Hybrid MHD/Particle Simulation Study of Sub-Cyclotron Alfvén Eigenmodes in NSTX

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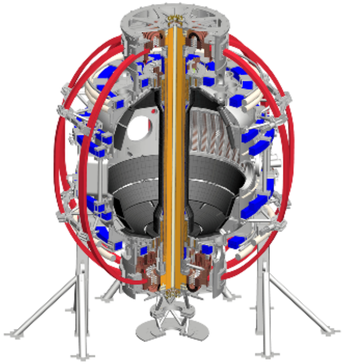
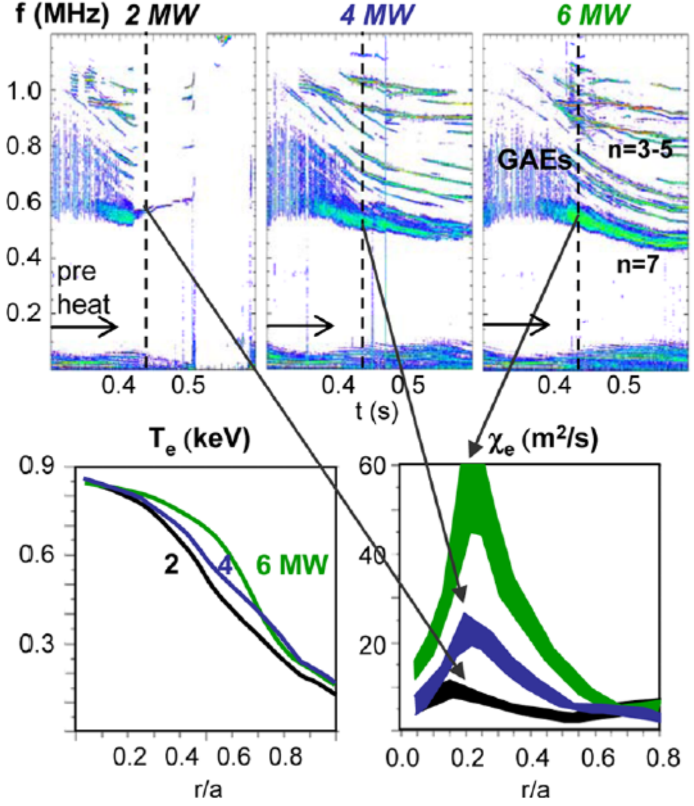


Abstract

High frequency compressional (CAE) and global (GAE) Alfvén Eigenmodes are often driven unstable by super-Alfvénic beam ions in NSTX. These modes have been identified as part of an energy channeling mechanism that may explain observed anomalous electron temperature profile flattening in beam-heated NSTX plasmas. 3D hybrid simulations using the HYM code are conducted to study the excitation and stability properties of such CAE and GAE modes in H-mode NSTX discharge 141398. HYM uses a delta-f particle treatment of the energetic beam ions coupled to a single fluid resistive MHD model of the bulk thermal plasma. Parameters in the beam ion distribution function are varied in order to explore mode stability in pitch-energy space.

CAE/GAE May Limit ST Performance

- Many beam-heated NSTX discharges exhibit anomalously flat T_e profile.
  - Correlates with increased beam power, strong CAE/GAE activity [Stutman, PRL 2009]
- Multiple potential flattening mechanisms
  - Enhanced electron transport due to orbit stochasticity from multiple GAE [Gorelenkov, NF 2010]
  - Energy channeling via mode conversion from core CAE to edge KAW [Belova, PRL 2015]



- Vital to understand preferential conditions for exciting (or suppressing) these allegedly deleterious modes.
  - Anomalously low T_e could threaten future ST development.

Review of CAE/GAE Mode Properties

- Common Properties
  - Typical frequency in range $0.3\omega_{ci} < \omega < \omega_{ci}$
  - Normal plasma mode (e.g exist with or without EP present)
  - May be driven unstable through energetic particle resonances
    - $\omega = k_{||}v_{||}$
    - $\omega = k_{||}v_{||} + \omega_{ci}$

- Compressional AE (CAE)/fast magnetosonic
  - Compressional polarization
    - $\delta B \sim \delta B_{||} >> \delta B_{\perp} \sim 0$
  - Disperson: $\omega \sim k\, v_A$
  - Core localized

- Global AE (GAE)
  - Shear polarization
    - $\delta B \sim \delta B_{\perp} >> \delta B_{||} \sim 0$
  - Dispersion: $\omega \sim k_{||}v_A$
  - Broader mode structure

HYM Hybrid MHD/Particle Code

- Allows investigation of kinetic effects on MHD modes in toroidal geometry
- 3D nonlinear, parallel
- Several physical models
  - Resistive MHD & Hall MHD
  - Hybrid (fluid electrons, particle ions)
  - **MHD/particle (one fluid bulk plasma, energetic particle ions)**
- Full-orbit kinetic ions
- Delta-f numerical scheme to reduce noise
- Self consistently solves for equilibrium including energetic particle effects

HYM Physical Equations

Background plasma - fluid:

$$\rho\frac{d\mathbf{V}}{dt} = -\nabla p + (\mathbf{j} - \mathbf{j}_b)\times\mathbf{B} - n_b(\mathbf{E} - \eta\mathbf{j})$$

$$\mathbf{E} = -\mathbf{V}\times\mathbf{B} + \eta\mathbf{j}$$

$$\mathbf{B} = \mathbf{B}_0 + \nabla\times\mathbf{A}$$

$$\partial\mathbf{A}/\partial t = -\mathbf{E}$$

$$\mathbf{j} = \nabla\times\mathbf{B}$$

$$\partial p^{1/\gamma}/\partial t = -\nabla\cdot(\mathbf{V}p^{1/\gamma})$$

$$\partial\rho/\partial t = -\nabla\cdot(\mathbf{V}\rho)$$

Fast ions – delta-F scheme:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \mathbf{E} - \eta\mathbf{j} + \mathbf{v}\times\mathbf{B}$$

$w = \delta F/F$ - particle weight

$$\frac{dw}{dt} = -(1-w)\frac{d(\ln F_0)}{dt}$$

$$F_0 = F_0(\varepsilon, \mu, p_\phi)$$

ρ, **V** and p are thermal plasma density, velocity and pressure, n_b and $\mathbf{j}_b$ are beam ion density and current, and $n_b << n_e$ – is assumed.

