Initial applications of the non-Maxwellian extension of the full-wave TORIC v.5 code in the mid/high harmonic and minority heating regimes

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PPPL

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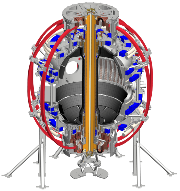


Outline

- Full wave code TORIC
  - Brief code description
  - TORIC's versions
- Non-Maxwellian extension of TORIC v.5 in minority heating regime
  - Test I: Numerical vs. analytical Maxwellian case in TORIC
  - Bi-Maxwellian distribution
- Non-Maxwellian extension of TORIC v.5 in HHFW heating regime
  - Test I: Numerical vs. analytical Maxwellian full hot dielectric tensor
  - Test II: Numerical vs. analytical Maxwellian case in TORIC
  - Bi-Maxwellian distribution
  - Slowing-down distribution
- Conclusions

TORIC code

- The TORIC code solves the wave equation

$$\nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2}\left[\mathbf{E} + \frac{4\pi i}{\omega}(\mathbf{J}^P + \mathbf{J}^A)\right]$$

  for the electric field $\mathbf{E}$.

-
  - $\mathbf{J}^A \rightarrow$ prescribed antenna current density
  - $\mathbf{J}^P \rightarrow$ plasma current density

$$\mathbf{J}^P(\mathbf{x}) = \int \mathrm{d}\mathbf{x}' \boldsymbol{\sigma}(\mathbf{x}, \mathbf{x}') \cdot \mathbf{E}(\mathbf{x}')$$

  - Conductivity tensor $\boldsymbol{\sigma}[f_0(\mathbf{x}, \mathbf{v})]$, is a functional of f_0, which is, in general, non-Maxwellian

- Spectral ansatz

$$\mathbf{E}(\mathbf{r}, t) = \sum_{m,n} \mathbf{E}^{mn}(\psi) e^{i(m\theta + n\phi - \omega t)}$$

  $m \rightarrow$ poloidal mode number; $n \rightarrow$ toroidal mode number

- Principal author M. Brambilla (IPP Garching, Germany)

TORIC code (2)

- For each toroidal component one has to solve a (formally infinite) system of coupled ordinary differential equations for the physical components of $E^{mn}(\psi)$, written in the local field-aligned orthogonal basis vectors.

- The Spectral Ansatz transforms the θ-integral of the constitutive relation into a convolution over poloidal modes.

- Due to the toroidal axisymmetry, the wave equations are solved separately for each toroidal Fourier component.

- A spectral decomposition defines an accurate representation of the “local" parallel wave-vector
  $$k_{\parallel}^{m} = (m\nabla\theta + n\nabla\phi)\cdot\hat{\mathbf{b}}$$

- The ψ variation is represented by Hermite cubic finite elements

TORIC's versions

- TORIC: IC frequency regime
  extended to include non-Maxwellian ions to the second order in $k_\perp v_\perp / \Omega_c$
  (part of the initial work shown by E. J. Valeo at APS-DPP 2011):
  ADDITIONAL TESTS & INITIAL APPLICATIONS in THIS WORK

- TORIC-HHFW: High Harmonic Fast Wave regime
  to extend to include non-Maxwellian ions: THIS WORK

- TORIC-LH: LH frequency regime
  extended to include non-Maxwellian electrons
  J. C. Wright *et al*, Nucl. Fusion **45** (2005) 1411 &
  J. C. Wright *et al*, Commun. Comput. Phys. **4** (2008) 545

Non-Maxwellian extension of TORIC v.5 in minority heating regime

Beyond Maxwellian

FLR non-Maxwellian susceptibility in a local coordinate (Stix) frame $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$, with $\hat{\mathbf{z}} = \hat{\mathbf{b}}$, $\mathbf{k} \cdot \hat{\mathbf{y}} = 0$, to second order in $k_\perp v_\perp / \omega_c$

$$\chi_{xx} = \frac{\omega_{\mathrm{p,s}}^2}{\omega}\left[\frac{1}{2}\left(A_{1,0}+A_{-1,0}\right) - \frac{\lambda}{2}\left(A_{1,1}+A_{-1,1}\right) + \frac{\lambda}{2}\left(A_{2,1}+A_{-2,1}\right)\right]$$

$$\chi_{xy} = -\chi_{yx} = i\frac{\omega_{\mathrm{p,s}}^2}{\omega}\left[\frac{1}{2}\left(A_{1,0}-A_{-1,0}\right) - \lambda\left(A_{1,1}-A_{-1,1}\right) + \frac{\lambda}{2}\left(A_{2,1}-A_{-2,1}\right)\right]$$

