Identification of multi-mode plasma response and extraction of plasma transfer function in tokamaks

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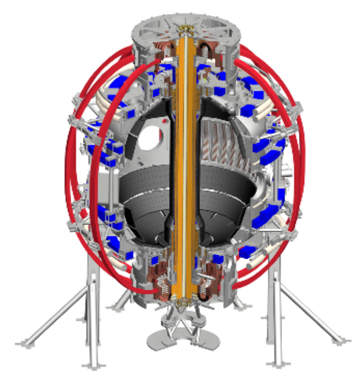


Motivation

- Externally applied non-axisymmetric magnetic perturbations strongly modify tokamak plasmas with perturbed plasma currents → plasma response (include magnetic perturbation and plasma displacement etc.)

- Physics understanding for plasma response closely relate to many subjects
  - Neoclassical toroidal viscosity, Error fields control, Resistive wall mode instability, ELM suppression etc.

- Nyquist analysis, combined with Padé approximation, can provide a deep physical understanding of the plasma response.
  - how kinetic effects fundamentally change eigenmodes in DIII-D and NSTX plasmas. Note plasma response is the result of the linear combination of stable eigenmodes
  - quantify contribution of each eigenmode in order to characterize details of the multi-modal plasma response
  - better understand the plasma response during ELM suppression

Linear Simulation of Plasma Response: Hybrid Drift-Kinetic MHD Formulation (MARS-K)

MARS-K extends MARS-F and solves linearized MHD equations with perturbed kinetic pressure.

Fluid Part (MHD equations):

$$i(\omega+n\Omega)\boldsymbol{\xi} = \mathbf{v} + (\boldsymbol{\xi}\cdot\nabla\Omega)R^2\nabla\phi$$

$$(\omega+n\Omega)\mathbf{v} = -\nabla\cdot\mathbf{p} + \mathbf{j}\times\mathbf{B}_0 + \mathbf{J}_0\times\mathbf{b} + \rho[2\Omega\mathbf{Z}\times\mathbf{v} - (\mathbf{v}\cdot\nabla\Omega)R^2\nabla\phi] - \nabla\cdot(\rho\boldsymbol{\xi})\Omega\mathbf{Z}\times\mathbf{V}_0$$

$$i(\omega+n\Omega)\mathbf{b} = \nabla\times(\mathbf{v}\times\mathbf{B}_0) + (\mathbf{b}\cdot\nabla\Omega)R^2\nabla\phi$$

$$i(\omega+n\Omega)p = -\mathbf{v}\cdot\nabla P_0 - \Gamma P_0\nabla\cdot\mathbf{v}$$

Applied field frequency

$$\mathbf{j} = \nabla\times\mathbf{b}$$

replaced by kinetic pressure

Coil equations:

$$\nabla\times\mathbf{b} = \mathbf{j}_{coil} \quad \nabla\cdot\mathbf{j}_{coil} = 0$$

Kinetic Model Y.Q. Liu et al, PoP 2008

$$\mathbf{p} = p\mathbf{I} + p_{\|}\mathbf{b}\mathbf{b} + p_{\perp}(\mathbf{I} - \mathbf{b}\mathbf{b})$$

Kinetic pressure $p_{\|}$ and $p_{\perp}$ couple with MHD equations

$$p_{\|}e^{-i\omega t+in\phi} = \sum_{e,i}\int d\Gamma M v_{\|}^{2} \ldots$$

$$p_{\perp}e^{-i\omega t+in\phi} = \sum_{e,i}\int d\Gamma \tfrac{1}{2} M \ldots$$

Resonant operator in f_L^1:

Diamagnetic drift

Applied field frequency

Drift-kinetic equation:

$$df_L^1/dt = f_\varepsilon^0\,\partial H^1/\partial t - f_{P_\phi}^0\,\partial H^1/\partial\phi \ldots n[\omega_{*N} + (\hat{\varepsilon} - 3/2)\omega_{*T} + \omega_E] - \omega \ldots$$

Precession drift

Bounce/Transit

EXB

Crook Collisions

H^1: perturbed Lagrangian

Ignore finite orbit width effect.

Drift kinetic effects can modify plasma response.

Nyquist Analysis and Padé Approximation

The standard Nyquist contour involves varying the field rotation frequency, f_{coil}, generated by external magnetic perturbation coils from -∞ to +∞, where the '+' and '-' denotes the co and counter plasma current directions.

The real and imaginary parts of the total radial perturbed magnetic fields δB^{tot} measured by toroidal arrays of magnetic sensors can be plotted in the complex plane to form the **Nyquist contour**.

Nyquist contour can be fitted by Padé approximation to extract the plasma transfer functions and the eigenvalue.

Considering the generalized form of linearized MHD equations,

$$f_{coil} AX = BX + Ce^{i f_{coil} t}$$

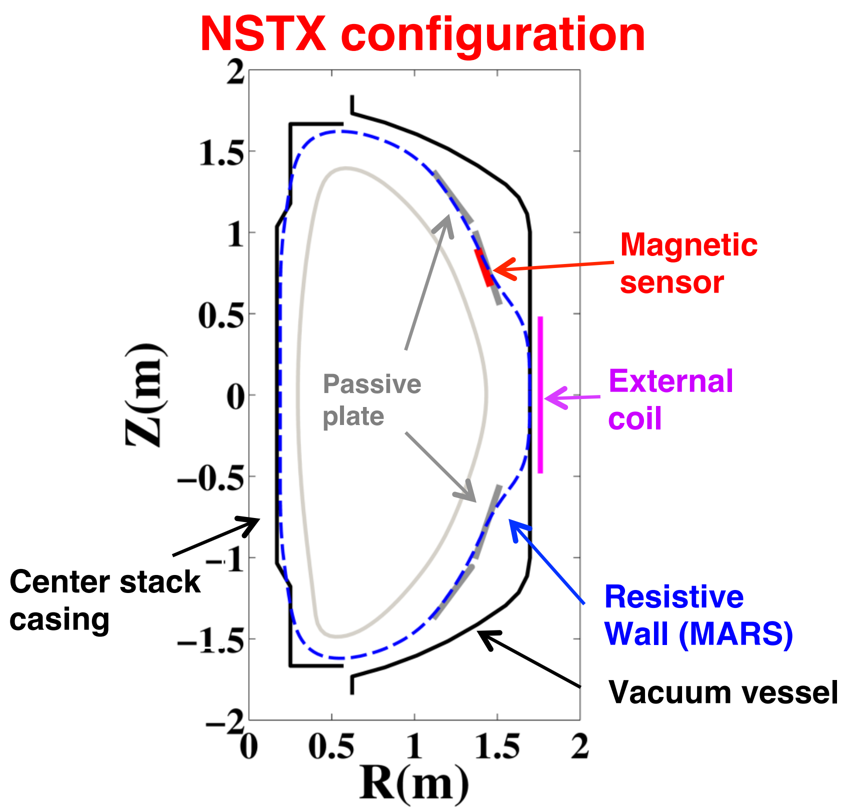
C is the driving term of coil, the form of transfer function on given sensor is

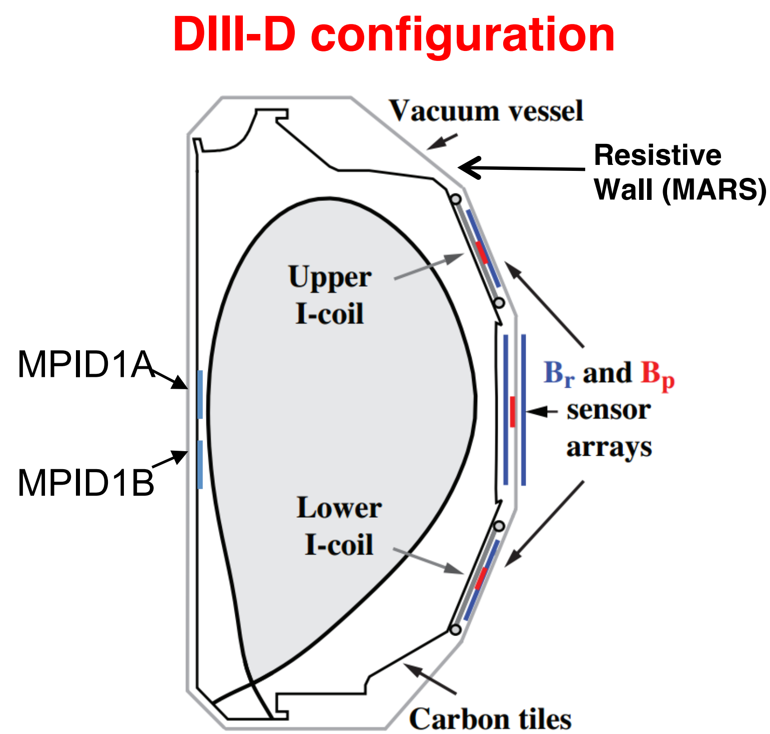
Sensor transfer function: $P = \sum_{i=1}^{N} a_i / (i f_{\mathrm{coil}} - \gamma_i)$

γ_i is the eigenvalue of each eigenmode.

Nyquist Analysis of Kinetic Plasma Response

Frequency Scan of Plasma Response in DIII-D and NSTX with Different Coil Configurations





Magnetic perturbation can be generated by I-coils in DIII-D and the external coil in NSTX.

By scanning the rotating frequency of external field, Nyquist contour can be formed by plotting the magnetic sensor measurement in complex plane.

