Application of the Finite Orbit Width Version of the CQL3D Code to NBI+RF Heating of NSTX Plasma

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Outline of CQL3D-FOW development in 2015

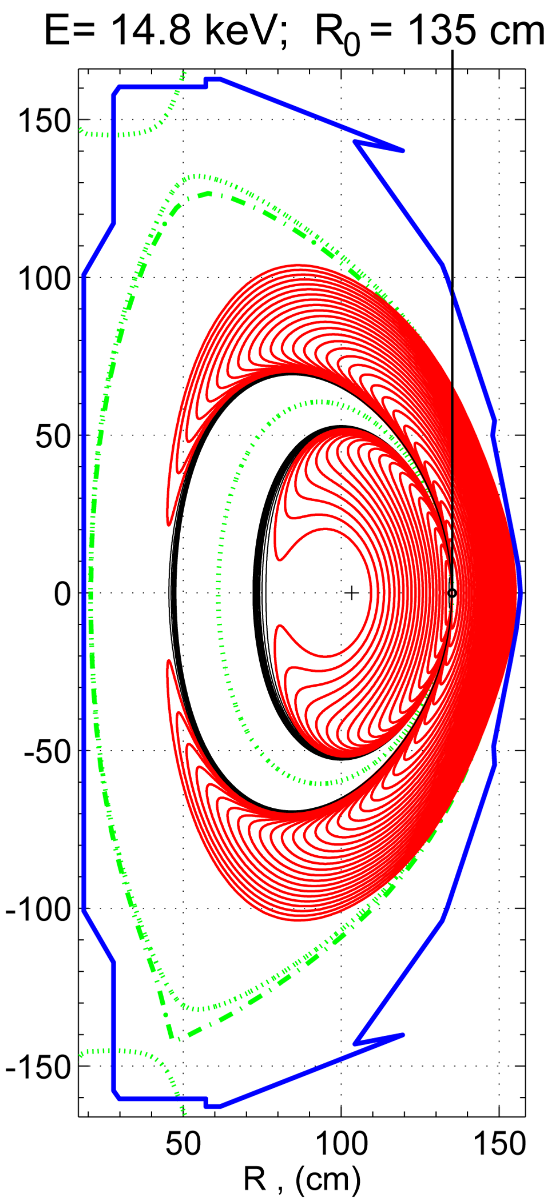
- Setting the initial distribution function for full-FOW.
- Modified method for rescaling the distribution at each time step.
- Re-running bootstrap current tests;

  Internal Boundary Conditions (IBC) + ”balancing term”.
- Full-FOW form of bounce-average QL rf operator.
- FOW to ZOW convergence tests.

Quick Review: The Essence of Full-FOW

Write FPE in canonical action-angle ($\mathbf{J}, \mathbf{\Theta}$), then average over periodical angles.

Then – transform to "convenient" coordinates $\mathbf{I}$.



$$\Im \frac{\partial f_0}{\partial t} = \frac{\partial}{\partial I_i} \Im \left(\left\langle \frac{\partial I_i}{\partial \mathbf{u}} \mathbf{D}^{\mathbf{uu}} \frac{\partial I_j}{\partial \mathbf{u}} \right\rangle \frac{\partial}{\partial I_j} - \left\langle \frac{\partial I_i}{\partial \mathbf{u}} \mathbf{F}^{\mathbf{u}} \right\rangle \right) f_0$$

Jacobian of the transformation from canonical $\mathbf{J}$ to $\mathbf{I}$.
(Summation over i and j)

- $\mathbf{D}^{\mathbf{uu}}$ is *local*, but it couples with radial transformation coefficients like $\partial R_0/\partial u$ and, after averaging over gyro and bounce periods + tor.symmetry, results in **appearance of neoclassical radial transport terms.**
- **Choice of $\mathbf{I}$ space** (computational grid): The midplane R coord (R_0), mom./mass (u_0), and pitch-angle at the midplane (θ_0).
- **Representation of f_0** at $R_{0,l}$: A set of orbits with ($u_{0,j}$, $\theta_{0,i}$) launched from $R = R_{0,l}$ grid point (computed g.c. orbits are used for bounce-averaging of coll. operator, QL operator, and setting the loss cone).

Based on: Kaufman, Phys. Fluids **15**, 1063 (1972); Bernstein and Molvig, Phys. Fluids **26**, 1488 (1983); Kupfer, IAEA TCM on FWCD in Reactor Scale Tokamaks, Arles, 1991; Westerhof and Peeters, 35th APS-DPP meeting, St. Louis, (1993); Eriksson and Helander, Phys. Plasmas **1**, 308(1994).

Setting the Initial Distribution and Rescaling: Approach

Want to set a Maxwellian-like distribution (before NBI sources are added):

$$f_M = \frac{n \exp\left(-mu^2/2T\right)}{\pi^{3/2}\left(2T/m\right)^{3/2}}$$

But f in the bounce-average FPE is a function of COM, so it must have same values at both legs of each orbit.

What to use for $n(R_0)$ and $T(R_0)$ at a given radial grid coord. R_0?

1. For each orbit find $<\Psi>_{BA}$ of pol. flux over complete turn.
2. Use $\rho \sim <\Psi>$ to find $n(\rho)$ and $T(\rho)$.
3. Attribute these values of $n(\rho)$ and $T(\rho)$ to both legs of given orbit, and set the value of "Maxwellian" distribution at R_0 grid point based on these $n(\rho)$ and $T(\rho)$.

Rescaling: In ZOW, the distr. function can be rescaled at each time step to maintain $<n>_{FSA} = const$ at each ρ.

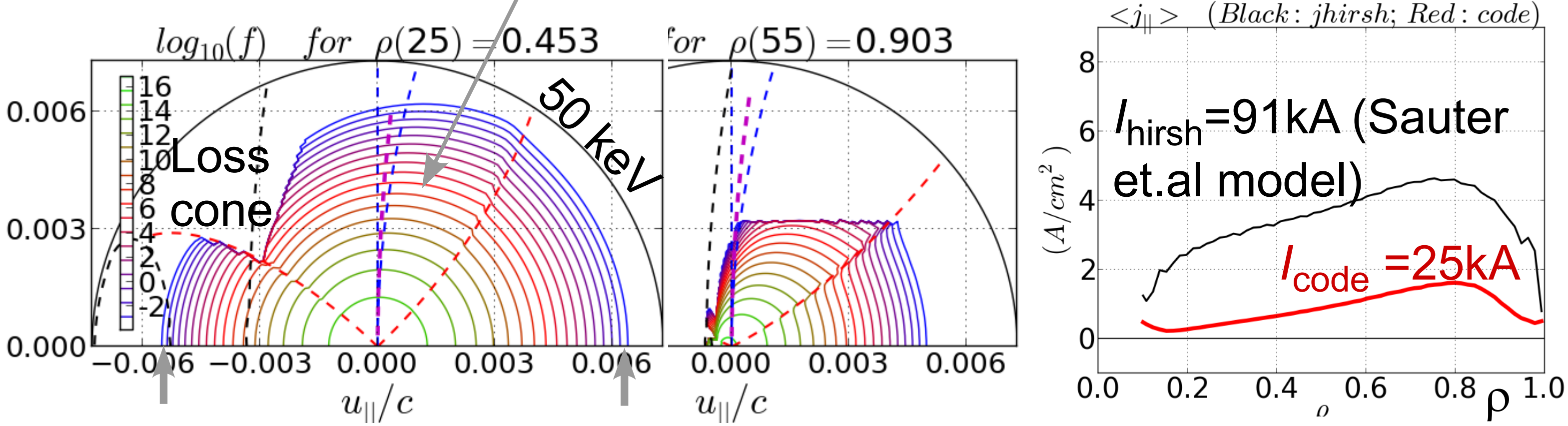
For full-FOW, before 2015, the rescaling was done to maintain $n_0(\rho) = const$ (the midplane density). It can create an imbalance of f at two legs of an orbit.

