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Non-axisymmetric effects in strongly driven Coaxial Helicity Injection in simulations in the NSTX geometry

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ABSTRACT:

Nonaxisymmetric effects become important in strongly-driven CHI in NSTX simulations using NIMROD. An n≥1, high m mode excited in simulations of injection and flux closure can significantly impact the injected poloidal flux evolution and closure.[1,2] In nonlinear simulations, the mode velocity and magnetic perturbations occur in “bursts;” in previous, lower temperature work the mode was weak and not bursting, with little effect on the injection.[1] The mode is excited just outside the poloidal flux bubble with axes of poloidal velocity vortexes and magnetic flux surfaces aligned along the magnetic field. Their width is approximately that of the current layer in the surface of the bubble. The instability significantly broadens the current layer and apparently is driven in part by currents resulting from expansion of the injected poloidal flux. Linear simulations starting from nonlinear, purely axisymmetric simulations or from the axisymmetric parts of nonaxisymmetric simulations yield the linear eigenmodes and sensitivity to plasma parameters. Ongoing analysis to identify the driving mechanism(s) for the instability is constrained by these linear results.

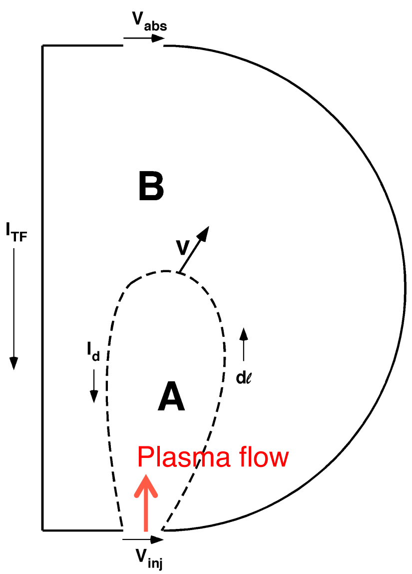
[1]E B Hooper, et al., Phys. Plasmas 20, 092510 (2013).
[2]F Ebrahimi, et al., Phys. Plasmas 20, 090702 (2013).

Field-line-following instability during simulations of CHI

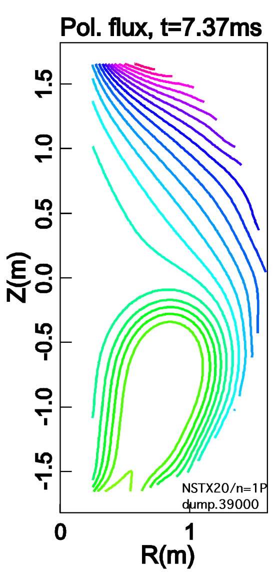
— Seen in resistive MHD simulations (NIMROD)
— Forms on the surface of flux-bubbles during injection

- The mode had little effect on injection in weakly-driven (I_{inj} = 2-10 kA), low temperature (T< 25 eV) plasmas used to compare with experimental results
- Studied here in simulated, strongly-driven ($I_{inj} \approx$ 15 kA) plasmas with reduced impurity radiation and reduced cross-field thermal conductivity
  — Higher T (~50-100 eV) in the current channel; lower (~5 eV) outside
  — The mode is stronger and affects the evolution and closure of the flux-bubble

