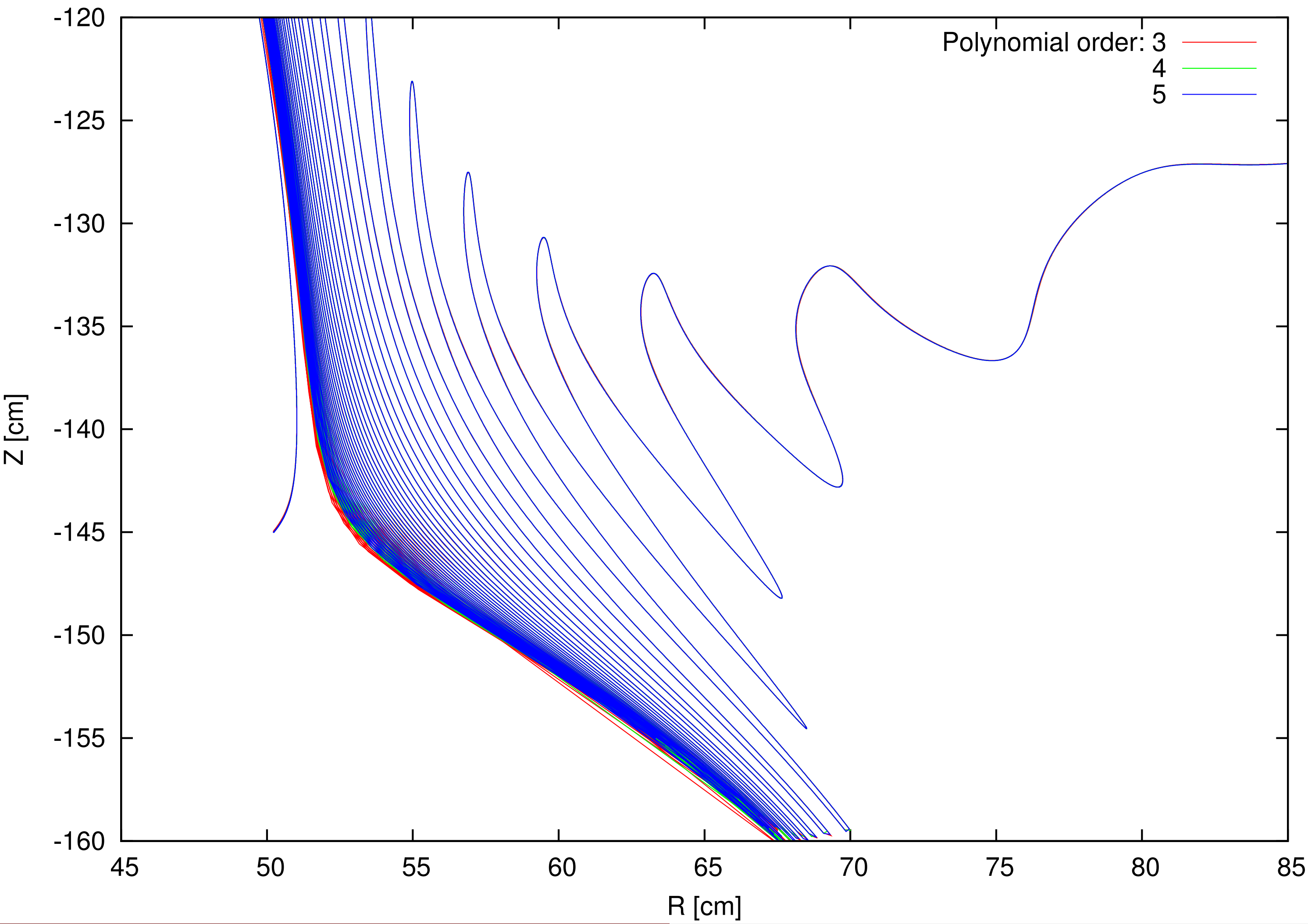
$$B_{\text{pol}} = -\frac{1}{R}\frac{\partial \Psi}{\partial Z}\,\mathbf{e}_R + \frac{1}{R}\frac{\partial \Psi}{\partial R}\,\mathbf{e}_Z$$