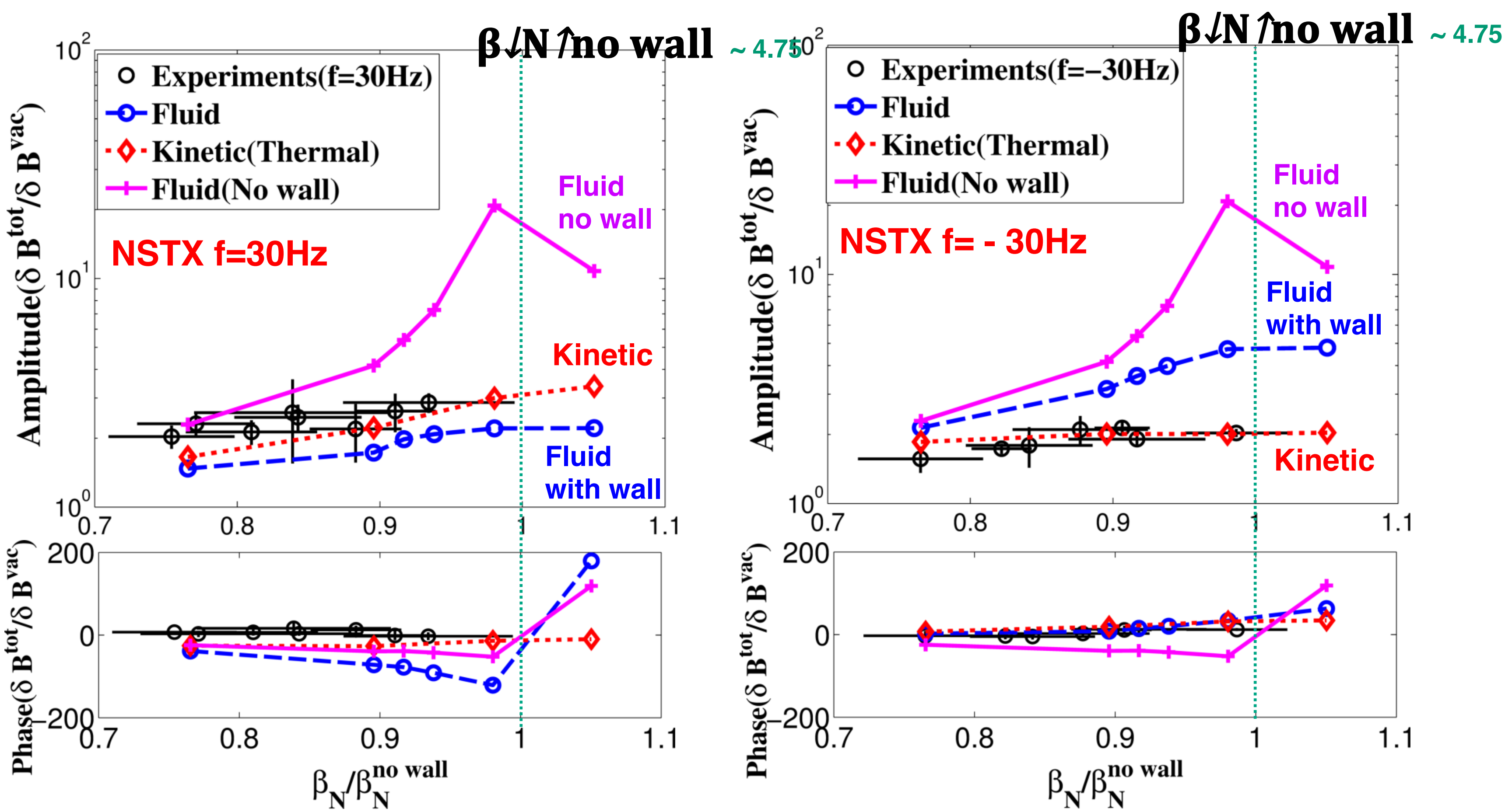
NSTX: Kinetic Plasma Response With Thermal Particles Shows Quantitative Agreement with Magnetic Sensor Measurements

The simulated plasma response is compared with experiments at <u>upper radial magnetic sensor</u>.

1) Fluid plasma response is solved by MARS-F

2) Kinetic plasma response is solved by MARS-K (include thermal ions and electrons).

3) Fluid plasma response is solved without resistive wall

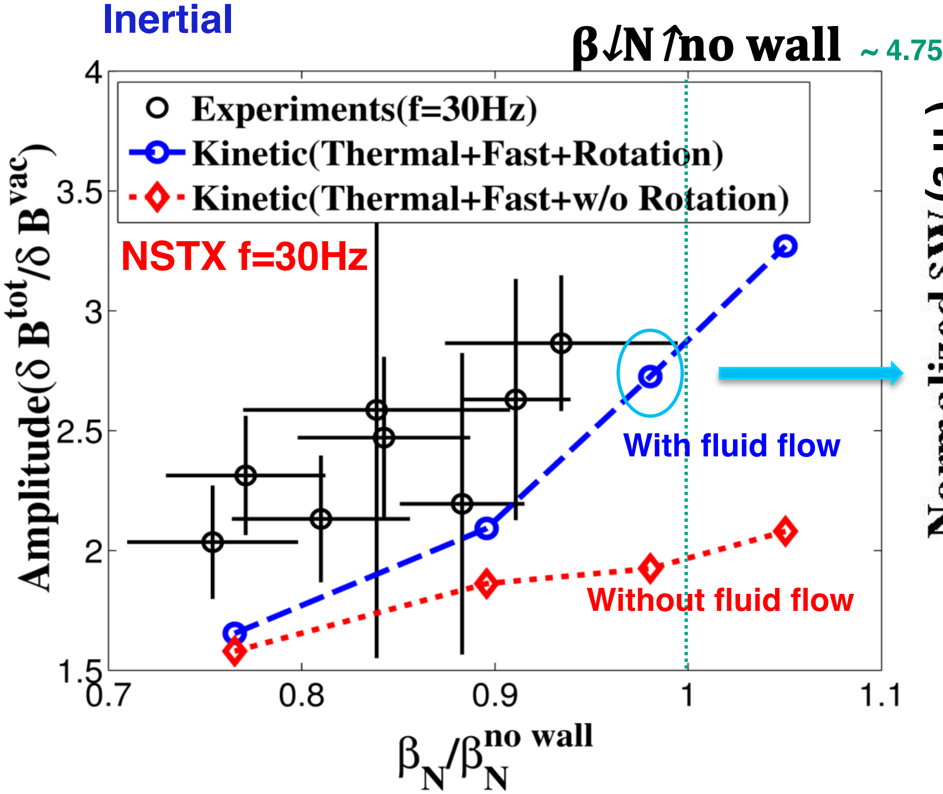


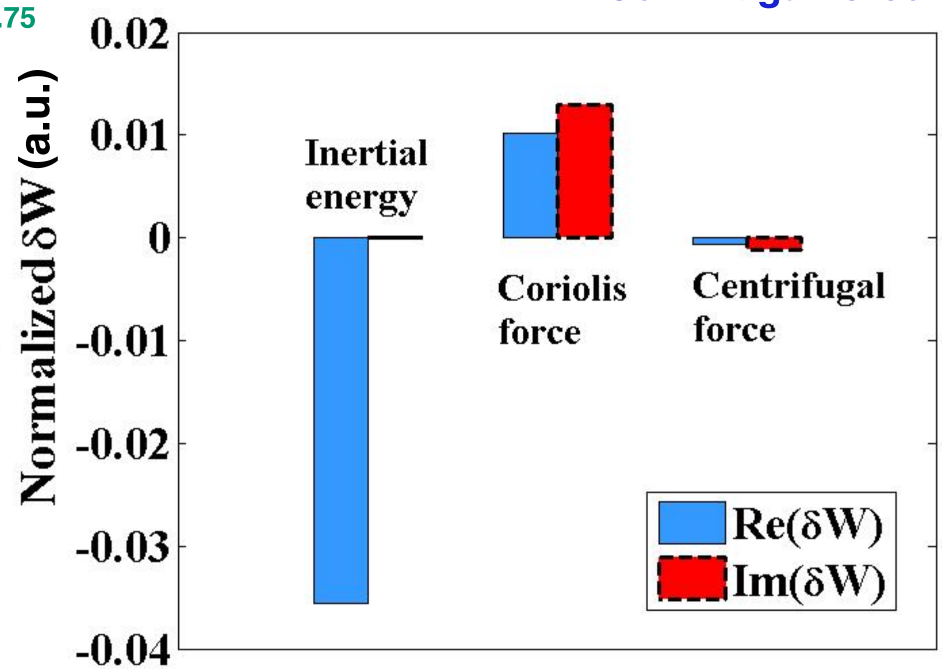
NSTX: Inertial Energy Plays a Destabilizing Role in NSTX Plasmas Response

Momentum equation:

$$\rho(\omega+n\Omega)\mathbf{v} = -\nabla\cdot\mathbf{p} + \mathbf{j}\times\mathbf{B}_0 + \mathbf{J}_0\times\mathbf{b} + \rho[2\Omega\mathbf{Z}\times\mathbf{v} - (\mathbf{v}\cdot\nabla\Omega)R^2\nabla\phi] - \nabla\cdot(\rho\boldsymbol{\xi})\Omega\mathbf{Z}\times\mathbf{V}_0$$