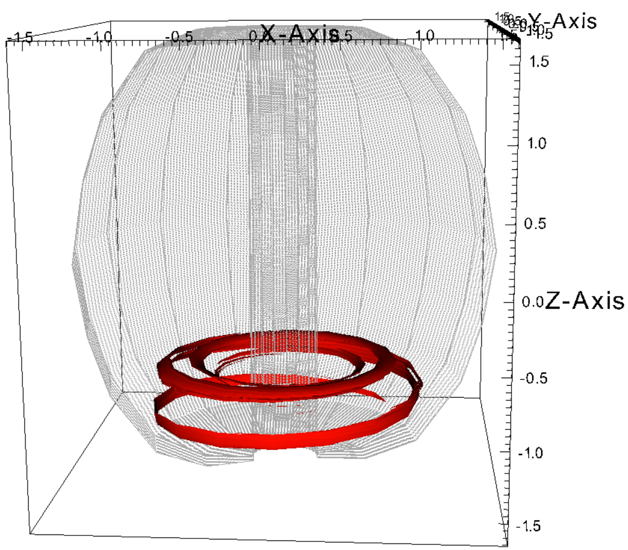
CHI schematic



Flux bubble



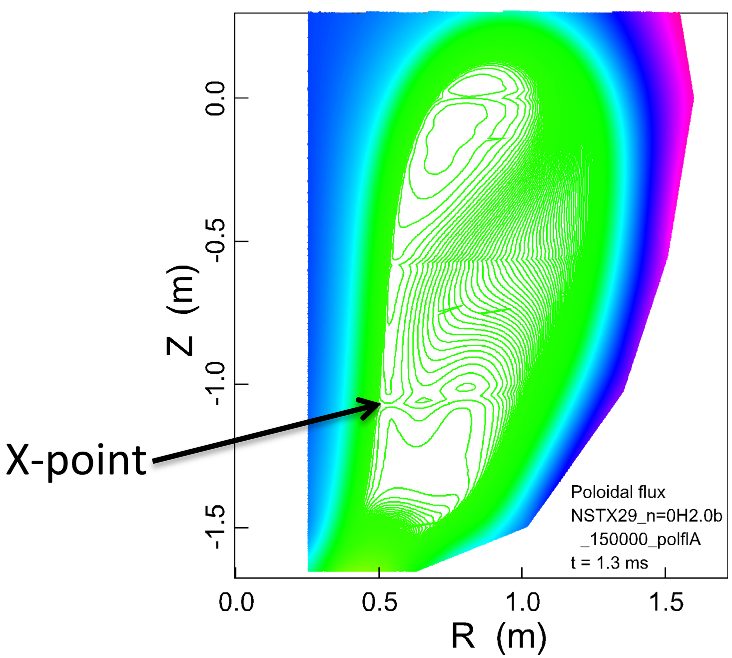
Instability



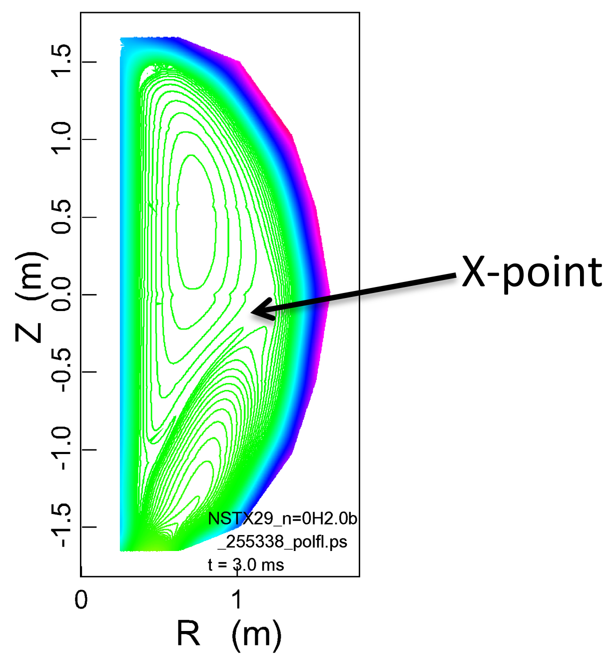
Axisymmetric (n=0), strongly-driven simulations with high T (in current channel), low thermal conductivity — X-point forms during injection

- — X-point is in cold (5 eV) plasma outside current channel
- — Plasma divides into two “lobes”

t = 1.3 ms after start of injection



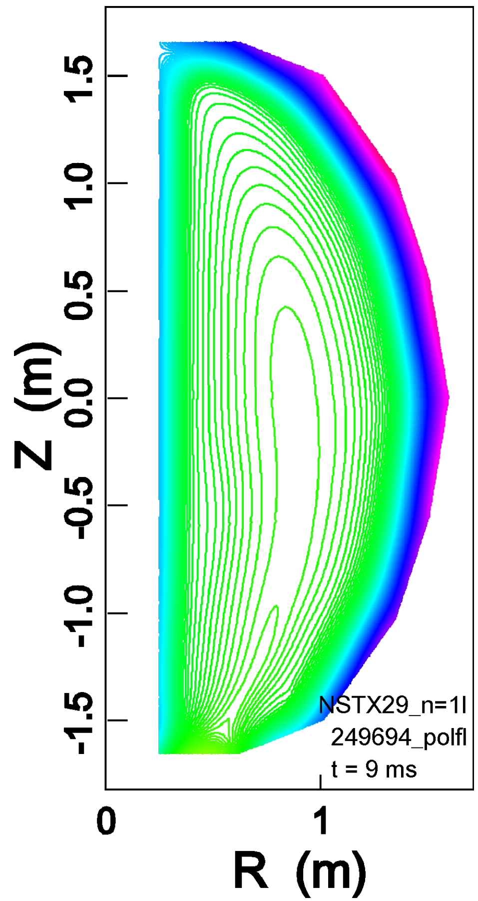
t = 3.0 ms after start of injection



A single, well-defined flux bubble does not form

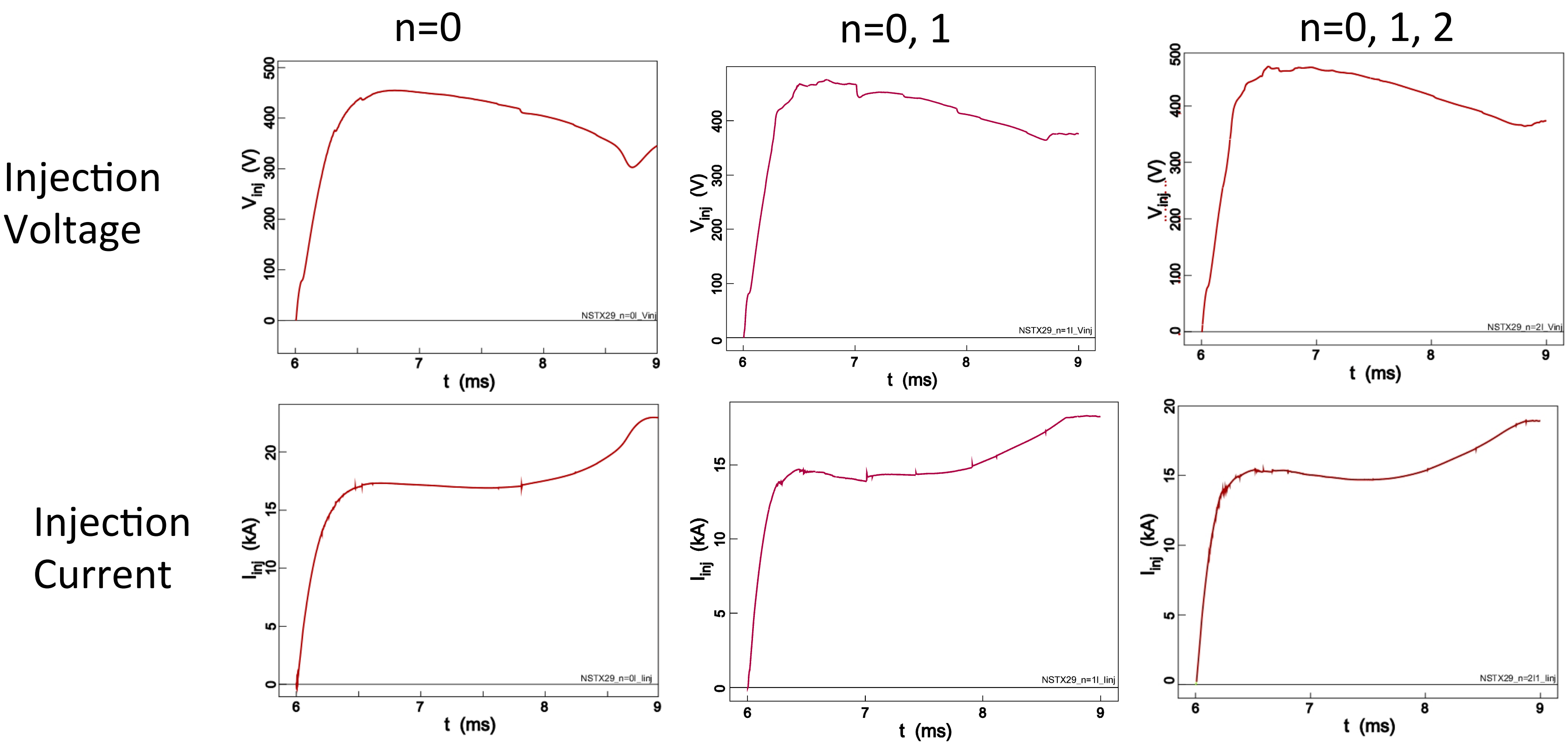
Non-axisymmetric (n=0, 1, 2) simulations – The axisymmetric poloidal flux distribution is more like that in low-temperature simulations