is intrinsically divergence-free

Spline order affects connectivity of X-points



The shape of the separatrix manifolds is robust



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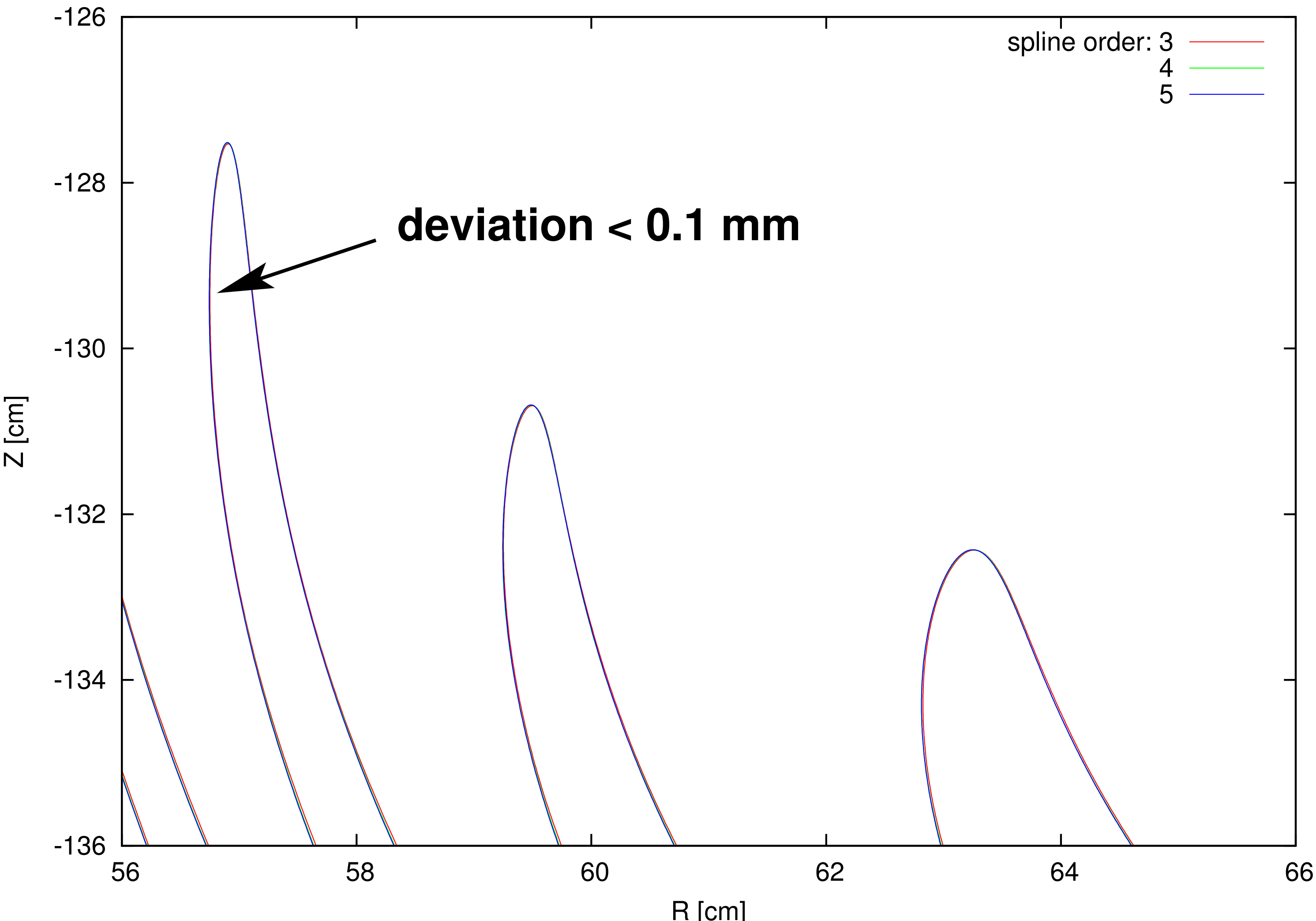


Analytical solutions to the Grad-Shafranov equation allow to address mesh resolution effects

- The following analysis is based on the 'One size fits all' approach (see A. Cerfon et al., Phys. Plasmas 17, 032502 (2010)). Non-dimensional form of GS with Solov'ev profiles ($r = R/R_0, z = Z/R_0$):

$$r \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right) + \frac{\partial^2 \psi}{\partial z^2} + (1 - A)\, r^2 + A \tag{1}$$

- Solution with polynomials $\psi_i(r, z)$ up to r^6 and z^6:

