Difficulties in solving for SOL main ion flow from impurity ion flow, *n*, and *T*

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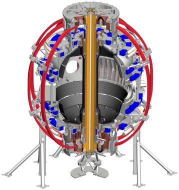
NOTE: much of the 'Difficulty' has been resolved due to the inclusion of an important term. SEE SLIDES AT THE END



Motivation

Measuring main ion velocity is important for understanding SOL heat transport:

- Transition between conduction and convection
- Study flows during detachment
- Flow reversal and stagnation points

Impurity ion velocities inform studies of core impurity uptake.

$\implies$ Need a way to measure v_i in the poloidal plane.

Three methods of measuring velocities

- Mach probes
  - Measures main ion flow directly
  - few-spatial-point measurements
  - difficult inside SOL due to heating [1]
  - perturbs plasma
- High-resolution spectrometer chords
  - line-integrated measurements
  - tomography may be possible with many chords
  - requires much alignment
- Coherence imaging system [2]
  - One camera: potentially easier setup
  - Imaging: much higher spatial resolution
  - Tomography may be possible:
    - requires assumption of axisymmetry and flow along fields only
  - One spectral line (multiplet) at a time
  - Sensitive to contamination from nearby lines

A wild scheme to measure n_e, T, n_{z1}, n_{z2}, $E_{\|}$, and v_i

With a coherence imaging camera, by measuring local emissivity ϵ of four spectral lines from two species, we could measure two main plasma variables n_e, T, and impurity densities n_{z1} and n_{z2}:

$$\epsilon_1 = n_e n_{z1} L_1(n_e, T_e)$$
$$\epsilon_2 = n_e n_{z1} L_2(n_e, T_e)$$
$$\epsilon_3 = n_e n_{z2} L_3(n_e, T_e)$$
$$\epsilon_4 = n_e n_{z2} L_4(n_e, T_e)$$

Use four cameras, or four similar shots each with a different filter.

(Or use three spectral lines from one species: get n_e, T, n_z.)

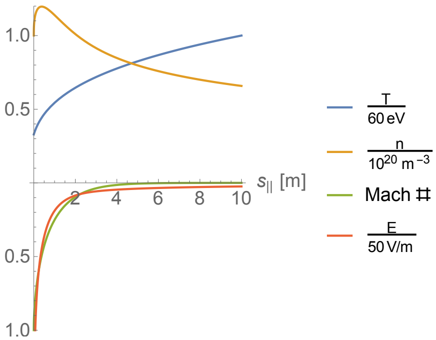
Since the coherence imaging camera also measures v_{z1} and v_{z2}, find a relation between impurity velocity and main ion velocity v_i, and find $E_{\|}$ along the way.

1D Conduction-limited SOL background

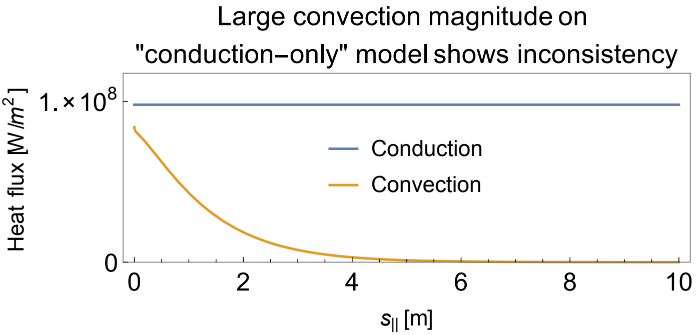
Use a 1D conduction-limited SOL model [3]. This gives an analytic form for the background plasma.