Poloidal flux



Non-axisymmetric (n=0, 1, 2) simulations with high T (in current channel), low thermal conductivity

— **Symmetry-breaking mode prevents X-point formation seen in axisymmetric simulations**



The jagged structures with n = 1 and n = 1, 2 result from relaxation oscillations of the n=1 mode

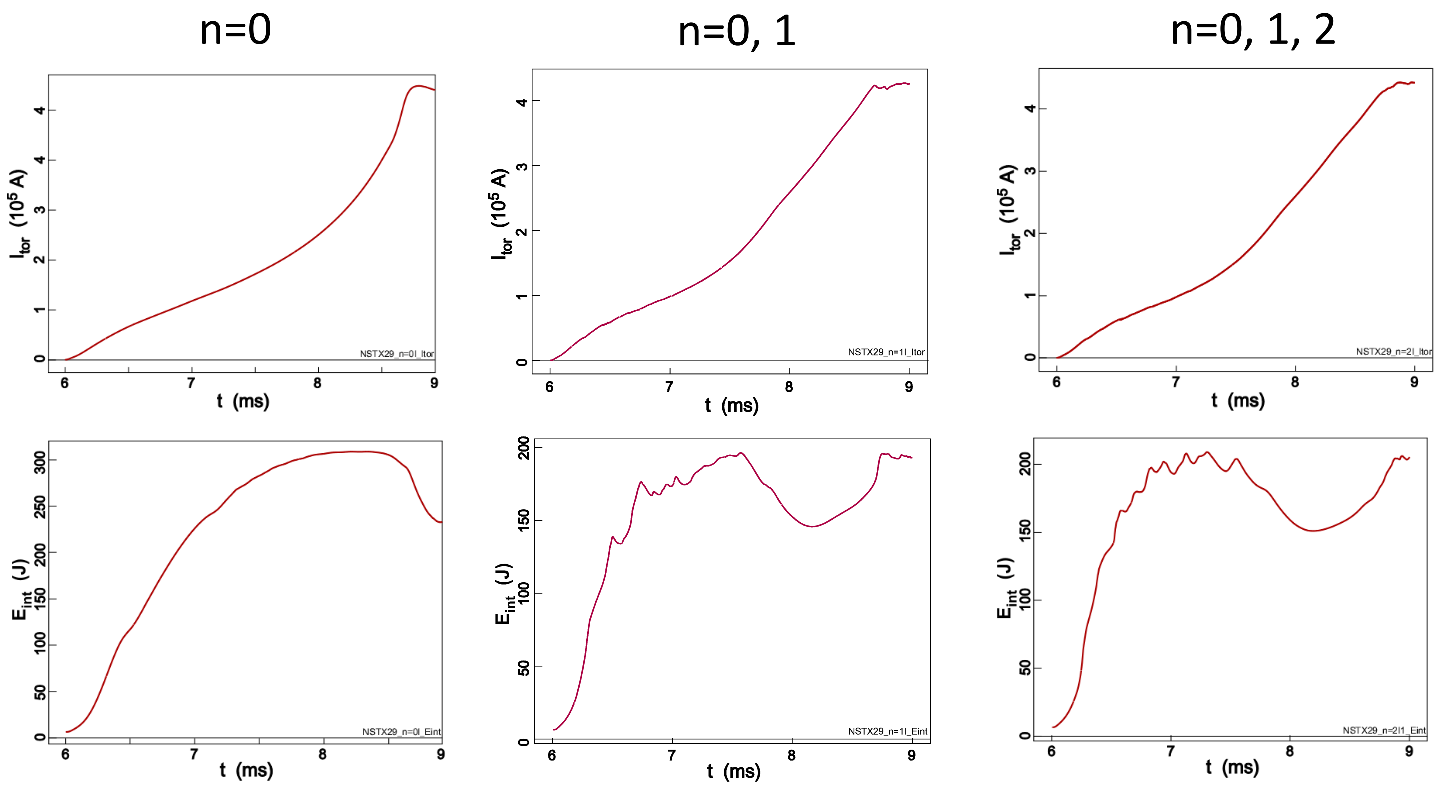
Time not extended beyond 9 ms due to long run time requirements at high resolution and numerical problems. At lower resolution I_{tor} drops to 40% at 14 ms

Non-axisymmetric (n=0, 1, 2) simulations with high T (in current channel), low thermal conductivity (cont.)

— Plasma evolution (toroidal current, internal energy)