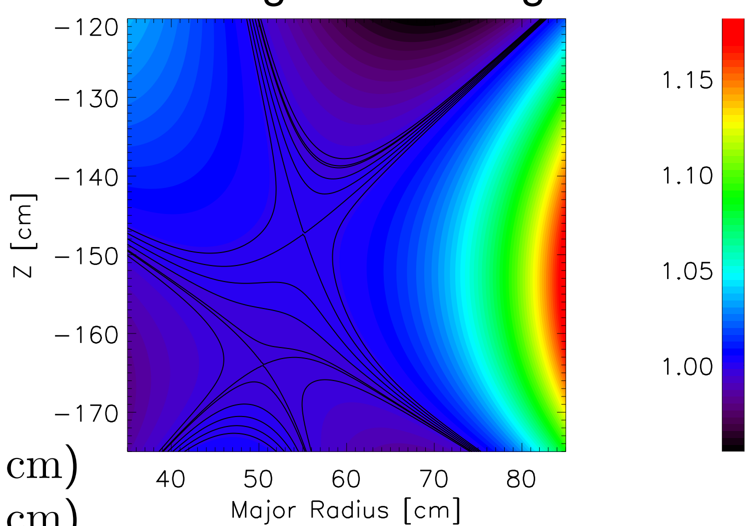
$$\psi(r, z) = \frac{r^4}{8} + A\left(\frac{1}{2} r^2 \ln r - \frac{r^4}{8}\right) + \sum_{i=1}^{12} c_i\, \psi_i(r, z) \tag{2}$$

- The coefficients c_i are determined by geometric constraints, which allows to recover a "D"-shape with elongation κ, triangularity δ, inverse aspect ratio ε and X-point coordinates (r_X, z_X). The parameter A allows to set a value for plasma beta β.

Next order polynomials allow access to (some) snowflake-configurations

- A general arbitrary degree polynomial solution with up-down symmetry has been presented in Zheng et al., Phys. Plasmas 3, 1176 (1996). $\rightarrow$ Evaluate this formula up to the next order to add constraints for the secondary X-point.
- Example: NSTX-U equilibrium based on discharge 116313/Qg

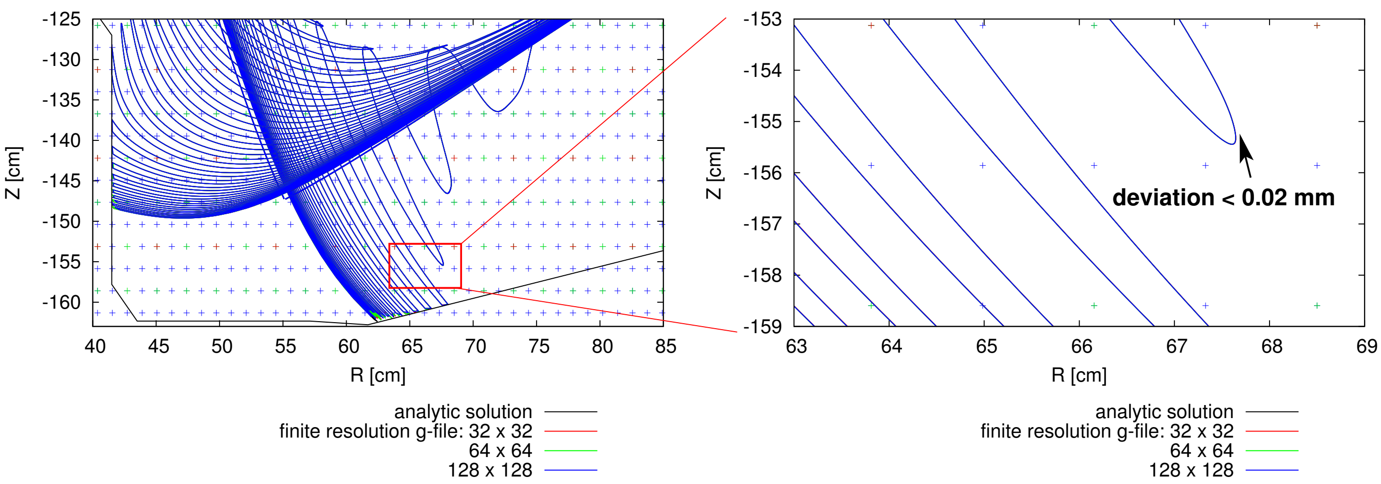
κ	2.36
δ	0.38
ε	0.60
R_0	92.7 cm
B_t	-1 T
I_p	2 MA
β	0.31
Primary X-point	(55.1 cm, -147.3 cm)
Secondary X-point	(50.5 cm, -164.0 cm)



- A g-file with arbitrary solution can be exported.

Separatrix manifolds are robust, even for low resolution g-files

- Resonant magnetic perturbations from midplane coils in $n = 3$ configuration, $I_c = 3\,\mathrm{kA}$.