- $T_i = T_e$
- $j = 0$
- Sonic flow at targt
- Particle source $\sim e^{-s_\parallel/\mathrm{m}}$
- Target conditions 20 eV, $n = 1 \times 10^{20}\ \mathrm{m}^{-3}$
- $E = -0.71 \frac{dT}{ds} - \frac{1}{n}\frac{dnT}{ds}$

A SOL background model



Aside: Convection is non-neglible near target



Convected heat flux is similar to conducted heat flux near the target:

$$\frac{1}{2} m_i n v^3 + 5Tnv, \quad \kappa_{oe} T^{5/2} \frac{dT}{ds}$$

Model for 1D impurity forces

Forces on impurities [3, 4]:

$$m_z v_z \frac{dv_z}{ds_{\|}} = m_z \frac{v_i - v_z}{\tau_s} + ZeE + (\alpha_e + \beta_i)\frac{dT}{ds_{\|}}$$

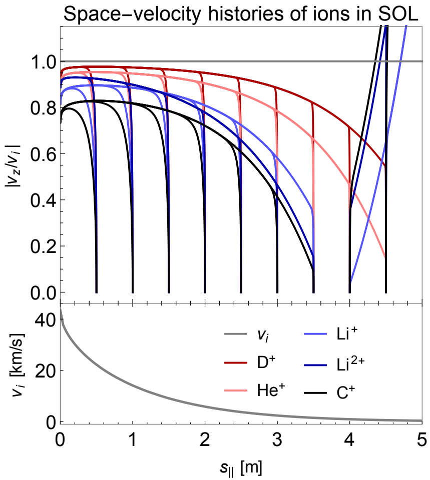
- Electric force $F_E = Z\ e\ E$
- Temperature gradient force $F_{\nabla T} = (\alpha_e + \beta_i)\frac{\partial T}{\partial s_{\|}}$
- Friction force $F_f = m_z \frac{v_i - v_z}{\tau_s}$
- No impurity pressure force — trace impurity limit

Where $\alpha_e = 0.71 Z^2$; $\quad \beta_i = \frac{3(\mu + 5\sqrt{2}Z^2(1.1\mu^{5/2} - 0.35\mu^{3/2}) - 1)}{2.6 - 2\mu + 5.4\mu^2}$

$$\mu = \frac{m_z}{m_z + m_i}; \quad \tau_s = \frac{1.47 \times 10^{13} \frac{m_z}{mp} T_z \left(\frac{T_i}{m_i/mp}\right)^{1/2}}{\left(1 + \frac{m_i}{m_z}\right) n Z^2 \log(\Lambda)}$$

Impurity ions reach "terminal" velocity

- Use impurity ion test particles
- Start with $v = 0$ from various points
- Numerically integrate with a Runge-Kutta method
- F_f and F_E sweep ions toward target
- ∇T pulls ions into core



Ions converge to attractors according to their Z and m.

Compute an ensemble velocity from many test particles

Drop one test ion per mm into the SOL, and record their velocity v_i as they pass 'checkpoints' every 1/8 m. Particles are assigned weights w_i by source location s_s with

$$w_i = e^{-s_s/\mathrm{m}}, \quad \text{c.f. particle source}$$

to simulate higher density near the target.

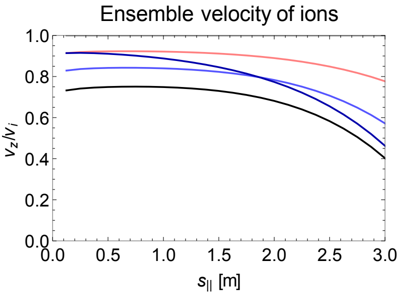
From Doppler-shifted emissions we measure the *fluid velocity* of the ensemble of particles at a point, which is not the same as the *attractor* velocity due to the source of $v = 0$ particles.