Fast Ion Distribution Function

Equilibrium distribution function $F_0 = F_1(v)\, F_2(\lambda)\, F_3(p_\varphi, v)$

$$F_1(v) = \frac{1}{v^3 + v_*^3}, \text{ for } \ v < v_0$$

$$F_2(\lambda) = \exp(-(\lambda - \lambda_0)^2 / \Delta\lambda^2)$$

$$F_3(p_\phi, v) = \frac{(p_\phi - p_0)^\beta}{(R_0 v - \psi_0 - p_0)^\beta}, \ \text{ for } \ p_\phi > p_0$$

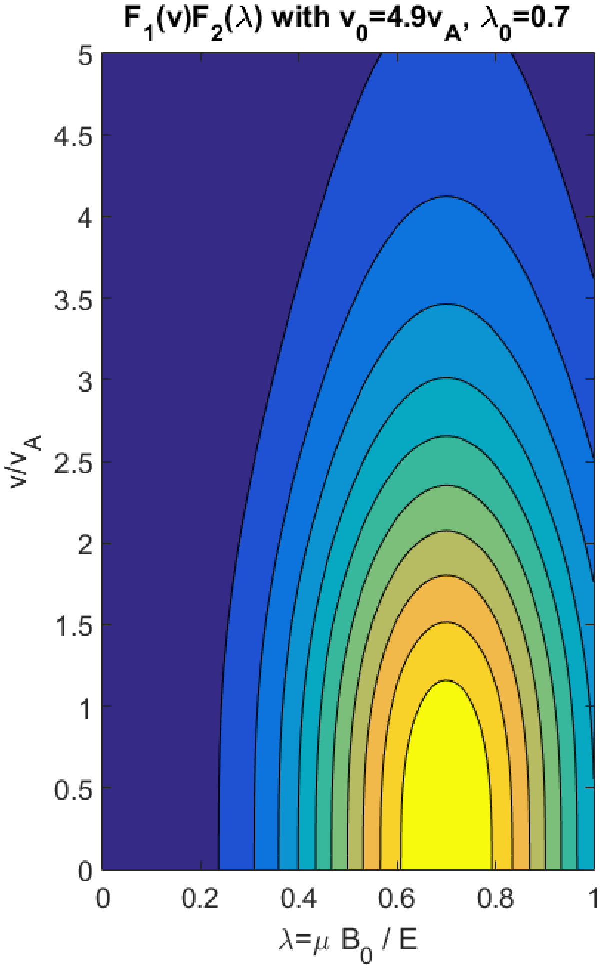
where v_0 =2 - 6 v_A, $v_* = v_0/2$, $\lambda = \mu B_0/\varepsilon$ – pitch angle parameter, λ_0= 0.1 - 0.9

and $\mu = \mu_0 + \mu_1$ includes first-order corrections [Littlejohn’81]:

$$\mu = \frac{(\mathbf{v}_\perp - \mathbf{v}_\mathbf{d})^2}{2B} - \frac{\mu_0 v_\parallel}{2\mathrm{B}}[\hat{b} \cdot \nabla \times \hat{b} - 2(\hat{a} \cdot \nabla \hat{b}) \cdot \hat{c}]$$

$\mathbf{v}_d$ is magnetic gradient and curvature drift velocity, $\hat{c} = \mathbf{v}_\perp / v_\perp$, $\hat{a} = \hat{b} \times \hat{c}$

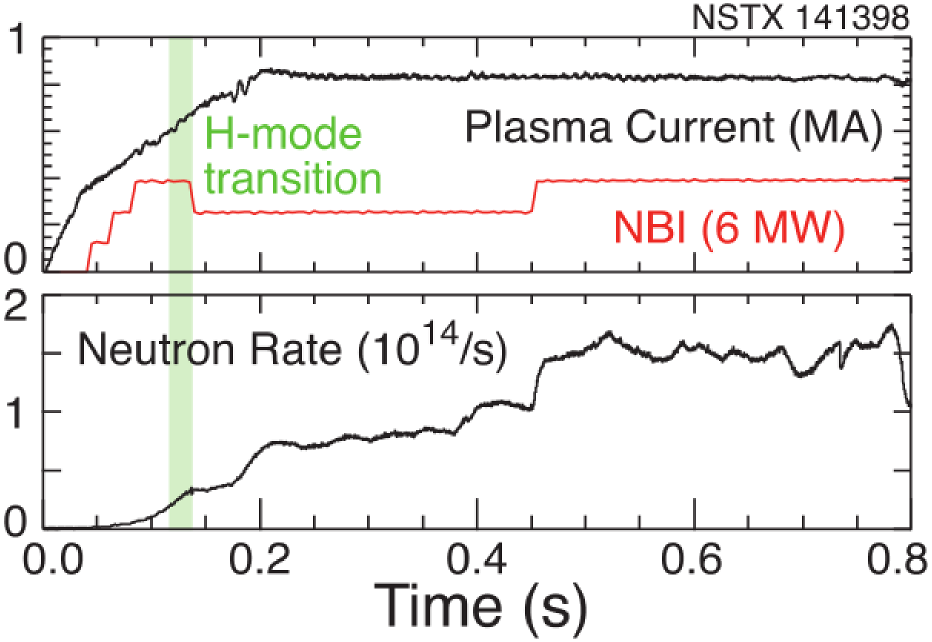
Parameters are chosen to match TRANSP beam profiles.