$$\chi_{xz} = +\chi_{zx} = -\chi_{yx} = \frac{\omega_{\mathrm{p,s}}^2}{\omega}\left(\frac{1}{2}\frac{k_\perp}{\omega}\right)\left[\left(B_{1,0}+B_{-1,0}\right) - \lambda\left(B_{1,1}+B_{-1,1}\right) + \frac{\lambda}{2}\left(B_{2,1}+B_{-2,1}\right)\right]$$

$$\chi_{yy} = \frac{\omega_{\mathrm{p,s}}^2}{\omega}\left[2\lambda A_{0,1} + \frac{1}{2}\left(A_{1,0}+A_{-1,0}\right) - \frac{3\lambda}{2}\left(A_{1,1}+A_{-1,1}\right) + \frac{\lambda}{2}\left(A_{2,1}+A_{-2,1}\right)\right]$$

$$\begin{aligned}\chi_{yz} &= -\chi_{zy} = i\frac{\omega_{\mathrm{p,s}}^2}{\omega}\left(\frac{k_\perp}{\omega}\right)\left[B_{0,0} - \lambda B_{0,1} - \frac{1}{2}\left(B_{1,0}+B_{-1,0}\right) - \lambda\left(B_{1,1}+B_{-1,1}\right)\right.\\ &\left. - \frac{\lambda}{4}\left(B_{2,1}+B_{-2,1}\right)\right]\end{aligned}$$

$$\begin{aligned}\chi_{zz} &= \frac{2\omega_{\mathrm{p}}^2}{k_\parallel w_\perp^2}\left[(1-\lambda)B_{0,0} + \int_{-\infty}^{+\infty} \mathrm{d}v_\parallel \int_0^{+\infty} \mathrm{d}v_\perp v_\perp \frac{v_\parallel}{\omega} f_0(v_\parallel, v_\perp)\right]\\ &+ \frac{\lambda}{2}\frac{\omega_{\mathrm{p}}^2}{\omega}\left[2\frac{\omega-\omega_{\mathrm{c}}}{k_\parallel w_\perp^2}B_{1,0} + 2\frac{\omega+\omega_{\mathrm{c}}}{k_\parallel w_\perp^2}B_{-1,0}\right]\end{aligned}$$

$$\lambda \equiv \frac{1}{2}\left(\frac{k_\perp v_\perp}{\omega_{\mathrm{c}}}\right)^2$$

Beyond Maxwellian (2)

Evaluations of the FLR susceptibility requires computation of two functions $A_{n,j}$ $B_{n,j}$, for $n = -2 \ldots 2$, $j = 0, 1$, which are $v_\perp$ moments of resonant integrals of $f_0(\psi, \frac{B}{B_{\min}}, v_\parallel, v_\perp)$

$$\left\{ \begin{array}{c} A_{n,j} \\ B_{n,j} \end{array} \right\} = \int_{-\infty}^{\infty} \mathrm{d}v_\parallel \left\{ \begin{array}{c} 1 \\ v_\parallel \end{array} \right\} \frac{1}{\omega - k_\parallel v_\parallel - n\omega_\mathrm{c}} \int_0^{+\infty} 2\pi v_\perp \mathrm{d}v_\perp H_j(v_\parallel, v_\perp)$$

with

$$\begin{aligned} H_0(v_\parallel, v_\perp) &= \frac{1}{2}\frac{k_\parallel w_\perp^2}{\omega}\frac{\partial f_0}{\partial v_\parallel} - \left(1 - \frac{k_\parallel v_\parallel}{\omega}\right) f_0(v_\parallel, v_\perp) \\ H_0(v_\parallel, v_\perp) &= \frac{1}{2}\frac{k_\parallel w_\perp^2}{\omega}\frac{\partial f_0}{\partial v_\parallel}\frac{v_\perp^4}{w_\perp^4} - \left(1 - \frac{k_\parallel v_\parallel}{\omega}\right) f_0(v_\parallel, v_\perp)\frac{v_\perp^2}{w_\perp^2} \end{aligned}$$

and

$$w_\perp^2 \equiv \int_{-\infty}^{\infty} \mathrm{d}v_\parallel \int_0^{+\infty} 2\pi v_\perp \mathrm{d}v_\perp^2 f_0(v_\parallel, v_\perp)$$

χ is pre-computed to reduce TORIC-IC runtime

- A set of N_ψ files is constructed, each containing the principal values and residues of χ for a single species on a uniform $(v_\parallel, \theta)$ mesh, for a specified flux surface ψ_j

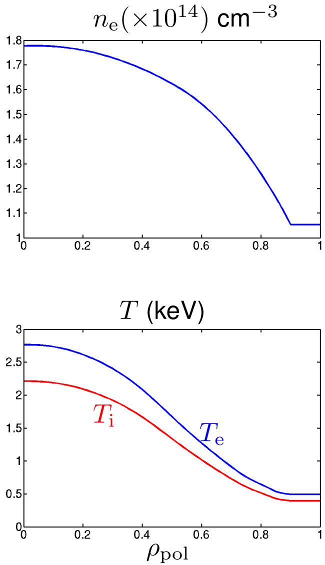
- The distribution, $f(v_\parallel, v_\perp)$, is specified in functional form at the minimum field strength point $B(\theta) = B_{\min}$ on ψ_j

- An interpolator returns the components of χ or ε (i.e., the dielectric tensor).