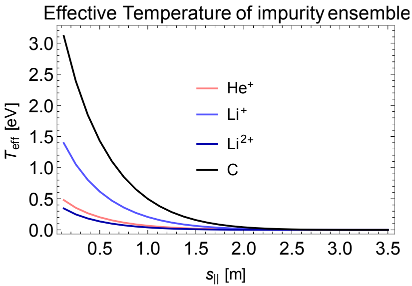
For each checkpoint, compute the fluid average, and effective temperature

$$\bar{v} = \frac{\int v f(v)\, dv}{\int f(v)\, dv} = \frac{\sum w_i}{\sum \frac{w_i}{v_i}}$$

Effective temperature due to distributed source $\ll T$

$$\frac{T_{eff}}{m_z} = \frac{\int f(v)(v-\bar{v})^2\,dv}{\int f(v)\,dv} = \frac{\sum \frac{w_i}{v_i}(v_i-\bar{v})^2}{\sum \frac{w_i}{v_i}}$$





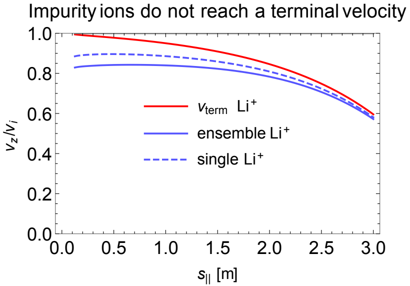
Effective temperatures are smaller than main ion temperatures by ~ 10: this T_{eff} would not be the dominant factor in spectral line broadening.

Particles' dynamic equilibrium is not a terminal velocity

Inertia is important for impurity ions: despite convergence to an attractor, they do not reach a 'terminal velocity':

$F_{\text{friction}}(v_{\text{term},z}) + F_{\text{other}} = ma = 0.$

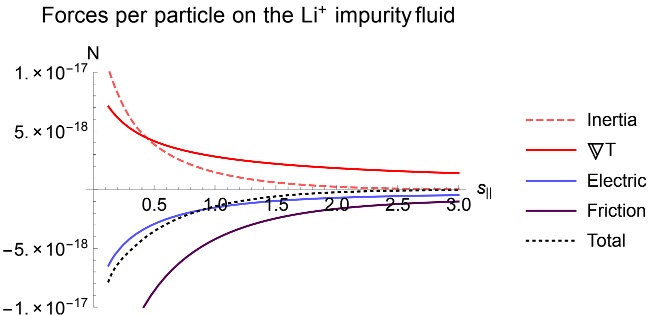
$$v_{\text{term},z} = v_i + \left(ZeE + (\alpha_e + \beta_i)\frac{dT}{ds_{||}} \right) \frac{\tau_s}{m_z}$$



Therefore when solving for v_i, we will need to consider the 'inertial' acceleration term $m_z v_z \frac{dv_z}{ds}$.

For the ensemble, $F \neq ma$

There is a residual force due the averaged nature of the fluid, perhaps because because particle number is not constant in space.

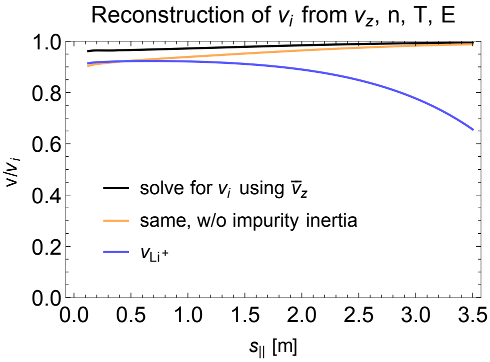


The total force is similar in magnitude and direction to the electric force.

Perfect knowledge of plasma still yields discrepancy in v_i from ensemble v_z

Under the assumption that n, T, and E are already known **perfectly**, solve for v_i using a measured ensemble v_z:

$$v_i = v_z - \left(-m_z v_z \frac{dv_z}{ds_{||}} + ZeE + (\alpha_e + \beta_i) \frac{dT}{ds_{||}} \right) \frac{\tau_s}{m_z}$$



