Development of the NSTX-U Advanced Divertor Control

P. Vail and E. Kolemen

Department of Mechanical and Aerospace Engineering
Princeton University, Princeton, NJ 08540

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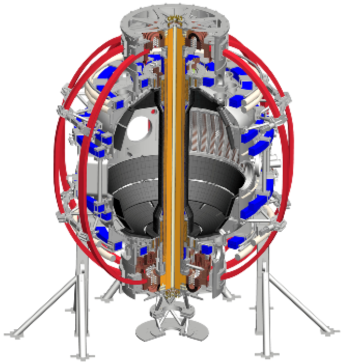
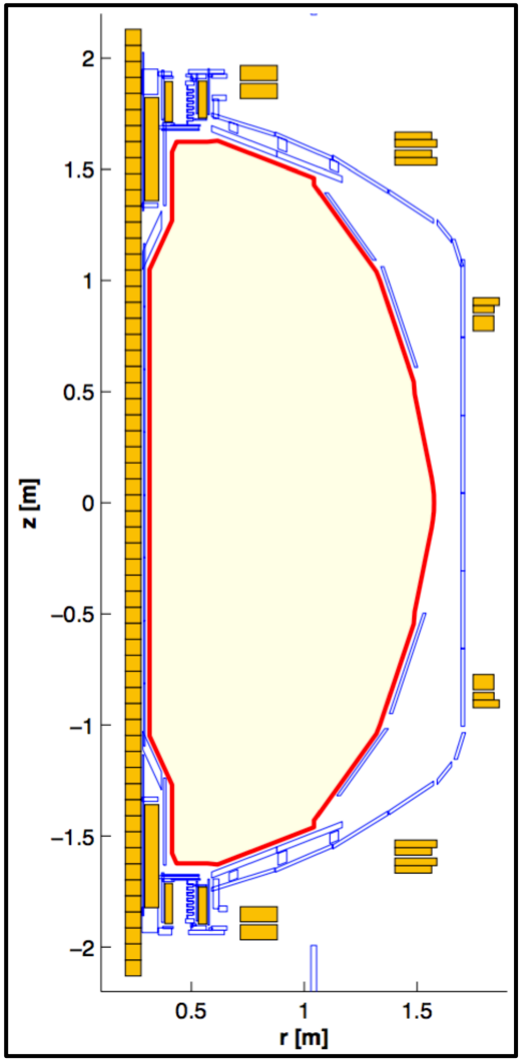


Introduction

- The determination of techniques for mitigating the heat flux onto plasma-facing surfaces is a major goal of current tokamak research.
- High heat fluxes to the walls push the limits of present-day engineering materials and active cooling technologies.
- **ITER** – expected that **70-90%** of power will need to be radiated with standard single x-point divertor.[1]
- **NSTX-U** – peak heat fluxes over **20 MW/m²** have been predicted in 2 MA, 12 MW NBI-heated discharges.[2] Long pulse limit for graphite PFCs is **10 MW/m²**.
- **Novel solutions to the heat flux problem are needed.**



NSTX-U Poloidal Cross-Section

[1] Kotschenreuther M. et al 2007. Phys. Plasmas. 14 072502.

[2] Gray T.K. et al 2011. J. Nucl. Mater. 415 S360.

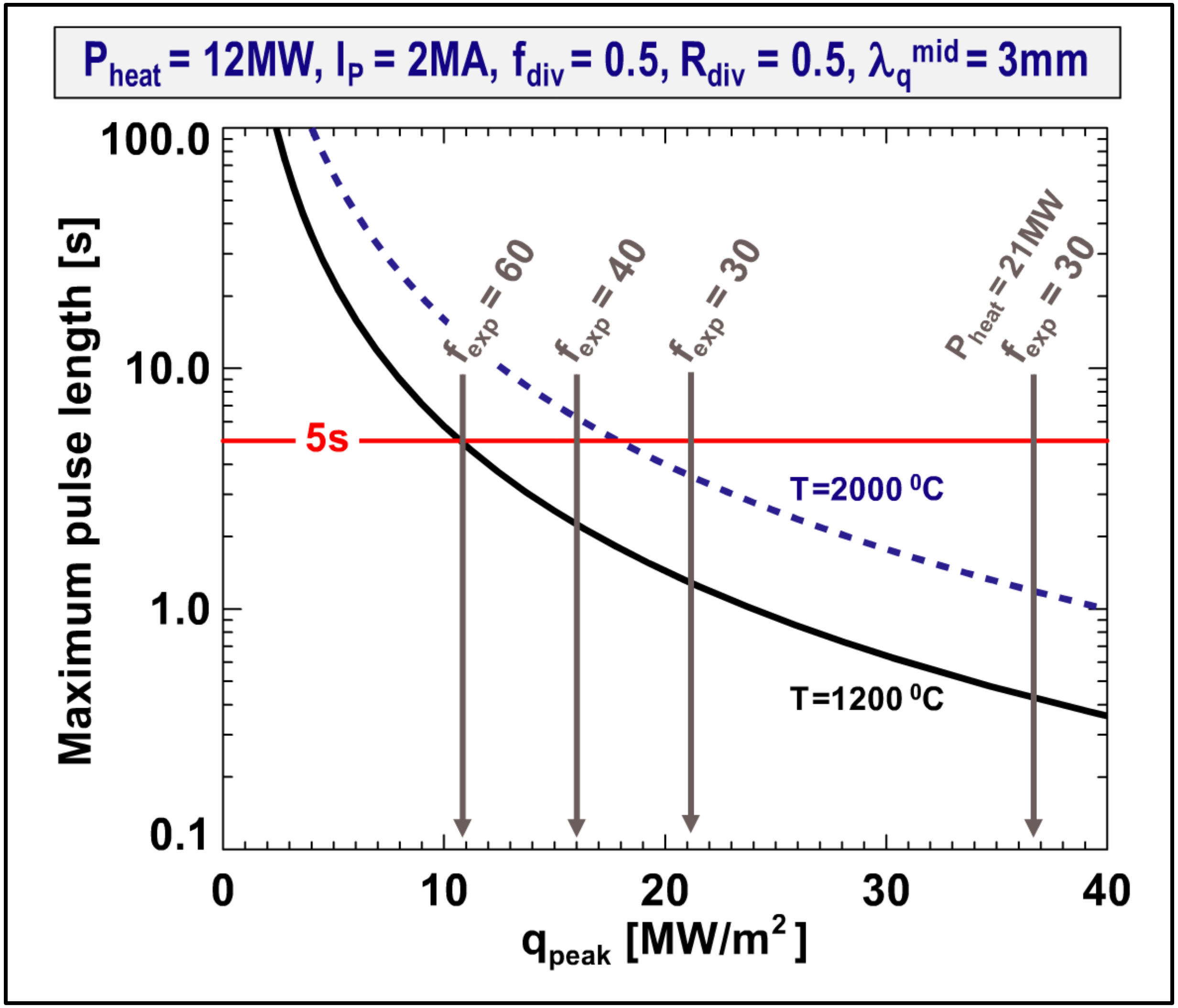
Divertor Heat Flux

$$q_{peak} = \frac{P_{heat}^{SOL}\left(1 - f_{rad}\right) f_{div} \sin\left(\theta_{plate}\right)}{2\pi R_{strike} f_{exp} \lambda_q}$$

q_{peak}	peak heat flux at the divertor strike point (P_{strike}/A_{wetted})
P_{heat}^{SOL}	input heating power to SOL
f_{rad}	fraction of radiated power
f_{div}	fraction of power to divertor leg of interest
θ_{plate}	poloidal angle between divertor plate and magnetic field line
R_{strike}	major radius of divertor strike point
f_{exp}	flux expansion
λ_q	width of heat flux profile in SOL

Stangeby, P.C. *The Plasma Boundary of Magnetic Fusion Devices*. Bristol: IOP Publishing Ltd, 2000.