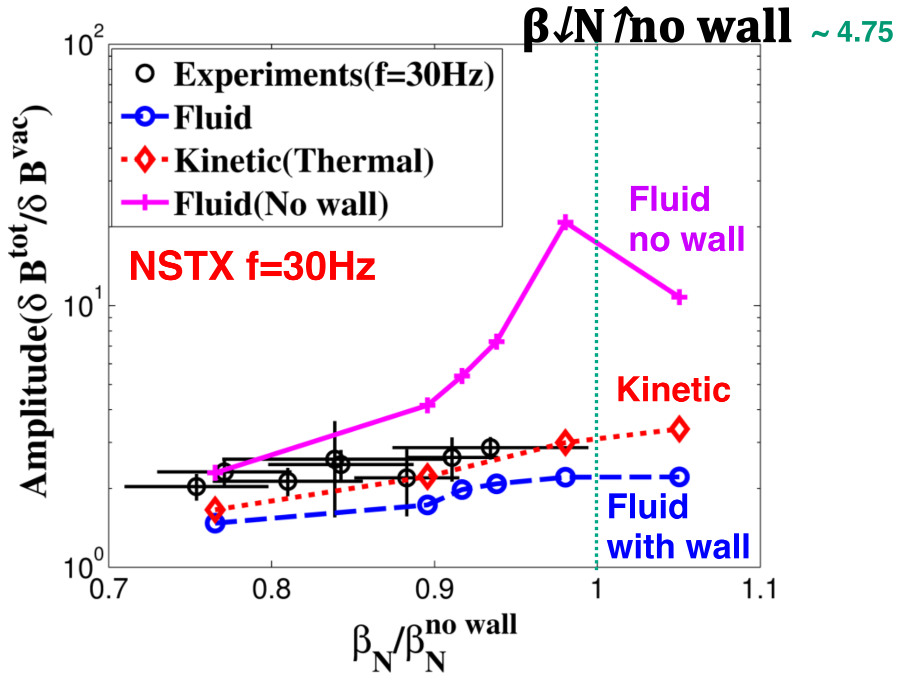
Inertial | Coriolis force | Centrifugal force





Inertial energy is negative and destabilizes the plasma which leads to larger amplification of plasma response.

NSTX Shows Strong Plasma-Wall Coupling Effect in n=1 Plasma Response



A simple analysis based on (s, α) model in the approximation with a single dominant mode:

Park, Boozer et al, PoP 2009

At marginal stability: $|\delta B^{tot}/\delta B^{vac}| = |1/2\pi n \tau_w f_{coil}|$

No wall: $\tau_w \to 0 \Rightarrow |\delta B^{tot}/\delta B^{vac}| \to +$

NSTX experiments

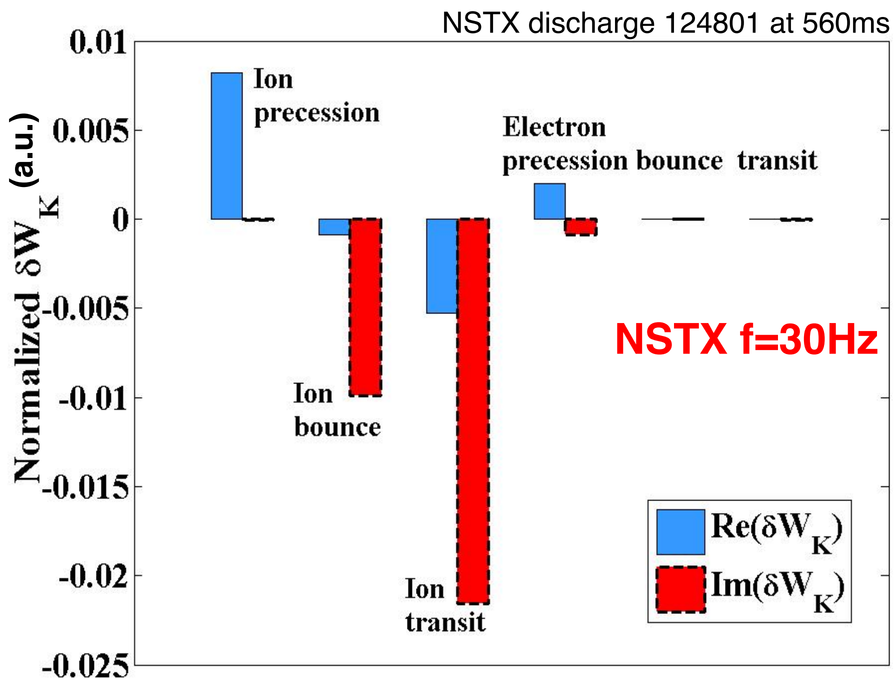
τ_w =3.5ms, f_{coil} =+/-30Hz, $|\delta B^{tot}/\delta B^{vac}|$ =1.52

$2\pi |f_{coil}| \sim 1/\tau_w \Rightarrow$ strong plasma-wall coupling

NSTX: Bounce and Transit Resonances of Thermal Ions Contributes Dominant Kinetic Energy to Kinetic Response

To understand which drift kinetic effect of thermal particles play a role to change the plasma response, the perturbed drift kinetic energy δW_K is analyzed near no-wall beta limit in NSTX plasmas.

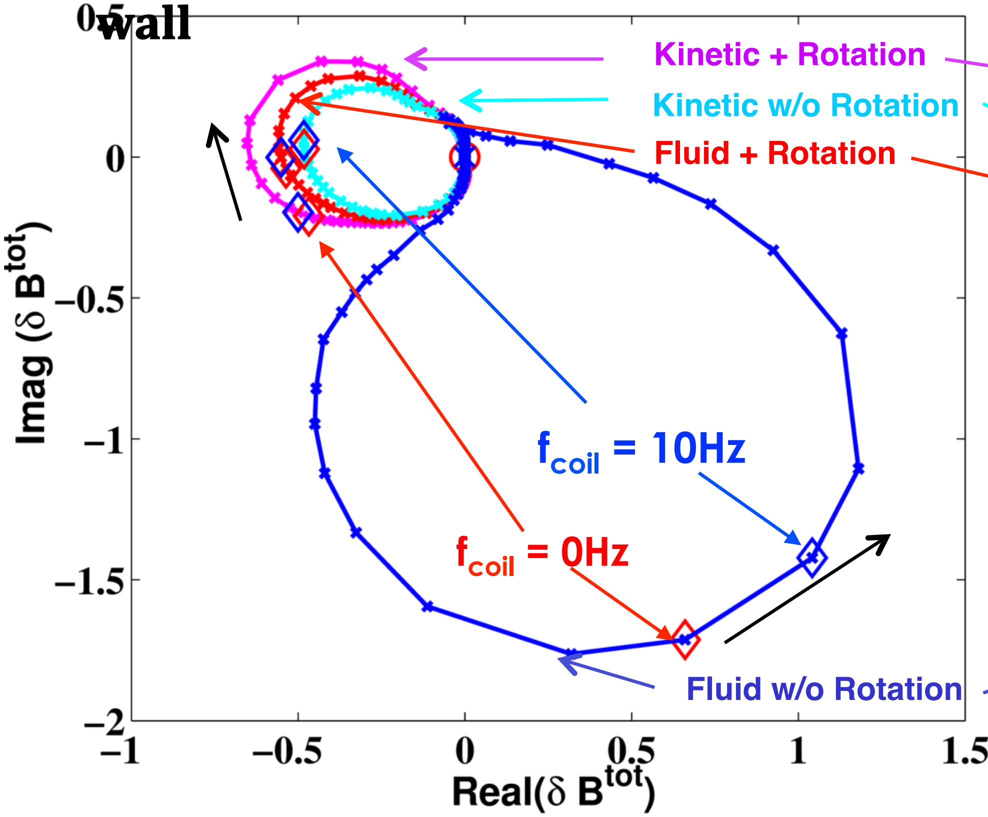
$$\delta W_K = -\frac{1}{2}\int (-\nabla \cdot \mathbf{p}^{kinetic} \cdot \boldsymbol{\xi}_\perp^*)$$

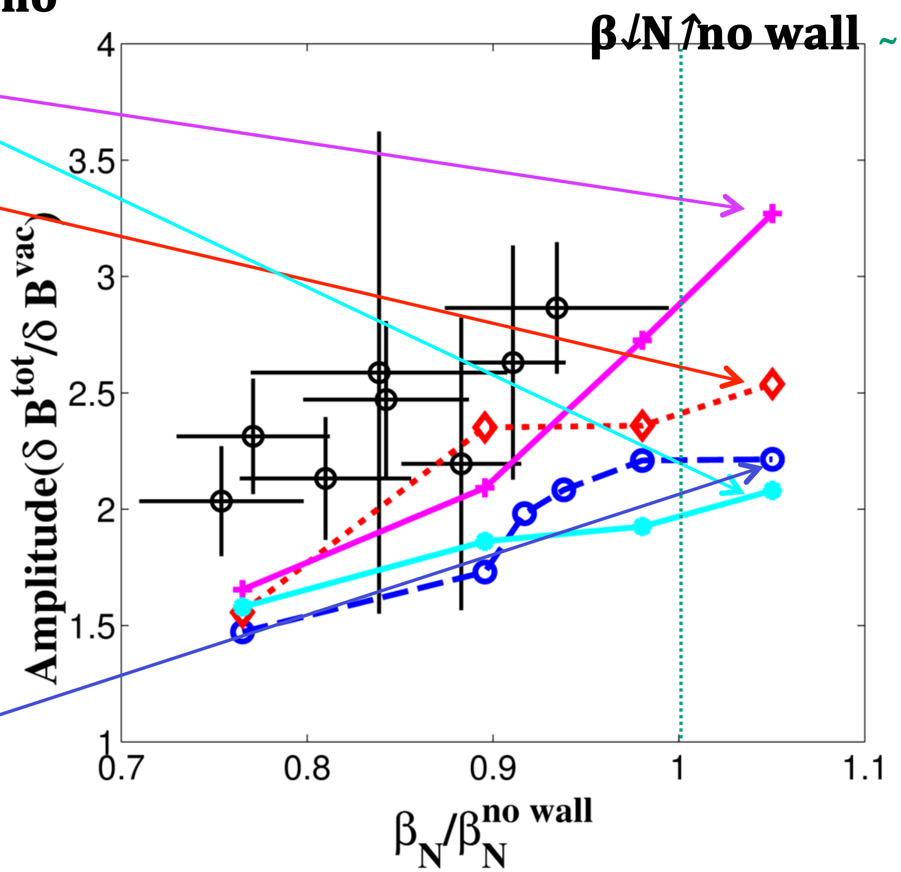


- Thermal electrons contribute much smaller δW_K due to high collision frequency, bounce frequency and transit frequency.
- **The fluid contour with rotation is similar to the kinetic contours since the physics of ion acoustic damping is dominant at high rotation.** The ion acoustic damping is the fluid description of the kinetic resonance of passing ions.