Toroidal current

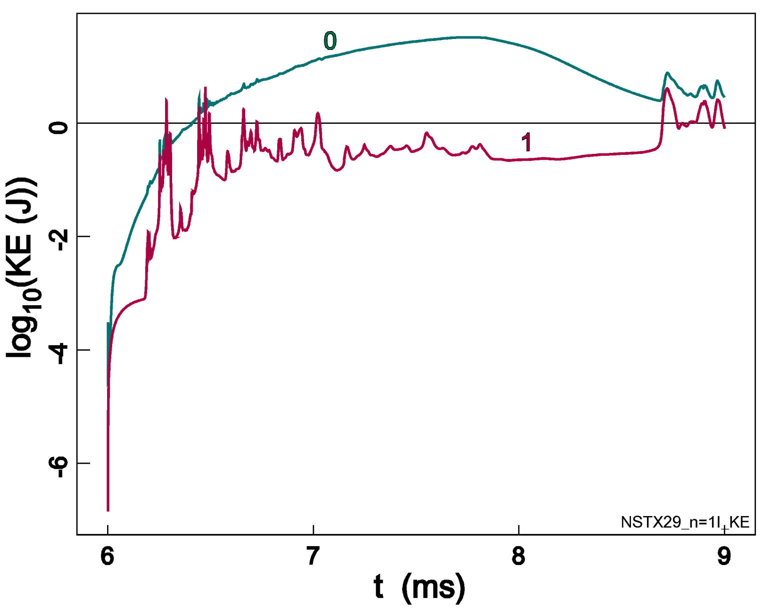
Internal energy



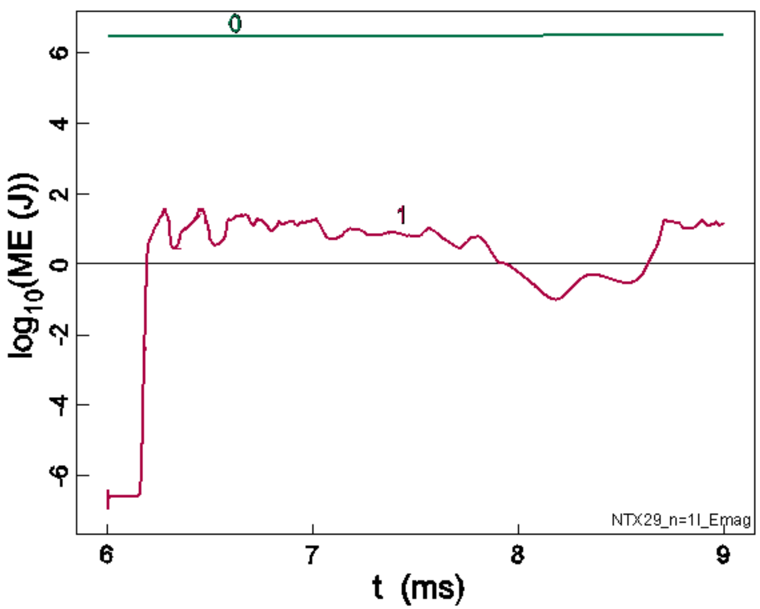
The jagged, internal energy structures are due to the instability. During dips in n=0,1 & n=0,1, 2 internal energy (at ≈ 8 ms) the mode is low-level and not bursting

Kinetic and magnetic energies show a bursting characteristic correlated with the injection current

Kinetic Energy



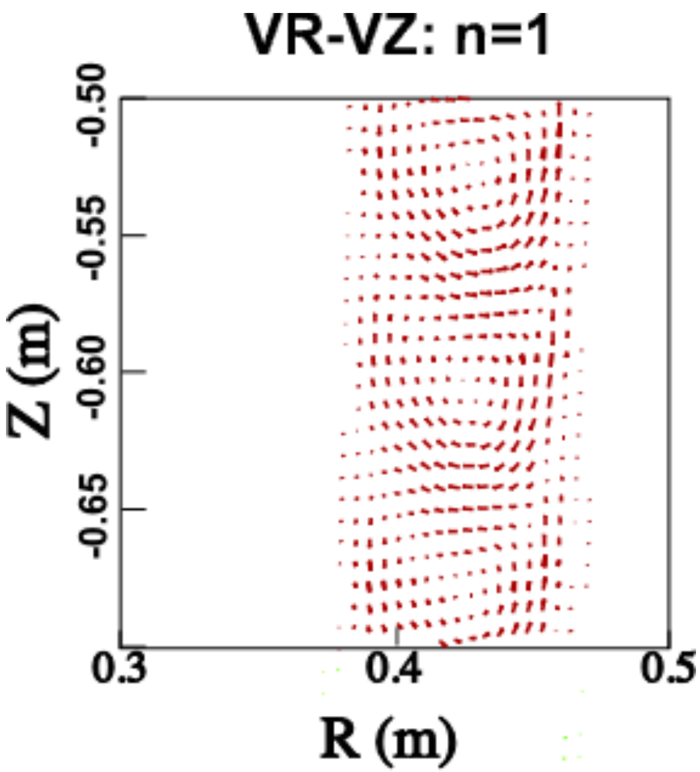
Magnetic Energy

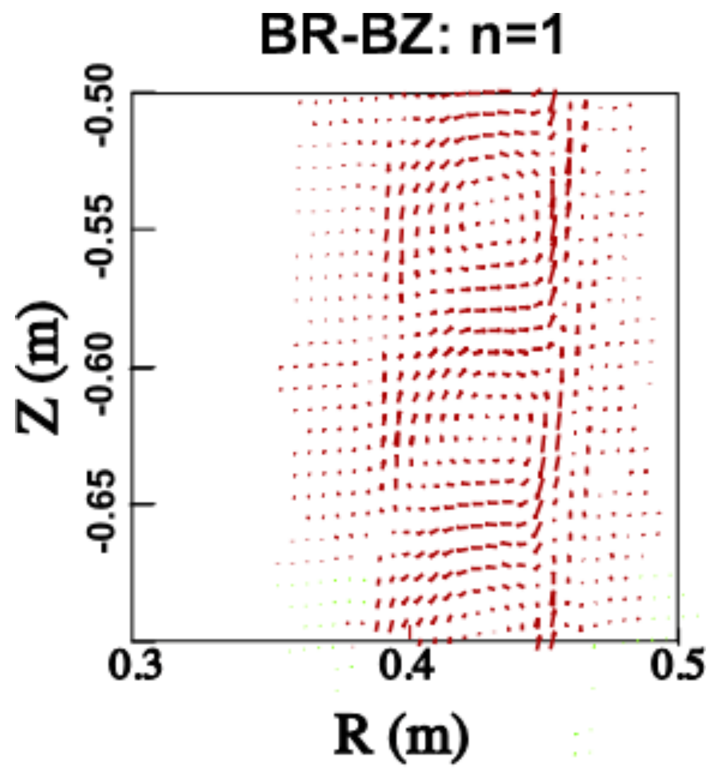


During the dips (at ≈ 8 ms) the mode is low-level and not bursting

The mode consists of eddies in velocity and magnetic field

The real parts are shown



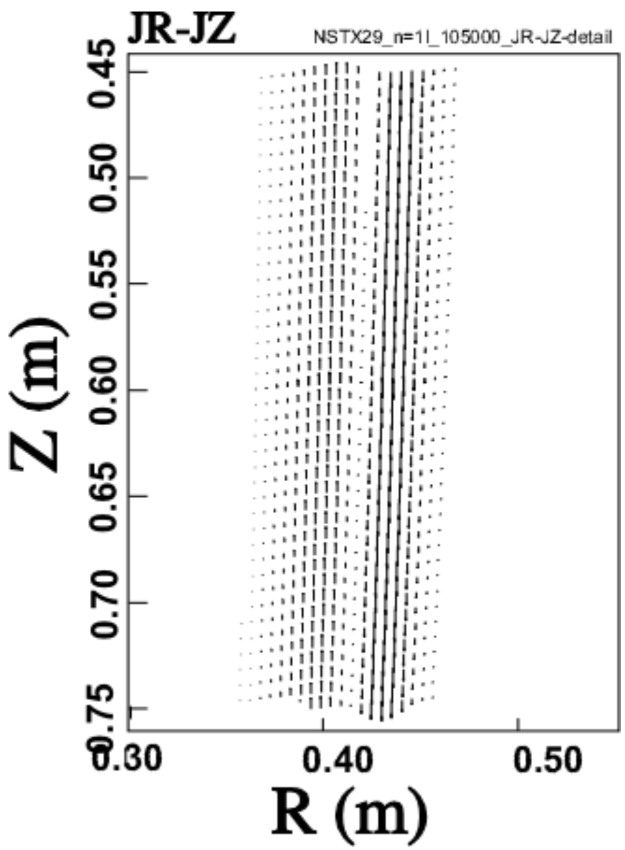


NSTX29/n=1I/201500
(t = 7.75 ms)