The Need for High Flux Expansion



Operational limit is 10 MW/m^2

Menard J.E. et al 2012. Nucl Fusion. 52 083015.

Advanced Divertors

- Investigation of high flux expansion divertors is a major near-term research goal of the NSTX-U program.

- Two novel magnetic geometries have proven to be attractive candidates for steady-state (and potentially ELM transient) heat flux mitigation – **snowflake** and **x-divertor**

- **These divertors have the following magnetic properties[1]:**
  - **higher poloidal flux expansion (compared to single x-point divertor)**
  - **longer x-point connection length**
  - **higher divertor flux tube volume**
  - **Four separatrix branches and strike points**

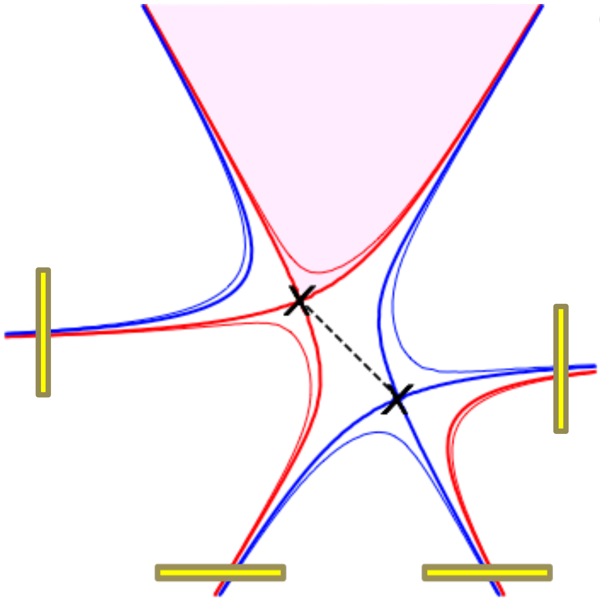
- **SNOWFLAKE** – characterized by a second-order poloidal field null (or two closely-spaced first-order nulls)

- **X-DIVERTOR** – snowflake-like with the additional property that the secondary null is located in the vicinity of the strike point

[1] Soukhanovskii V.A. et al 2012. Phys. Plasmas. 19 082504.

Snowflake Divertor Configurations

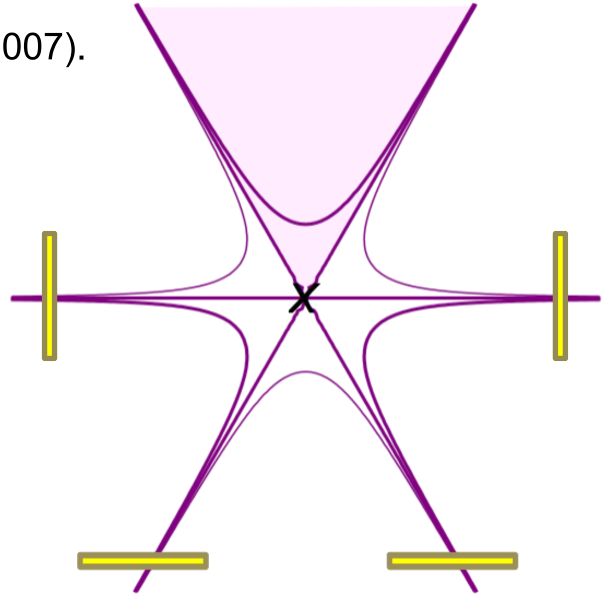
D.D. Ryutov. *Phys*. *Plasmas*. 14 (2007).



Snowflake Minus

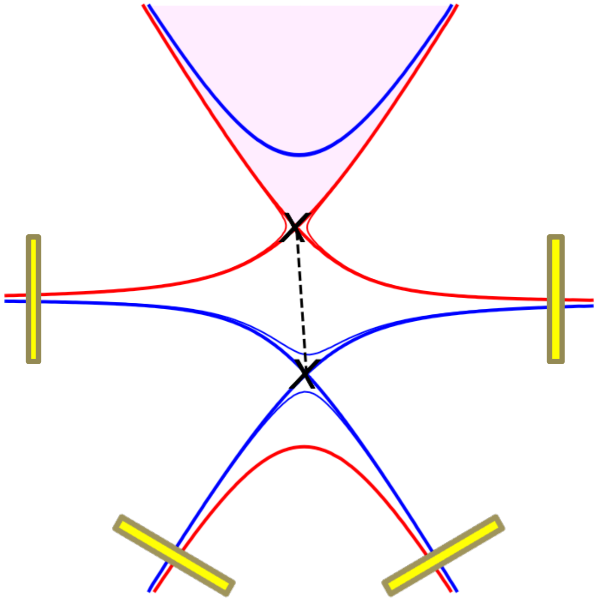
First-Order Primary Null on Separatrix

Secondary Null in Scrape-off Layer



Perfect Snowflake

Second-Order Null on Separatrix



Snowflake Plus

First-Order Primary Null on Separatrix

Secondary Null in Private Flux

Advanced Divertor Control

- NSTX experiments demonstrated the need for active magnetic control of snowflake divertor configurations.[1]

