



U.S. DEPARTMENT OF  
**ENERGY**

Office of  
Science



# Initial Development of the NSTX-U Snowflake Divertor Control

**Patrick Vail, Egemen Kolemen**

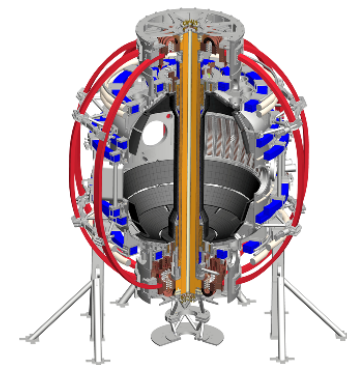
Princeton University, Princeton, NJ 08540

**Anders Welander, Matthew Lanctot**

General Atomics, P.O. Box 85608, San Diego, CA 92186

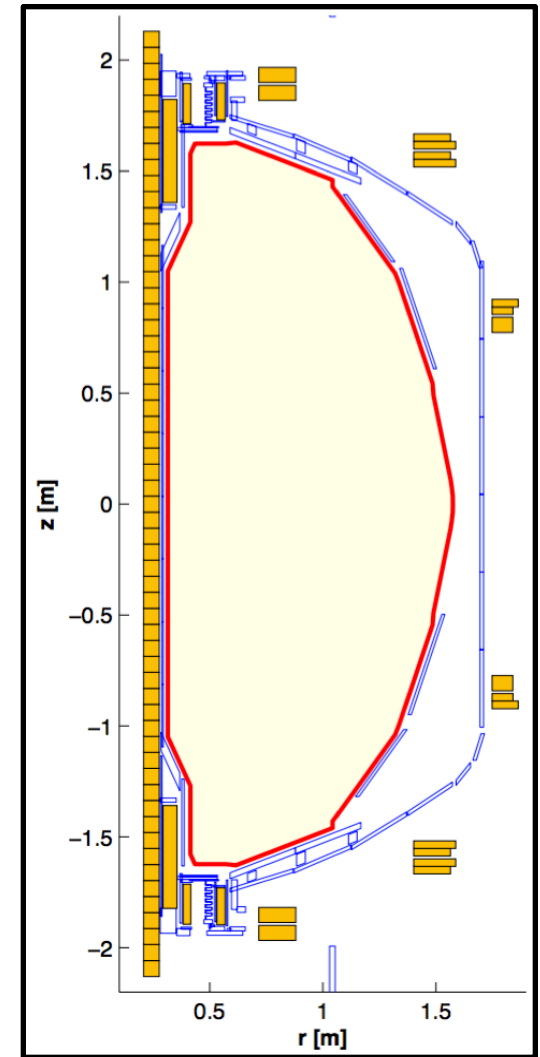


57<sup>th</sup> Annual Meeting of the APS Division of Plasma Physics  
Savannah, GA, November 16-20, 2015



# Introduction

- The determination of techniques for mitigating the heat flux onto plasma-facing surfaces is a major goal of current tokamak research.
- High heat fluxes to the walls push the limits of present-day engineering materials and active cooling technologies.
- **ITER** – expected that **70-90%** of power will need to be radiated with standard single x-point divertor.<sup>1</sup>
- **NSTX-U** – peak heat fluxes over **20 MW/m<sup>2</sup>** have been predicted in 2 MA, 12 MW NBI-heated discharges.<sup>2</sup> Long pulse limit for graphite PFCs is **10 MW/m<sup>2</sup>**.
- **Novel solutions to the heat flux problem are needed.**



*NSTX-U Poloidal Cross-Section*

<sup>1</sup>Kotschenreuther M. et al 2007. Phys. Plasmas. 14 072502.

<sup>2</sup>Gray T.K. et al 2011. J. Nucl. Mater. 415 S360.

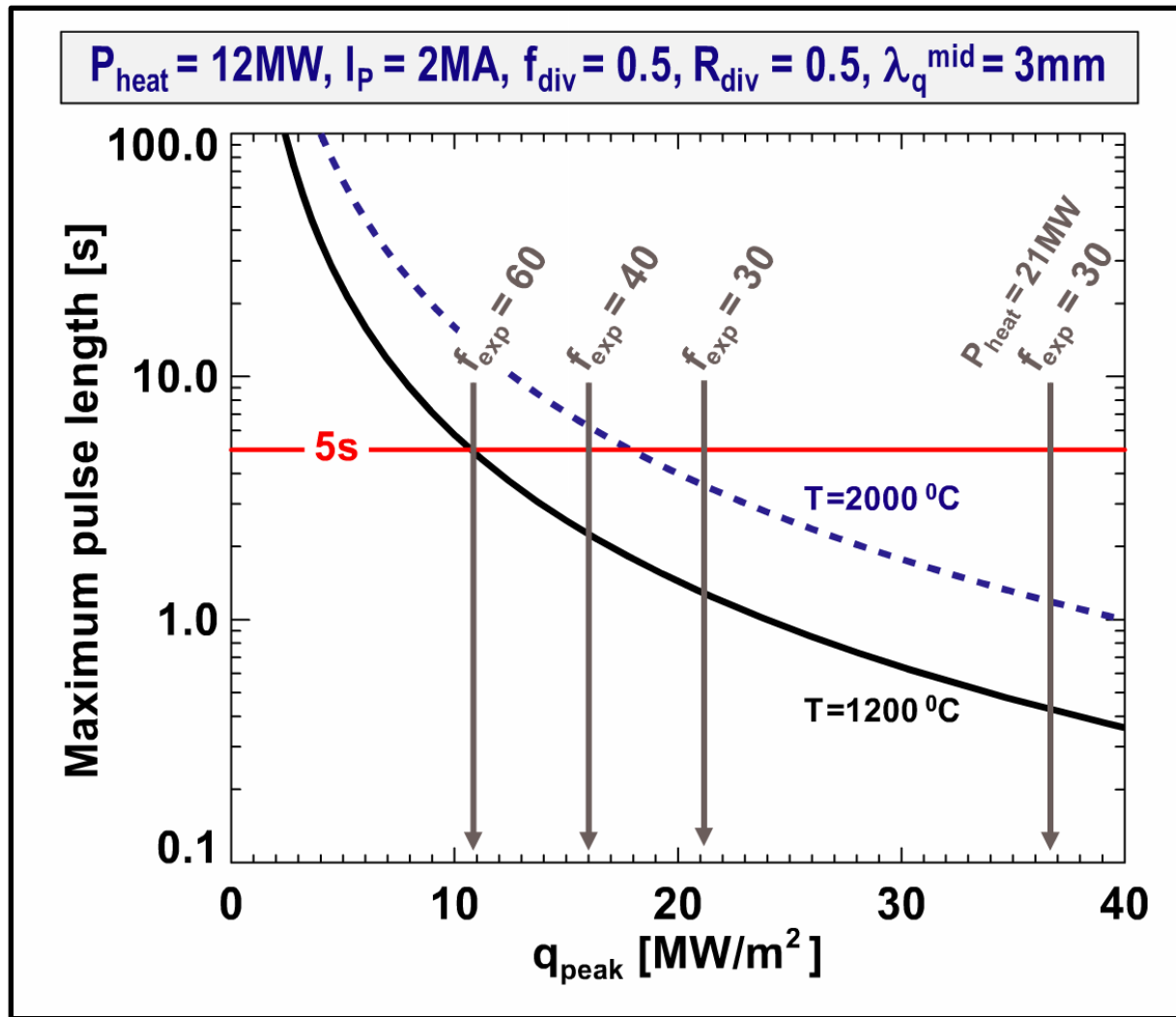
# Divertor Heat Flux

$$q_{peak} = \frac{P_{heat}^{SOL} (1 - f_{rad}) f_{div} \sin(\theta_{plate})}{2\pi R_{strike} f_{exp} \lambda_q}$$

$q_{peak}$	peak heat flux at the divertor strike point ( $P_{strike}/A_{wetted}$ )
$P_{heat}^{SOL}$	input heating power to SOL
$f_{rad}$	fraction of radiated power
$f_{div}$	fraction of power to divertor leg of interest
$\theta_{plate}$	poloidal angle between divertor plate and magnetic field line
$R_{strike}$	major radius of divertor strike point
$f_{exp}$	flux expansion
$\lambda_q$	width of heat flux profile in SOL

Stangeby, P.C. *The Plasma Boundary of Magnetic Fusion Devices*. Bristol: IOP Publishing Ltd, 2000.