NSTX: Nyquist Contour Directly Identifies Kinetic Stabilization of Least Stable Mode in Plasma Response

Nyquist plot with highest beta above $\beta_N^{no\ wall}$





Padé approximation: $P = \frac{c_1}{s-\gamma_1} + \frac{c_2}{s-\gamma_2}$

Fluid w/o Rotation: γ_{max} = 13.66 Hz **Unstable**

$\vee$ **Fluid ion acoustic damping**

Kinetic w/o Rotation: γ_{max} = -34.3 Hz **Stable**

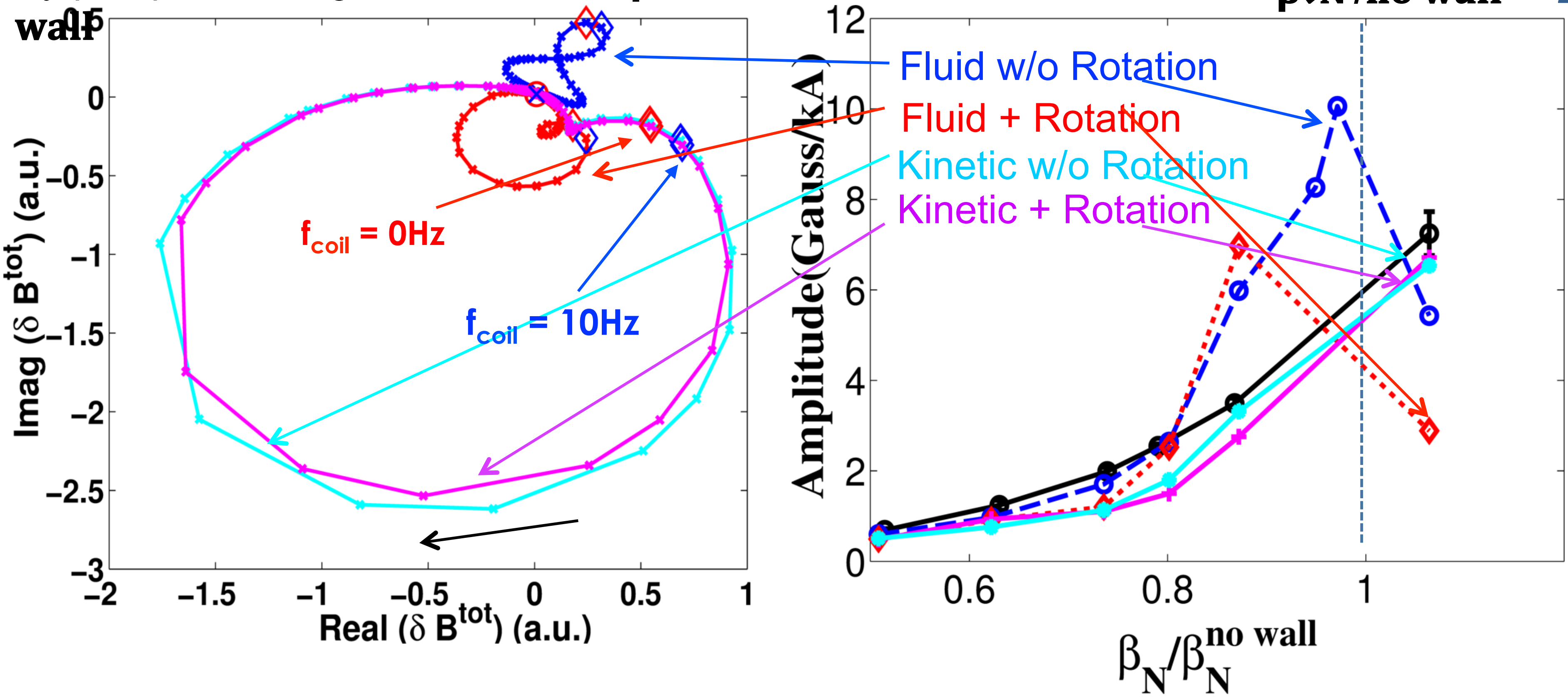
$\vee$ **Inertial energy**

Fluid + Rotation: γ_{max} = -28.97 Hz **Stable**

Kinetic + Rotation: γ_{max} = -13.22 Hz **Stable**

DIII-D: Different Response Physics Form Different Nyquist Contour

Nyquist plot with highest beta above $\beta_N^{no\ wall}$ $\qquad$ $\beta_N^{no\ wall}$ ~ 2



Padé approximation: $P = \frac{c_1}{s-\gamma_1} + \frac{c_2}{s-\gamma_2} + \frac{c_3}{s-\gamma_3}$ $\qquad$ Z. R. Wang, M. J. Lanctot et al PRL 2015

Fluid w/o Rotation: γ_{max} = 79.24 Hz Unstable

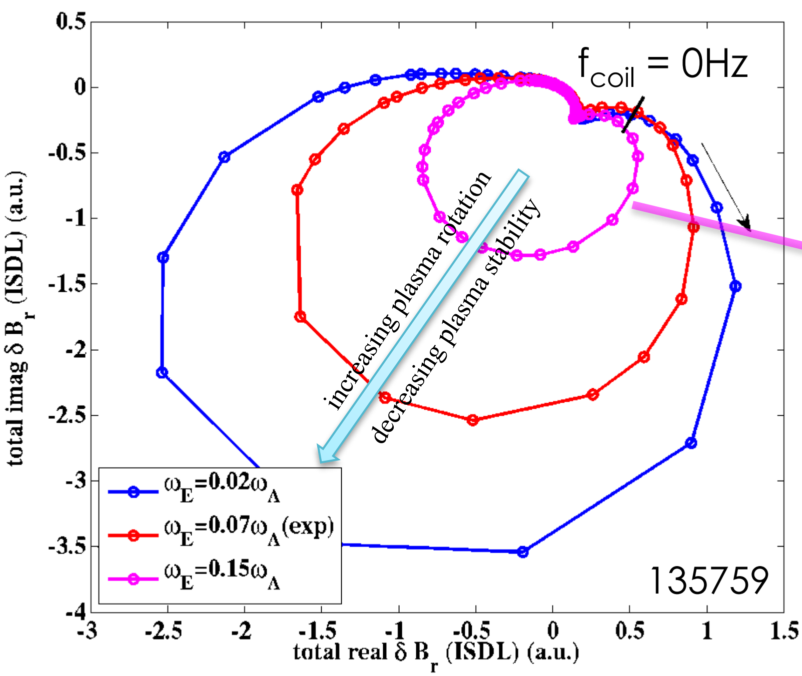
V Fluid ion acoustic damping

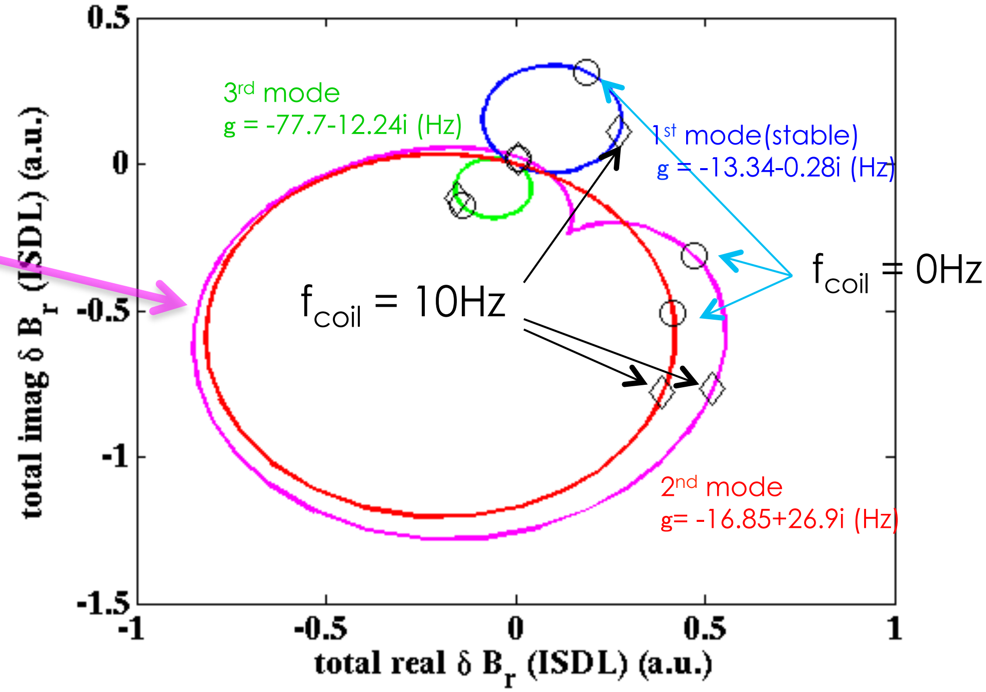
Fluid + Rotation: γ_{max} = -20.44 Hz Stable

Kinetic w/o Rotation: γ_{max} = -9.19 Hz Stable

Kinetic + Rotation: γ_{max} = -8.69 Hz Stable

DIII-D: Multi-Mode Plasma Response in Kinetic-MHD Simulation (MARS-K) Quantified using Nyquist Contour Analysis





Scan rotation & calculate radial magnetic field at low field side of the wall

- Low rotation => Single-mode response
- High rotation => Multi-mode response

Use Padé approximation to extract eigenmodes

- Yields damping rate for each stable mode
- Quantifies contribution of each eigenmode to the total sensor signal

Instead of first least stable mode, contribution from second least stable mode can dominate the kinetic plasma response when largest response observed (~10Hz).