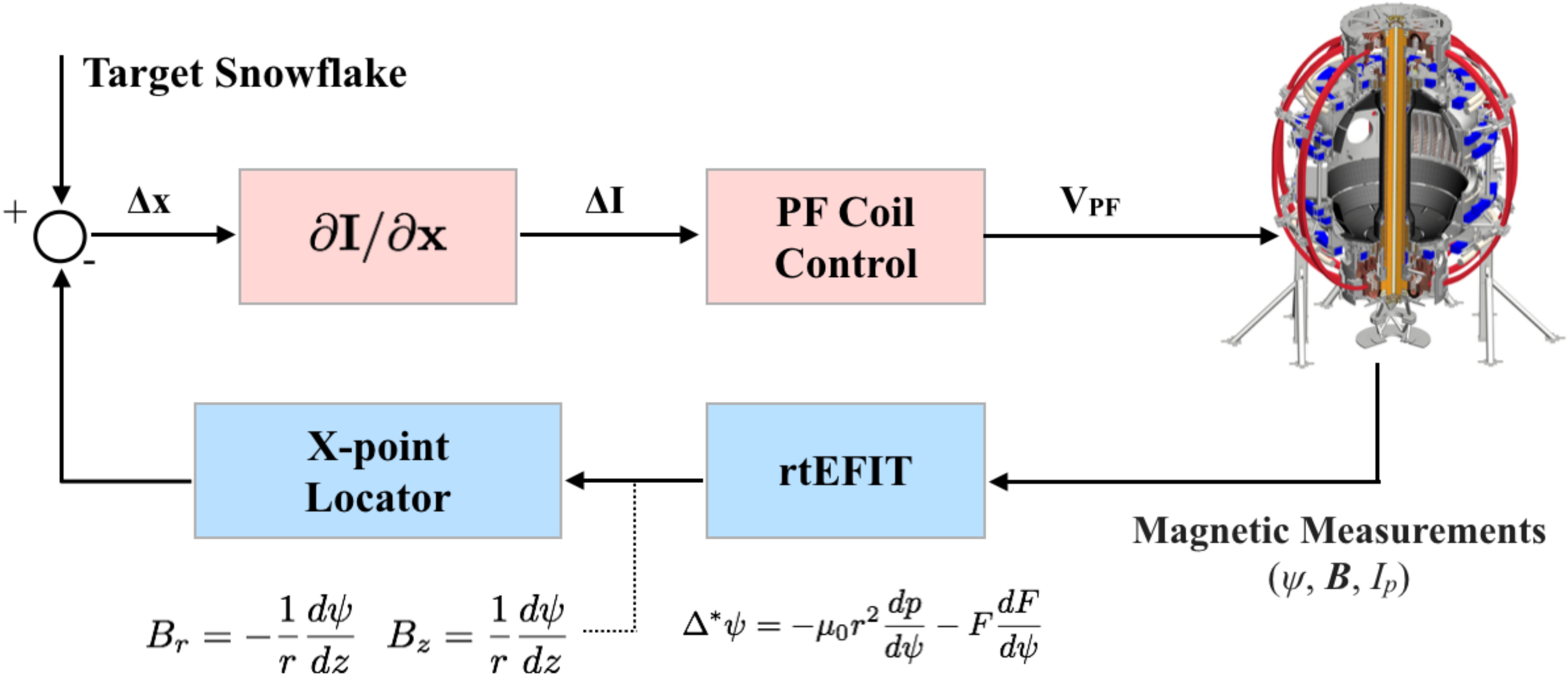
- Advanced divertor control presents new challenges – identification and control of multiple x-points and other parameters

- Snowflake / x-divertor tend to be topologically unstable – sensitive to changes in OH coil current, plasma profiles. (to be further characterized)

- Aim is for near-term implementation of the following control capabilities at NSTX-U:
  - **multiple x-point locations (for snowflake divertor)**
  - **strike point location (for x-divertor)**
  - **flux expansion independent of x-point locations**

[1] Soukhanovskii V.A. et al 2012. Phys. Plasmas. 19 082504.

Snowflake Control System



Target Snowflake

+

-

Δx

$\partial I / \partial x$

ΔI

PF Coil Control

V_{PF}

rtEFIT

X-point Locator

Magnetic Measurements
$(\psi, \boldsymbol{B}, I_p)$

$$B_r = -\frac{1}{r}\frac{d\psi}{dz} \quad B_z = \frac{1}{r}\frac{d\psi}{dz}$$

$$\Delta^{*}\psi = -\mu_0 r^2 \frac{dp}{d\psi} - F\frac{dF}{d\psi}$$

X-Point Location Algorithm

- Consider the Grad-Shafranov equation with zero current density:

$$(R+x)\frac{\partial}{\partial x}\left(\frac{1}{R+x}\frac{\partial \psi}{\partial x}\right)+\frac{\partial^2\psi}{\partial v^2}=0$$

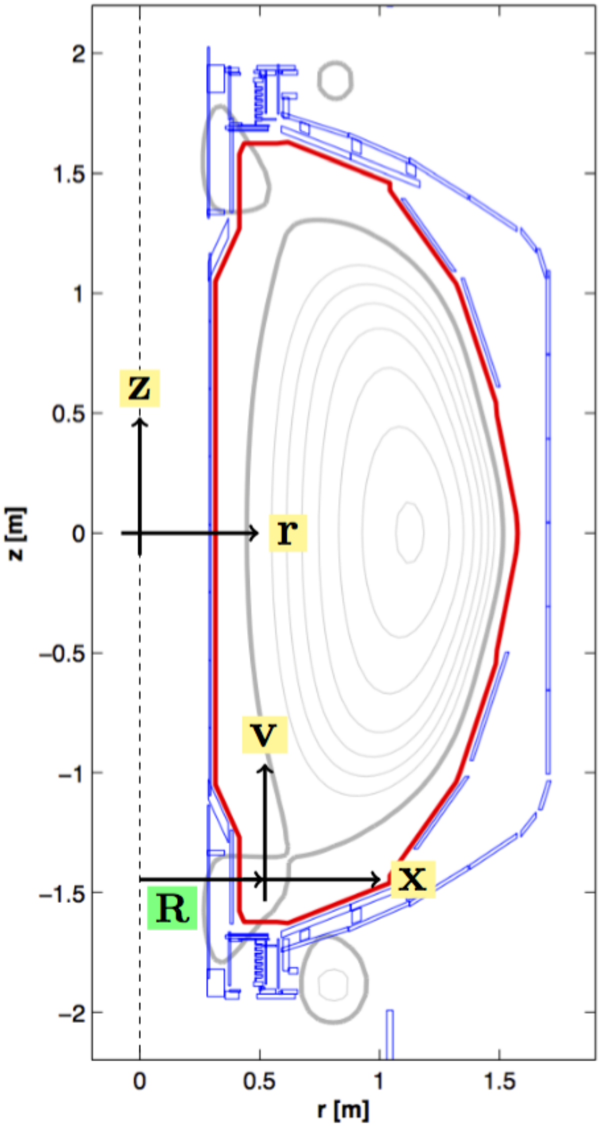
where the coordinate system has been shifted to be local to the null point(s) – new variables ***x*** and ***v***

$$B_x=-\frac{1}{R+x}\frac{\partial\psi}{\partial v}\qquad B_v=\frac{1}{R+x}\frac{\partial\psi}{\partial x}$$

- Then expand the magnetic flux function to third order in the new coordinates

$$\begin{aligned}\psi = l_1x+l_2v\\ +q_1x^2+2q_2xv+q_3v^2\\ +c_1x^3+c_2x^2v+c_3xv^2+c_4v^3\end{aligned}$$

Ryutov, D.D. et al 2010. Plasma Phys. Control. Fusion. 52 105001.



X-Point Location Algorithm

- The goal is to determine the expansion coefficients under the constraint that $\boldsymbol{\psi}$ approximately satisfies the current-free G-S equation.