# The Need for High Flux Expansion



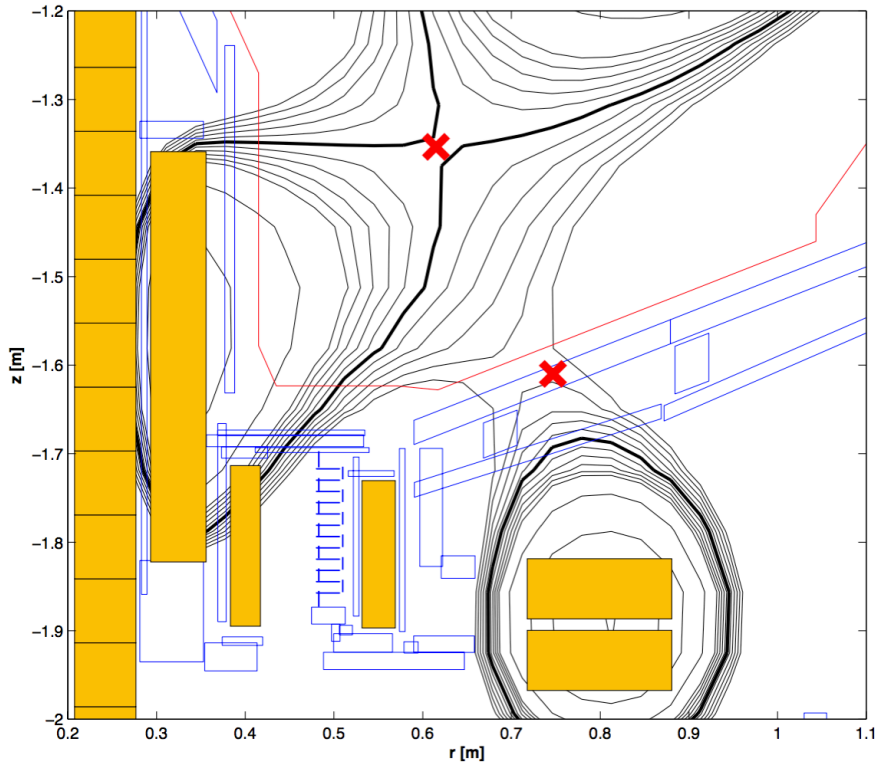
Menard J.E. et al 2012. Nucl Fusion. 52 083015.

# Advanced Divertors (1)

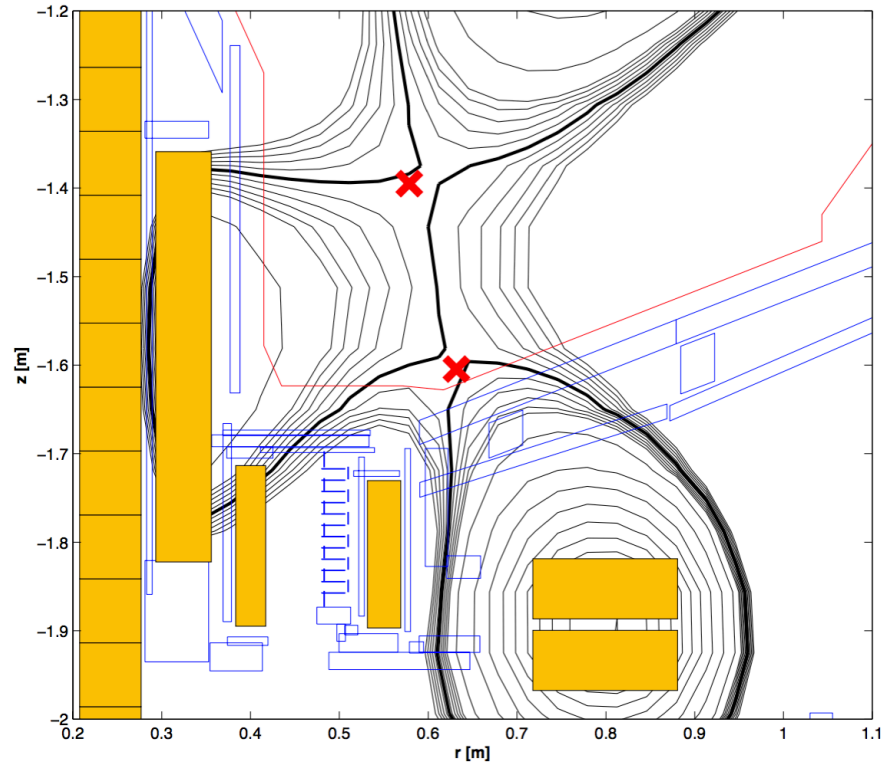
- Investigation of high flux expansion divertors is a major near-term research goal of the NSTX-U program.
- Two novel magnetic geometries have proven to be attractive candidates for steady-state (and potentially ELM transient) heat flux mitigation – **snowflake** and **x-divertor**
- **These divertors have the following magnetic properties<sup>1</sup>:**
  - higher poloidal flux expansion (compared to single x-point divertor)
  - longer x-point connection length
  - higher divertor flux tube volume
  - Four separatrix branches and strike points
- **SNOWFLAKE** – characterized by a second-order poloidal field null (or two closely-spaced first-order nulls)
- **X-DIVERTOR** – snowflake-like with the additional property that the secondary null is located in the vicinity of the strike point

<sup>1</sup>Soukhanovskii V.A. et al 2012. Phys. Plasmas. 19 082504.