Alcator C-Mod case

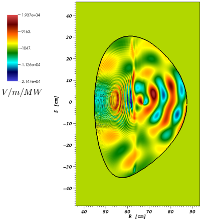
Main parameters:

- Plasma species: electron, D, and minority H (4%)
- $B_\mathrm{T} = 5$ T
- $I_\mathrm{p} = 1047$ kA
- $q(0)$ = 0.885
- q at plasma edge = 4.439
- $T_\mathrm{e}(0) = 2.764$ keV
- $n_\mathrm{e}(0) = 1.778 \times 10^{14}$ cm^{-3}
- $T_\mathrm{D,H}(0) = 2.212$ keV
- TORIC resolution: $n_\mathrm{mod} = 255$, $n_\mathrm{elm} = 480$

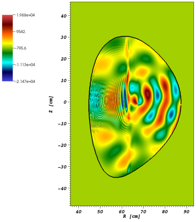


Excellent agreement between numerical and analytical evaluation of the electric field and in terms of absorbed power

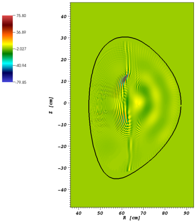
Maxw. analytical: Re(E_-)



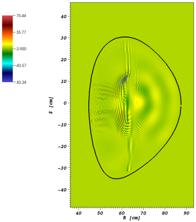
Maxw. numerical: Re(E_-)



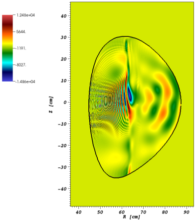
Maxw. analytical: Re($E_{||}$)



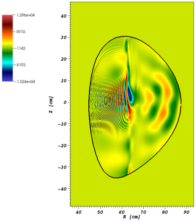
Maxw. numerical: Re($E_{||}$)



Maxw. analytical: Re(E_+)



Maxw. numerical: Re(E_+)



Absorbed fraction	Maxw. analytical	Maxw. numerical
2nd Harmonic D	10.18	9.91
Fundamental H	69.95	70.50
Electrons - FW	11.35	11.21
Electrons -IBW	8.53	8.38

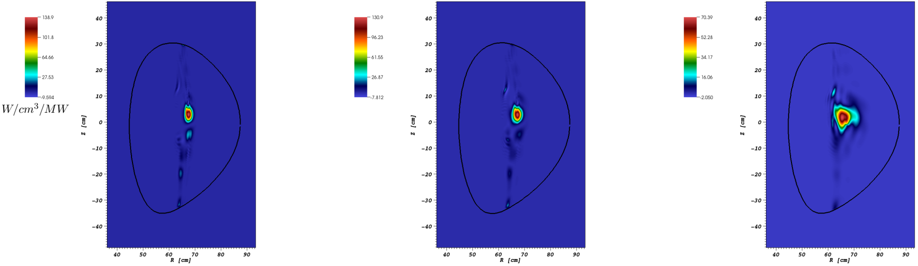
Bi-Maxwellian distribution

$$f_{\mathrm{H}}(v_{\|}, v_{\perp}) = (2\pi)^{-3/2}(v_{\mathrm{th},\|}v_{\mathrm{th},\perp}^2)^{-1}\exp[-(v_{\|}/v_{\mathrm{th},\|})^2 - (v_{\perp}/v_{\mathrm{th},\perp})^2]$$

with $v_{\mathrm{th},\|} = \sqrt{2C_{\|}T(\psi)/m_{\mathrm{H}}}$, $v_{\mathrm{th},\perp} = \sqrt{2C_{\perp}T(\psi)/m_{\mathrm{H}}}$, with constants $C_{\|}$ and $C_{\perp}$

- For $C_{\|} = 1$ and $C_{\perp} = \{.5, 1., 3., 5.\}$, P_{H}, varied by less than 2%
- For $C_{\perp} = 1$ and $C_{\|} = \{.5, 1., 3., 5.\}$, the corresponding $P_{\mathrm{H}} = \{61.27\%, 70.50\%, 90.46\%, 94.18\%\}$
  - for small $C_{\|}$, the absorption profile is localized to the resonant layer
  - for large $C_{\|}$, the absorption profile is significantly broadened radially