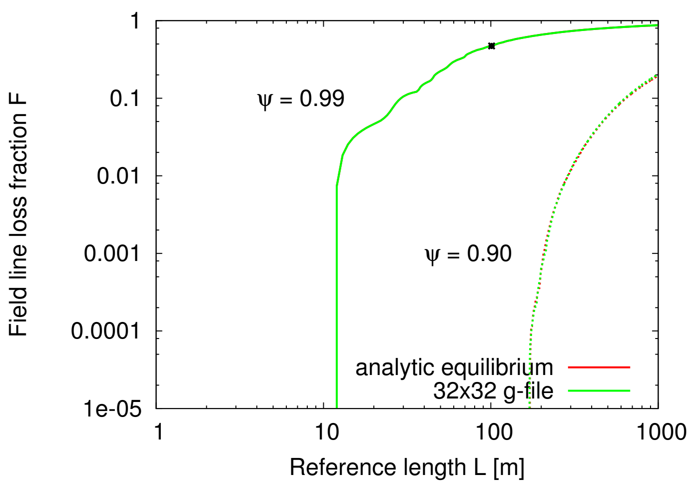


- No noticable difference between the original configuration and the reconstructed configuration from a g-file with 32×32, 64×64 and 128×128 grid points.

The field lines loss fraction $\mathcal{F}(\psi)$ is an indication for degradation of particle and energy confinement

- The loss fraction $\mathcal{F}(\psi)$ of field lines from a reference (unperturbed) flux surface is a function of the radial coordinate ψ.

- It also depends on the reference length L - the numerical cut-off length for field line tracing. In relation to the mean free path length, this parameter becomes relevant for transport physics as well.



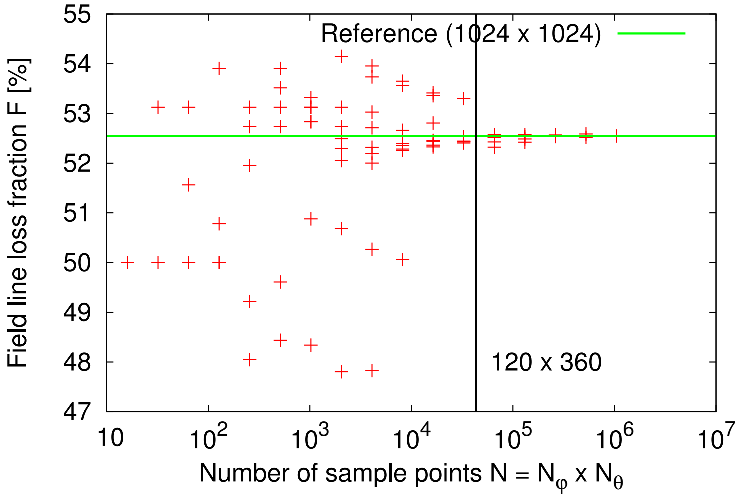
- No noticable impact of a finite resolution g-file up to $L = 1000\,\mathrm{m}$!

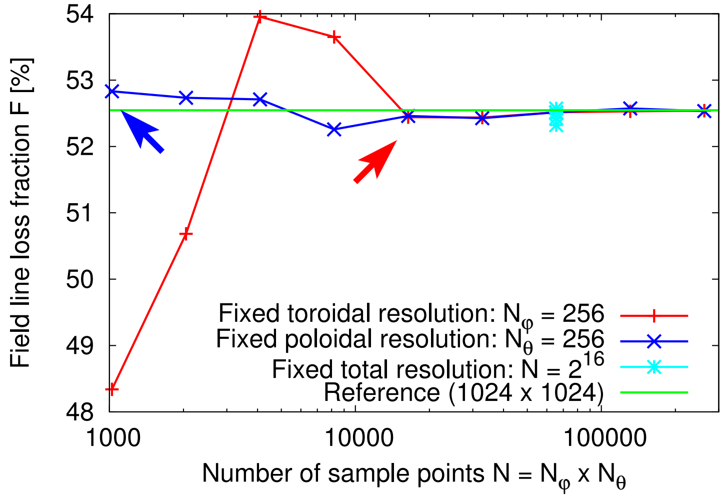
$$\mathcal{F}(\psi = 0.99, L = 100\,\mathrm{m}) = 52.66\,\%, \qquad \rightarrow \Delta\mathcal{F} = 0.06\,\%$$

- Further numerical parameters are the toroidal and poloidal sample resolutions N_φ and N_ϑ, respectively, for initial points on the flux surface ψ.