# Advanced Divertors (2)



**SNOWFLAKE**



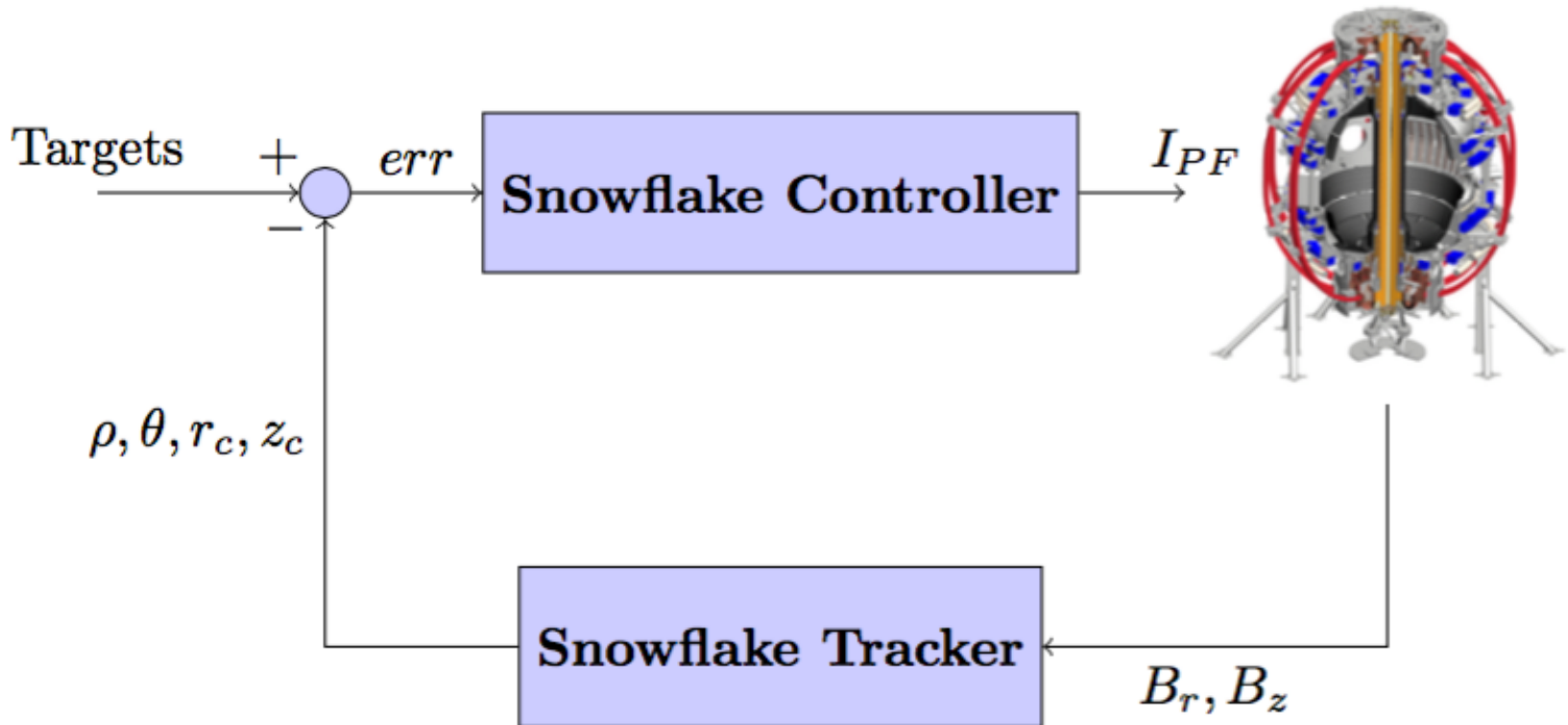
**X-DIVERTOR**

# Advanced Divertor Control

- NSTX experiments demonstrated the need for active magnetic control of snowflake divertor configurations.<sup>1</sup>
- Advanced divertor control presents new challenges – identification and control of multiple x-points and other parameters
- Snowflake / x-divertor tend to be topologically unstable – sensitive to changes in OH coil current, plasma profiles. (to be further characterized)
- Aim is for near-term implementation of the following control capabilities at NSTX-U:
  - **multiple x-point locations (for snowflake divertor)**
  - **strike point location (for x-divertor)**
  - **flux expansion independent of x-point locations**

<sup>1</sup>Soukhanovskii V.A. et al 2012. Phys. Plasmas. 19 082504.

# Snowflake Control System





# Snowflake Tracker (1)

- Consider the Grad-Shafranov equation with zero current density:

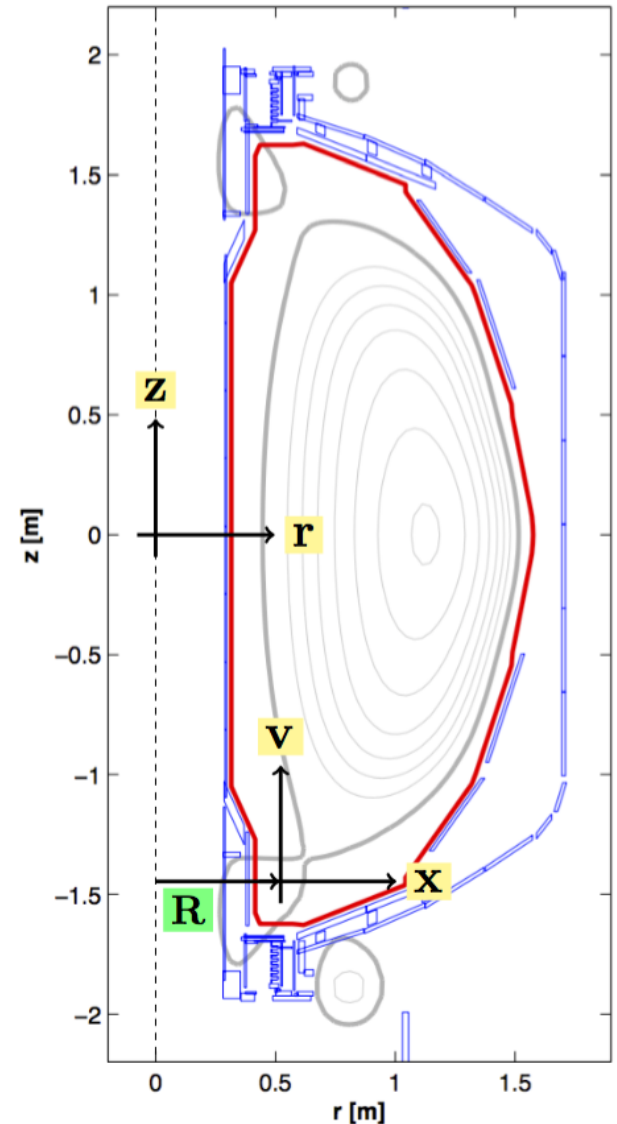
$$(R + x) \frac{\partial}{\partial x} \left( \frac{1}{R + x} \frac{\partial \psi}{\partial x} \right) + \frac{\partial^2 \psi}{\partial v^2} = 0$$

where the coordinate system has been shifted to be local to the null point(s) – new variables  $x$  and  $v$

$$B_x = -\frac{1}{R + x} \frac{\partial \psi}{\partial v} \quad B_v = \frac{1}{R + x} \frac{\partial \psi}{\partial x}$$

- Then expand the magnetic flux function to third order in the new coordinates

$$\begin{aligned} \psi = & l_1 x + l_2 v \\ & + q_1 x^2 + 2q_2 x v + q_3 v^2 \\ & + c_1 x^3 + c_2 x^2 v + c_3 x v^2 + c_4 v^3 \end{aligned}$$



Ryutov, D.D. et al 2010. Plasma Phys. Control. Fusion. 52 105001.

# Snowflake Tracker (2)

- The goal is to determine the expansion coefficients under the constraint that  $\psi$  approximately satisfies the current-free G-S equation.
- Magnetic field components in terms of 9 expansion coefficients:

$$B_x = -\frac{1}{R+x} (l_2 + 2q_2x + 2q_3v + c_2v^2 + 2c_3xv + 3c_4v^2)$$

$$B_v = \frac{1}{R+x} (l_1 + 2q_1x + 2q_2v + 3c_1x^2 + 2c_2xv + c_3v^2)$$

- Three constraints come from substituting  $\psi$  into the G-S equation:

$$-l_1 + 2q_1R + 2q_3R = 0$$

$$-2q_1 + 6c_1R + 2c_3R = 0$$

$$-2q_2 + 2c_2R + 6c_4R = 0$$

- For a second-order null at the origin of the  $x, v$  coordinate system:  $l$ 's and  $q$ 's = 0.
- For a snowflake with two closely separated x-points:  $l$ 's and  $q$ 's are small.