Choose the coil frequency to amplify the preferred eigenmode's response.

Modified Nyquist Analysis
(3D MHD Spectroscopy)

Modified Padé Approximation Include Both Coil Phasing and Frequency Dependence

Standard Nyquist analysis requires a frequency scan $f \in (-\infty, +\infty)$. Measurements at high frequency can be weak and noisy due to ELM etc.

Modified Nyquist Analysis: transfer function of Padé approximation is modified to include both frequency and phase dependence with two coils.

$$P_j(\Delta\phi)=\sum_{i=1}^{N} \frac{a_i^j e^{i\phi_{up}} + b_i^j e^{i\phi_{low}}}{if-\gamma_i}$$

$$P_j(\Delta\phi)=\sum_{i=1}^{N} \frac{a_i^j + b_i^j e^{-i\Delta\phi}}{if-\gamma_i}, \quad \Delta\phi=\phi_{up}-\phi_{low}$$

Varying both coil phase difference and coil frequency reduces the requirement of frequency scan, no need to reach the high coil frequency.

Different sensor has the same eigenvalue (linear theory).

Avoid the resonance with mode and weak sensor signal at high frequency.

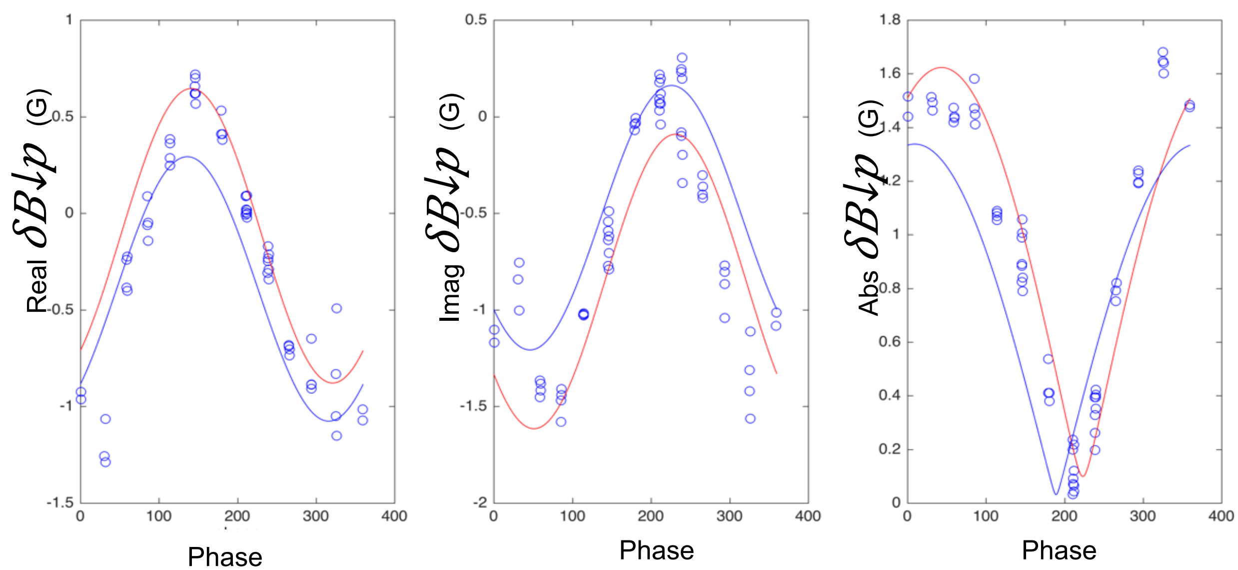
DIII-D: Application of Modified Nyquist Method in n=2 plasma response experiments

The data of n=2 magnetic measurements in [Paz-Soldan, Nazikian et al, PRL 114, 105001 (2015)] is used to exam modified Pade Approximation.

The static field makes $f_{coil}=0$ at each sensor j: $P_j(\Delta\phi)=\sum_{i=1}^{N} a_i^j + b_i^j e^{i\Delta\phi}/\gamma_i$

The measurements of three sensors(MPID1A, MPID66M and MPID1B) are fitted by one pole (N=1) and three pole (N=3) models.

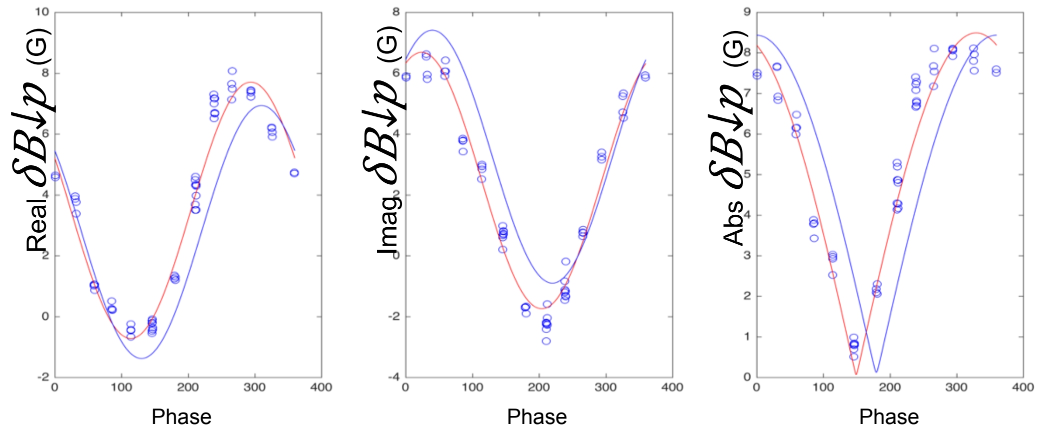
MPID1A sensor



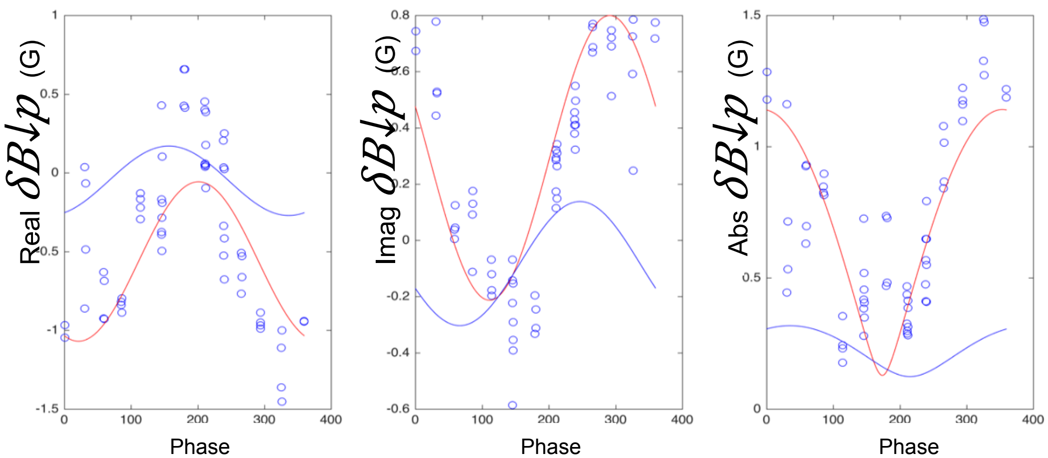
DIII-D: Application of Modified Nyquist Method in n=2 plasma response experiments

The measurements of three sensors(MPID1A, MPID66M and MPID1B) are fitted by one pole (N=1) and three pole (N=3) models.

MPID66M sensor



MPID1B sensor



Three poles model fits three sensors and confirms the multi-mode response.

Three eigenvalue can be found but not be able to determine the dimension since f=0.