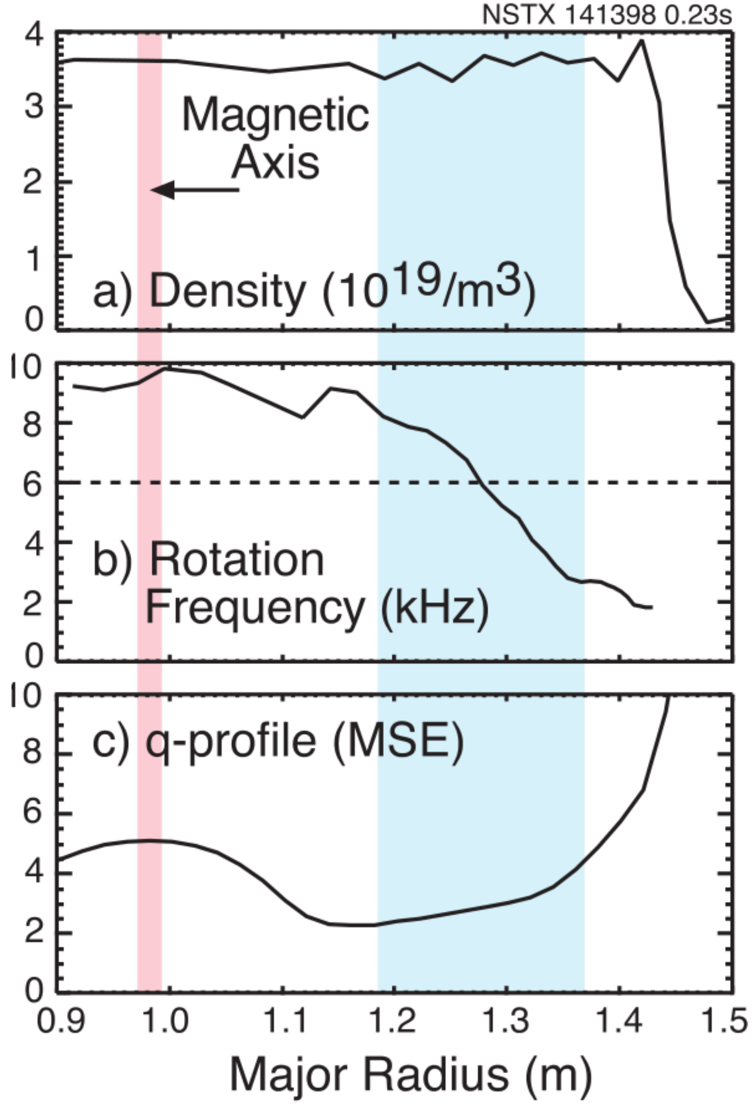

$F_1(v)F_2(\lambda)$ with v_0=4.9v_A, λ_0=0.7

NSTX H-mode Shot 141398

- $n_e = 6 \times 10^{19}\ m^{-3}$
- $B_{tor,0}=0.325$ T
- $I_p = 0.8$ MA

- 90 keV NBI at 6 MW
- $v_0 = 4.9\ v_A$
- $N_b/n_e \sim 5\%$



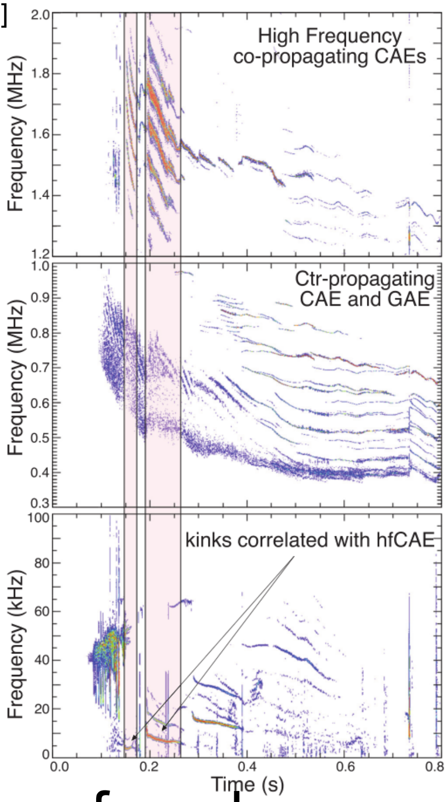


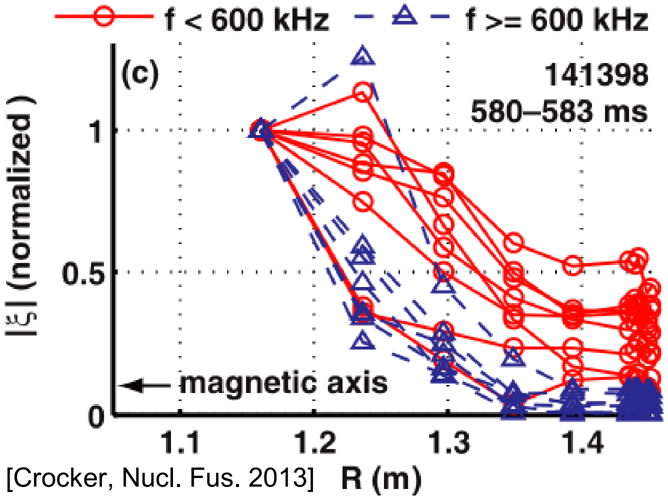
[Figures from Fredrickson, PoP 2013]

CAE & GAE Observed in Experiment

- Fredrickson observes 3 groups of modes
  - Co-propagating CAE
    - n=6-14
    - $0.5\omega_{ci} < \omega < 0.75\omega_{ci}$
  - Counter-propagating CAE & GAE
    - $0.15\omega_{ci} < \omega < 0.35\omega_{ci}$
  - Kinks correlated with high frequency co-CAE
    - $\omega \sim .005\omega_{ci}$

[Fredrickson, PoP. 2013]