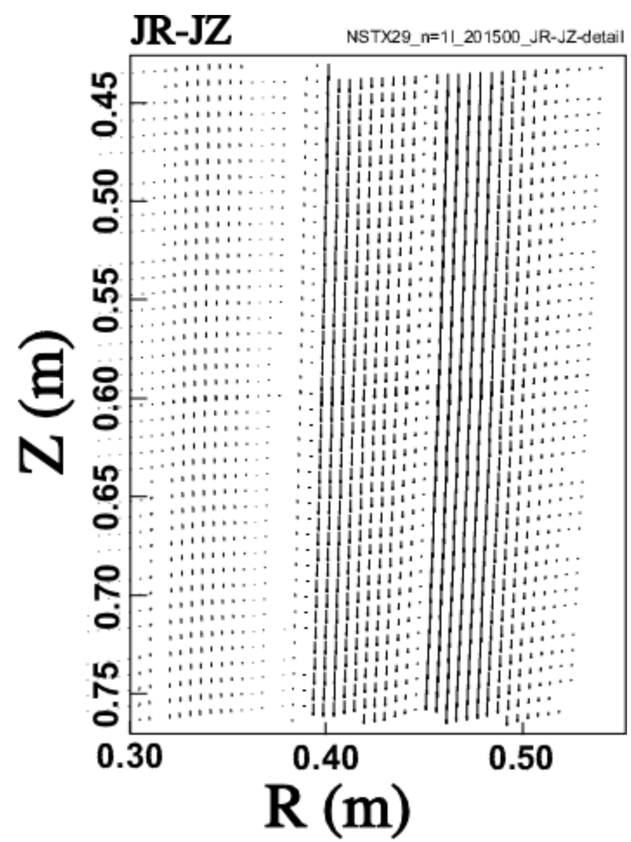
The instability broadens the n=0 current flow

— **Current is generated outside the flux-bubble where the field lines are bent by the expanding bubble**

n = 0 only



n = 0 & 1



Linear analysis — used to identify the instability driving force

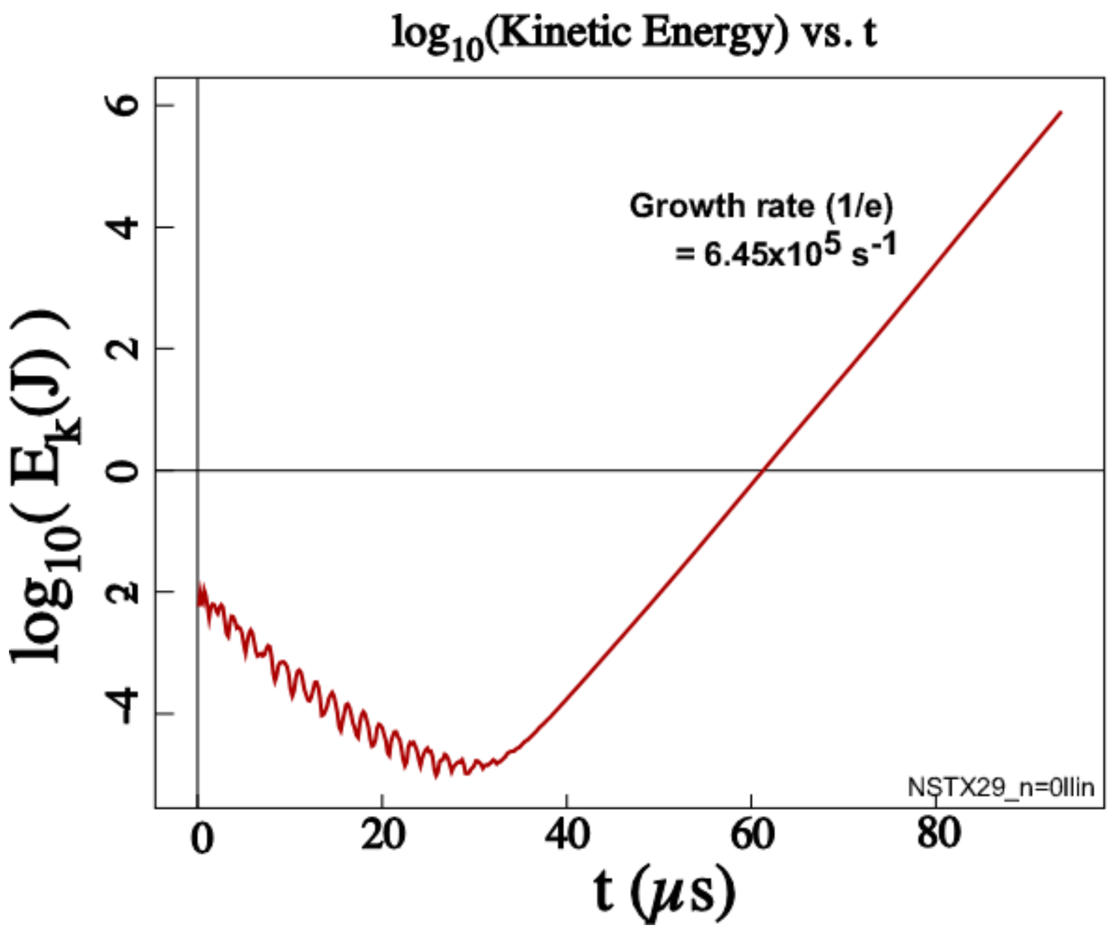
Two approaches have been used to determine the linear-mode characteristics