Solution: Calculate total number of particles $N(t)$ in the volume, then rescale $f(R_0,u_0,\theta_0)$ by the same factor at all radial points, to maintain $N(t) = const$.

(With this setup, the profile of density is allowed to evolve now.)

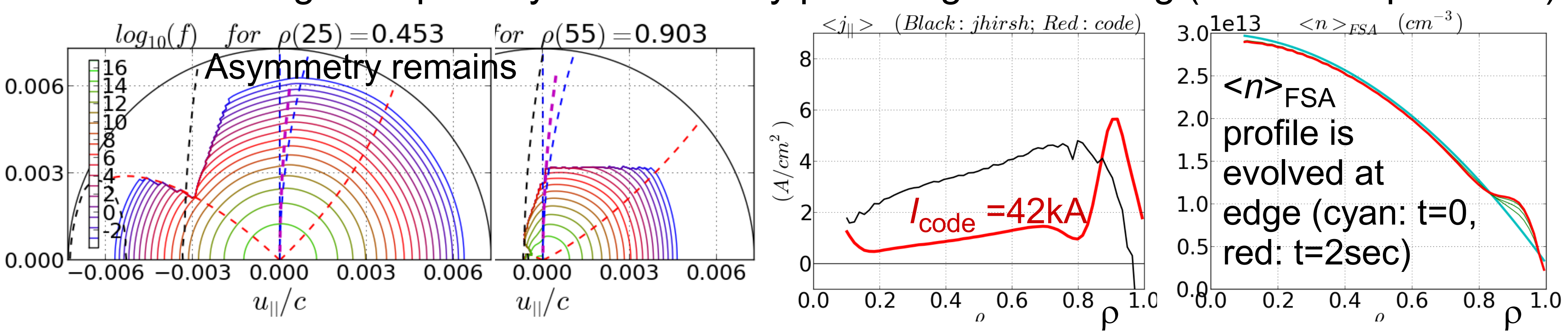
Initial Distribution and its Evolution [NSTX, parab $n(\rho)$ and $T(\rho)$; D^+]

t=0: Asymmetry for trapped, also for passing particles

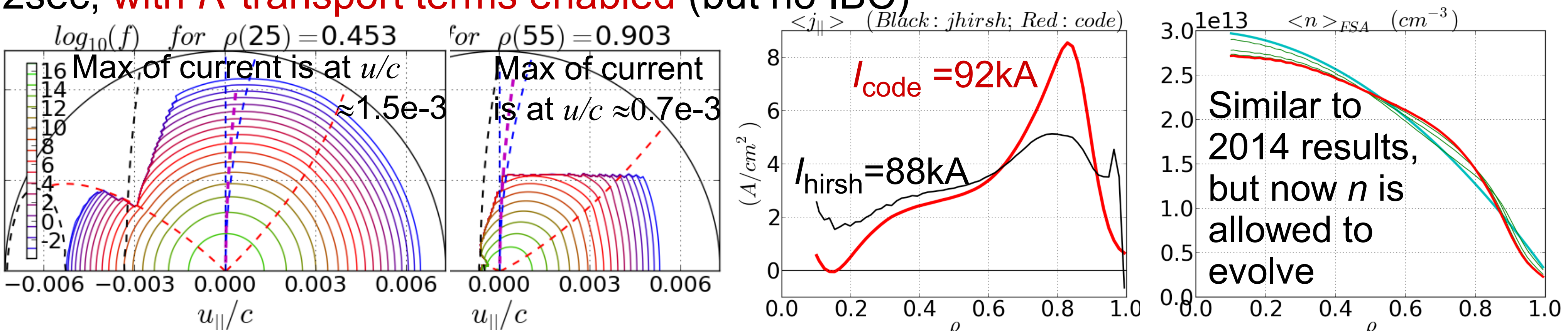


The initial asymmetry already accounts for 27% of expected bootstrap current.

t=2sec: The ridge at t-p bndry is diffused by pitch-angle scattering (no *R*-transport here)

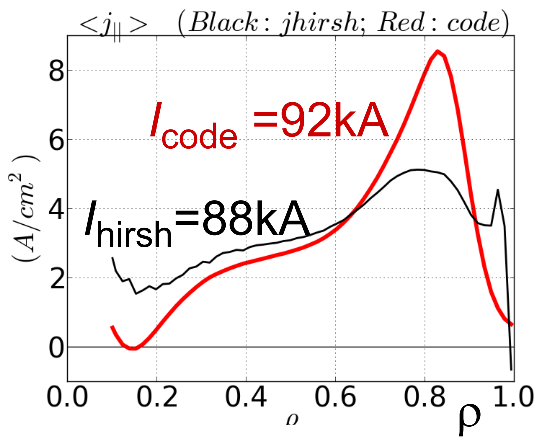


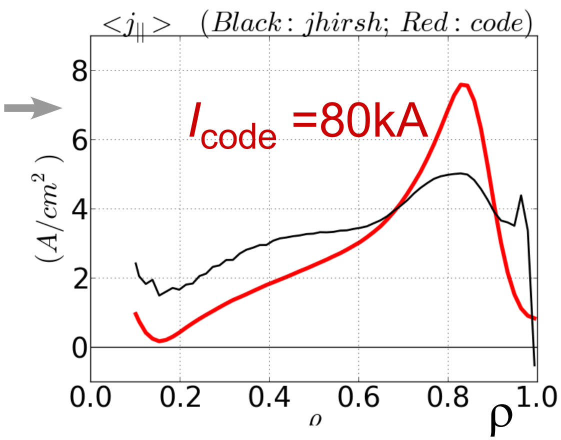
t=2sec, with *R*-transport terms enabled (but no IBC)



Bootstrap Current Test: IBC and "Balancing Term"

No IBC → With IBC





Last year, enabling IBC resulted in 24% drop of the bootstrap current.

⇐ Now, the drop is 13% (because of proper initial f and proper rescaling).

Can the result be improved?