$C_{\perp} = 1, C_{\|} = 0.5$ $\quad$ $C_{\perp} = 1, C_{\|} = 1$ $\quad$ $C_{\perp} = 1, C_{\|} = 5.0$



Non-Maxwellian extension of TORIC v.5 in HHFW heating regime

The susceptibility for a hot plasma with an arbitrary distribution function

Local coordinate frame $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ with $\hat{\mathbf{z}} = \hat{\mathbf{b}}$ and $\mathbf{k} \cdot \hat{\mathbf{y}} = 0$ (Stix)

$$\begin{aligned}
\boldsymbol{\chi}_{\rm s} &= \frac{\omega_{\rm ps}^2}{\omega} \int_0^{+\infty} 2\pi v_\perp {\rm d}v_\perp \int_{-\infty}^{+\infty} {\rm d}v_\parallel \hat{\mathbf{z}}\hat{\mathbf{z}} \frac{v_\parallel^2}{\omega} \left(\frac{1}{v_\parallel} \frac{\partial f}{\partial v_\parallel} - \frac{1}{v_\perp} \frac{\partial f}{\partial v_\perp} \right)_{\rm s} + \\
&+ \frac{\omega_{\rm ps}^2}{\omega} \int_0^{+\infty} 2\pi v_\perp {\rm d}v_\perp \int_{-\infty}^{+\infty} {\rm d}v_\parallel \sum_{n=-\infty}^{+\infty} \left[\frac{v_\perp U}{\omega - k_\parallel v_\parallel - n\Omega_{\rm cs}} \mathbf{T}_n \right]
\end{aligned}$$

where

$$U \equiv \frac{\partial f}{\partial v_\perp} + \frac{k_\parallel}{\omega} \left(v_\perp \frac{\partial f}{\partial v_\parallel} - v_\parallel \frac{\partial f}{\partial v_\perp} \right) \qquad \text{and}$$

$$\mathbf{T}_n = \begin{pmatrix} \frac{n^2 J_n^2(z)}{z^2} & \frac{i n J_n(z) J_n'(z)}{z} & \frac{n J_n^2(z) v_\parallel}{z v_\perp} \\ -\frac{i n J_n(z) J_n'(z)}{z} & (J_n'(z))^2 & -\frac{i J_n(z) J_n'(z) v_\parallel}{v_\perp} \\ \frac{n J_n^2(z) v_\parallel}{z v_\perp} & \frac{i J_n(z) J_n'(z) v_\parallel}{v_\perp} & \frac{J_n^2(z) v_\parallel^2}{v_\perp^2} \end{pmatrix}, \qquad z \equiv \frac{k_\perp v_\perp}{\Omega_{\rm cs}}$$

Numerical evaluation of χ needed for arbitrary distribution function

- The "best" approach for a complete extension of the code is to implement directly the general expression for $\boldsymbol{\chi}$ (previous slide)
  - Plemelj's formula $\rightarrow \frac{1}{\omega-\omega_0 \pm i0} = \wp \frac{1}{\omega-\omega_0} \mp i\pi\delta(\omega-\omega_0)$
- Integrals in the expression for $\boldsymbol{\chi}$ are computed numerous times in TORIC-HHFW so **an efficient evaluation is essential**
- Strong similarity of the generalization done in TORIC-IC regime
  - integrals in the $v_{||}$-space with the singularity function $(\omega - k_{||}v_{||} - n\Omega_{\rm cs})^{-1}$
  - general integrals' structure and logic of the implementation
  - for TORIC-HHFW: to reconstruct the new integrands including the sum over the harmonic number n and the $k_\perp$ dependence in the argument of the Bessel functions
- Precomputation of χ

The susceptibility for a hot plasma with a Maxwellian distribution function can be evaluated analitically

$$\chi_{\mathrm{s}} = \left[\hat{\mathbf{z}}\hat{\mathbf{z}} \frac{2\omega_{\mathrm{p}}^2}{\omega k_\parallel v_{\mathrm{th}}^2} \left\langle v_\parallel \right\rangle + \frac{\omega_{\mathrm{p}}^2}{\omega} \sum_{n=-\infty}^{+\infty} e^{-\lambda} \mathbf{Y}_n(\lambda) \right]_s$$

where

$$\mathbf{Y}_n = \begin{pmatrix} \frac{n^2 I_n}{\lambda} A_n & -in(I_n - I_n') A_n & \frac{k_\perp}{\omega_c} \frac{n I_n}{\lambda} B_n \\ in(I_n - I_n') A_n & \left(\frac{n^2}{\lambda} I_n + 2\lambda I_n - 2\lambda I_n' \right) A_n & \frac{i k_\perp}{\omega_c} (I_n - I_n') B_n \\ \frac{k_\perp}{\omega_c} \frac{n I_n}{\lambda} B_n & -\frac{i k_\perp}{\omega_c} (I_n - I_n') B_n & \frac{2(\omega - n\omega_c)}{k_\parallel v_{\mathrm{th}}^2} I_n B_n \end{pmatrix}$$

$$A_n = \frac{1}{k_\parallel v_{\mathrm{th}}} Z_0(\zeta_n), \quad B_n = \frac{1}{k_\parallel} (1 + \zeta_n Z_0(\zeta_n)), \quad Z_0(\zeta_n) \equiv \text{plasma dispersion func.}$$