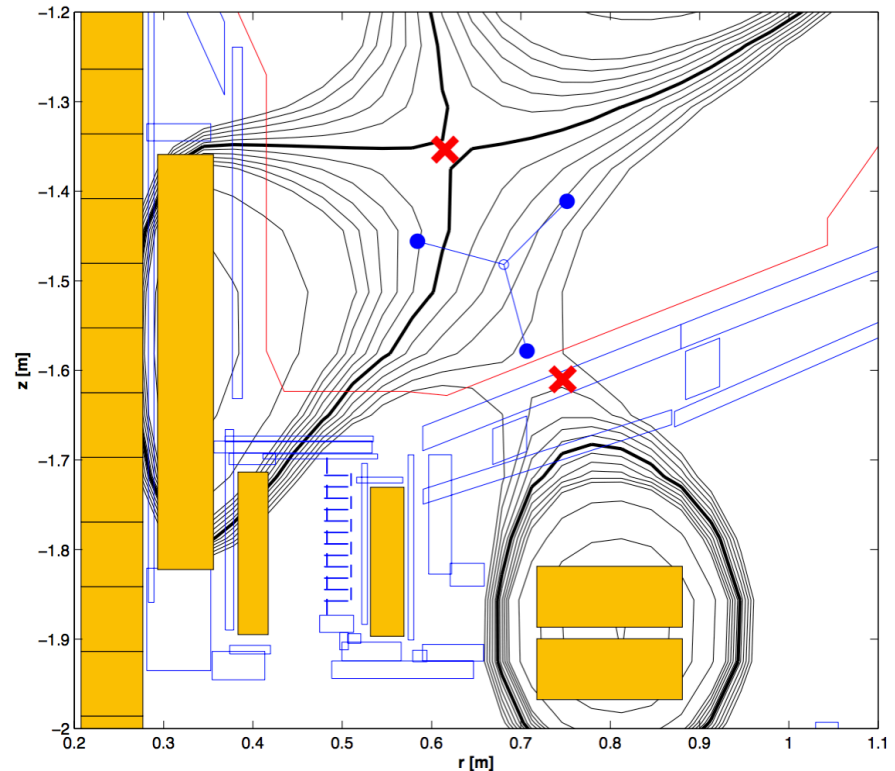
# Finding the expansion coefficients

- 6 free expansion coefficients are found by sampling  $\mathbf{B}_x$  and  $\mathbf{B}_v$  at three points and then solving the following linear equations for  $i = \{1, 2, 3\}$ .

$$-(R + x_i) B_{x,i} = l_2 + 2q_2x_i + 2q_3v_i - 3c_4 (x_i^2 - v_i^2) - 6c_1x_iv_i$$

$$(R + x_i) B_{v,i} = l_1 - 2q_3x_i + 2q_2v_i + 3c_1 (x_i^2 - v_i^2) - 6c_4x_iv_i$$

- $\mathbf{B}_x$  and  $\mathbf{B}_v$  obtained in real-time from rtEFIT.



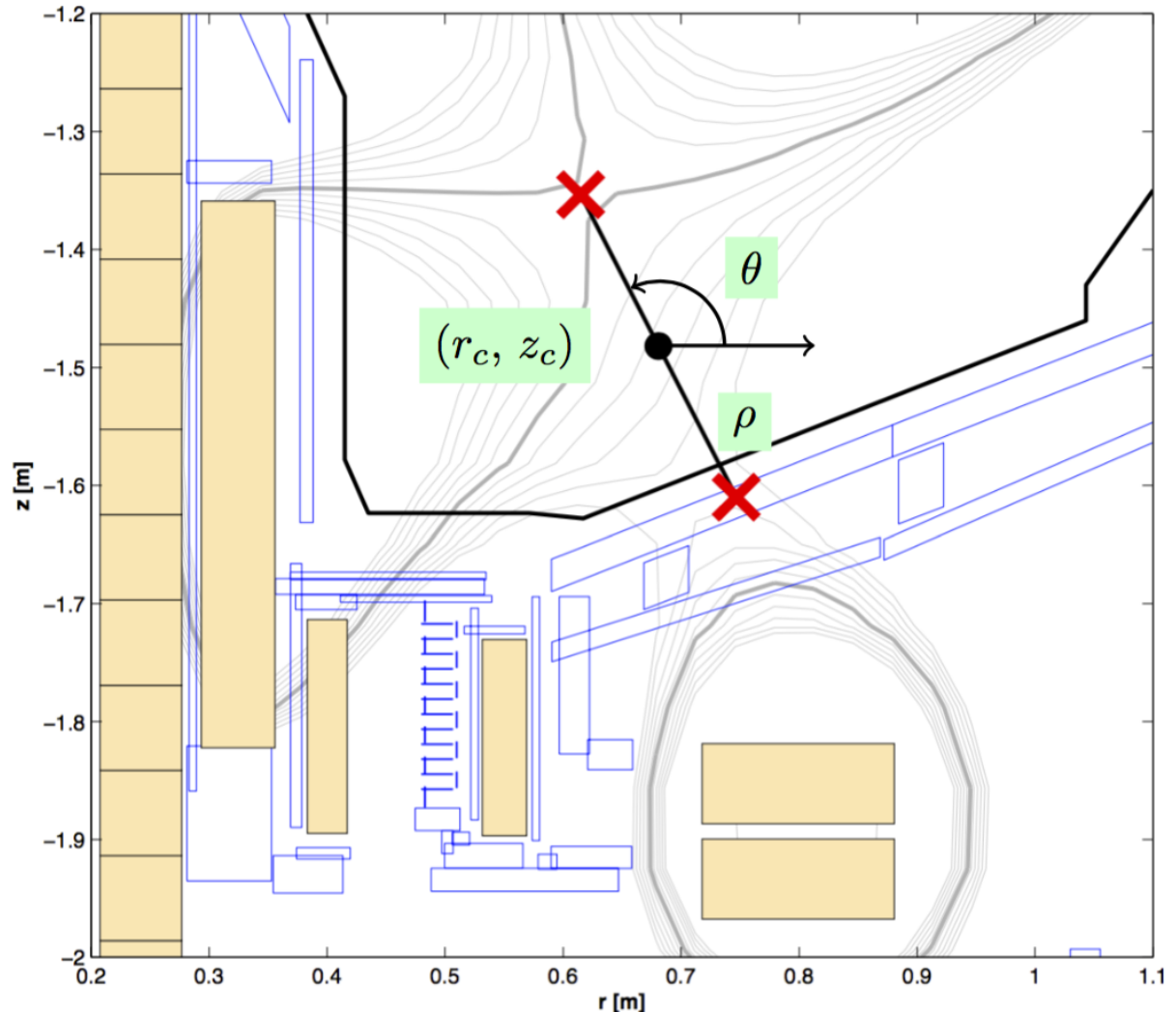
# Snowflake Geometry

Consider a polar coordinate system local to the x-points.

Snowflake geometry then defined by four parameters:

- $r_c$  and  $z_c$  – coordinates of the polar coordinate system origin (centroid) relative to the tokamak
- $\rho$  – radial distance between x-point and centroid
- $\theta$  – angle as defined in the figure (right)

Goal is independent control of each parameter.



# Snowflake Controller

1. Calculate Jacobian matrix  $\mathbf{J}$  that maps changes in coil currents to changes in snowflake location.

$$\begin{bmatrix} \delta\rho \\ \delta\theta \\ \delta r_c \\ \delta z_c \end{bmatrix} = \mathbf{J} \begin{bmatrix} \delta I_{PF2L} \\ \delta I_{PF1CL} \\ \delta I_{PF1AL} \end{bmatrix}$$

2. Take the pseudoinverse of  $\mathbf{J}$  and compute the set of coil currents that minimizes error between current and target snowflake location.

$$\begin{bmatrix} \delta I_{PF2L} \\ \delta I_{PF1CL} \\ \delta I_{PF1AL} \end{bmatrix} = (\mathbf{J}^T \mathbf{J})^{-1} \mathbf{J}^T \begin{bmatrix} \delta\rho \\ \delta\theta \\ \delta r_c \\ \delta z_c \end{bmatrix}$$