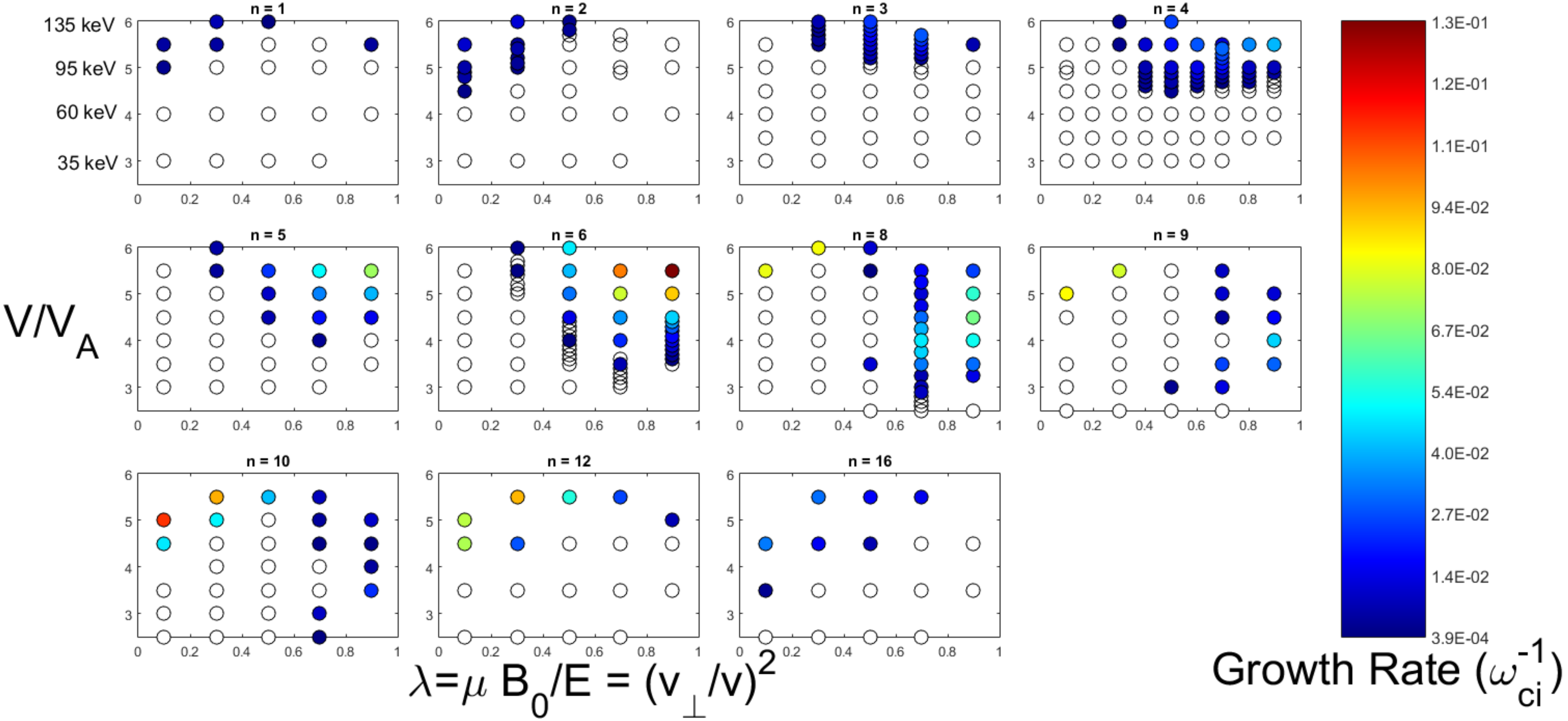


[Crocker, Nucl. Fus. 2013]

- Crocker identifies 2 groups of modes
  - $\omega < 0.25\omega_{ci}$ CAE
    - $-6 < n < -8$
  - $\omega > 0.25\omega_{ci}$ GAE
    - $|n| \leq 5$, mostly counter-propagating