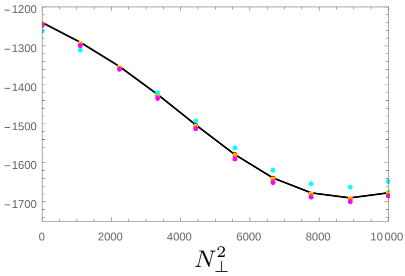
$$\zeta_n \equiv \frac{\omega - n\omega_c}{k_\parallel v_{\mathrm{th}}}, \quad \lambda \equiv \frac{k_\perp^2 v_{\mathrm{th}}^2}{2\Omega_{\mathrm{c}}^2}$$

Good agreement between numerical and analytical evaluation of the full hot dielectric tensor

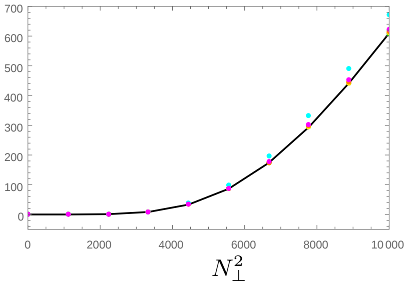
	$N_{v_\parallel}$	$N_{v_\perp}$
	100	50
	200	100
	324	150
	650	300
	1300	600
	2600	1200

Parameters:
$f = 30 \times 10^6$ Hz; $n_{\rm dens} = 5 \times 10^{13}$ cm^{-3},
$N_\parallel = 10$, $B = 0.5$ T, $T_i = 20$ keV
$N_{\rm harmonics} = 10$
Ion species: Deuterium
Black curve: analytical solution

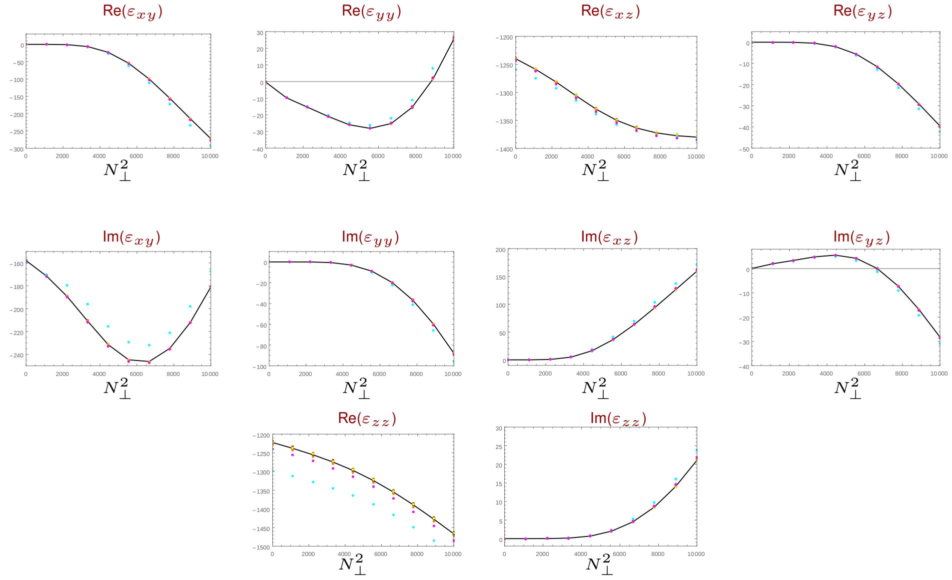
Re(ε_{xx})



Im(ε_{xx})



Good agreement between numerical and analytical evaluation of the full hot dielectric tensor (2)

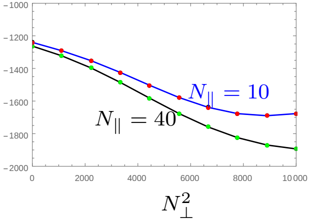


Good agreement between numerical and analytical evaluation of the full hot dielectric tensor: Test on $N_\parallel$