More uncertainty from flux surface sampling than from equilibrium resolution

- Analysis of finite resolution effects: $N_\varphi = 2^i, N_\vartheta = 2^j, \quad i, j = 2, \ldots, 10$
  $F(N) = \mathcal{F}(\psi = 0.99, L = 100\,\mathrm{m}, N_\varphi, N_\vartheta), \quad N = N_\varphi \cdot N_\vartheta.$

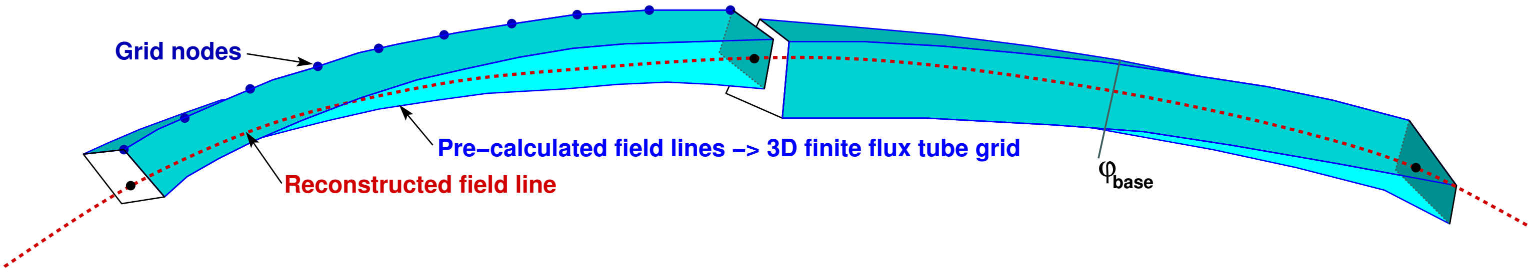




- $F(2^{15}) = 52.59 \pm 0.35\,\%, \quad F(2^{16}) = 52.47 \pm 0.10\,\%$
- A descent poloidal resolution $N_\vartheta \geq 64$ is required, but a low toroidal resolution $N_\varphi \geq 4$ may be sufficient for reasonable accuracy.

Field lines are reconstructed from a finite flux-tube grid

- A **reversible field line mapping** technique allows a continuous, non-diffusive field line representation (see Y. Feng et al., Phys. Plasmas 12, 052505 (2005))
- Grid nodes are constructed from initial field line tracing from 2D base grid(s) at φ_{base} for a finite length $\pm\Delta\varphi$.



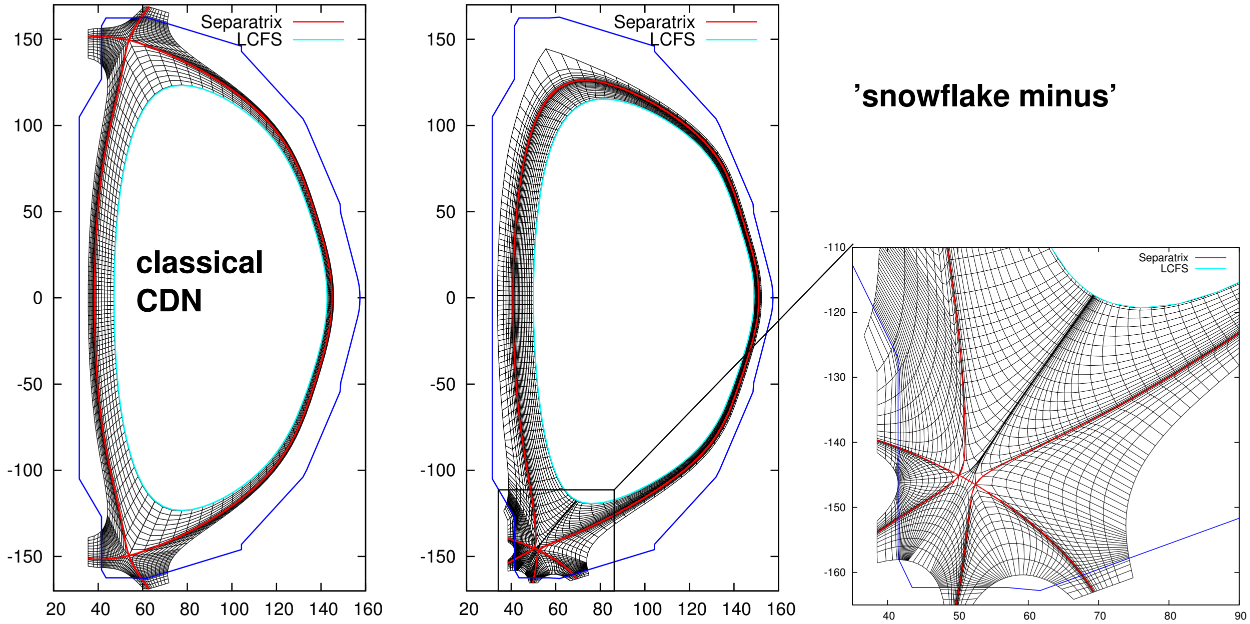
- A bilinear interpolation between pre-calculated field lines $\mathbf{F}_i(\varphi) = (R_i(\varphi), Z_i(\varphi))$ allows a fast reconstruction

$$\mathbf{F}^*_{\xi,\eta}(\varphi) = \sum_{i=1}^{4} \mathbf{F}_i(\varphi)\, N_i(\xi,\eta), \qquad N_i = \frac{1}{4}(1+\xi_i\,\xi)(1+\eta_i\eta)$$

- The shape of the flux-tubes must remain **convex** throughout the simulation domain in order to provide a unique relation $(R, Z) \leftrightarrow (\xi, \eta)$.