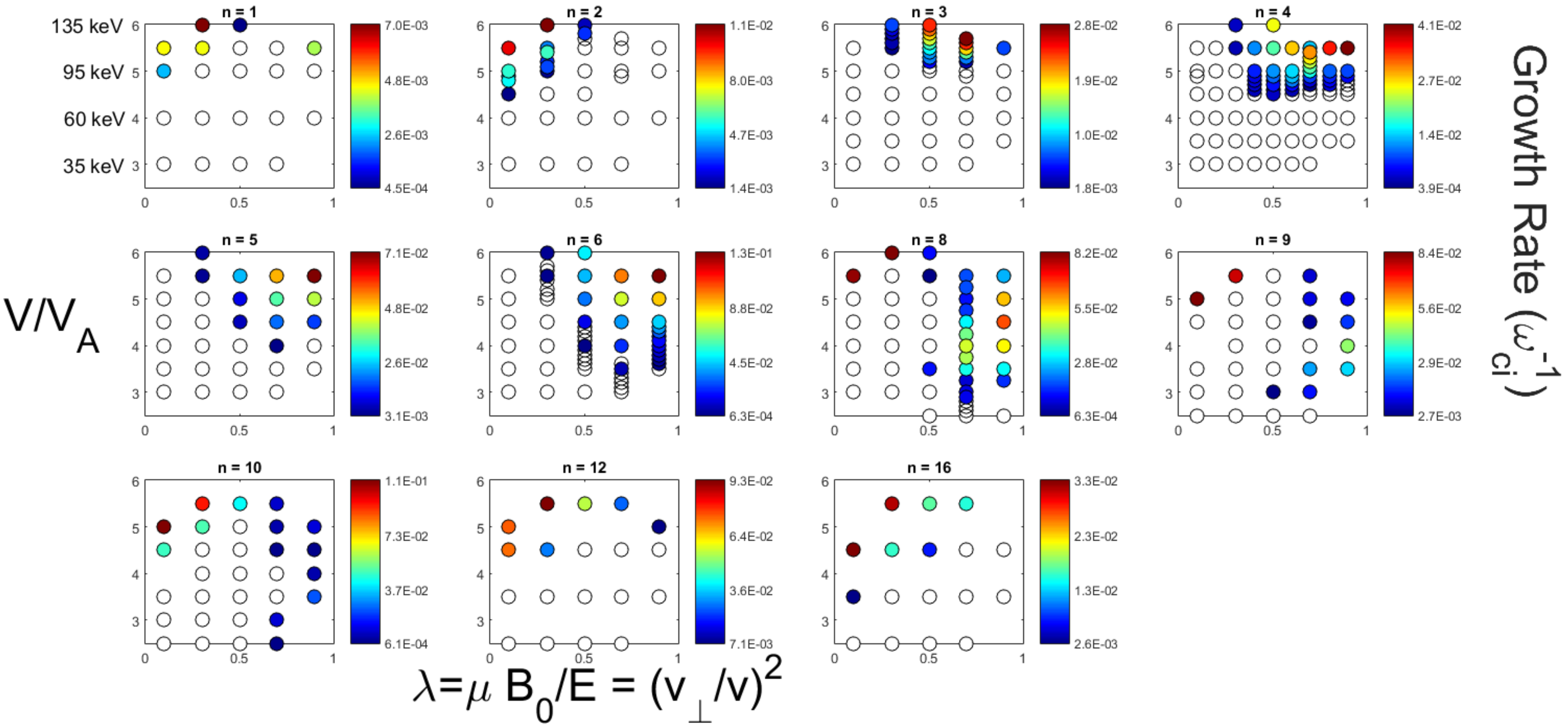
Stability Results

- Most unstable modes found with n ~ 5-12
- White circles are stable modes in simulation
- n=7,11,13-15,>16 not yet analyzed



Stability Results

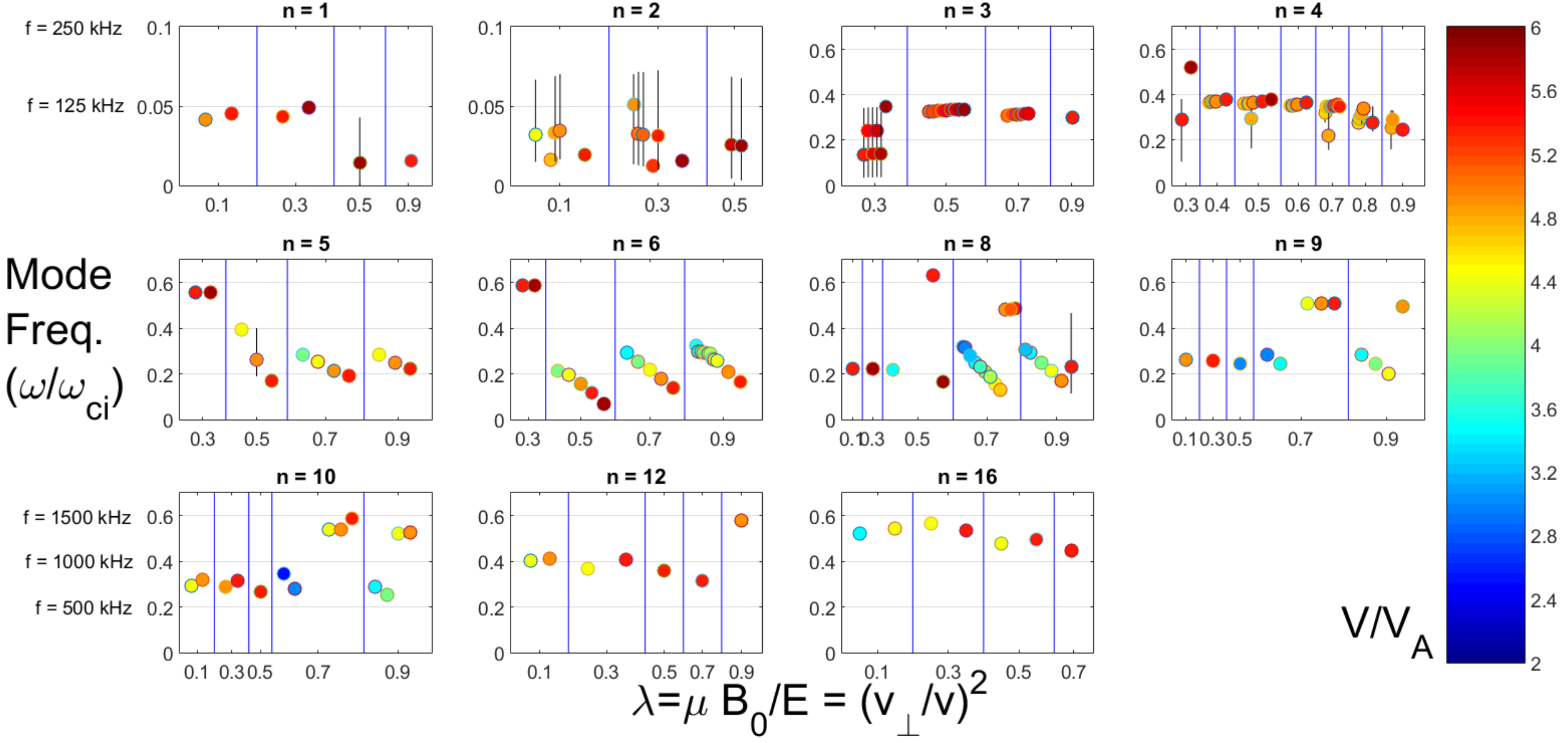
- In general, growth rate increases with beam energy
  - Notable exception: n=8,9 have local maxima
- Stability boundaries are not simple v, $v_{||}$, $v_{\perp}$, or λ contours
  - n=3-6 growth rates trend with $v_{\perp}$
  - n=1,2,16 growth rates trend with $v_{||}$
  - n=8-10,12 show mixed behavior