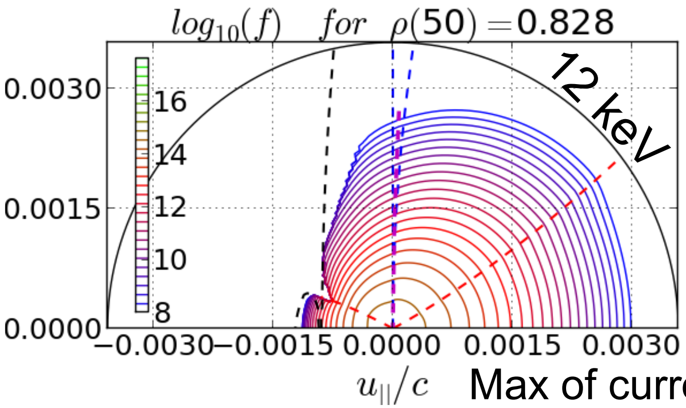
Introducing the "balancing term" :

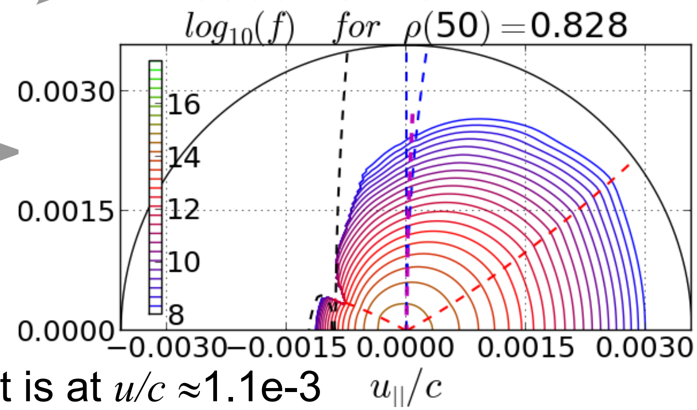
$$\frac{\partial f_a}{\partial t} = C(f_a) + QL(f_a) + S + \frac{-(f_a - f_b)}{\tau_B}$$

where f_b is the value at other leg of orbit.

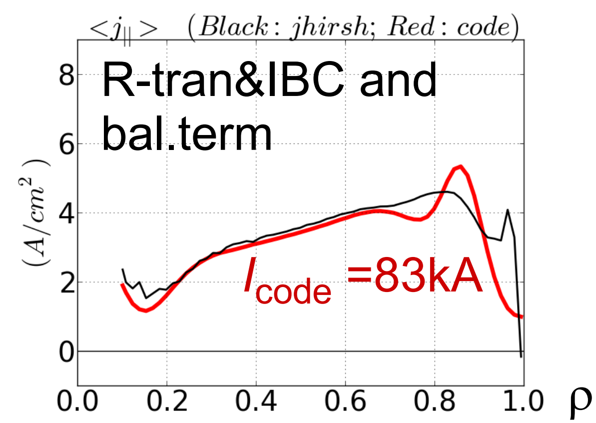
At leg "b", $\partial f_b/\partial t = \ldots - (f_b - f_a)/\tau_B$

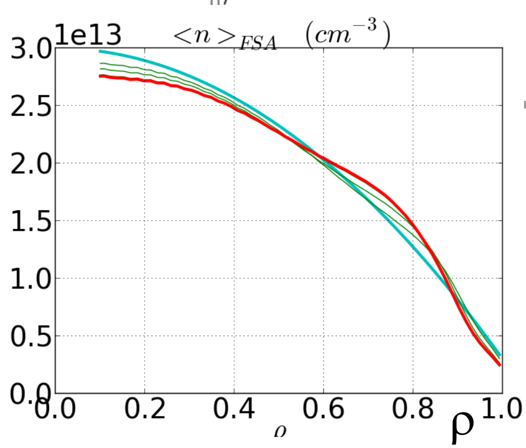
R-tran&IBC but no bal. term → Added bal. term