- Magnetic field components in terms of 9 expansion coefficients:

$$\begin{aligned} B_x &= -\frac{1}{R+x}\left(l_2 + 2q_2 x + 2q_3 v + c_2 v^2 + 2c_3 xv + 3c_4 v^2\right) \\ B_v &= \frac{1}{R+x}\left(l_1 + 2q_1 x + 2q_2 v + 3c_1 x^2 + 2c_2 xv + c_3 v^2\right) \end{aligned}$$

- Three constraints come from substituting $\boldsymbol{\psi}$ into the G-S equation:

$$\begin{aligned} -l_1 + 2q_1 R + 2q_3 R &= 0 \\ -2q_1 + 6c_1 R + 2c_3 R &= 0 \\ -2q_2 + 2c_2 R + 6c_4 R &= 0 \end{aligned}$$

- For a second-order null at the origin of the $\boldsymbol{x}, \boldsymbol{v}$ coordinate system: $\boldsymbol{l}$’s and $\boldsymbol{q}$’s $= 0$.

- For a snowflake with two closely separated x-points: $\boldsymbol{l}$’s and $\boldsymbol{q}$’s are small.

Ryutov, D.D. et al 2010. Plasma Phys. Control. Fusion. 52 105001.

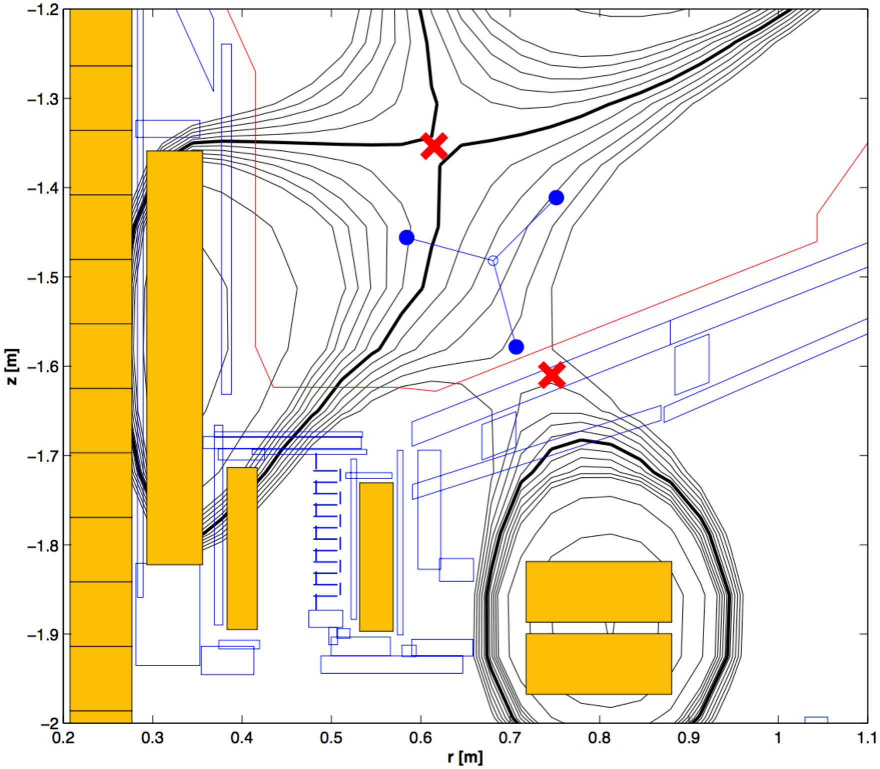
Finding the expansion coefficients

- 6 free expansion coefficients are found by sampling $\boldsymbol{B_x}$ and $\boldsymbol{B_v}$ at three points and then solving the following linear equations for ***i*** = {1, 2, 3}.

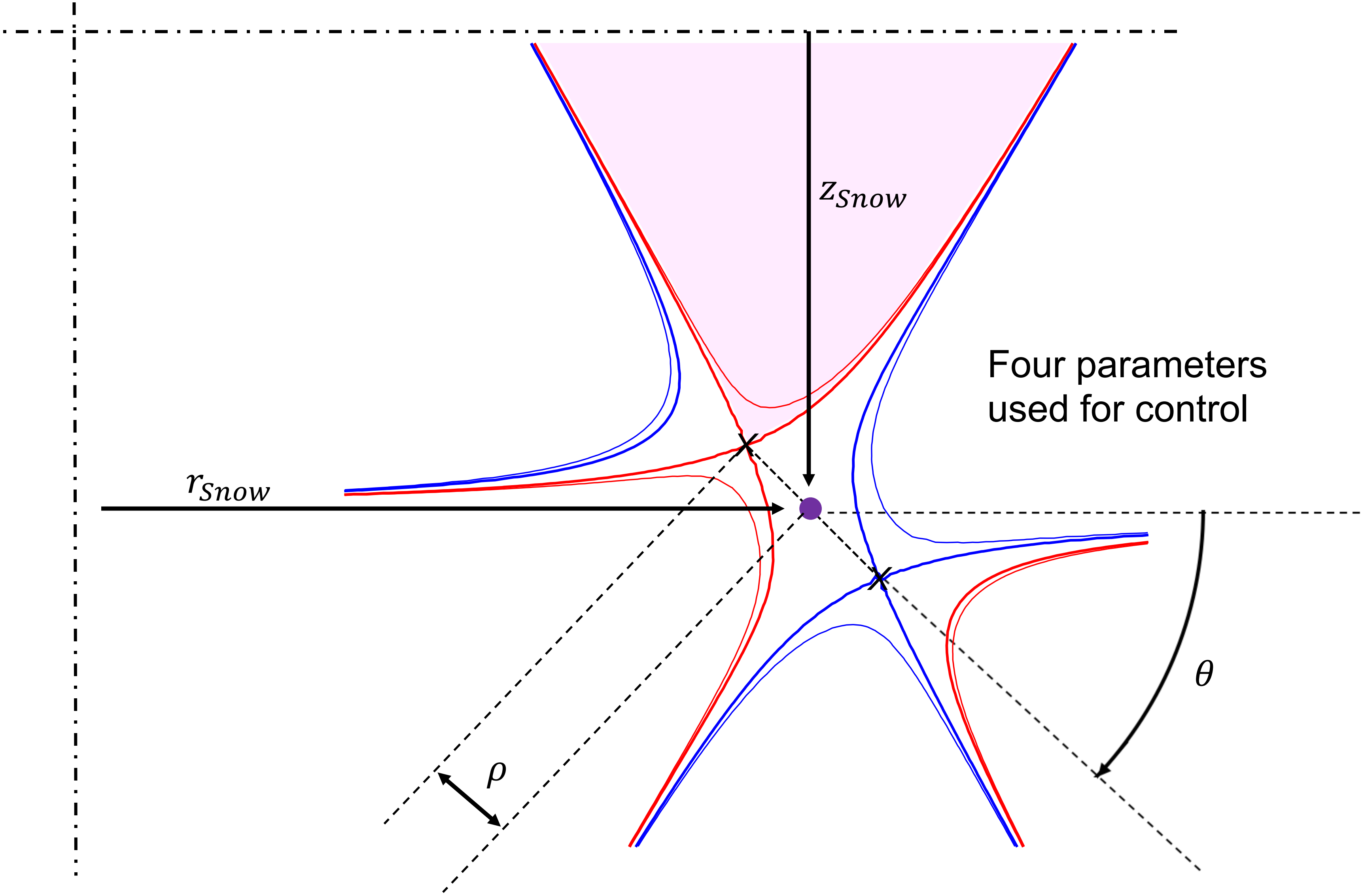
$$-\left(R+x_i\right)B_{x,i} = l_2 + 2q_2x_i + 2q_3v_i - 3c_4\left({x_i}^2 - v_i^2\right) - 6c_1x_iv_i$$

$$\left(R+x_i\right)B_{v,i} = l_1 - 2q_3x_i + 2q_2v_i + 3c_1\left({x_i}^2 - v_i^2\right) - 6c_4x_iv_i$$

- $\boldsymbol{B_x}$ and $\boldsymbol{B_v}$ obtained in real-time from rtEFIT.