Mode Frequencies Shift with λ,E

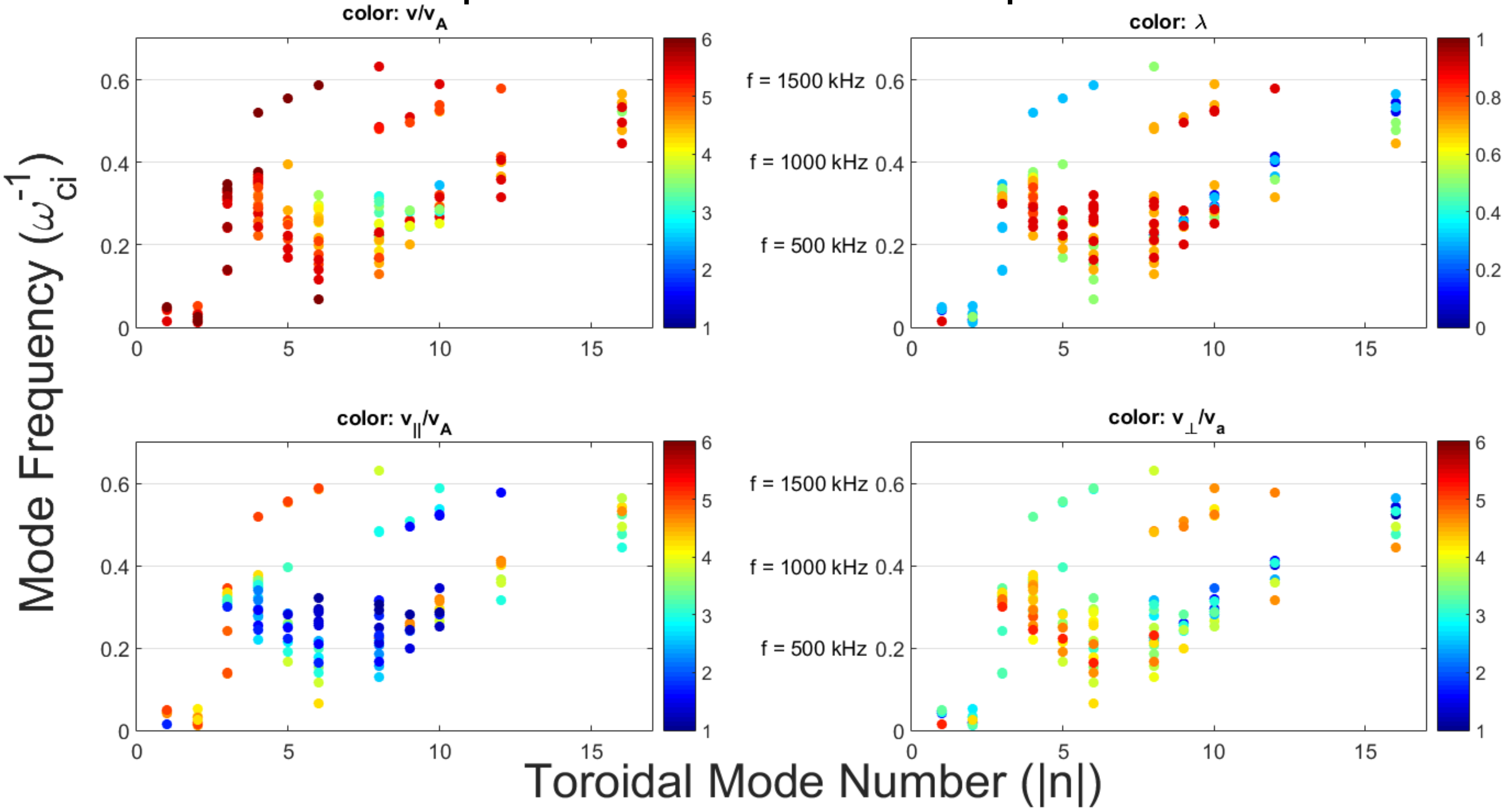
- Three groups of frequencies identified
  - $\omega < .05\omega_{ci}$ (n=1,2)
  - $0.1\omega_{ci} < \omega < 0.4\omega_{ci}$ (n=3-6,8-10,12)
  - $\omega > 0.5\omega_{ci}$ (n=4-6,8-10,12,16)

- Fixing λ, many modes have frequency decrease with energy
  - Most dramatically n=5, 6, 8, 9
- A few exhibit the reverse: frequency increasing with energy
  - n=4,10