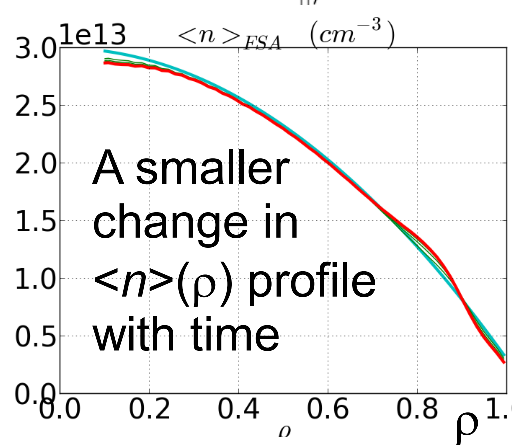


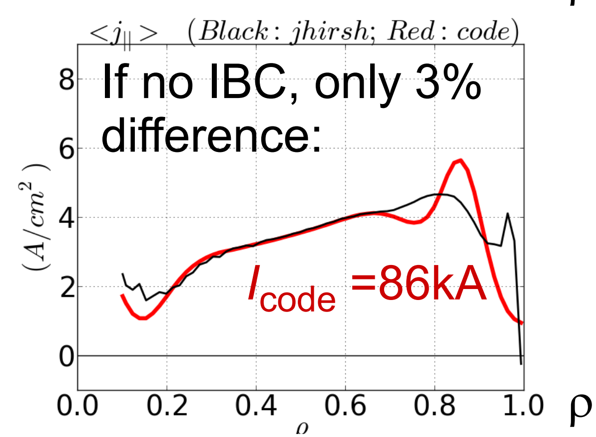


Max of current is at $u/c \approx$1.1e-3





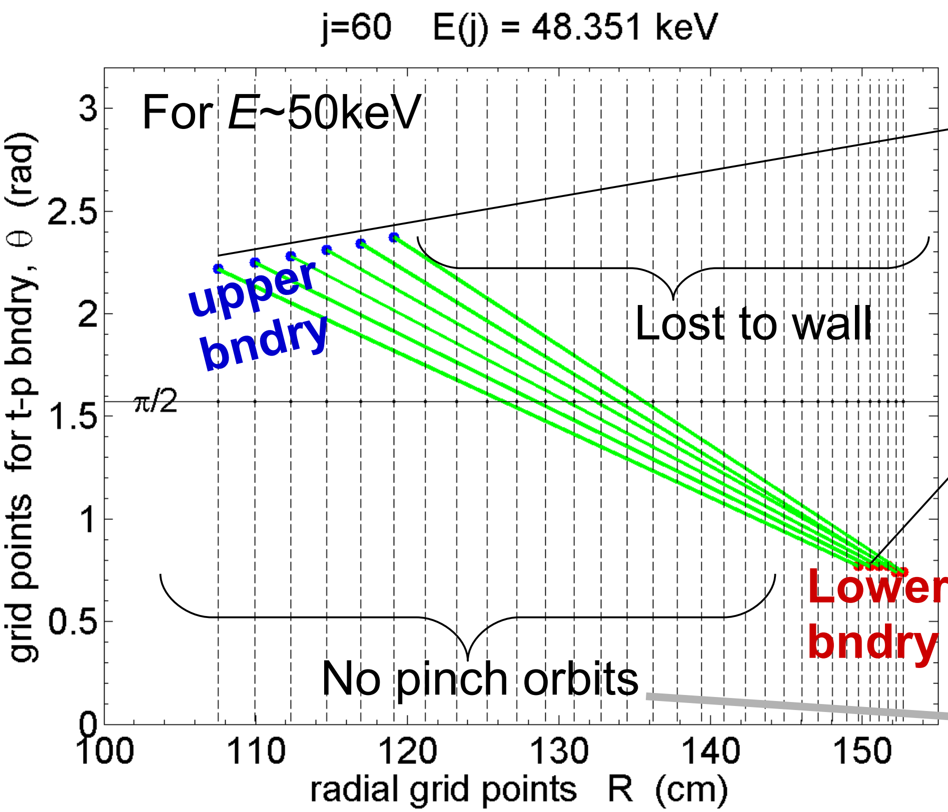


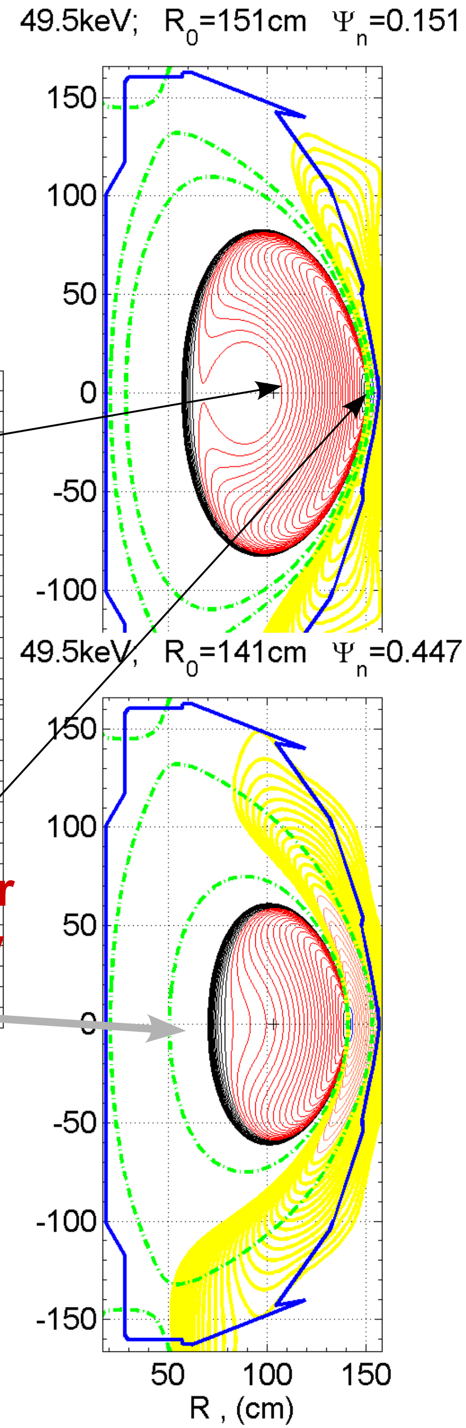


Full-FOW: Details on Internal Boundary Conditions

IBC "links" in the plot over (R_0, θ_0):

They connect two sides of a pinch orbit. (The picture depends on particle energy.)





At each boundary the value of the distr function is projected into virtual points on the other side of physical boundary (green terms below).
IBC conditions are:

1. Continuity of f at <u>lower</u> t-p boundary, at $l = l\_a$ radial index:
$$f_{ipl+1} - f_{itl} = f_{itl-1} - f_{ipl} \; .$$

2. Continuity of f at <u>upper</u> t-p boundary, at $l = l\_b$ radial index:
$$f_{ipu-1} - f_{itu} = f_{itu+1} - f_{ipu} \, .$$

3. Symmetry of pitch-angle flux in trapped region:
$$H_{itu+0.5}\big|_{l\_b} = -H_{itl-0.5}\big|_{l\_a}$$

4. Conservation of total flux:
$$(H_{ipu-0.5} - H_{itu+0.5})\big|_{l\_b} + (H_{itl-0.5} - H_{ipl+0.5})\big|_{l\_a} = 0.$$

At higher energies there is no IBC – the pinch orbits are lost to wall or become a D-shape trapped orbits. ⇒

Full-FOW: RF Quasi-Linear Operator (Added in 2015)

The structure is similar to the collision operator, except there are no drag terms A and D, and, consequently, no radial convection term R_{03} :

$$\left(\frac{\partial f}{\partial t}\right)_{QL} = \frac{1}{\lambda u_0^2}\frac{\partial}{\partial u_0}\left(\boldsymbol{B}\frac{\partial f}{\partial u_0} + \boldsymbol{C}\frac{\partial f}{\partial \theta_0} + \boldsymbol{R}_{13}\frac{\partial f}{\partial R_0}\right)$$
$$+\frac{1}{\lambda u_0^2 \sin\theta_0}\frac{\partial}{\partial \theta_0}\left(\boldsymbol{C}\sin\theta_0\frac{\partial f}{\partial u_0} + \boldsymbol{F}\frac{\partial f}{\partial \theta_0} + \boldsymbol{R}_{23}\sin\theta_0\frac{\partial f}{\partial R_0}\right)$$
$$+\frac{1}{\lambda u_0^2}\frac{\partial}{\partial R_0}\left(\boldsymbol{R}_{13}\frac{\partial f}{\partial u_0} + \boldsymbol{R}_{23}\frac{\partial f}{\partial \theta_0} + \boldsymbol{R}_{33}\frac{\partial f}{\partial R_0}\right)$$

$\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{F}$ were present in ZOW, but now they are modified by transformation coeffs., e.g.

$\boldsymbol{C} = \lambda\langle B_{QL}\, \partial\theta_0/\partial u + C_{QL}\, \partial\theta_0/\partial\theta \rangle_{\mathrm{BA}}$

In ZOW, only $\partial\theta_0/\partial\theta$ is not zero.

θ = local pitch-angle at orbit (ray el)

θ_0 - at the midplane;

and $\lambda = |\partial \boldsymbol{J}_{\mathbf{can}}/\partial \boldsymbol{I}| / (u_0^2 \sin\theta_0)$.

The new terms, $\boldsymbol{R}_{13}$, $\boldsymbol{R}_{23}$, and $\boldsymbol{R}_{33}$ (e.g. $\boldsymbol{R}_{13} = \lambda\langle B_{QL}\, \partial R_0/\partial u + C_{QL}\, \partial R_0/\partial\theta \rangle$) correspond to the radial transport caused by resonant interaction of ions with RF.

In the above, B_{QL}, etc., are the **local** QL coeffs, e.g. :

$$B_{QL} = u^2 D_{uu} = u^2(\cos^2\theta\, D_{||} + 2\cos\theta\sin\theta\, D_{\perp||} + \sin^2\theta\, D_{\perp})$$

$$C_{QL} = u\, D_{u\theta} = u\, D_{\theta u} = u(\cos\theta\sin\theta\,(D_{\perp} - D_{||}) + (\cos^2\theta - \sin^2\theta)\, D_{\perp||})$$

The bounce-averaging of these local QL coeffs (and their combinations as in $\boldsymbol{C}$, $\boldsymbol{F}$ and $\boldsymbol{R}_{\#\#}$ coeffs) is done by mapping the local (u,θ) space at a given ray element to the corresponding set of (R_0, u_0, θ_0) grid points at the midplane using the COM table.

For the FOW runs, the $\boldsymbol{R}_{\#\#}$ terms can be omitted ("no R-transport run").