Discretization in the ’cross-field’ direction can be adapted to the magnetic configuration at hand

- The snowflake minus configuration is topologically equivalent to a connected double null configuration, and this can be exploited for grid generation

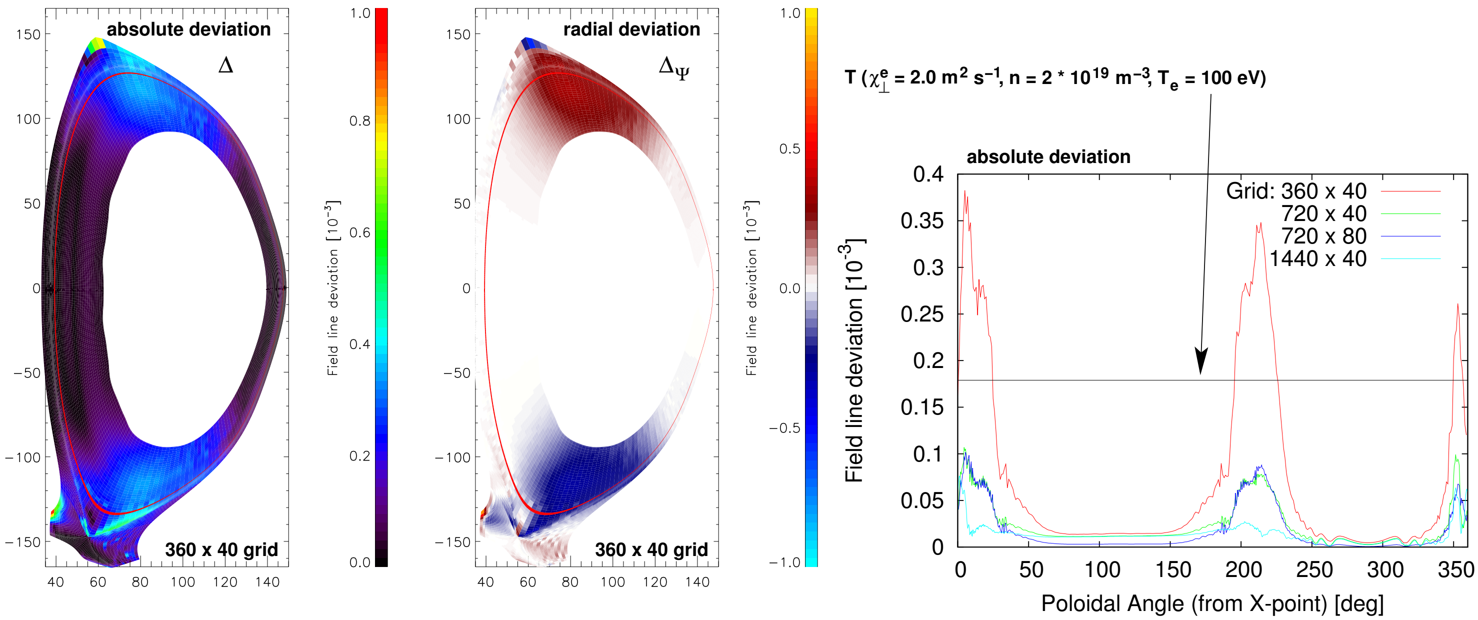


- Unlike grids for 2D transport modeling, the actual information about the magnetic configuration is not stored in the 2D base grid(s), but carried by the third toroidal component only

There are several means to characterize the quality of magnetic field line reconstruction