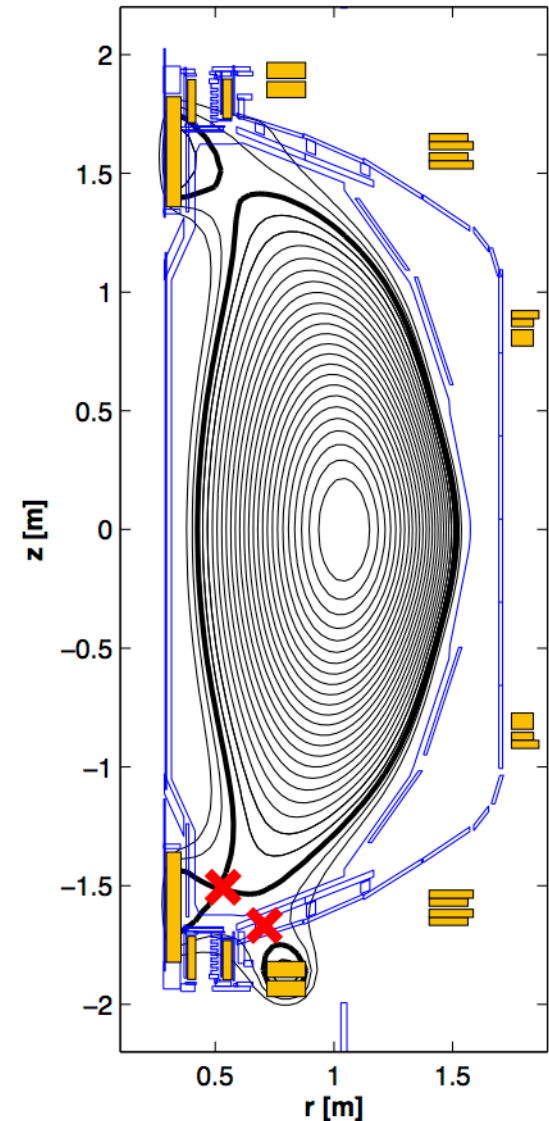
3. Pass  $\delta\mathbf{I}$ 's through PID controller and then output voltage commands to PF coils.

$$V_j(t) = R_j \times \left[ I_j(t-1) + K_{p,j} \left( \Delta I_j(t) + \frac{1}{T_{i,j}} \int_0^t \Delta I_j(\tau) d\tau + T_{d,j} \frac{d\Delta I_j(t)}{dt} \right) \right]$$

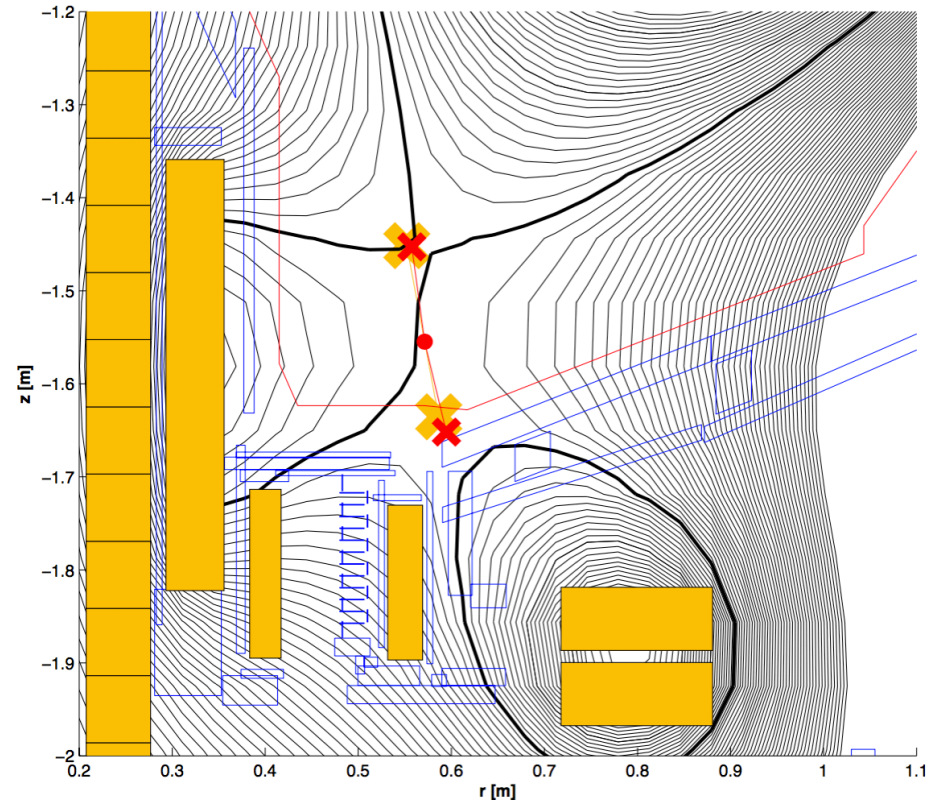
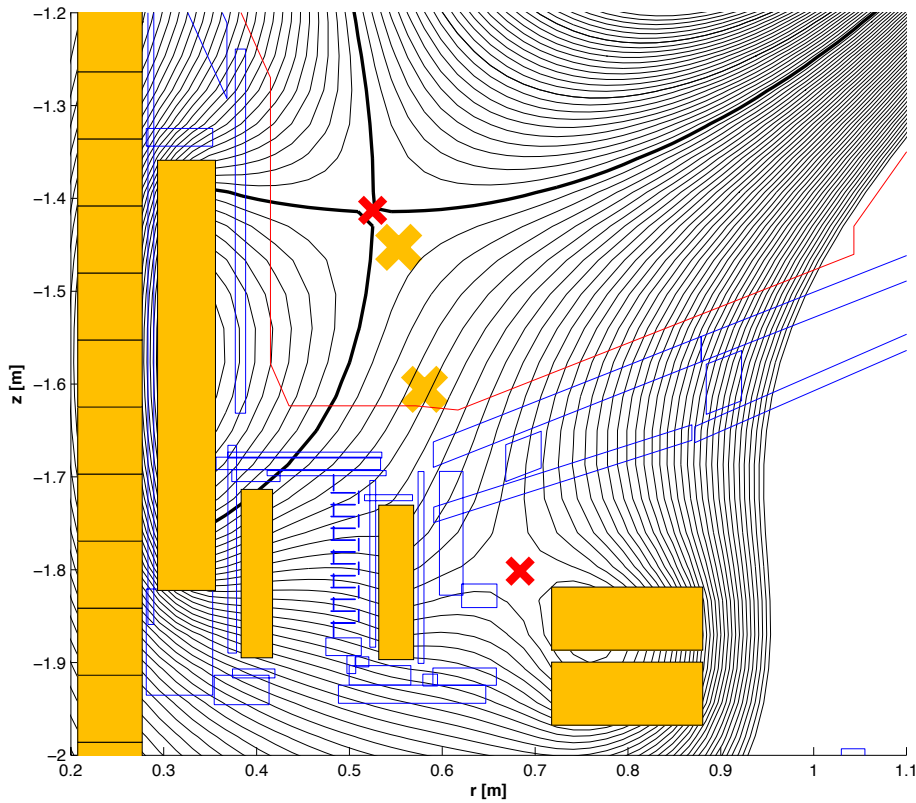
# Simulation

- Snowflake tracking and coil current computations have been simulated in TokSys for expected FY2016 NSTX-U plasmas.
- PF2L, PF1CL, and PF1AL coils used for control.