Full-FOW: QL Operator

Questions for tests on QL operator:

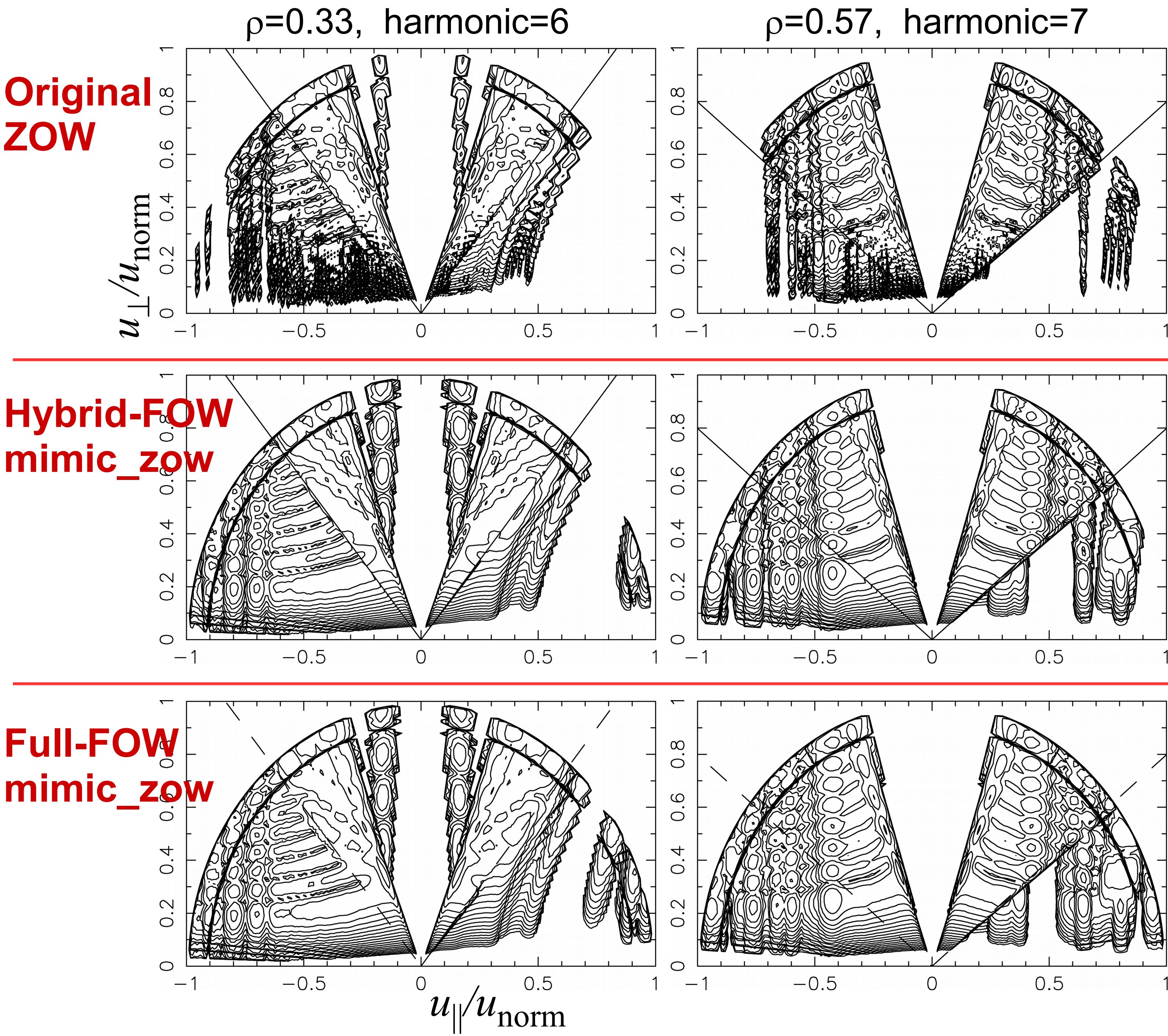
- Will it give convergence to ZOW results?
  (For Hybrid-FOW and Full-FOW, there is an option *mimic_zow='enabled'*, which makes the mapping from the local particle sources and ray elements to the midplane *along the surface*.)
- What is the role of the new radial $R_{\#\#}$ terms?

Test conditions:

NSTX equilibrium, parabolic profiles of $T(\rho)$ and $n(\rho)$ (ρ="sqtorflx"),
with $T(0)/T(1)$ = 1.0/0.01 keV, $n(0)/n(1)$ = 3e13/3e12 cm^{-3},
D ions (FP'd species),
Sources: Deuterium NBI (0.4 MW).
HHFW: 30 MHz, 1.1 MW (0.4-0.7 MW to D^+, the rest to e).

The deposition of power in plasma volume is based on ray-tracing (Genray). The locally absorbed power is mapped along the orbit legs to the midplane where the QL diffusion coefficients are formed (bounce-averaged).

ZOW convergence tests: RF Quasi-Linear Operator

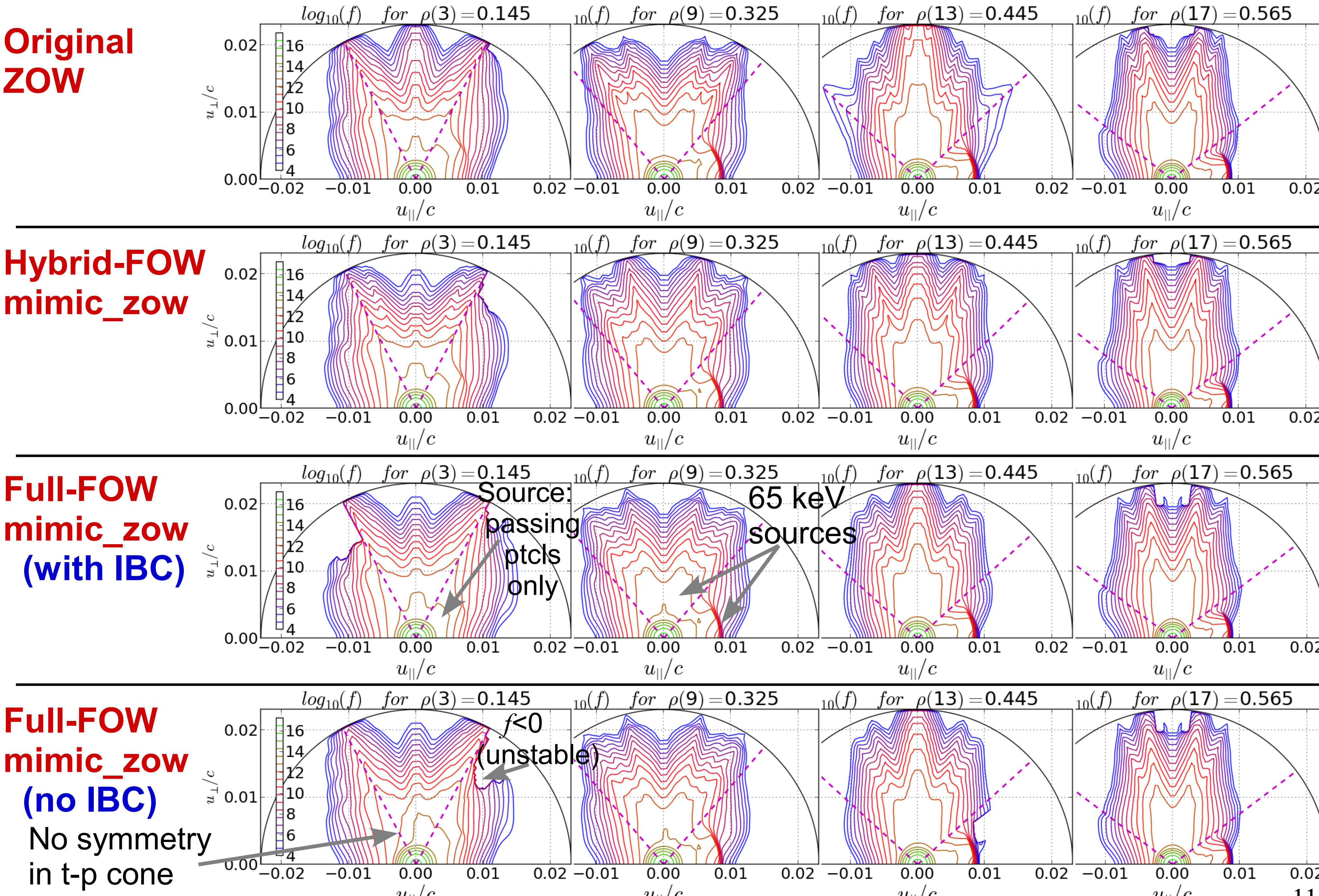


Shown:
B = $\lambda\langle B_{QL}\rangle_{BA}$ corresponding to D_{uu} diffusion.