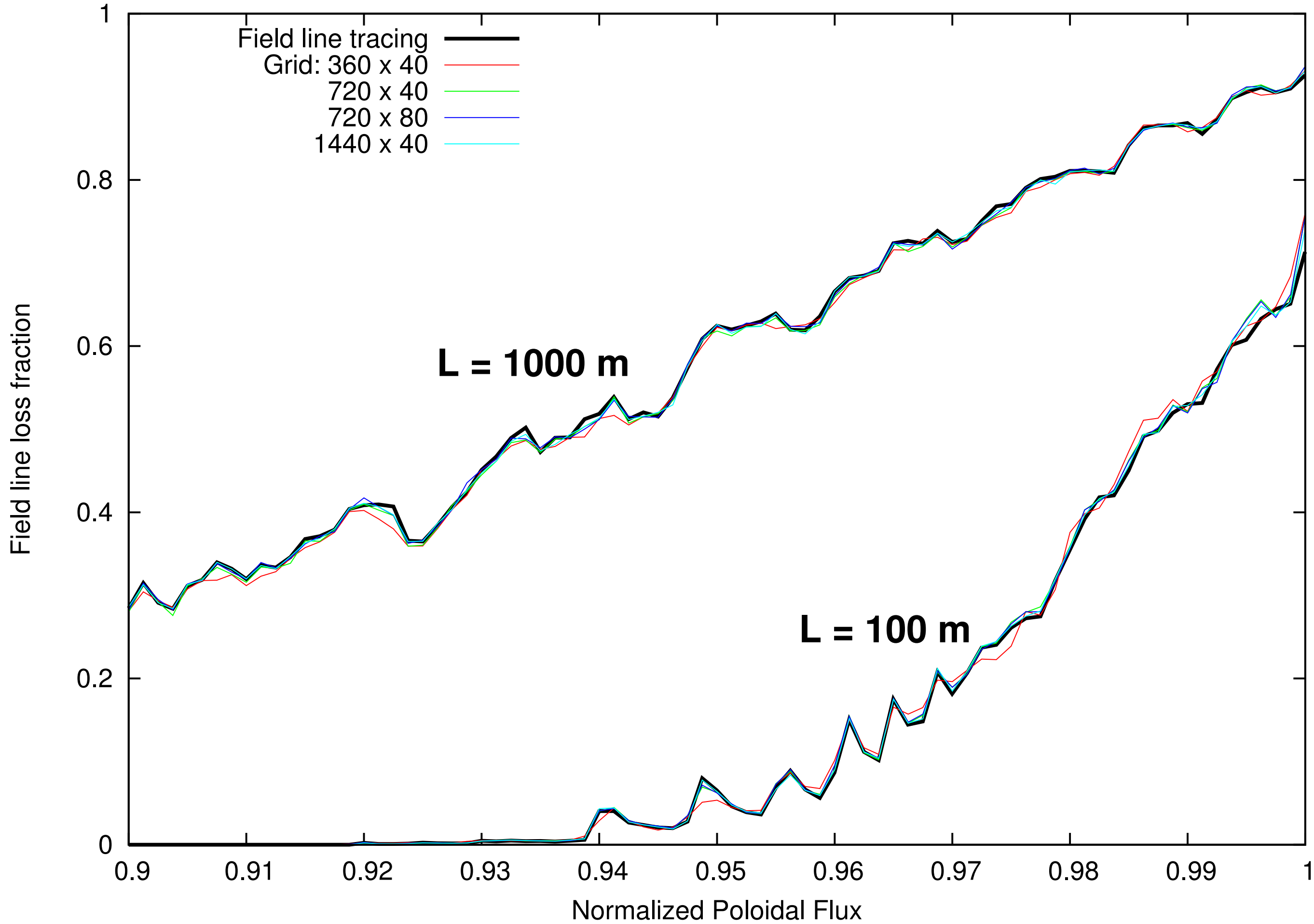
- Local interpolation error (within finite flux tube length $\Delta\varphi$)
  - field line deviation: $\Delta = \mathbf{F}^*_{\xi,\eta}(\varphi) - \mathbf{F}_{\xi,\eta}(\varphi)$
    $\rightarrow$ evaluate Δ for a reference field line ($\xi = \eta = 0$, i.e. cell center) for each cell at both ends of the finite flux-tube at $\varphi_{\text{base}} \pm \Delta\varphi$.
  - magnetic flux error: $\delta\Phi = (\max\Phi - \min\Phi)/\overline{\Phi}$
    $\rightarrow$ Φ is evaluated in each cell along a finite flux-tube
- Accumulated error (after long distance field line tracing and in transport simulations)
  - Evaluate field line loss fraction
  - Evaluate divertor footprints (particle and heat loads)