$I_p$	1 MA
$B_T$	0.65 T
$\beta_p$	1.07
$l_i$	0.65
<b>Configuration</b>	lower-null
<b>Inner Gap</b>	11.1 cm
<b>Outer Gap</b>	5.9 cm
$\kappa$	2.62



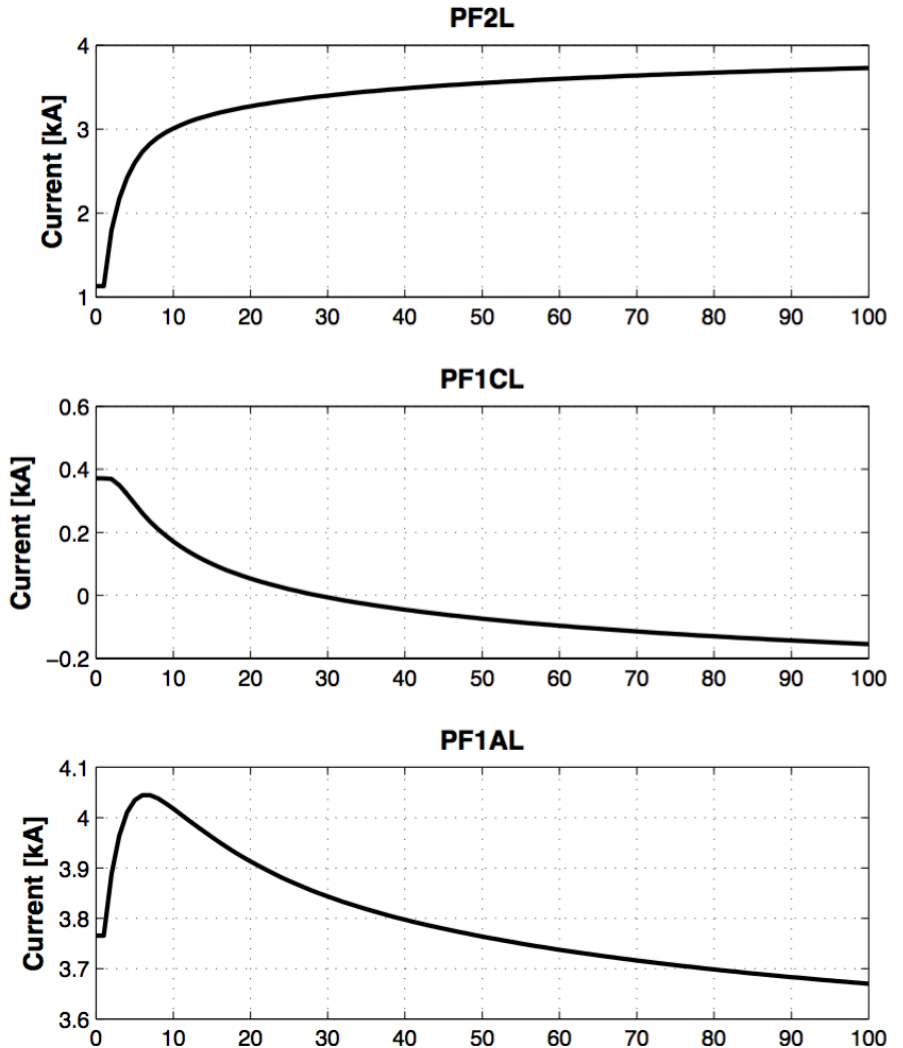
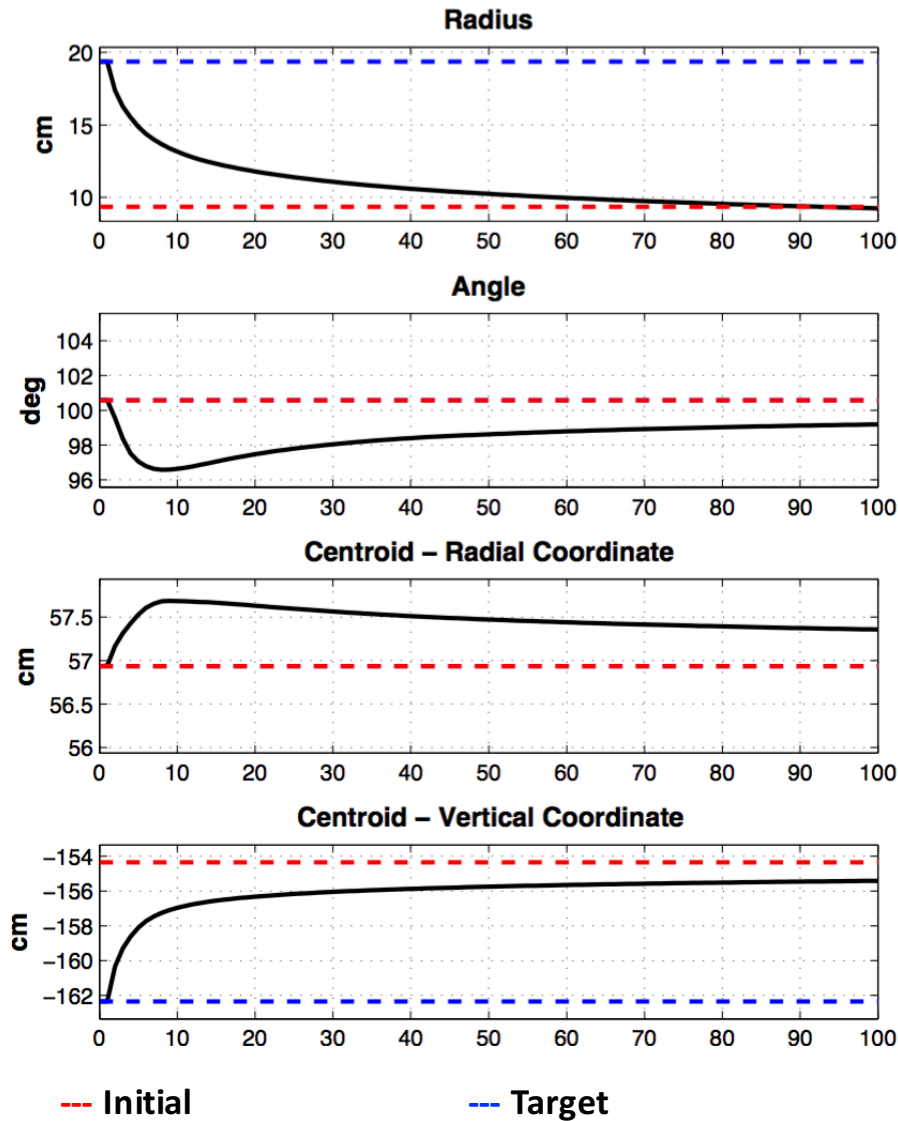
# Single X-Point to Snowflake Minus