Snowflake Shape Descriptors



Four parameters used for control

Control Algorithm

X-point Locator

Compute series solution $\psi_{exp}(r,z)$ to the homogenous Grad-Shafranov equation,

$$r\partial_r\left(r^{-1}\partial_r\psi\right)+\partial_{zz}\psi=0.$$

Coefficients of series are found using magnetic field values (B_r, B_z) at 3 points. We then find the x-point locations by solving,

$$-r^{-1}\partial_z\psi_{exp}=0, r^{-1}\partial_r\psi_{exp}=0.$$

$\partial\mathbf{I}/\partial\mathbf{x}$

Compute matrix $\partial\boldsymbol{x}/\partial\boldsymbol{I}$ and its pseudoinverse. We then compute the required coil currents,

$$\Delta I=\left(\partial\mathbf{x}/\partial\mathbf{I}\right)^{\dagger}\Delta x.$$

Coil current requests

Snowflake position errors

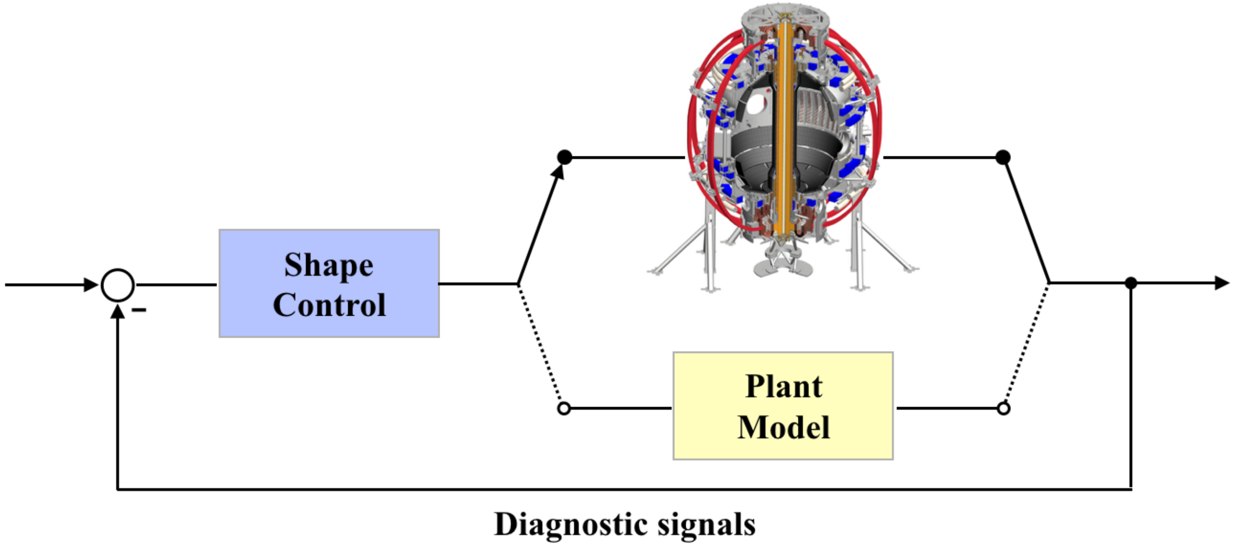
PF Coil Control

PID controller computes voltage requests for the PF coils.

$$V(t)=V(t-1)+R\cdot\mathrm{PID}(\Delta I)$$

Motivation for Plant Modeling

- For the purposes of control design and simulation, we need a ***plant model*** that can serve as a proxy for the ***tokamak and plasma***.
- Model should adequately capture the physics of interest.



- We have developed a model of NSTX-U which describes the electromagnetic interactions between the poloidal magnetic field coils, passive conductors, and plasma.
- **Useful for shape control development and simulation.**

Vacuum Modeling

- In the absence of plasma, the relevant equations are as follows,

$$M_{cc}\dot{I}_c + R_c I_c + M_{cv}\dot{I}_v = V_c$$ ***coils***

$$M_{vv}\dot{I}_v + R_v I_v + M_{vc}\dot{I}_c = 0$$ ***vessel***

- The equations can be written in standard state-space form,

$$\dot{x} = \mathbf{A}x + \mathbf{B}v \qquad y = \mathbf{C}x \longleftarrow$$

Map from currents to diagnostic signals

where

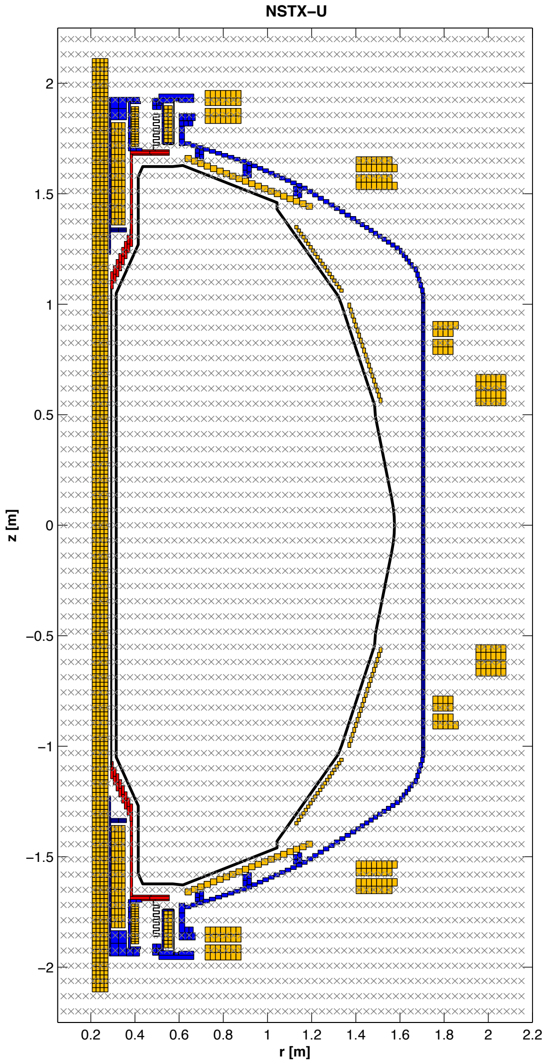
$$x = [I_c \;\; I_v] \qquad \mathbf{A} = -\mathbf{M}^{-1}\mathbf{R} \qquad \mathbf{B} = \mathbf{M}^{-1}\mathbf{V}$$

Mutual Inductance Matrix

Resistance Matrix

Map from input voltages to coils

- Mutual inductances and resistances are computed by discretizing the coils and passive conductors (vessel).