Local interpolation error associated with finite cell size

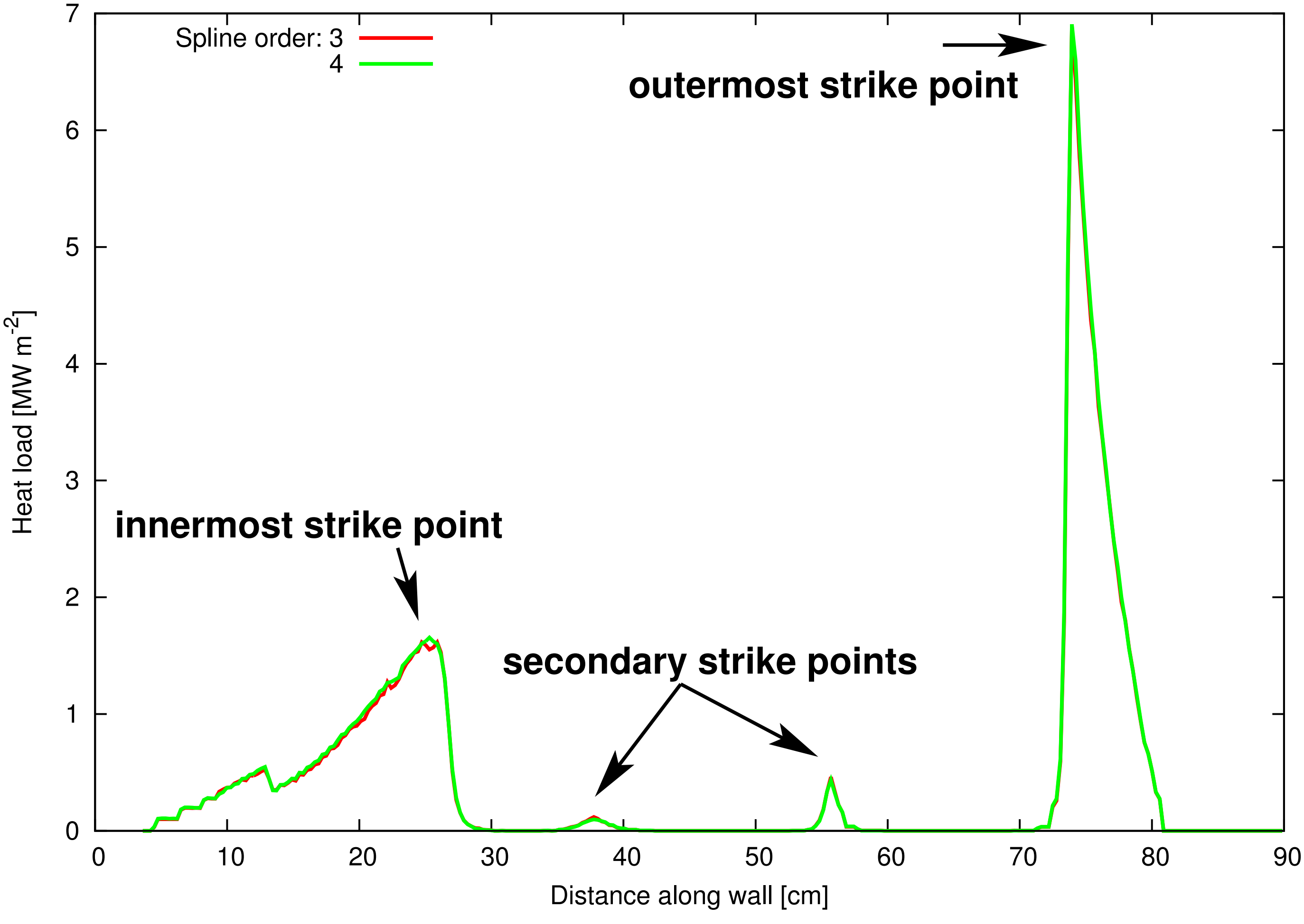


- The field line deviation Δ and its radial component Δ_Ψ depend on the cell size and poloidal position
- Electron heat transport imposes the limit $T = \sqrt{\chi_\perp^e / \chi_\parallel^e}$ for local accuracy, but it may be less impoartant in the long run for the radial deviation Δ_Ψ

Low accumulated error: Good representation of field line losses in transport code



Spline order related uncertainty in separatrix position is too small to be relevant for divertor heat loads



Conclusions

- The equilibrium is well represented even by low resolution g-files, and the separatrix manifolds can be generated with high accuracy
- The FLARE code provides tools to characterize the magnetic field configuration, and to generate and analyze grids for transport simulations
- The deviation of reconstructed field lines depends on cell size and poloidal position; its analysis allows to fine tune the grid for optimal resolution
- The average long-term behavior of reconstructed field lines is very robust even at lower grid resolution; divertor heat loads are not affected by uncertainties in the separatrix position
- See poster by I. Waters (GP12.00089) for more details on the magnetic configuration of NSTX-U advanced divertors with RMPs, including first results from transport modeling