In Hybrid and Full-FOW, the mapping is done differently: $\delta(u_{||}-u_{||res})$ function is represented by a hat function using at least 6 points across resonant strip. In ZOW: often only 1-3 points.

ZOW convergence tests: Distribution Function

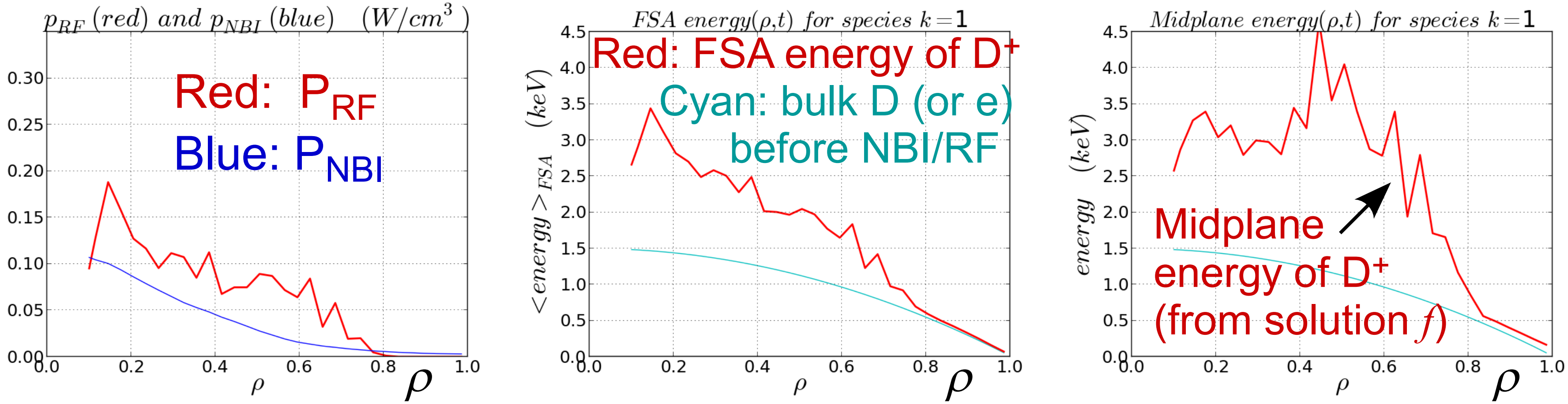
(steady-state solution at the midplane)



ZOW convergence tests: Profiles (SS solution after 0.5 s/step x 4 steps)

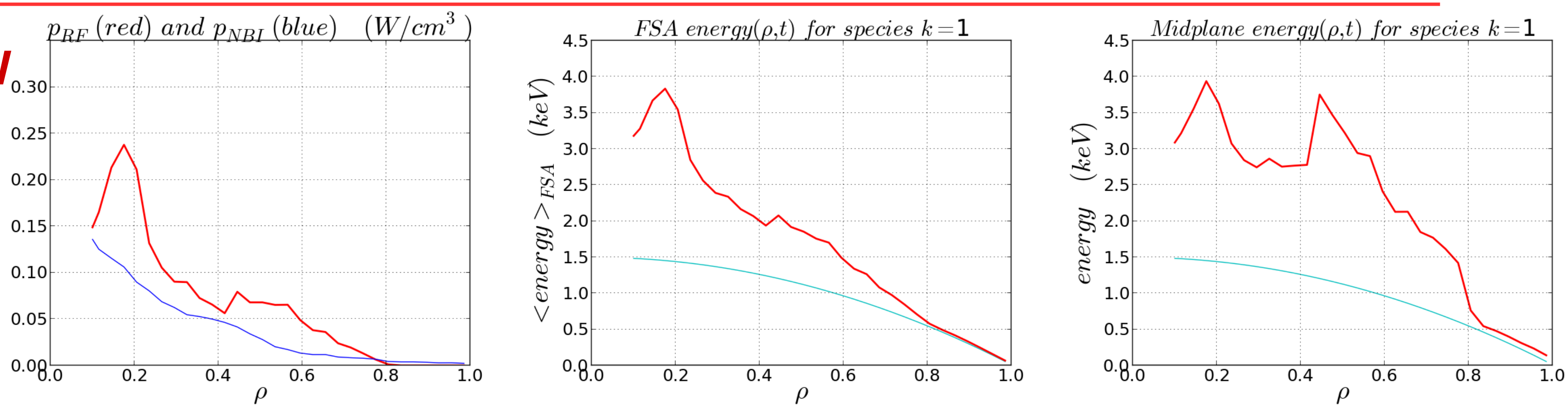
Original ZOW

P_{RF}(MW)=
0.79(D)+
0.31(e)



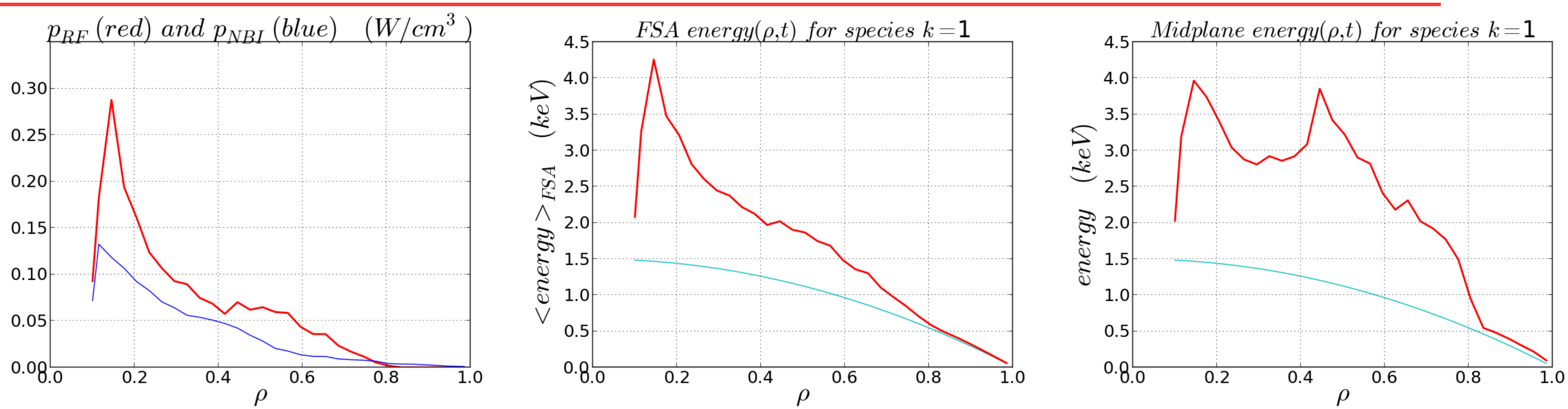
Hybrid-FOW mimic_zow

P_{RF}(MW)=
0.73(D)+
0.37(e)