- **Linear characteristics have been determined by using NIMROD**
  - **Starting from a non-linear axisymmetric simulation, a linear mode is excited from noise**
  - **The code parameters (e.g. viscosity, resistivity) and results (e.g. flow) were varied to determine the sensitivity to simulation parameters**

- **Simple slab models were used to examine the physics driving the mode**

Conclusion: The mode is primarily an ideal, current-driven instability

NIMROD — The linearized n=1 mode has a rapid growth rate ($t_{1/e}$ ≈ 1.5 μs)



The n=1 mode grows without the n=0 flow velocity

Option in the code: The n=0 velocity field can be set to zero in the linear study

- **Mode grows when the n=0 flow is zeroed**
  – $v_{max} \approx 10^4$ m/s (<< v_A, c_s) away from the injection slot
- **Growth rate — reduced from $6.45\times10^5\ s^{-1}$ to $2.88\times10^5\ s^{-1}$**

Also: The mode structure lies on the outer edge of the n=0 flow field

- **This is where the n=0 component of current is strongest**

Conclusions:

- **The mode does not need the n=0 velocity field to grow, although it does contribute to the mode growth rate**
- **The mode is primarily current driven**

Linear-instability calculation: MHD input parameters (e.g. viscosity, resistivity) can be changed to evaluate the sensitivity of the linear mode

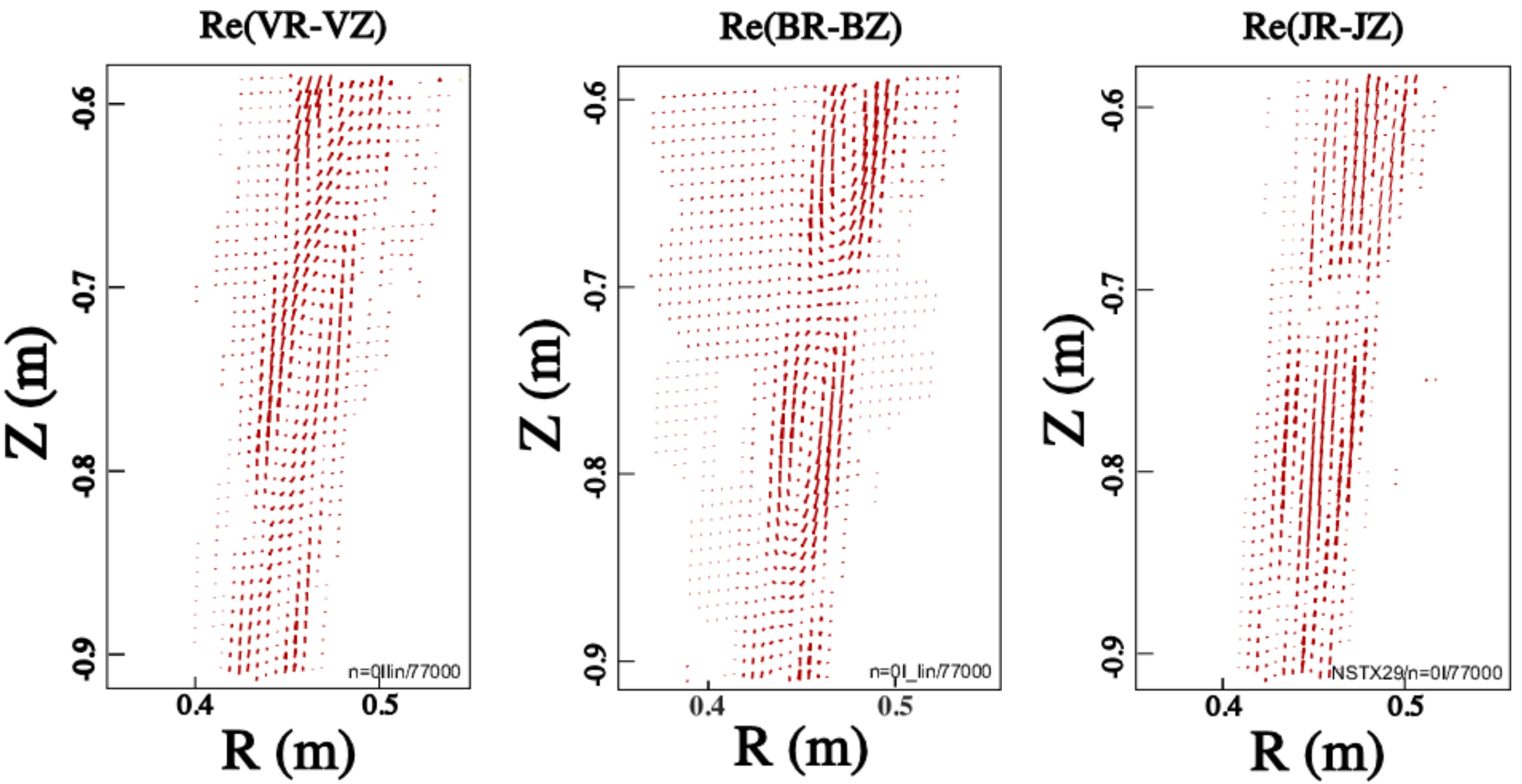
Parameter	Base value	Test value	Growth rate (s^{-1})
Base calculation*			6.45×10^5
Kinematic viscosity	150 m^2/s	15 m^2/s	8.79×10^5
Resistivity (magnetic diffusion)	$411/T_e^{3/2}$ m^2/s	$1 \times 10^4/T_e^{3/2}$ m^2/s	9.48×10^5
Particle diffusivity (holds n ≈ const)	10^5 m^2/s	10 m^2/s	5.30×10^5

*All n=0 plasma quantities are constant (slides 5-8) except as listed above

Conclusions:

- **Instability growth is insensitive to the numerical parameters used in the calculation**
- **Dissipation is not needed — The mode is primarily ideal**

The linear mode structure is similar to the non-linear — eddies in velocity and magnetic field are generated by the instability