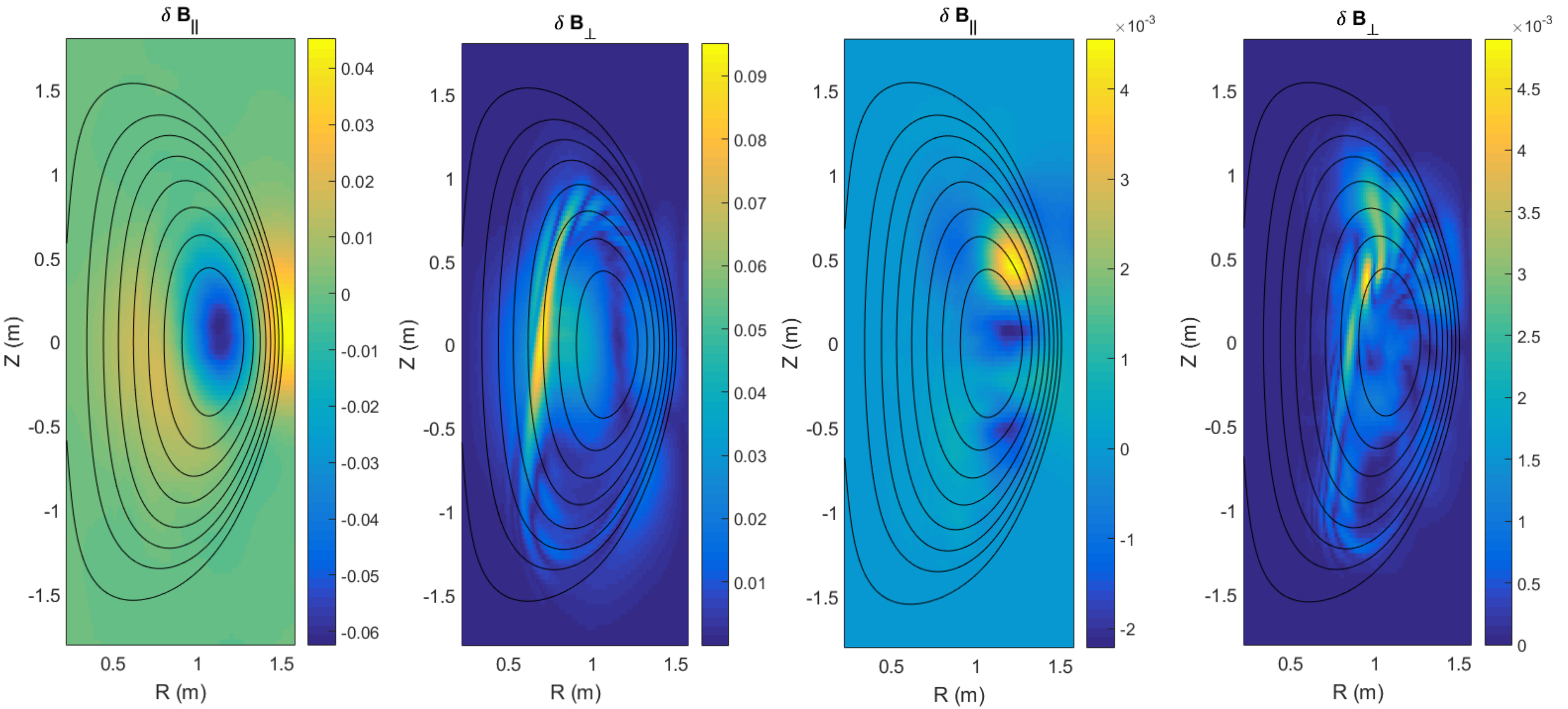
3 Groups of Mode Frequencies

- Moderate frequencies ($0.2\omega_{ci}$-$0.4\omega_{ci}$) decrease for n<7, increase for n>7
- High frequency modes require larger beam energy, otherwise all simple parameters are well represented across all frequencies



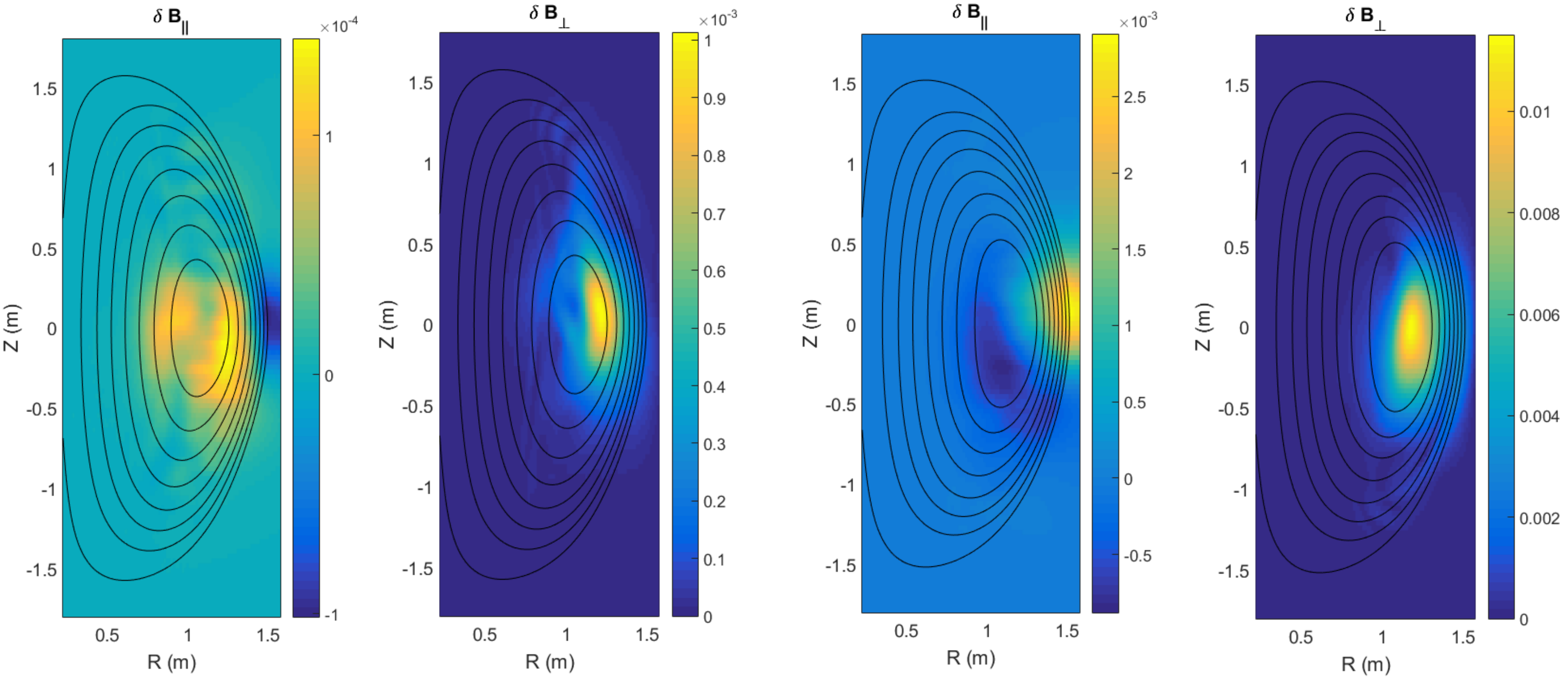
CAE/KAW Polarization

- Very core-localized, compressional, typically low poloidal mode number (m=0-2)
- KAW visible in $\delta B_\perp$, comparable or larger in magnitude than CAE
- Left: $n=4, \lambda=0.7, v=5.0, f \sim 870$kHz. Right: $n=10, \lambda=0.7, v=5.0, f \sim 1350$kHz