γ_1 =1.8696e+00 + 1.6963e+00i
γ_2 = 8.5712e+00 - 1.9080e+00i
γ_3 = 1.4569e-01 - 2.7639e-01i
No physical meaning

$$P_j(\Delta\phi)=\sum_{i=1}^{N} a_i^j +$$

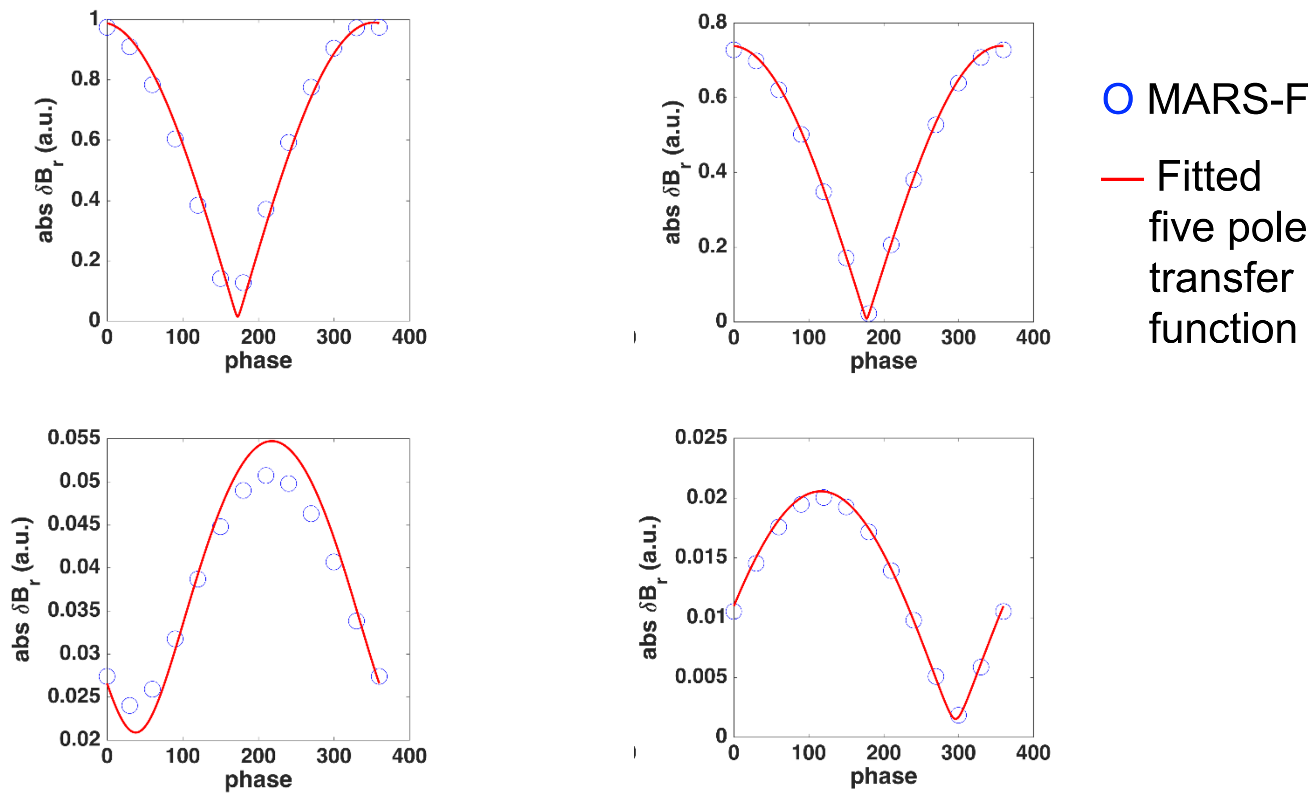
DIII-D: Modified Transfer Function Fits Simulated n=2 Resistive Response at Different Phasing and Frequency

- n=2 plasma response in DIII-D discharge 158103 at 3796ms is simulated by MARS-F.
- Five pole transfer function fits the LFS and HFS sensor measurements simultaneously.

f_{coil}=10Hz | f_{coil}=90Hz

Radial perturbed fields at LFS (ISL)

Radial perturbed fields at HFS (Radial MPID1B)



DIII-D: Extracted Transfer Functions Identify Modification of n=2 Plasma Stability Due to Rotation and Resistivity

- Nyquist method can investigate how the damping rates of these stable eigenmodes ir both simulation and experiments.
- The extracted damping rates of least stable mode for ideal response with/without rotation and resistive response are compared.

DIII-D discharge 158103 at 3796ms

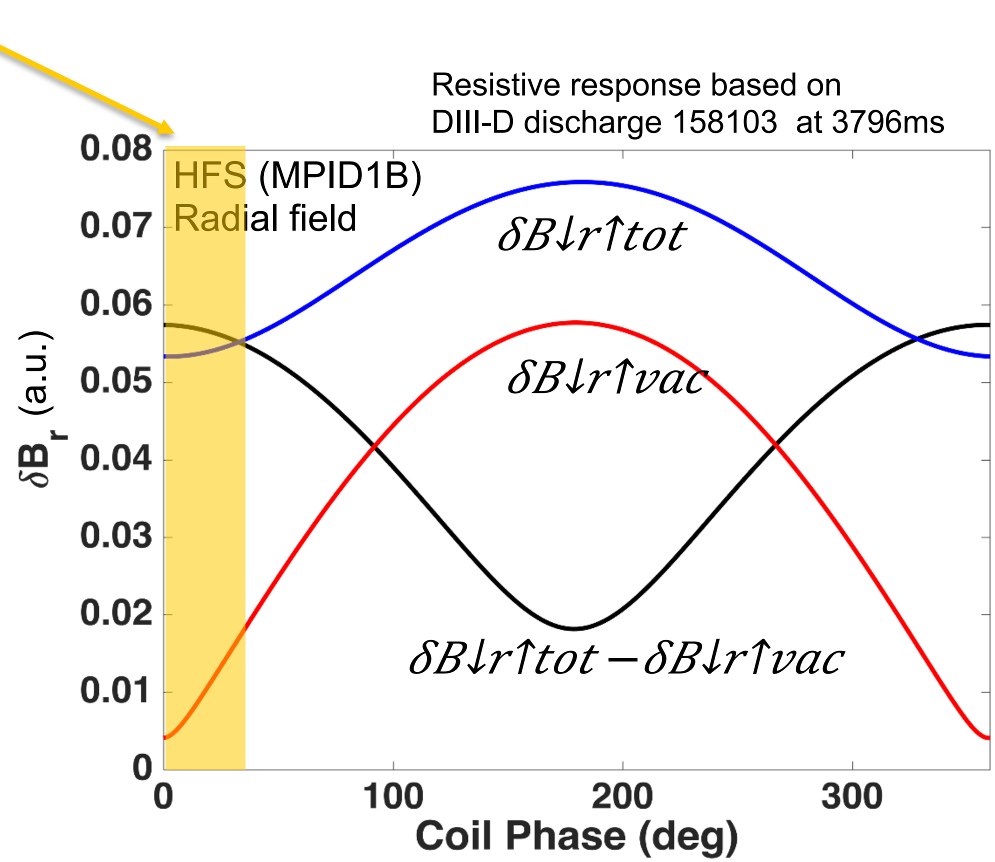
Case	Eigenvalue of Least Stable Mode
Ideal Response + no Rotation	-3.017 - 9.92E-03i Hz
Ideal Response + Rotation	-2.03 - 1.85E-03i Hz
Resistive Response + Rotation	-6.53 + 1.075E-02i Hz

More negative real part more stable plasma

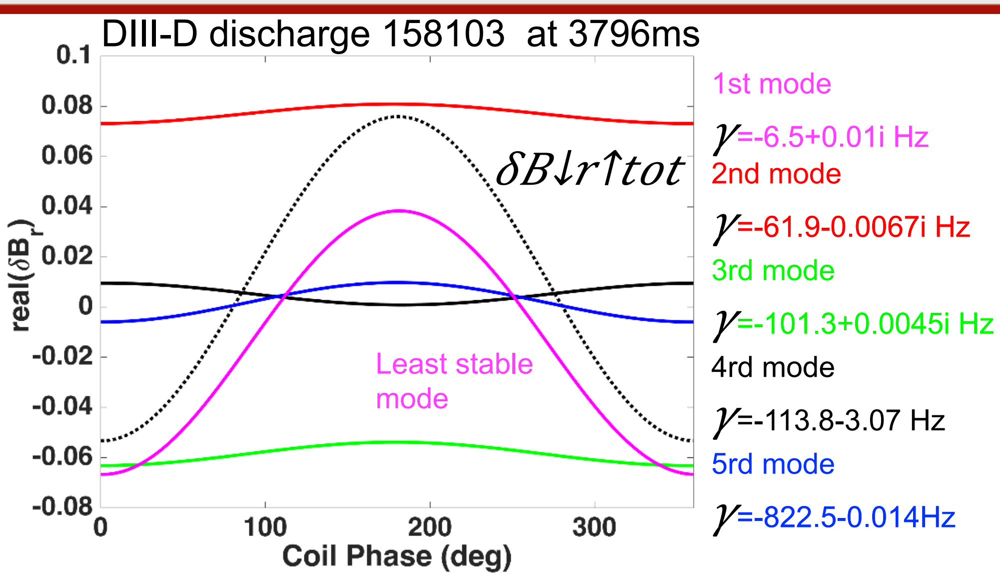
- In the simulated DIII-D equilibrium, rotation slightly destabilize the plasma.
- Resistivity plays a stabilizing effect on least stable mode.

DIII-D: n=2 Response at HFS Purely Contributed by Plasma with Zero Degree Coil Phasing (Even Parity)

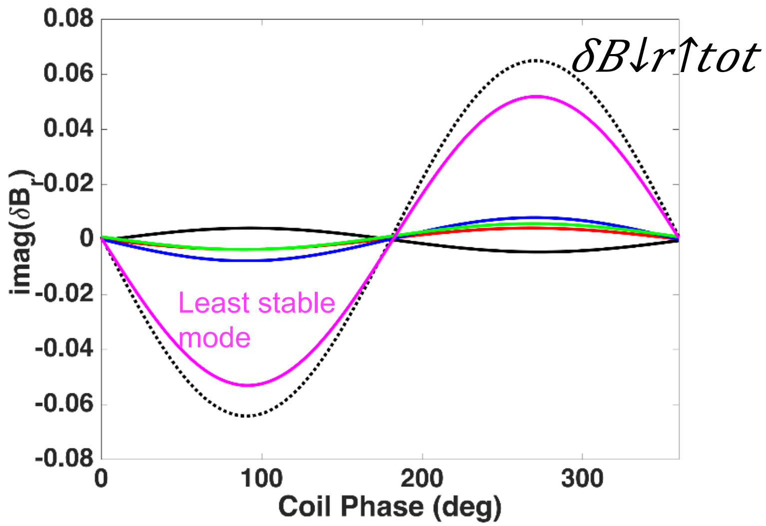
- Extracted transfer functions from simulated vacuum response and total total response indicate pure plasma response $\delta B\downarrow r\uparrow plasma = \delta B\downarrow r\uparrow tot - \delta B\downarrow r\uparrow vac$ has dominant contribution with even parity of I-coils in experiments.
- ELM suppression appears near zero coil phasing.