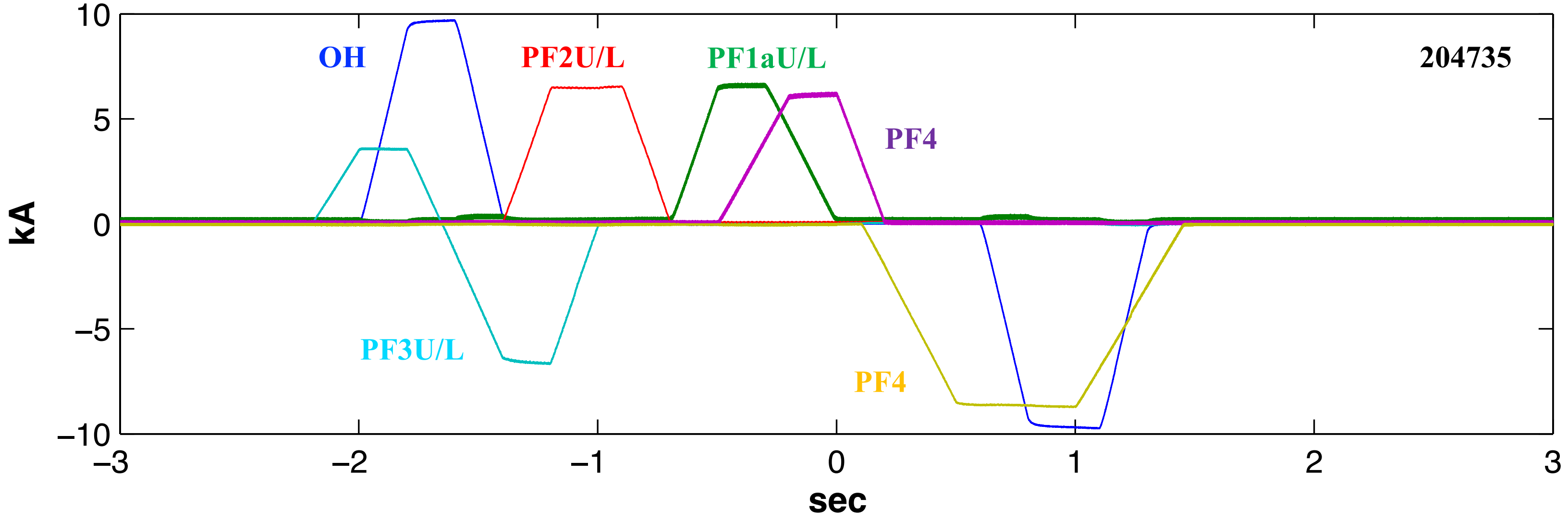


Conductor discretization

Vacuum Model Validation

- Model of the coil and vessel current dynamics (in the absence of plasma) can be validated using data from NSTX-U magnetics calibration or power supply test shots.

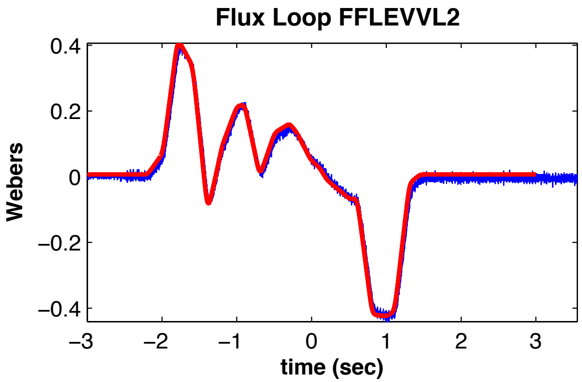
- Sample test shot (204735)

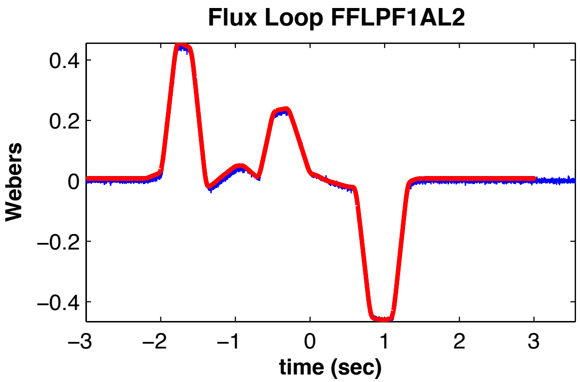


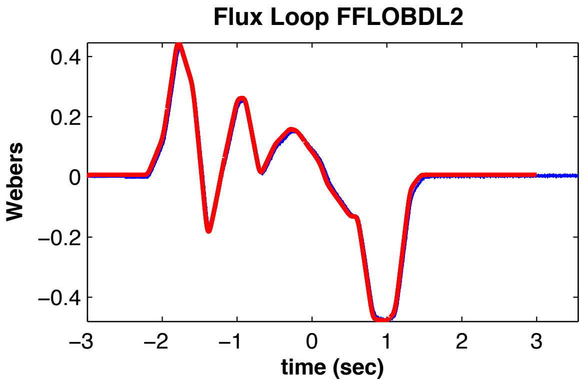
- ***Validation Method***: Given prescribed set of OH + PF coil current waveforms (such as above), integrate the model to determine vessel currents and flux loop signals.

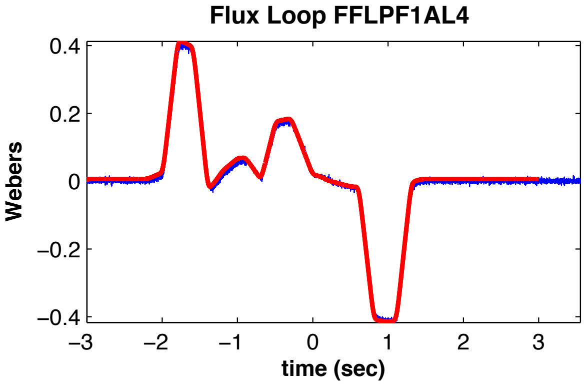
Flux Loop Signal Comparison

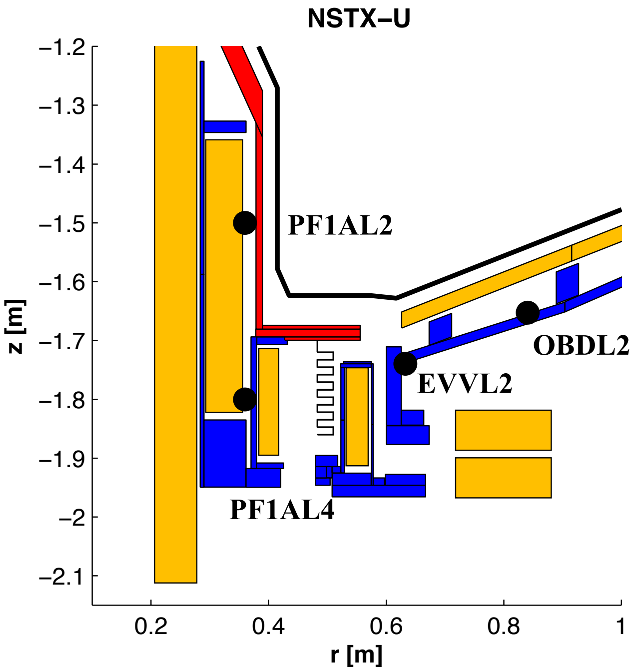
Sample flux loop signals (65 total)