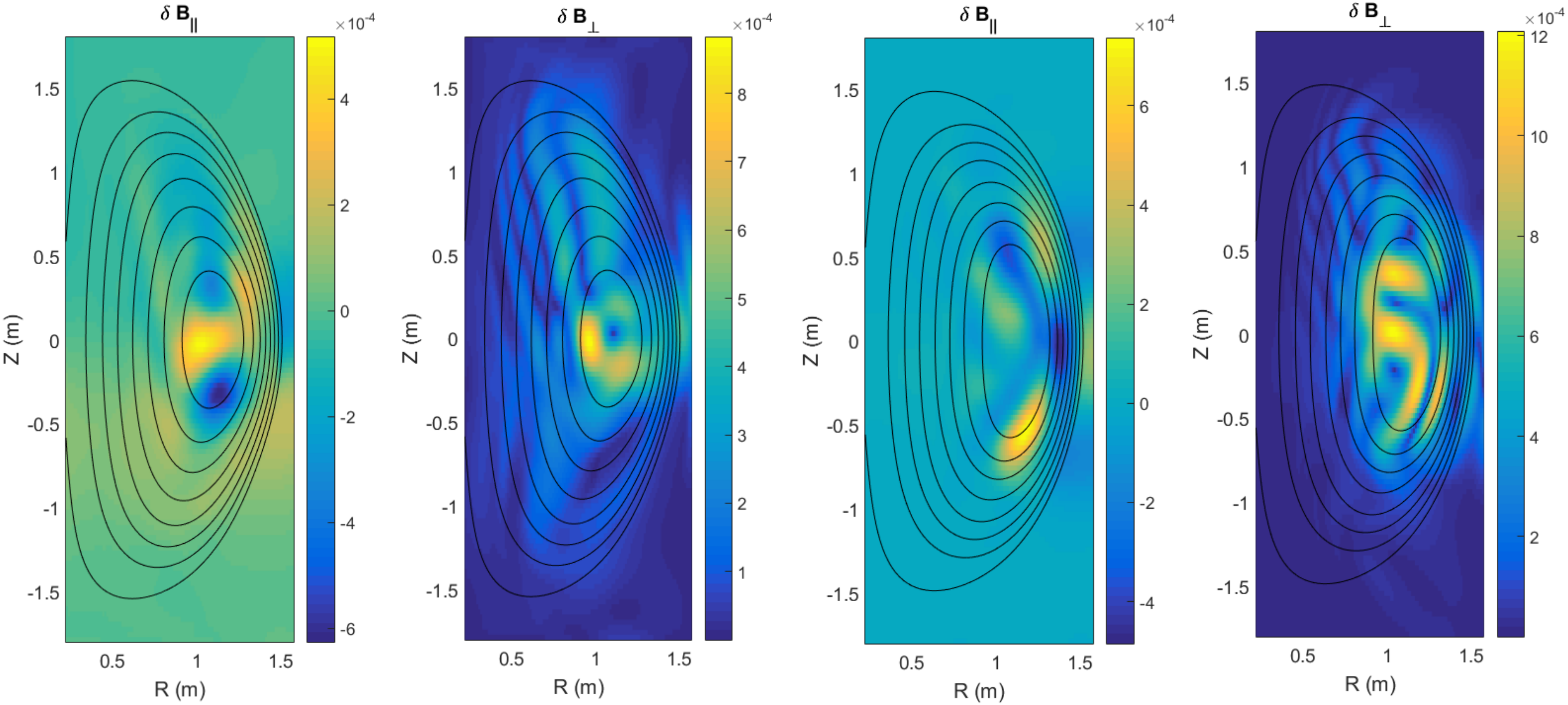
GAE Polarization

- Core-localized, typically low poloidal mode number (m=0-1)
- Shear polarization with $\delta B_{||}/\delta B_{\perp}$~10, but $\delta B_{||}$ not entirely negligible
- Left: n=8,λ=0.7,v=4.0,f~520kHz. Right: n=10,λ=0.1,v=5.0,f~800kHz



Low Frequency Mode Polarization

- Mixed polarization, though often $\delta B_\perp > \delta B_{||}$
- Higher poloidal mode number (m=3-4) than CAE/GAE (m=0-2)
- Left: $n=1, \lambda=0.9, v=5.5, f\sim 40$kHz. Right: $n=2, \lambda=0.3, v=6.0, f\sim 40$kHz



Summary and Conclusions

- Hybrid simulations conducted to investigate dependence of CAE/GAE stability on beam ion parameters
  - n=1-6,8-10,12,16 simulated with $2.5 \leq v_0/v_A \leq 6.0$, $0.1 \leq \lambda_0 \leq 0.9$
- 3 frequency groups of unstable modes found
  - Low ($0.01 \sim 0.05\omega_{ci}$), moderate ($0.2 \sim 0.4\omega_{ci}$), high ($0.5 \sim 0.65\omega_{ci}$)
- 3 groups of modes identified
  - CAE, GAE in n>2, broad frequency range (f=300-1625kHz)
  - TAE/kink/??? only in n=1,2, very low frequency (f=25-125kHz)
- Stability boundaries do not correspond to level surfaces of $v, v_{||}, v_\perp$, or λ. Almost all modes grow faster with larger v.
- At fixed n,λ, some mode frequencies decrease significantly
- In general, $v/v_A \lesssim 4$ turns off most unstable modes
  - Easily accessibly in NSTX-U with B_{tor} increasing by factor of 2

Further Work in Progress

- Wealth of simulation data remains for further analysis
  - ~200 unstable modes to be systematically classified in each case by...
    - Co- or counter-propagating relative to beam
    - Compressional or shear polarization – CAE vs GAE vs ???
    - Identify groups of resonant particles
- Further investigate low n, low frequency modes
  - TAE or kink or something else? Why not seen in n>3?
- Develop theory to explain simulation results
  - Can the marginal stability boundary be predicted?
  - For fixed n & λ: why the occasionally large change in frequency?
- Comparison of simulation results to experimental data
  - At the parameters used in this discharge ($v/v_A=4.9$, $\lambda=0.7$), do we see the same modes?
  - Do similar discharges agree with other points in parameter space?

References and Further Reading

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