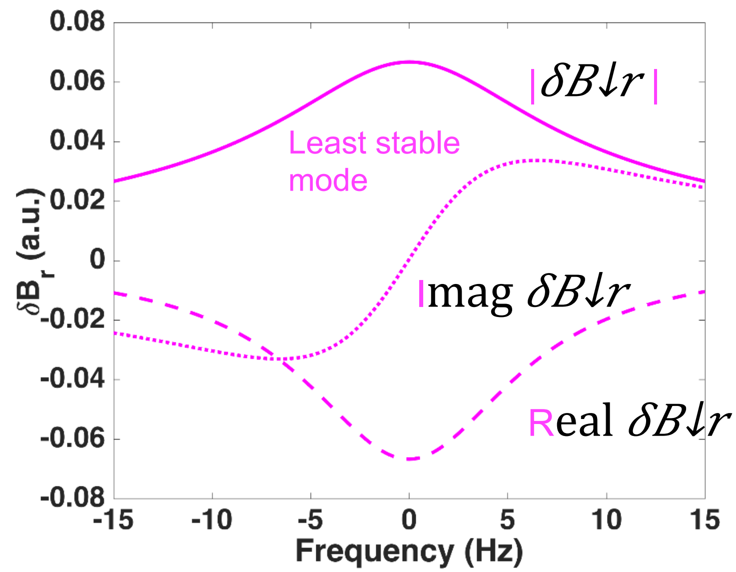


DIII-D: Least Stable Mode is Dominant in n=2 Resistive Plasma Response with Zero Degree Coil Phasing (Even Parity)



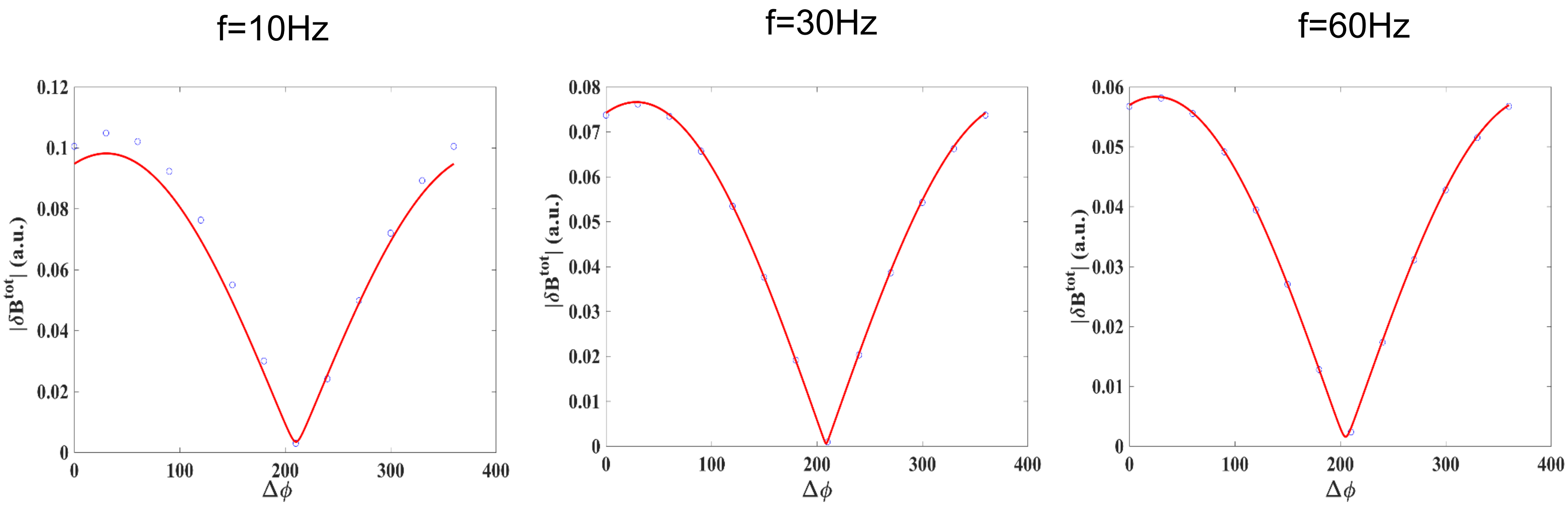
- The least stable provides dominant contribution of plasma response.
- Least stable mode has strongest amplification at $f_{coil} \sim 0$ Hz when coil phasing is 0 deg.





MARS-F Simulation of Response for Modified Pad$e$ Approximation

- **EAST equilibrium (shot No. 52340)** is used in the MARS-F simulation.
- Phase scan of n=1 ideal response is simulated and measured by the assumed radial sensor at mid-plane of HFS.
- Three contours with different coil frequencies (10Hz, 30Hz, 60Hz) are fitted at the same time.



Three poles model well fit the sensor signals to extract plasma transfer function.

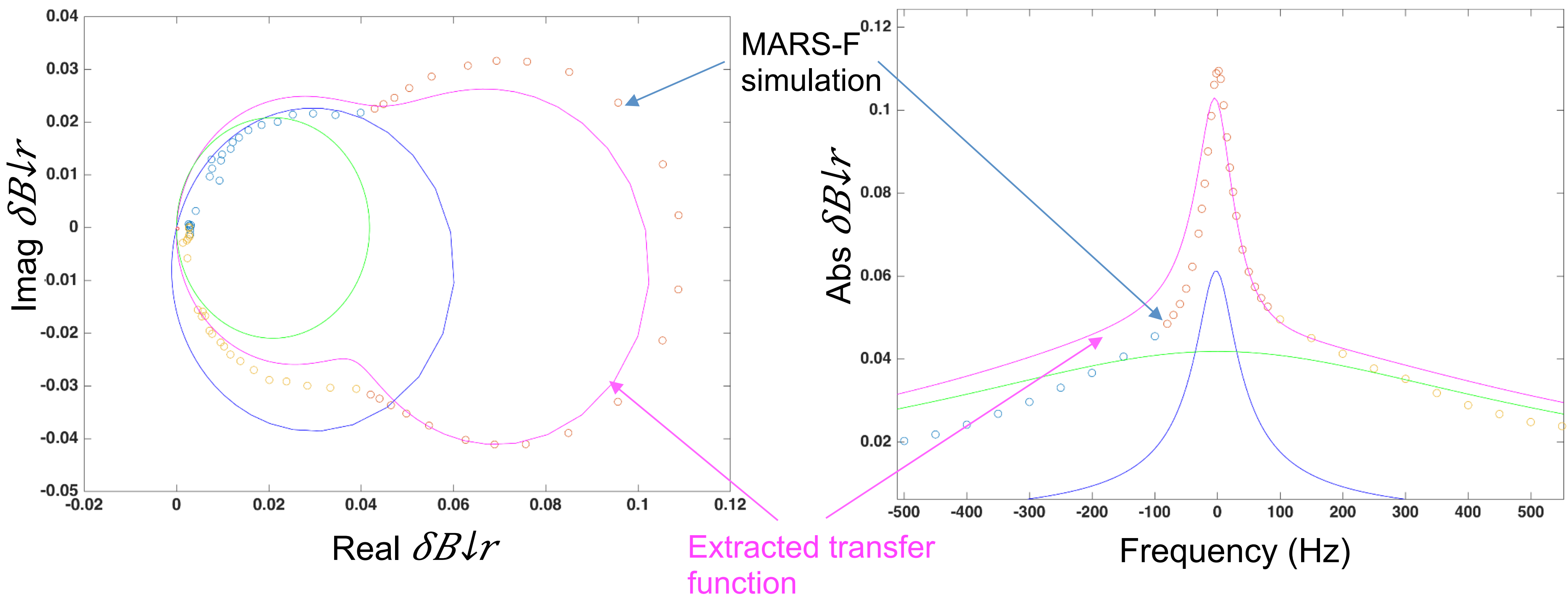
Two Modes Are Dominant on HFS with Zero Phase Degree

The eigenvalues of three poles：

γ_1 =-30.714 - 2.2163i Hz γ_2 =-458.71 + 0.2993i Hz γ_3 =-1624.8 + 3.02i Hz

The transfer function extracted from the low frequency and coil phase scan, can recover the simulated Nyquist contour at all frequency range.