Analytic Modeling: Instability drive

Ideal MHD Energy Principle — term proportional to $j_{||}$

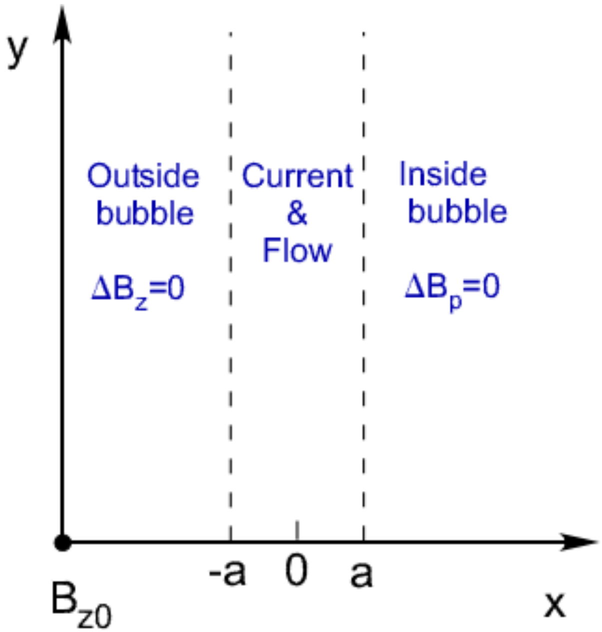
$$\delta W_F = -\frac{1}{2}\int_P d\mathbf{r} J_{\|}\left(\left(\xi_\perp^* \times \hat{\mathbf{b}}\right)\cdot \mathbf{B}_{1\perp}\right)$$
$$= \frac{1}{2}\int_P d\mathbf{r} J_{\|}\hat{\mathbf{b}}\cdot\left(\xi_\perp^* \times \mathbf{B}_{1\perp}\right)$$

The term in parenthesis is proportional to the negative of the electric field:

$$\tilde{\mathbf{E}} = -\frac{\partial \xi}{\partial t}\times \tilde{\mathbf{B}}_{1\perp}$$

This term is negative when the volume-averaged $J_{\|}\hat{\mathbf{b}}\cdot\tilde{\mathbf{E}}$ is positive —— In the direction to extract energy from the current

Slab Model — example of current sheet



Assume current is force-free and constant in the slab

Model the downward current along the inside leg of the flux-bubble ($j_{y0} < 0$)

$$B_z = B_{zo} \qquad x < -a$$

$$B_z = B_{zo} - \mu_0 j_{oy}\left(x + a\right) \qquad -a \le x \le a$$

$$B_z = B_{zo} - \mu_0 j_{oy}\left(2a\right) \qquad x > -\Delta/2$$

$$\frac{\partial B_y}{\partial x} = \frac{B_{zo}}{B_{yo}}\mu_0 j_{yo}$$

$$B_y = \sqrt{-2\mu_0 j_{yo} B_{zo}\left(2a\right)} \qquad x < -a$$

$$B_y = \sqrt{2\mu_0 j_{yo} B_{zo}\left(x - a\right)} \qquad -a \le x \le a$$

$$B_y = 0 \qquad x > a$$

Slab Model — Equilibrium and perturbed equations

Equilibrium

$$\rho_0 = const.$$

$$\mathbf{v}_0 = 0 \qquad \text{Neglect flow}$$

$$\mathbf{B}_0 = B_{0z}\hat{\mathbf{z}} + B_{0y}(x)\hat{\mathbf{y}} \qquad B_{0y} << B_{0z}$$

$$\mathbf{j}_0 = j_{0z}(x)\hat{\mathbf{z}} + j_{0y}(x)\hat{\mathbf{y}} \qquad \mathbf{j}_0 \cdot \mathbf{B}_0 = \mathbf{0}$$

Perturbation

$$\delta\rho = 0$$

$$\nabla \cdot \delta\mathbf{v} = 0 \qquad \text{incompressible}$$

$$\rho_0 \frac{\partial \delta\mathbf{v}}{\partial t} = \delta\mathbf{j} \times \mathbf{B}_0 + \mathbf{j}_0 \times \delta\mathbf{B}$$

$$\frac{\partial \delta\mathbf{B}}{\partial t} = \nabla \times \left(\delta\mathbf{v} \times \mathbf{B}_0\right)$$

$$\nabla \times \delta\mathbf{B} = \mu_0 \delta\mathbf{j}$$

Slab Model — Eigenmode equation

After considerable algebra:

$$\left(\omega^2 - \frac{\left(\mathbf{k}\cdot\mathbf{B}_0\right)^2}{B_{0z}^2} v_A^2\right)\xi = -v_A^2 \nabla\left[\frac{B_{0y}}{B_{0z}}\left(-i\frac{\mathbf{k}\cdot\mathbf{B}_0}{B_{0z}}\xi_y + \frac{\partial}{\partial x}\left(\frac{B_{0y}}{B_{0z}}\right)\xi_x\right) - i\frac{\mathbf{k}\cdot\mathbf{B}_0}{B_{0z}}\xi_z\right]$$

with

$$\delta\mathbf{v} = -i\omega\xi\left(x\right)\exp\left[-i\left(\omega t - k_y y - k_z z\right)\right]$$

$$k_\parallel = \mathbf{k}\cdot\mathbf{B}_0 / B_{0z} = k_z + k_y B_{0y}\left(x\right) / B_{0z} \qquad k_z << k_y,\ B_{0y} << B_{0z}$$

$$v_A^2 = B_{0z}^2 / \mu_0\rho_0$$

Set

$$\nabla\cdot\xi = 0 \text{ and } \xi_z << \xi_x,\ \xi_y \text{ — and eliminate } \xi_x$$

$$\frac{\partial}{\partial x}\frac{1}{\Omega^2}\frac{\partial}{\partial x}\left(\Omega^2\xi_y\right) - k_y^2\xi_y = 0 \quad \text{with} \quad \Omega^2 = \omega^2 - k_\parallel^2$$

Slab Model — Eigenmode equation (cont)

Define $\psi = \Omega^2 \xi_y$ so

$$\Omega^2 \frac{\partial}{\partial x}\left\{\frac{1}{\Omega^2}\frac{\partial \psi}{\partial x}\right\} - k_y^2 \psi = 0$$

or

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2k_{\|} k_y v_A^2}{\omega^2 - k_{\|}^2 v_A^2}\frac{\mu_0 j_{0z}}{B_{0z}}\frac{\partial \psi}{\partial x} - k_y^2 \psi = 0$$

Assume j_{0z} is constant and neglect the x-dependence of $k_{||}$

Normalize: $x = a\tilde{x},\ k = \tilde{k}/a,\ \omega = \tilde{\omega}\, v_A/a$ and $\mu_0 j_{0z}/B_{0z} = \tilde{j}/a$