Flux Loop FFLEVVL2


Flux Loop FFLPF1AL2


Flux Loop FFLOBDL2


Flux Loop FFLPF1AL4

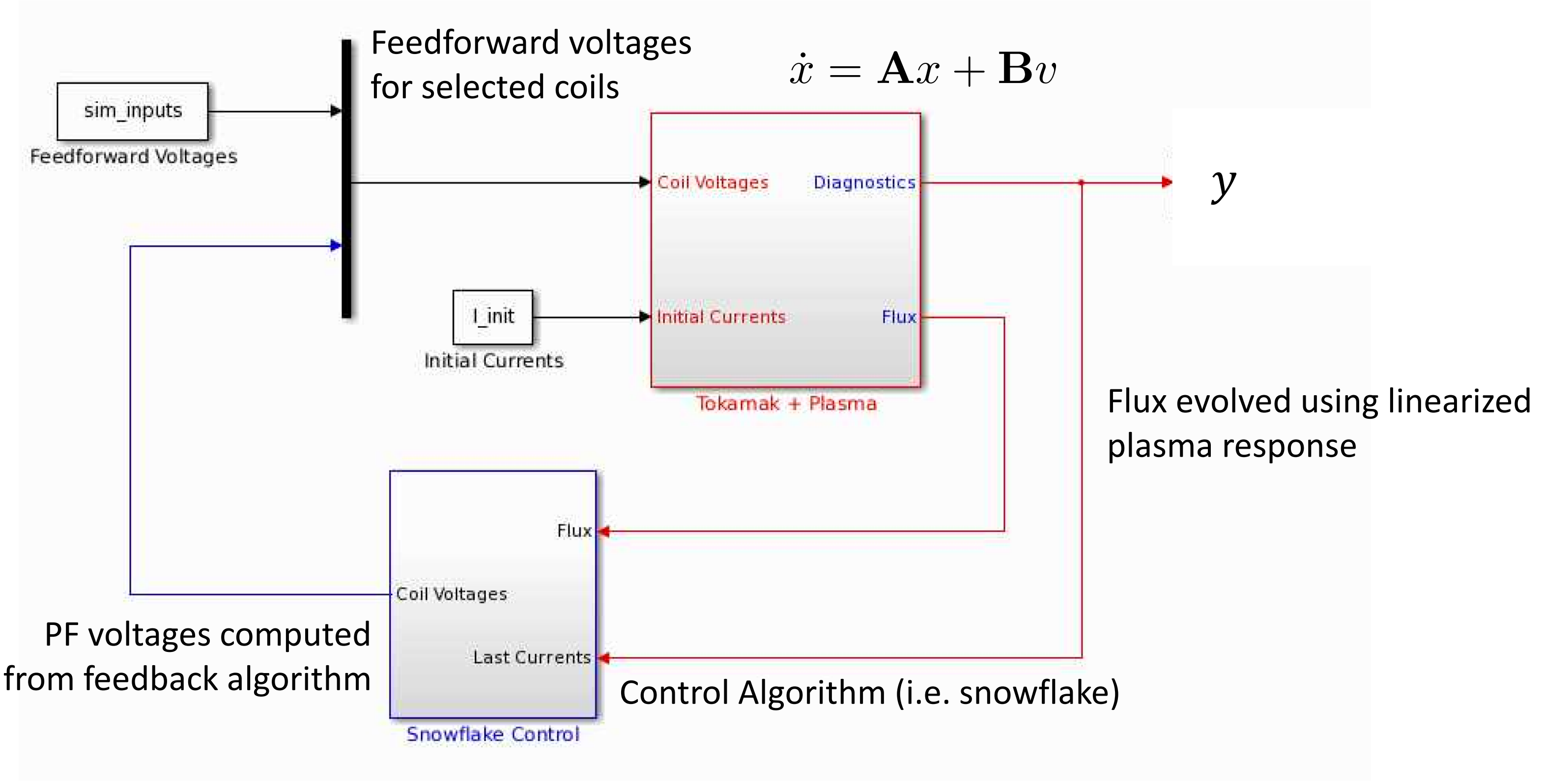

NSTX−U

204735

Simulated FL signal

Closed-Loop Simulation Overview

- We use the validated model of the tokamak + plasma for closed-loop control simulations.



Feedforward voltages for selected coils

$$\dot{x} = \mathbf{A}x + \mathbf{B}v$$

y

Flux evolved using linearized plasma response

PF voltages computed from feedback algorithm

Control Algorithm (i.e. snowflake)

- Implemented in MATLAB/Simulink.

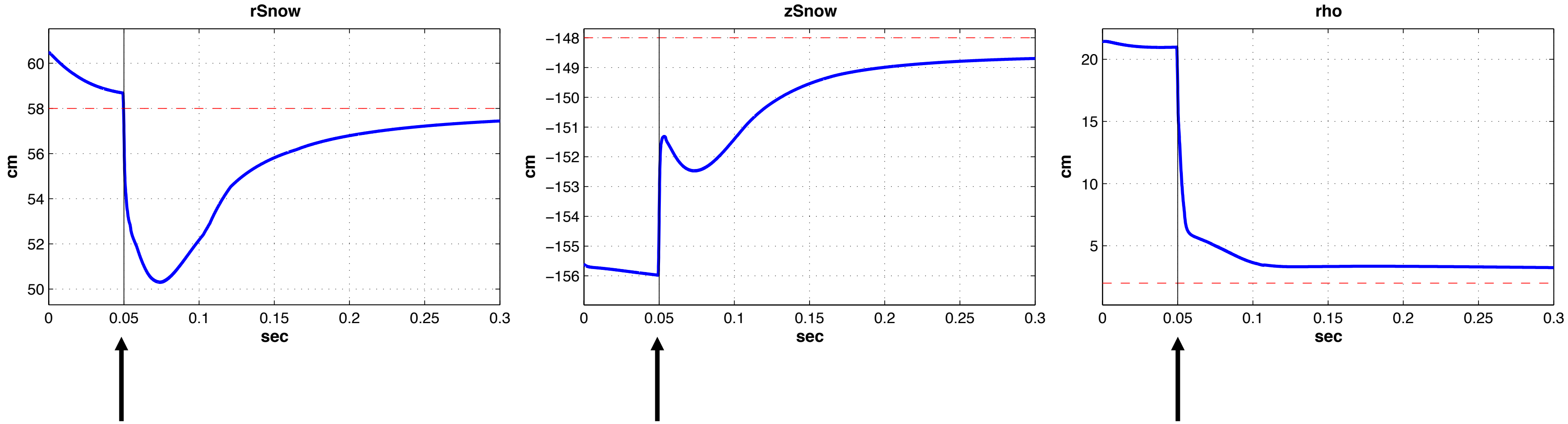
Closed-Loop SFD Development

The simulation tool was used to demonstrate closed-loop control of snowflake divertor configurations in NSTX-U.