Two modes are dominant (first and secondary modes)



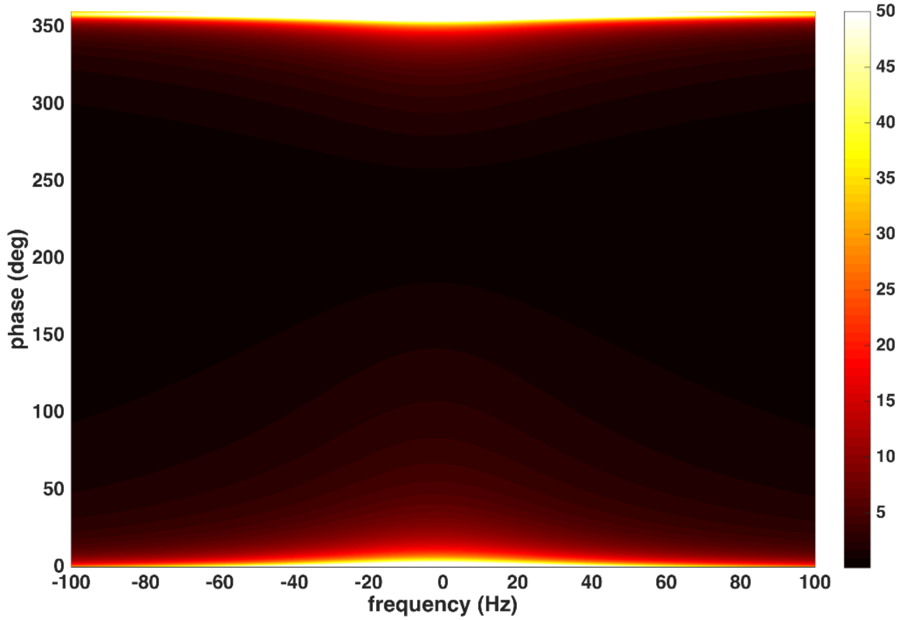
Use Transfer Functions to Optimize Coil Phase and Frequency for amplifying preferred eigenmode

Response transfer function: $P(\Delta\phi)=\sum_{i=1}^{N} \frac{a_i^j + b_i^j e^{-i\Delta\phi}}{if-\gamma_i}$

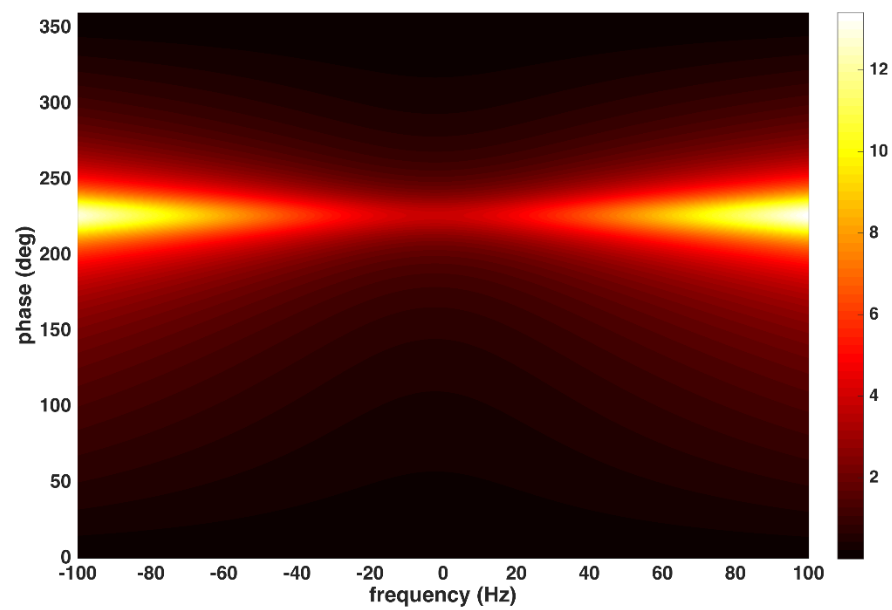
Two dominant modes in fitted P:

$P_1 = 0.9+0.17i+(0.92-0.66i)e^{i\Delta\phi}/(if-(-30.71-2.216i))$ $P_2 = 2.27-0.094i+(16.9+0.027i)e^{i\Delta\phi}/\ldots$

$Ratio = |P_1/P_2|$



$Ratio = |P_2/P_1|$

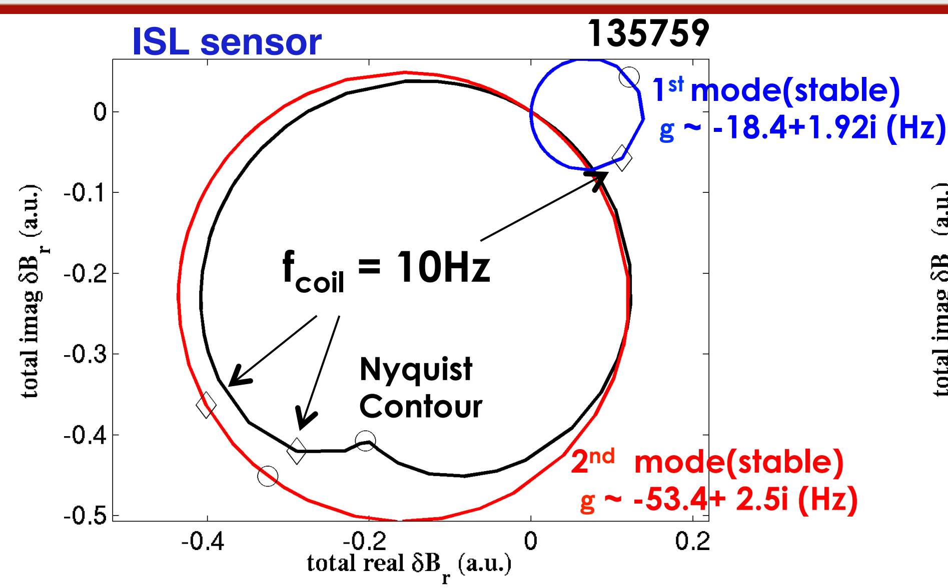


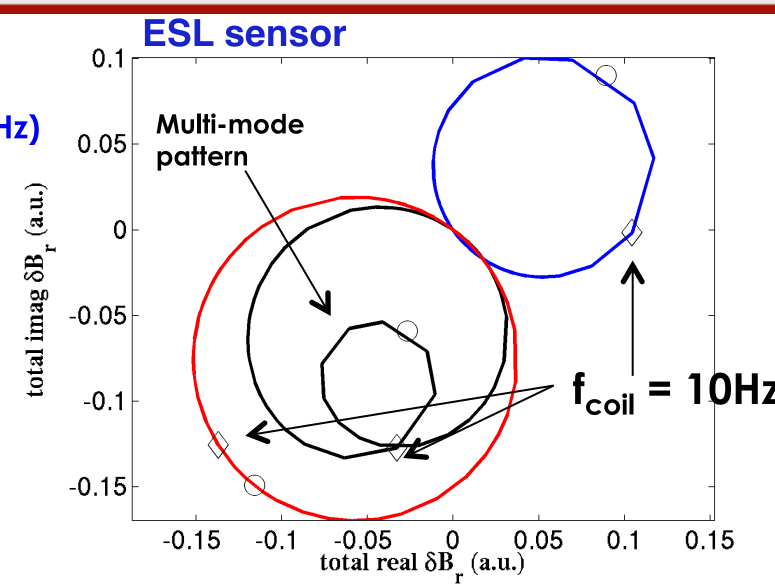
P_1 is amplified at 0 deg coil phasing. P_2 is amplified at 240 deg coil phasing. Increasing coil frequency further enhances the amplification of mode which can be important during ELM suppression.

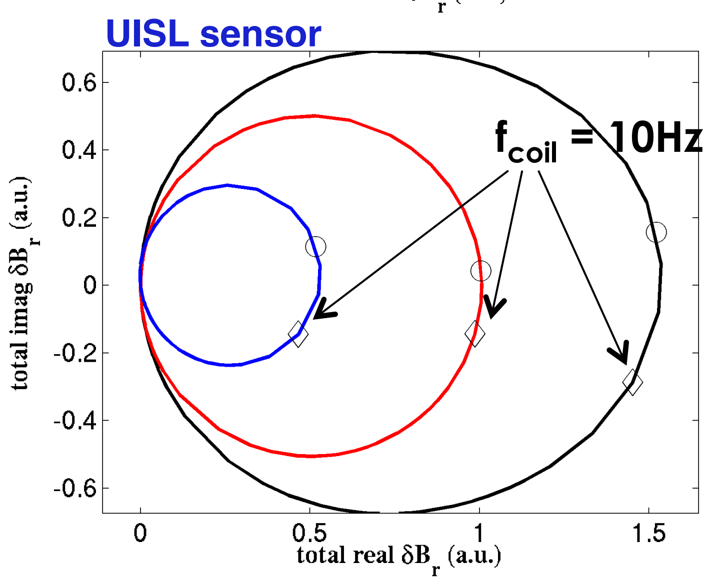
Nyquist Analysis Reveals Fundamental Contribution of Eigenmodes on Kinetic Plasma Response and Multi-Mode Response

- Nyquist analysis is a powerful tool that can help to reveal a range of underlying physics associated with 3D fields. It finds
  - dissipation of kinetic effects makes plasma more stable through stabilization of the eigenmodes.
  - plasma in NSTX is destabilized by flow rotation and stabilized by eddy current in the wall.
  - increasing EXB rotation can decrease the plasma stability in kinetic-MHD approach and make kinetic plasma response multi-modal.
  - secondary mode can be dominant in n=1 kinetic response.
- Modified Nyquist method including both coil phase and frequency dependences can
  - indicate least stable mode plays a dominant role with zero degree coil phase in DIII-D n=2 ELM suppression experiments.
  - give the capability to optimize coil phase and frequency for amplifying the preferred eigenmode during ELM control experiments e.g. amplify secondary mode with 240 degree coil phasing and finite coil frequency.
  - be a candidate for 3D MHD spectroscopy and real-time monitoring of the plasma stability in long-pulse fusion reactors (new need high frequency scan to form Nyquist contour).

DIII-D: n=2 Plasma Response Results Show Multi-Mode Structure with Nyquist Analysis in Ideal MHD simulation (MARS-F)







- Padé approximation indicates plasma is stable to n=2 modes in ideal MHD simulation.
- Two mode contribute to fluid response.
- Second least stable mode also dominates n=2 fluid plasma response.
- Nyquist contour measured by different sensor can have different pattern.

Frequency Scan of Each Eigenmode (n=2), no full cancellation among different modes