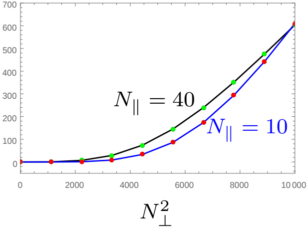
Black and **Blue** lines: **analytical solutions**

	$N_{v_\parallel}$	$N_{v_\perp}$
	2600	1200

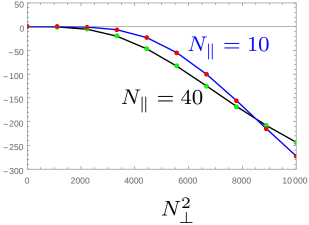
Re(ε_{xx})



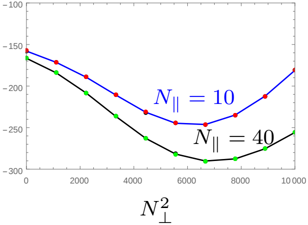
Im(ε_{xx})



Re(ε_{xy})



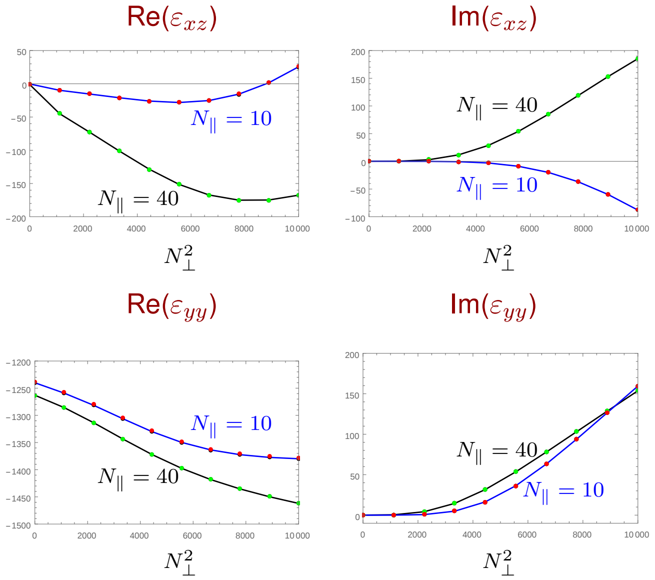
Im(ε_{xy})



Good agreement between numerical and analytical evaluation of the full hot dielectric tensor: Test on $N_\parallel$

Black and **Blue** lines: **analytical solutions**

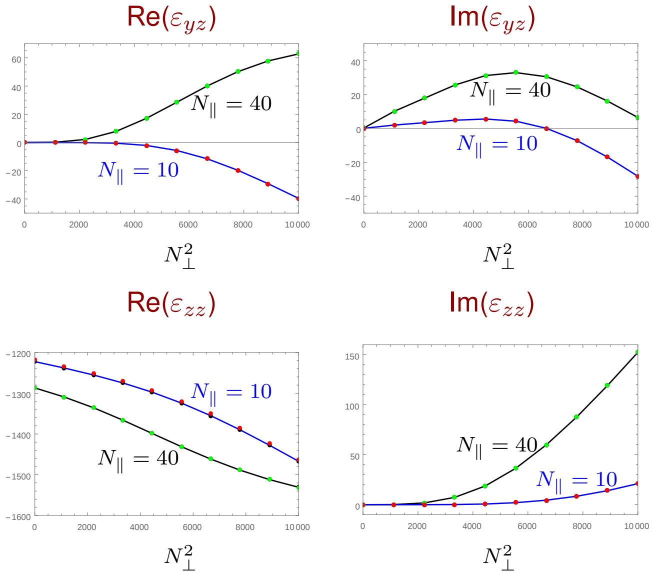
	$N_{v_\parallel}$	$N_{v_\perp}$
●●	2600	1200



Good agreement between numerical and analytical evaluation of the full hot dielectric tensor: Test on $N_\parallel$

Black and **Blue** lines: **analytical solutions**

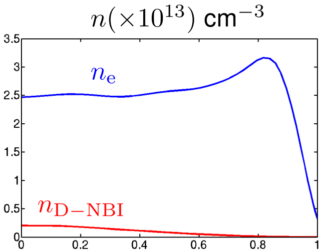
	$N_{v_\parallel}$	$N_{v_\perp}$
●●	2600	1200