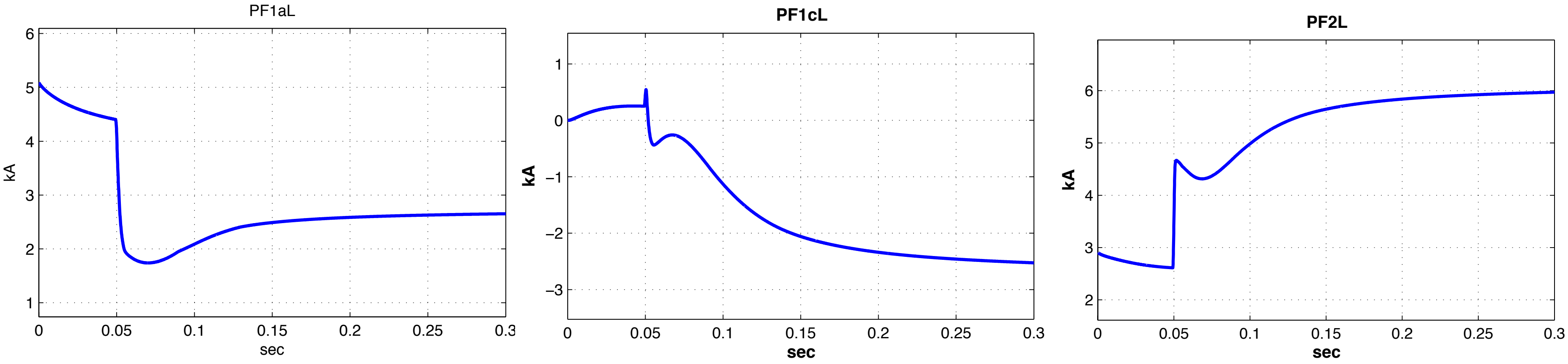
Two scenarios are used to demonstrate the capabilities of the control system:

1. **Scan from a standard single x-point divertor to a near-exact snowflake.**
   - **Controlled Parameters**: rSnow, zSnow, x-point separation (rho)
   - Move secondary x-point from outside the machine into close proximity to the primary x-point.
2. **Scan from SFD-minus to SFD-plus configuration at constant x-point separation.**
   - **Controlled Parameters**: x-point separation (rho), orientation (theta)

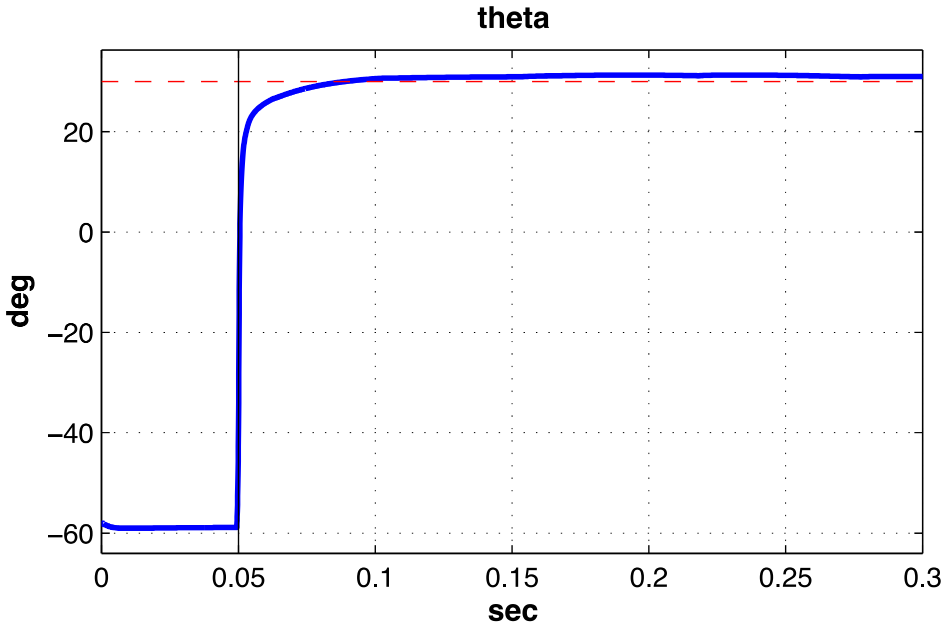
Standard Divertor to Near-Exact SFD



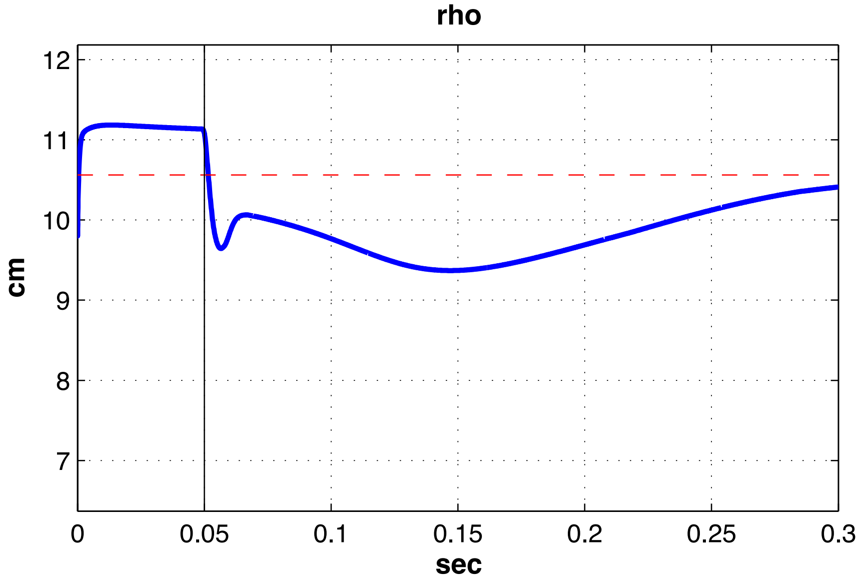
Control is enabled at 50 ms



Scan of Snowflake Angle



Control of the snowflake angle (theta) is heavily-weighted relative to the x-point separation (rho). The snowflake angle therefore quickly approaches its target.



The snowflake x-point separation (rho) begins to approach its target only when the angle reaches a steady-state (150 ms).

The algorithm allows the physics operator to tune the relative performance of the four SFD parameters (rSnow, zSnow, theta, rho).

Summary

- Advanced divertors (ADs) such as the snowflake divertor are promising candidates for management of high power exhaust.

- ADs require most sophisticated control schemes to regulate the magnetic configurations.

- A algorithm has been developed for control of the snowflake divertor configuration in NSTX-U.

- Closed-loop simulations demonstrate that the algorithm can successfully steer divertors from initial to target configurations.

- In addition, we have developed a time-dependent model of the NSTX-U coil, vessel, and plasma system