**Red** – current location

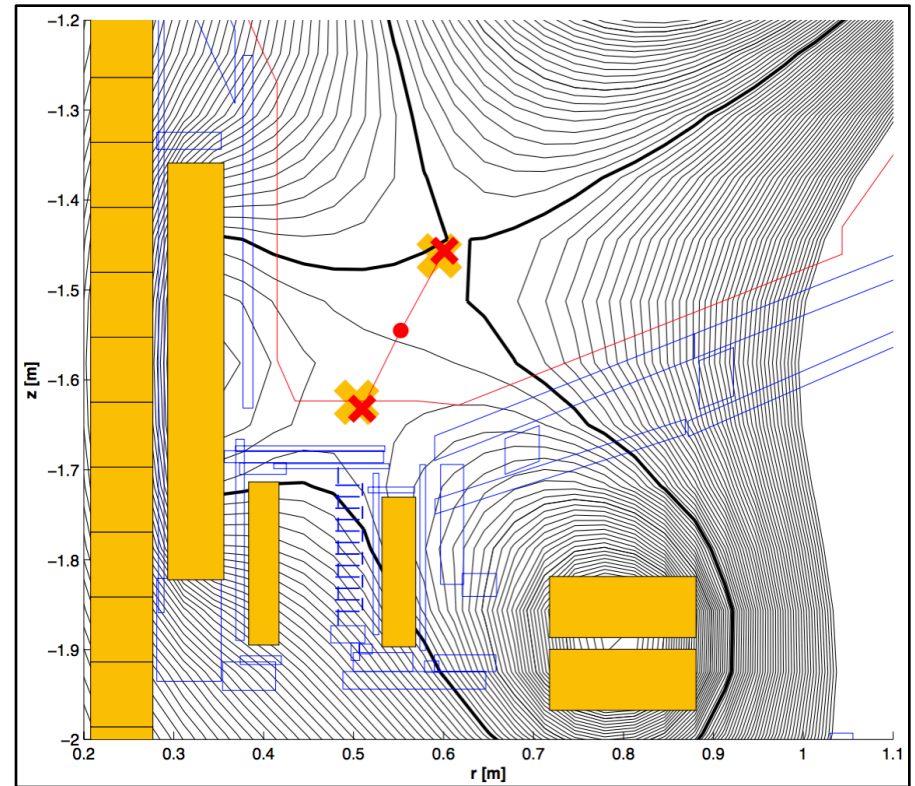
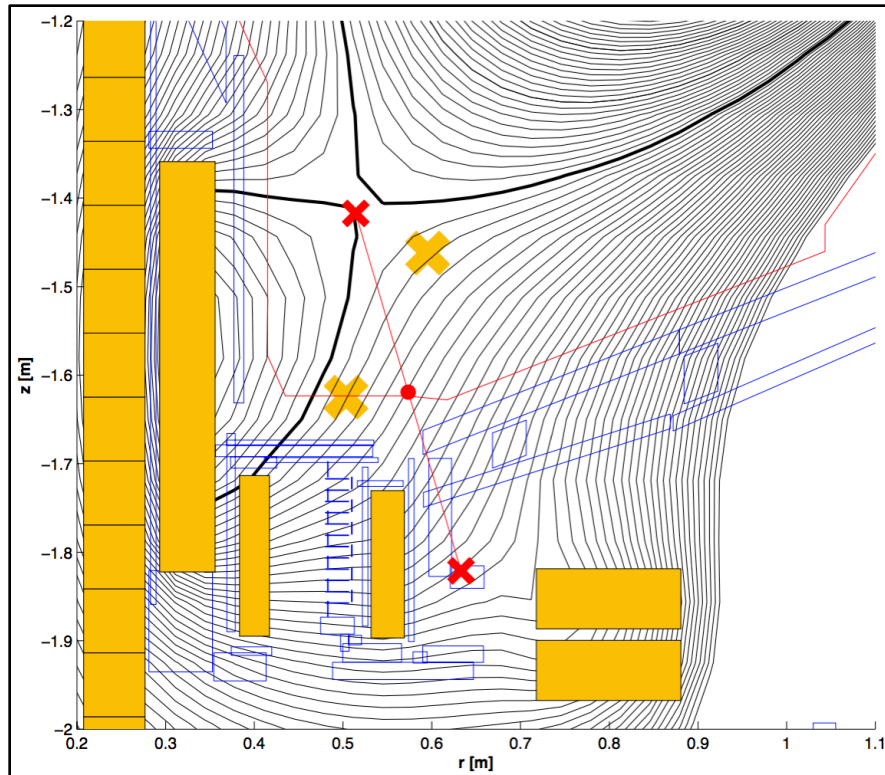
**Gold** – target location

# Single X-Point to Snowflake Minus





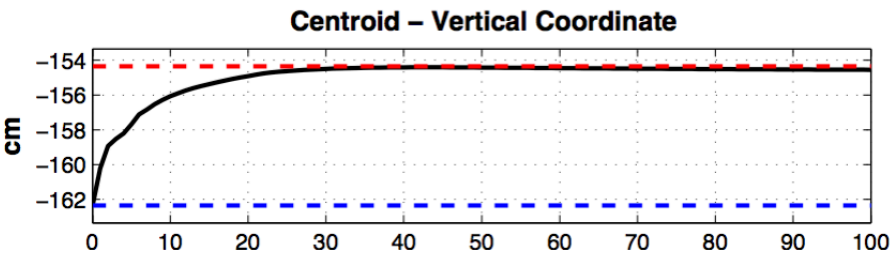
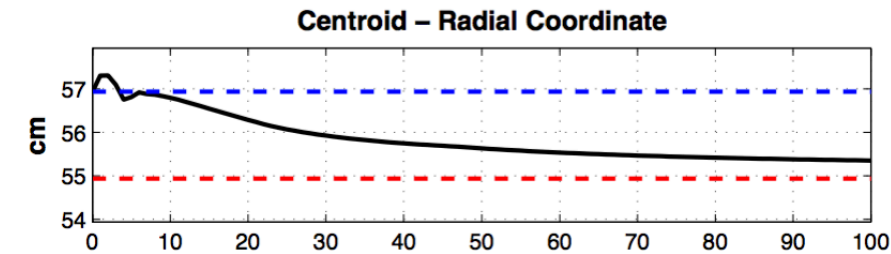
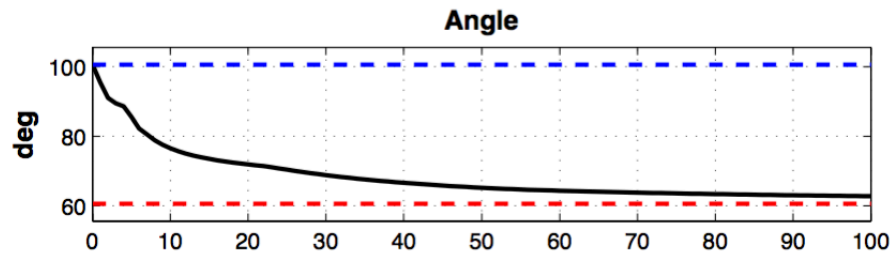
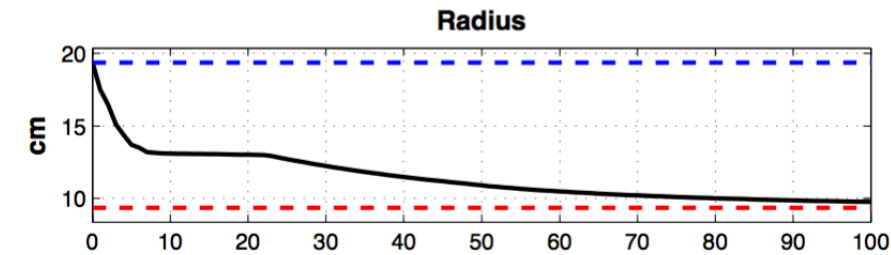
# Single X-Point to Snowflake Plus