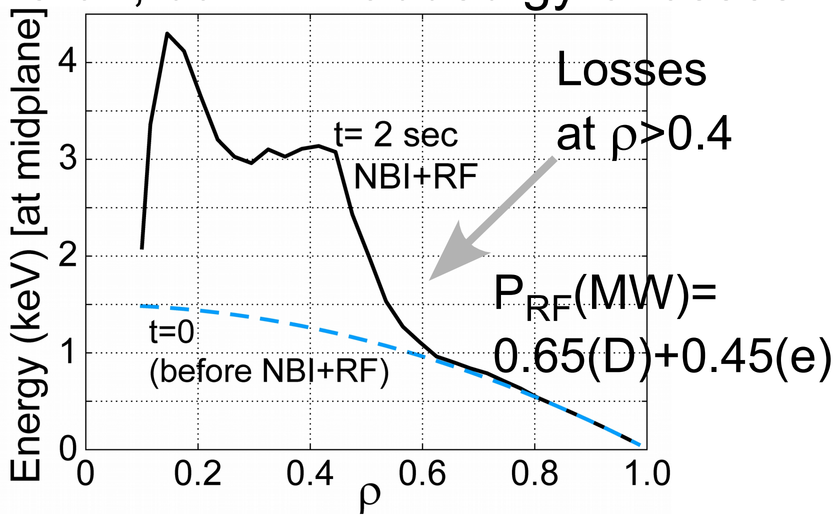
Full-FOW mimic_zow

P_{RF}(MW)=
0.70(D)+
0.40(e)



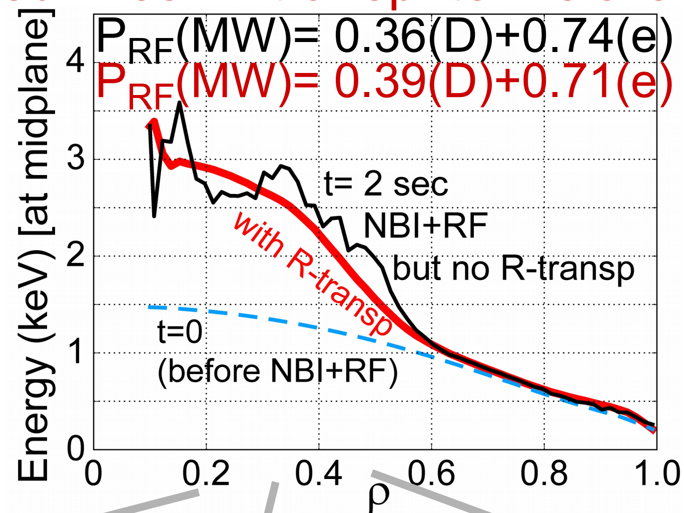
Full-FOW: NBI+HHFW (Test conditions: NSTX, same as in mimic_zow runs)

For a reference: mimic_zow rerun, but with added gyro-losses



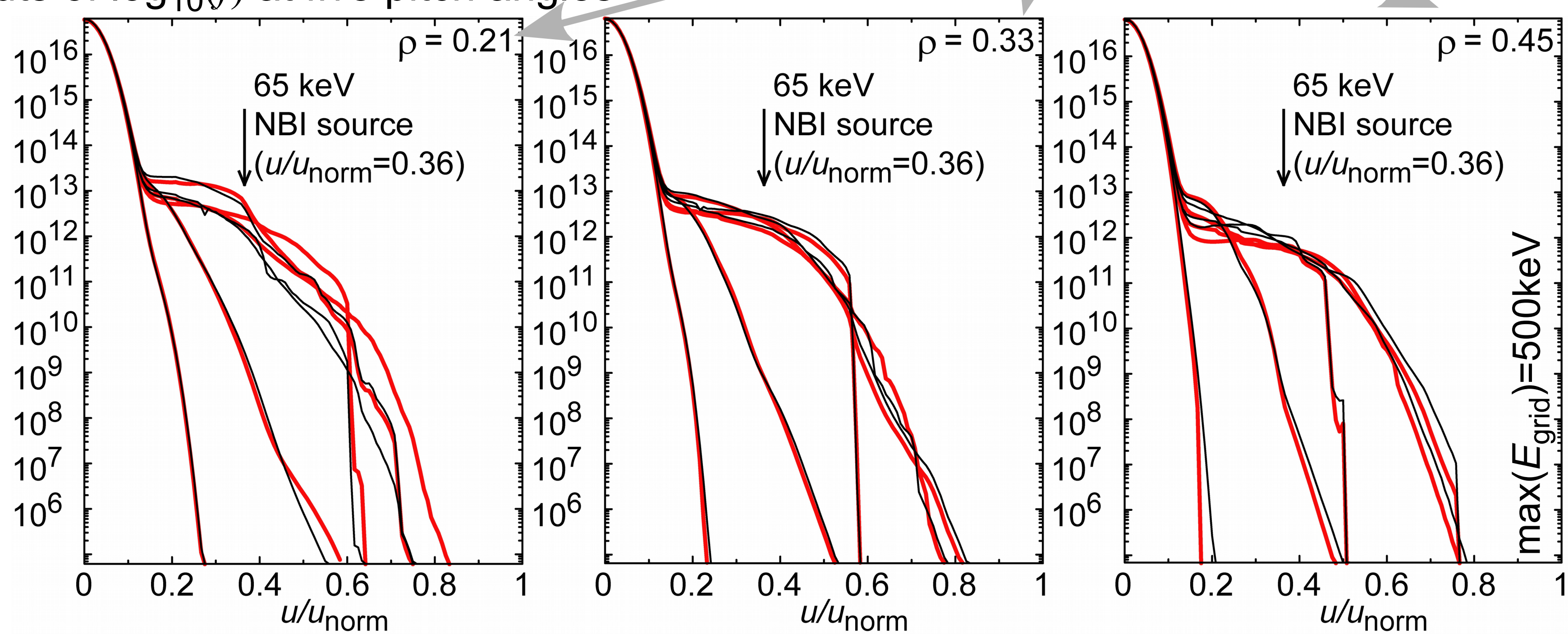
Full-FOW run, losses: orbits+gyro.

Red lines: *R*-transp. terms are added.



At some ρ, the "tail" is reduced by *R*-transport terms, but at some - enhanced.