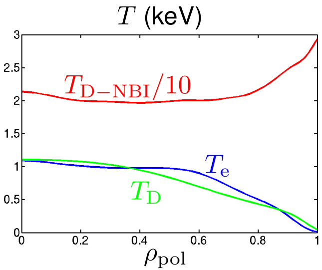


NSTX case

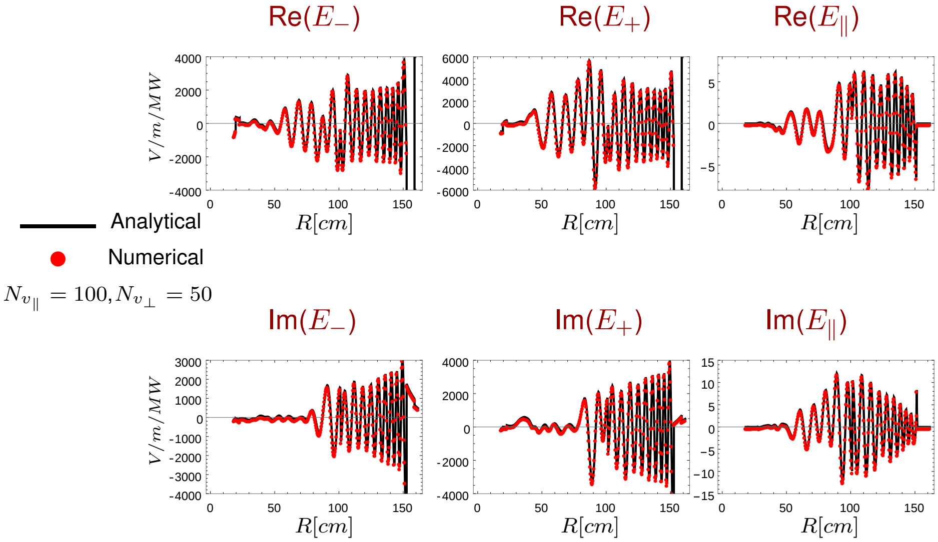
Main parameters:

- Plasma species: electron, D, D-NBI (4%)
- $B_\mathrm{T} = 0.529$ T
- $I_\mathrm{p} = 868$ kA
- $q(0)$ = 1.52
- q at plasma edge = 18.151
- $T_\mathrm{e}(0) = 1.093$ keV
- $n_\mathrm{e}(0) = 2.467 \times 10^{13}$ cm^{-3}
- $T_\mathrm{D}(0) = 1.104$ keV
- $T_\mathrm{D-NBI}(0) = 21.374$ keV
- $n_\mathrm{D-NBI}(0) = 2.011 \times 10^{12}$ cm^{-3}
- TORIC resolution: $n_\mathrm{mod} = 31$, $n_\mathrm{elm} = 200$



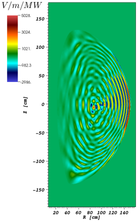


Excellent agreement between numerical and analytical evaluation of HHFW fields in the midplane

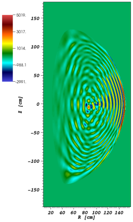


Excellent agreement between numerical and analytical evaluation of the 2D HHFW fields and in terms of absorbed power

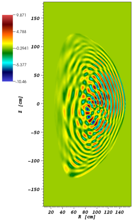
Maxw. analytical: Re(E_-)



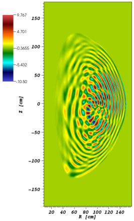
Maxw. numerical: Re(E_-)



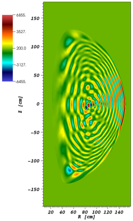
Maxw. analytical: Re($E_{||}$)



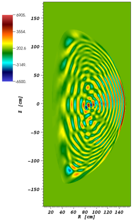
Maxw. numerical: Re($E_{||}$)



Maxw. analytical: Re(E_+)



Maxw. numerical: Re(E_+)



Absorbed fraction	Maxw. analytical	Maxw. numerical
D	0.22	0.22
D-NBI	73.88	73.58
Electrons	25.90	26.21

Resolution used for χ: $N_{v_{||}} = 100$ and $N_{v_{\perp}} = 50$

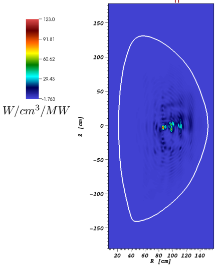
Bi-Maxwellian distribution

$$f_{\rm D}(v_\parallel, v_\perp) = (2\pi)^{-3/2}(v_{{\rm th},\parallel}v^2_{{\rm th},\perp})^{-1}\exp[-(v_\parallel/v_{{\rm th},\parallel})^2 - (v_\perp/v_{{\rm th},\perp})^2]$$

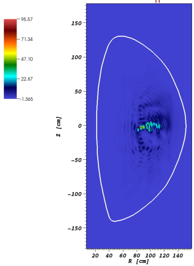
with $v_{{\rm th},\parallel} = \sqrt{2C_\parallel T(\psi)/m_{\rm D}}$, $v_{{\rm th},\perp} = \sqrt{2C_\perp T(\psi)/m_{\rm D}}$, with constants $C_\parallel$ and $C_\perp$