**Red** – current location

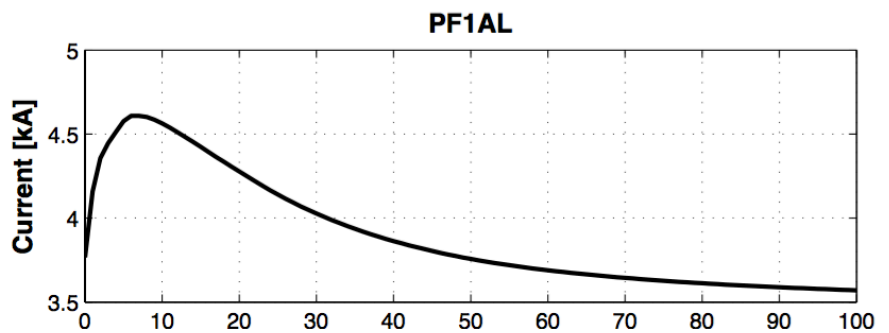
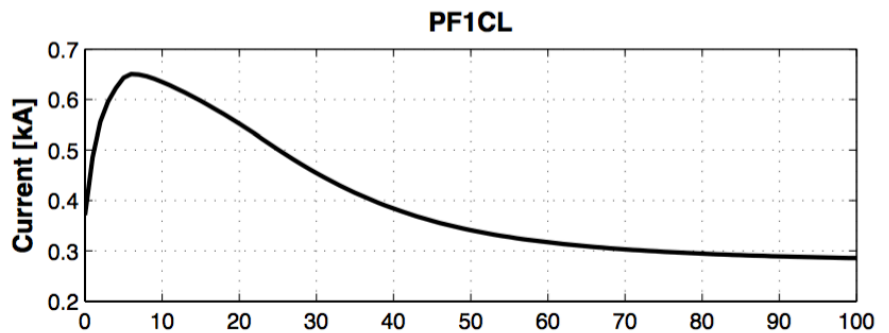
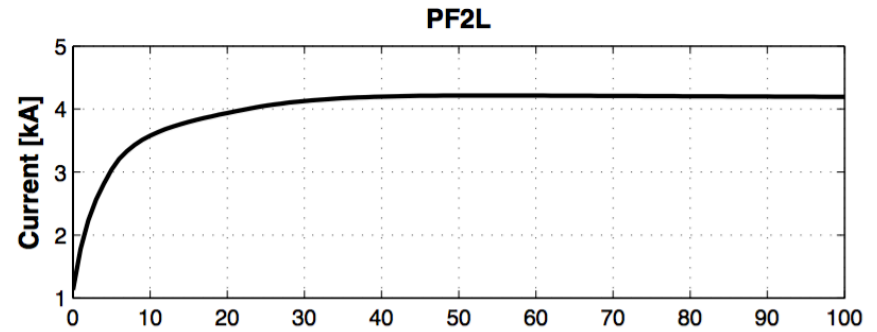
**Gold** – target location

# Single X-Point to Snowflake Plus



--- Initial

--- Target



# Strike Point Control (1)

1. Locate and calculate the flux at the primary x-point, calculate flux at the target strike point location, and then calculate the flux error as follows:

$$\Delta\psi = \psi_{strike,targ} - \psi_{xpt}$$

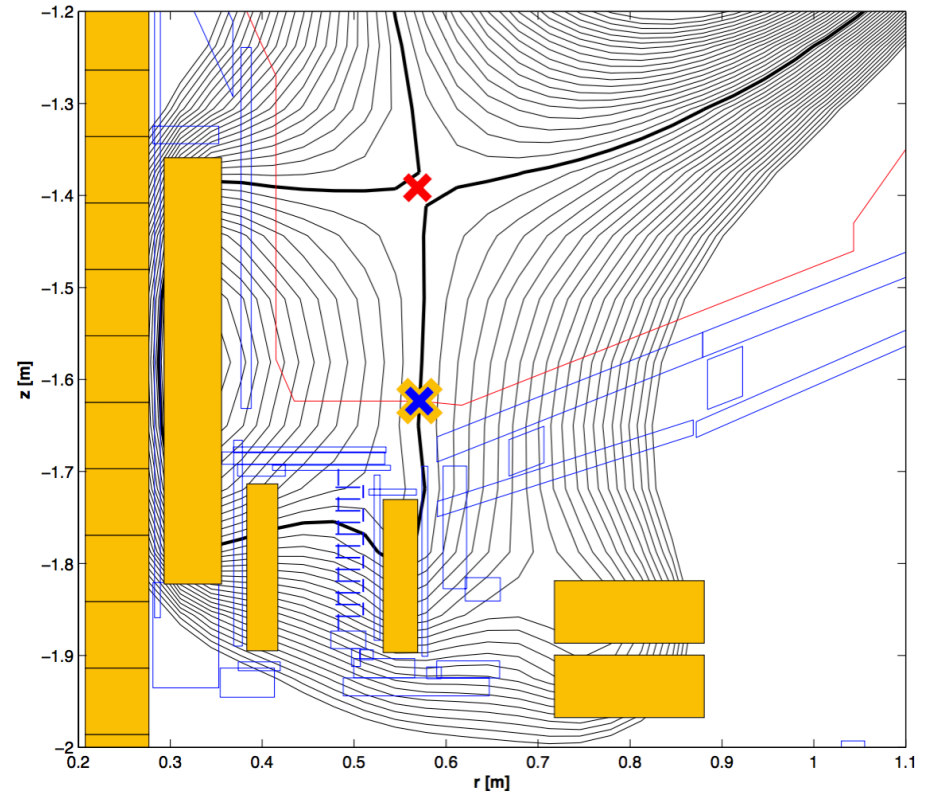
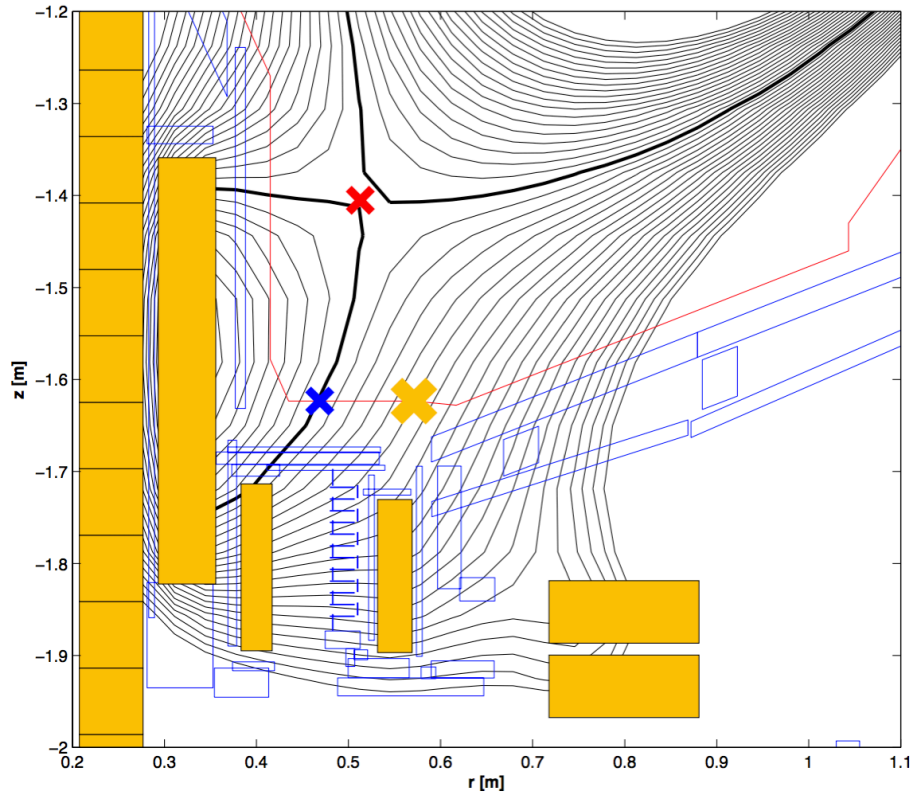
2. Calculate the Jacobian matrix  $\mathbf{J}$  that maps changes in coil currents to changes in  $\psi$  at the target strike point location, where the entries of  $\mathbf{J}$  are the mutual inductances  $\mathbf{M}_{ij}$  between coils and target strike point:

$$\Delta\psi = \mathbf{J} \begin{bmatrix} \delta I_{PF2L} \\ \delta I_{PF1CL} \\ \delta I_{PF1AL} \end{bmatrix}$$

3. Take the pseudoinverse of  $\mathbf{J}$  and compute the set of coil currents that minimizes the error between current and target strike point location:

$$\begin{bmatrix} \delta I_{PF2L} \\ \delta I_{PF1CL} \\ \delta I_{PF1AL} \end{bmatrix} = (\mathbf{J}^T \mathbf{J})^{-1} \mathbf{J}^T \Delta\psi$$

# Strike Point Control (2)



**Red** – x-point location

**Blue** – current strike point location

**Gold** – target strike point location

# Strike Point Control (3)