Cuts of $\log_{10}(f)$ at five pitch angles

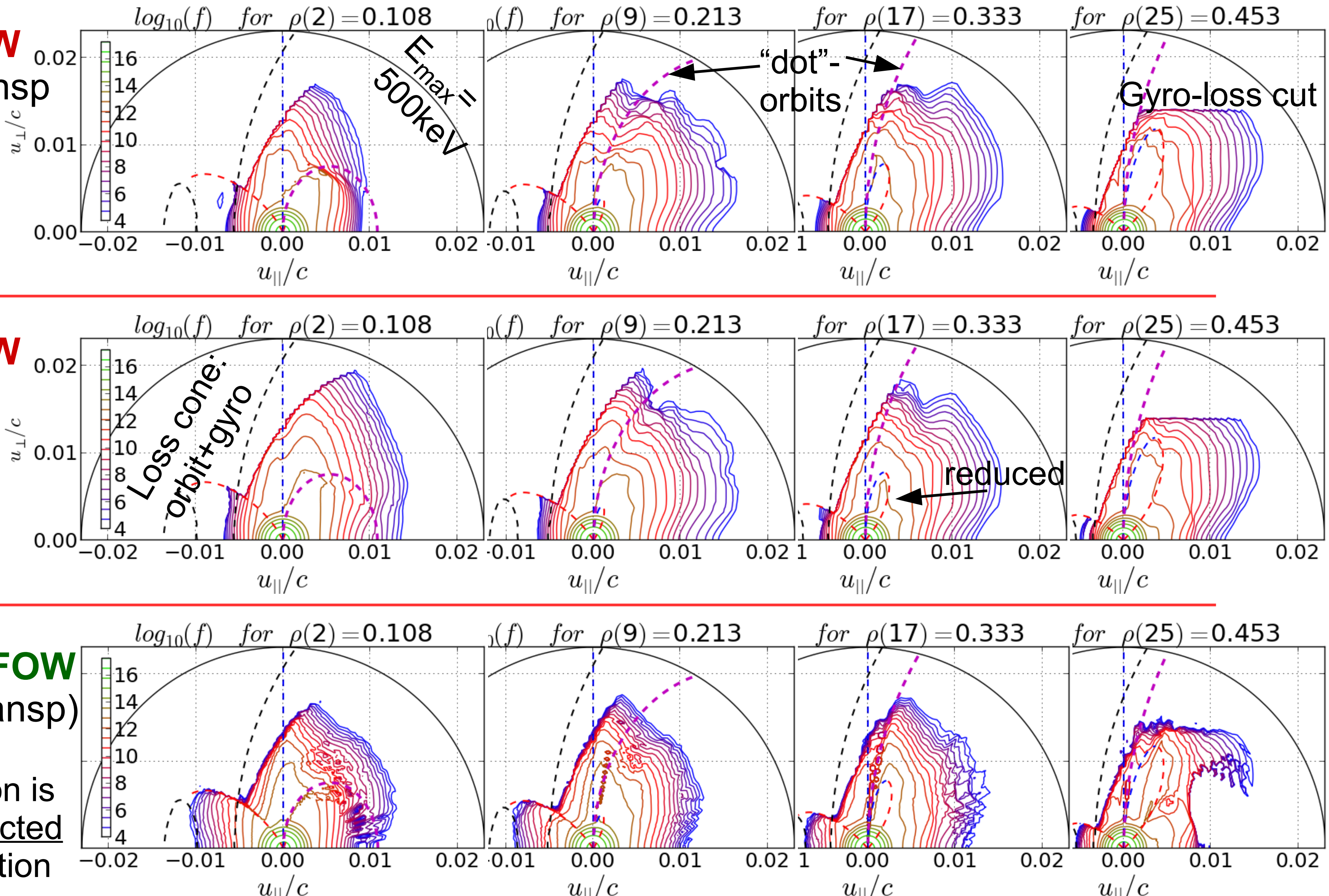


NBI+HHFW: Effect of *R*-terms; Also Full-FOW vs Hybrid-FOW

Full-FOW
no *R*-transp

Full-FOW
R-transp
added

Hybrid-FOW
(no *R*-transp)
The local distribution is <u>reconstructed</u> from solution

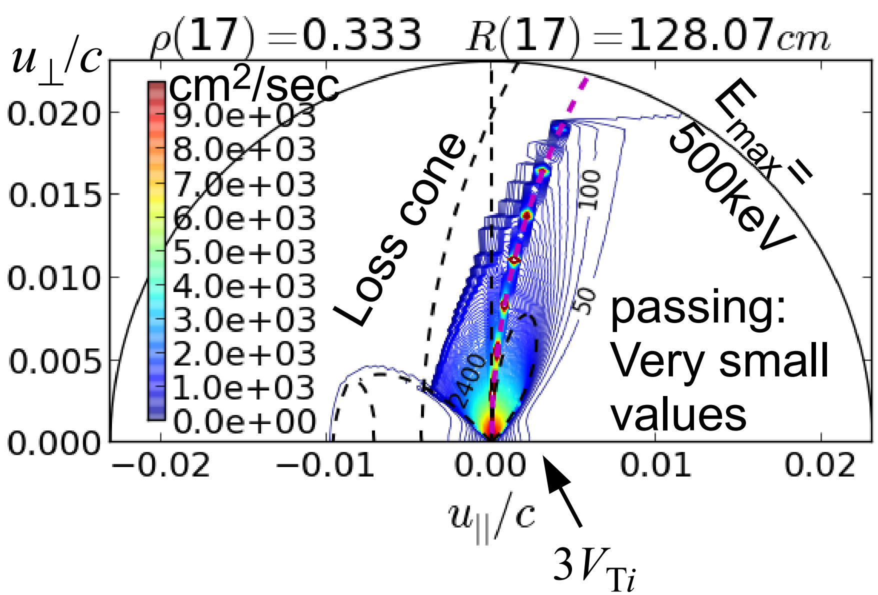


The features are similar in the Hybrid-FOW and Full-FOW

Full-FOW NBI+HHFW: Diffusion Coefficients

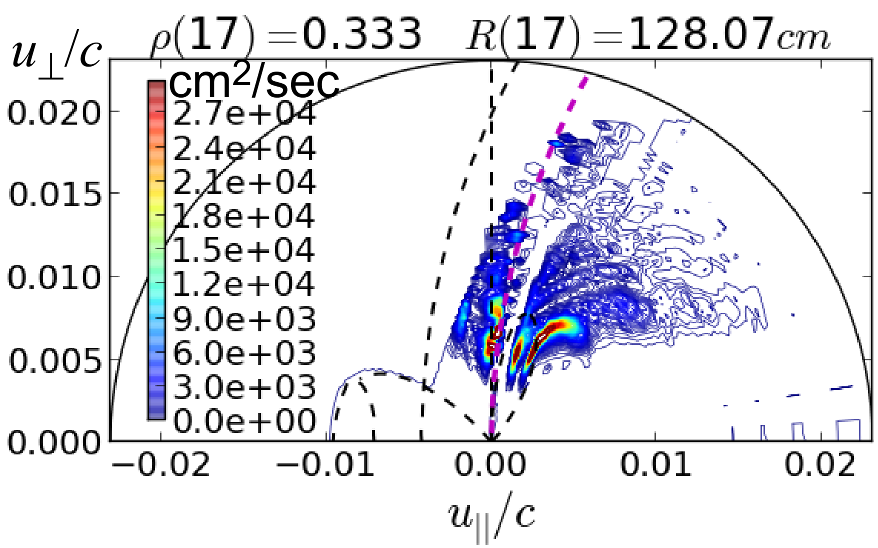
Compare Collision D_{RR} radial diffusion coeff. with RF QL D_{RR} coeff.

Collision
D_{RR} coeff.
$R_{33}/(\lambda u_0^2)$



The collision D_{RR} is largest at near-thermal energies ($\sim 10^4$ cm^2/sec for trapped particles).
At about $3V_{Ti}$, it drops to 2×10^3 cm^2/sec.

RF QL
D_{RR} coeff.



In contrast, the RF QL D_{RR} is largest at $E \sim 50$ keV energies (several peaks 3×10^4 cm^2/sec).

At large energies, the radial diffusion via RF prevails, while at near-thermal energies the radial diffusion is set by collisions

SUMMARY

- All essential parts for the Full-FOW CQL3D version are in place.
- It is demonstrated that the neoclassical radial transport is affected by RF heating.
- Need more synthetic diagnostics for characterization of the radial transport.
- Need independent verification tests for RF heating: Compare with NUBEAM? ORBIT-RF?
- Run more test cases:
  – AORSA + DC + CQL3D-full-FOW
  – Re-test FIDA profiles (was done with Hybrid-FOW)