A second impurity v_z measurement eliminates need for one of n, T, or E

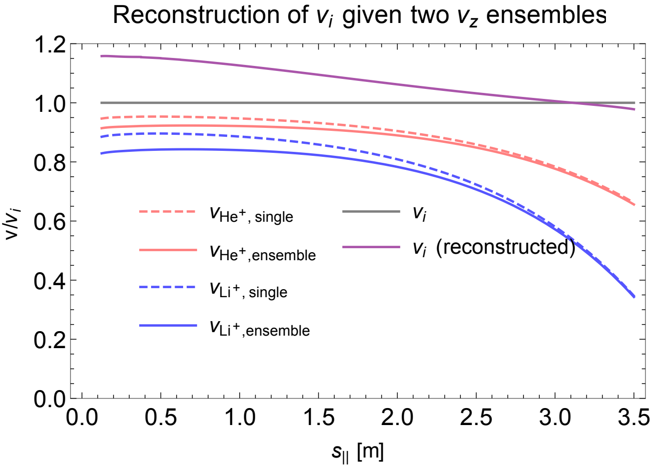
$E_{\|}$ may be difficult to measure: instead, use a second set of v_z data along with n and T to determine v_i.

$$m_1 v_1 \frac{dv_1}{ds_{\|}} = m_1 \frac{v_i - v_1}{\tau_1} + Z_1 eE + (\alpha_1 + \beta_1) \frac{dT}{ds_{\|}}$$
$$m_2 v_2 \frac{dv_2}{ds_{\|}} = m_2 \frac{v_i - v_2}{\tau_2} + Z_2 eE + (\alpha_2 + \beta_2) \frac{dT}{ds_{\|}}$$

Taking the case where $Z_1 = Z_2$, we can subtract the first equation from the second and solve for v_i. E and the α_e terms are eliminated.

$$v_i = \frac{m_1 v_1 \tau_2 - m_2 v_2 \tau_1 + \tau_1 \tau_2 \left(m_1 v_1 \frac{dv_1}{ds_{\|}} - m_2 v_2 \frac{dv_2}{ds_{\|}} + (\beta_2 - \beta_1) \frac{dT}{ds_{\|}} \right)}{m_1 \tau_2 - m_2 \tau_1}$$

But, reconstruction from two v_z *increases* errors



15% errors in reconstruction of v_i using v_{ensemble} of two He^+ and Li^+. Errors are 30% when using Li^+ and C^+.

Conclusions and future work

- Behavior of an 'ensemble' of test particles is not the same as of the individual particles.
- Reconstruction of v_i from Doppler-shift measurements of (multiple) v_z is difficult, even with perfect knowledge of the other background plasma parameters n, T, $E_{\|}$.

Future work:

- Understand the impurity ion $s_{\|} - v$ space attractors.
- Incorporate impurity source term into the ensemble force balance and v_i-reconstruction equations
  - Could this solve the $F \neq ma$ issue?

References

[1] Cedric Tsui.
Measuring the inboard side scrape-off layer of DIII-D plasmas using Swing-Probes.
PhD thesis, University of Toronto, 2014.

[2] Scott Alan Silburn.
A Doppler Coherence Imaging Diagnostic for the Mega-Amp Spherical Tokamak.
PhD thesis, Durham University, 2014.

[3] P. C. Stangeby.
The plasma boundary of magnetic fusion devices.
Plasma physics series. Institute of Physics Pub, Bristol ; Philadelphia, 2000.

[4] Filippo Scotti.
Modifications of Impurity Transport and Divertor Sources by Lithium Wall Conditioning in the NSTX.
PhD thesis, Princeton University, January 2014.

Post-meeting correction slides

After presenting the poster at APS, I have realized that an important term was left out of the impurity ensemble momentum equation, leading to errors in the reconstruction of v_i, as in Slides 13 and 15.