- For $C_\perp = 1$ and $C_\parallel = \{.5, 1., 3., 5.\}$, $P_{\rm D-NBI}$, varied by less than 1%
  - Opposite behavior w.r.t. the IC minority heating regime
  - however, for small (large) $C_\parallel$, the absorption profile is localized to the resonant layers (significantly broadened radially), as found in the IC minority heating regime
- For $C_\parallel = 1$ and $C_\perp = \{.5, 1., 3., 5.\}$, the corresponding $P_{\rm D-NBI} = \{70.06, 73.56, 62.84, 48.48\}$

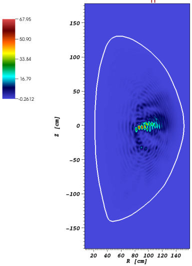
$C_\perp = 1, C_\parallel = 0.5$



$C_\perp = 1, C_\parallel = 1$



$C_\perp = 1, C_\parallel = 5.0$



Slowing-down distribution

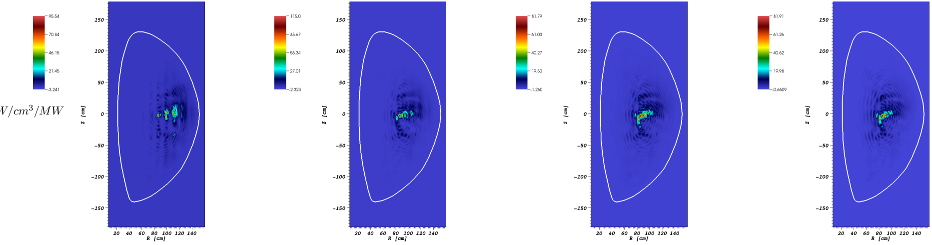
$$f_{\rm D}(v_{\parallel}, v_{\perp}) = \begin{cases} \frac{A}{v_{\rm c}^3} \frac{1}{1+(v/v_{\rm c})^3} & \text{for } v < v_{\rm m}, \\ 0 & \text{for } v > v_{\rm m} \end{cases} \qquad v_{\rm m} \equiv \sqrt{2E_{\rm D-NBI}/m_{\rm D}}$$

$$A = 3/[4\pi \ln(1+\delta^{-3})], \quad \delta \equiv \frac{v_{\rm c}}{v_{\rm m}}, \quad v_{\rm c} = 3\sqrt{\pi}(m_{\rm e}/m_{\rm D}) Z_{\rm eff} v_{\rm th}^3, \quad Z_{\rm eff} \equiv \sum_{\rm ions} \frac{Z_{\rm i}^2}{A_{\rm i}} \frac{n_{\rm i}}{n_{\rm e}}$$

For $Z_{\rm eff} = 2$ and $E_{\rm D-NBI} = 30, 60, 90, 120$ keV $\Longrightarrow P_{\rm D-NBI} = \{77.84\%, 75.85\%, 70.97\%, 64.71\%\}$

- Similar behavior when varied $C_{\perp}$ in the bi-Maxwellian case
- Fast ions absorption, in fact, should decrease with something like $T_{\rm fast\ ions}^{-3/2}$
  - This is due to the behavior of the function $f(\lambda) = \lambda I_n e^{-\lambda}$, which reached a maximum value at $\lambda = n^2/3$. For large λ, $f(\lambda) \propto \lambda^{-3/2}$ [See Stix's book & Ono, PoP 1995]

$E_{\rm D-NBI}$ = 30 KeV $\quad E_{\rm D-NBI}$ = 60 KeV $\quad E_{\rm D-NBI}$ = 90 KeV $\quad E_{\rm D-NBI}$ = 120 KeV



Conclusions

- Non-Maxwellian extension of TORIC in minority heating regime reproduces previous simulations
  - Excellent agreement of the 2D electric field and in terms of absorbed power
- Implementation of the full hot dielectric tensor reproduces the analytic Maxwellian case
- Non-Maxwellian extension of TORIC in HHFW regime reproduces previous simulations
  - Excellent agreement of the 2D electric field and in terms of absorbed power
- For bi-Maxwellian distribution found two different behaviors:
  - for IC minority heating regime, the absorbed power at the H fundamental is insensitive to variations in $T_\perp$, but varies with changes in $T_{||}$
  - for HHFW heating regime, the fast ions absorbed power is insensitive to variations in $T_{||}$, but varies with changes in $T_\perp$
    - however, absorption profile varies with changes in $T_{||}$
- For slowing down distribution in the HHFW heating regime, the fast ions absorbed power varies with changes in E_{NBI}
  - $P_{\mathrm{D-NBI}}$ decreases with increasing E_{NBI}
    - similar behavior found at large $T_\perp$ in the bi-Maxwellian case