The solutions for ψ are

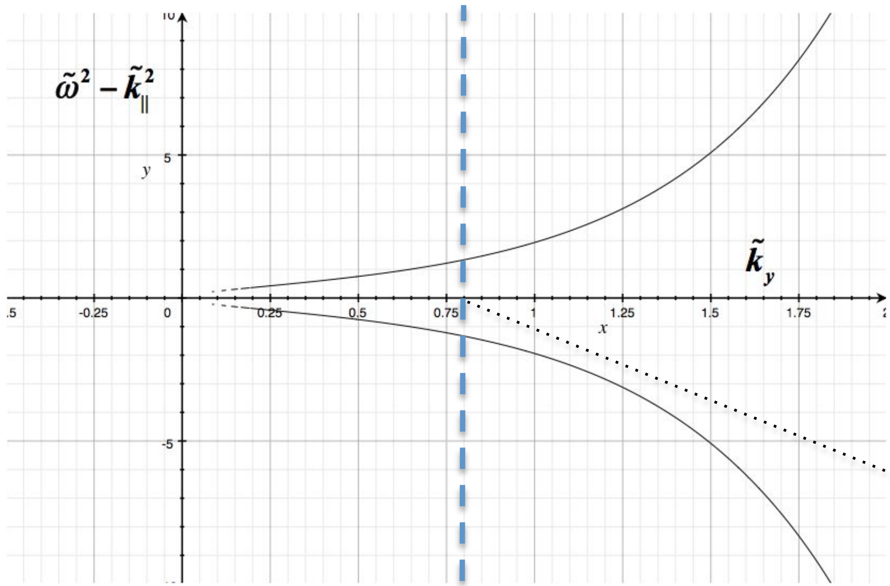
$$\psi = A_1 e^{\tilde{k}_y(\tilde{x}+1)} \qquad \tilde{x} \le -1$$

$$\psi = A_2 e^{\beta_1 \tilde{k}_y \tilde{x}} + A_3 e^{\beta_2 \tilde{k}_y \tilde{x}} \qquad -1 < \tilde{x} \le 1 \quad \text{with } \beta_{1,2} \text{ found from the Eq for } \psi$$

$$\psi = A_4 e^{-\tilde{k}_y(\tilde{x}-1)} \qquad \tilde{x} > 1$$

Slab Model — Solution

Apply continuity and jump conditions at $\tilde{x} = \pm 1$ to find the dispersion condition



Linear simulations: inner leg

$a \approx 0.04$ m, $R \approx 0.4$ m

$\tilde{k}_z = a/R \approx 0.1$

$B_{z0} \approx 1.2$ T

$B_{y0} \approx 0.15$ T

$\tilde{k}_y = \tilde{k}_z B_{0z}/B_{0y} \approx 0.8$

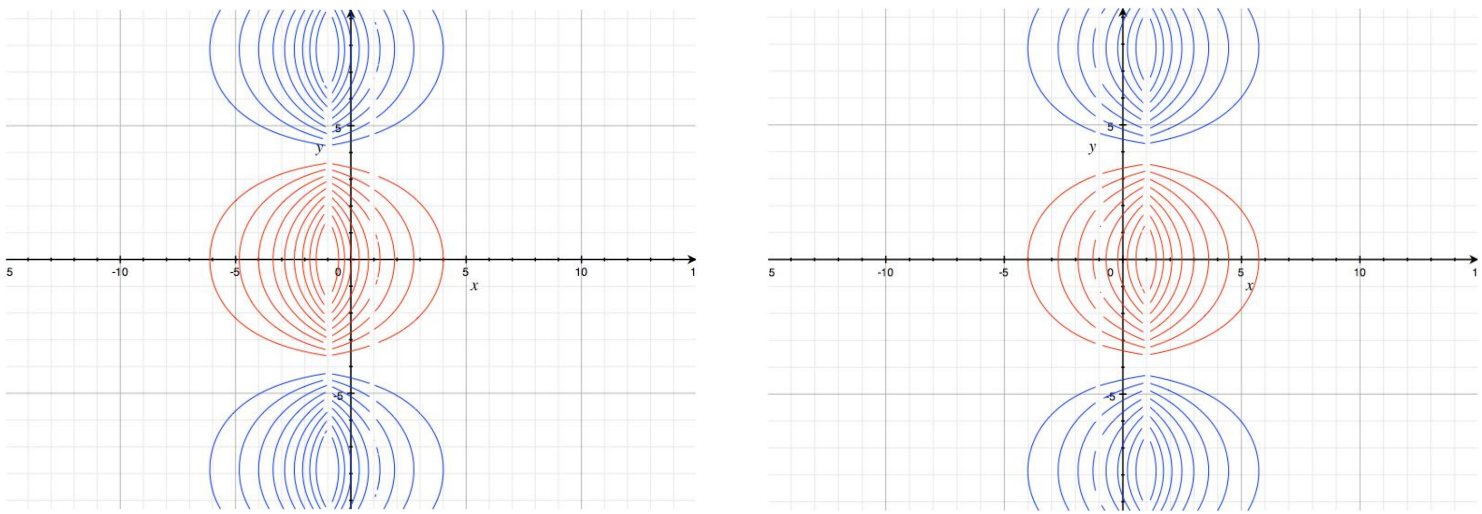
$\tilde{k}_\parallel \approx 0.2$

$$\tilde{\omega}^2 - \tilde{k}_\parallel^2 = \left(\omega^2 - k_\parallel^2 v_A^2\right) \frac{B_{0z}/\mu_0 j_{0z}}{2k_\parallel v_A^2}$$

In this simple model: Instability for all $\tilde{k}_y$ > a minimum value where $\tilde{\omega}^2 = \tilde{k}_\parallel^2$

- For strongly-driven injection the mode is clearly unstable
- The value of *j* determines the minimum value of k_y for instability

Slab Model –– Eigenmode –> 2 solutions with off-set eddies



Parameters as in previous slide for $\tilde{k}_y = 0.8; \tilde{j} = 0.1.$

Summary

At conditions where the plasma current is confined to a narrow channel at the surface of the expanding flux-bubble:

- A field-aligned instability is generated
- Linear analysis and simulations strongly suggest it is predominately an ideal, MHD current-driven mode

When the mode is low amplitude, it has little effect on the plasma evolution

At large amplitude the mode undergoes relaxation oscillations and significantly affects the evolution of the injected poloidal flux

- It prevents the formation of closed flux regions during injection
- It expands the current channel outside that due directly to the injection