The single-impurity-ion momentum equation is correct; it is a term for the ‘fluid’ ensemble part that was missing:

$$\frac{d}{ds_{\parallel}}\left(n(s_{\parallel})v(s_{\parallel})\right) = \text{Source}(s_{\parallel}) \neq 0$$

where Source is the impurity recycling source $e^{-s_{\parallel}/\mathrm{m}}$

Post-meeting correction slides

Therefore when computing the R.H.S. of $F = ma$,

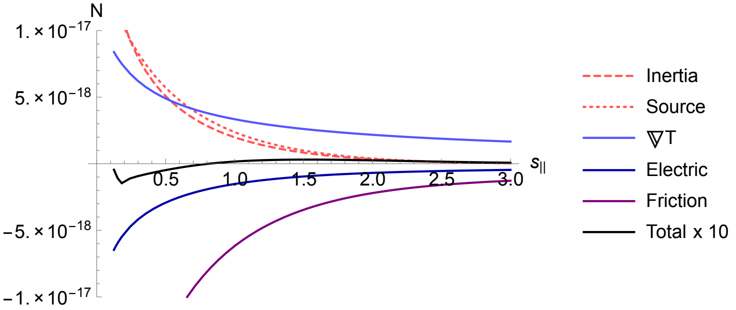
$$\frac{1}{n_z}\frac{d}{ds_{\|}}(m_z n_z v_z^2) = m_z v_z \frac{dv_z}{ds_{\|}} + m_z \frac{v_z}{n_z}\text{Source}$$

This problem did not show up when checking that the single-particle momentum balance equation holds, since then the Source is $\delta(s_s)$, where s_s is the starting position of a given particle.

We now show corrected versions of several plots...

Correct momentum balance

Forces per particle on the Li^+ impurity fluid



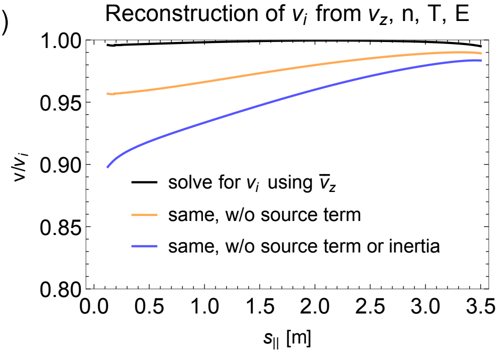
The residual $F \neq ma$, shown multiplied by 10 and in black, is much smaller than previously. It may be not identically zero due to the discrete nature of the test particles.

Good reconstruction with one impurity

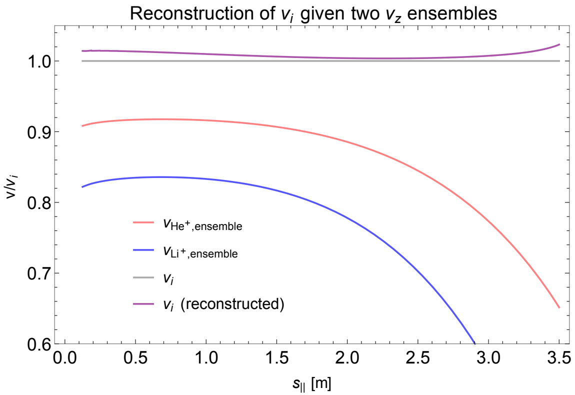
Under the assumption that n, T, and E are already known **perfectly**, solve for v_i using a measured ensemble v_z **and** measured density n_z.

$$v_i = v_z - \left(-m_z v_z v_z' - m_z \frac{v_z}{n_z} (n_z v_z)' + ZeE + (\alpha_e + \beta_i) T' \right) \frac{\tau_s}{m_z}$$

(with $\frac{d}{ds_{||}}$ as $'$)



Reconstruction from two impurities— few % errors



1.5% errors in reconstruction of v_i using v_{ensemble} of two He^+ and Li^+. Errors are 5% when using Li^+ and C^+.

Once again, I'm not sure where the residual errors are from—it may be the discrete nature of the impurity test particles.

New conclusions

- When averaging the histories of many test particles together, remember all the terms in the continuity and momentum balance equations—some may apply to the ensemble but not to the invidual particles.
- Reconstruction of v_i to with a few % from measurements of impurity n_z and v_z is possible with perfect knowledge of the background plasma parameters n, T, $E_{||}$, in a simple 1D SOL model.