# Comparison of simulations, experiments, and theory of sub-cyclotron Alfvén eigenmodes in NSTX

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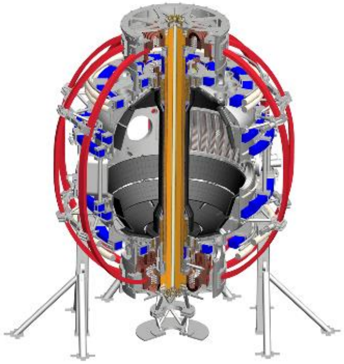
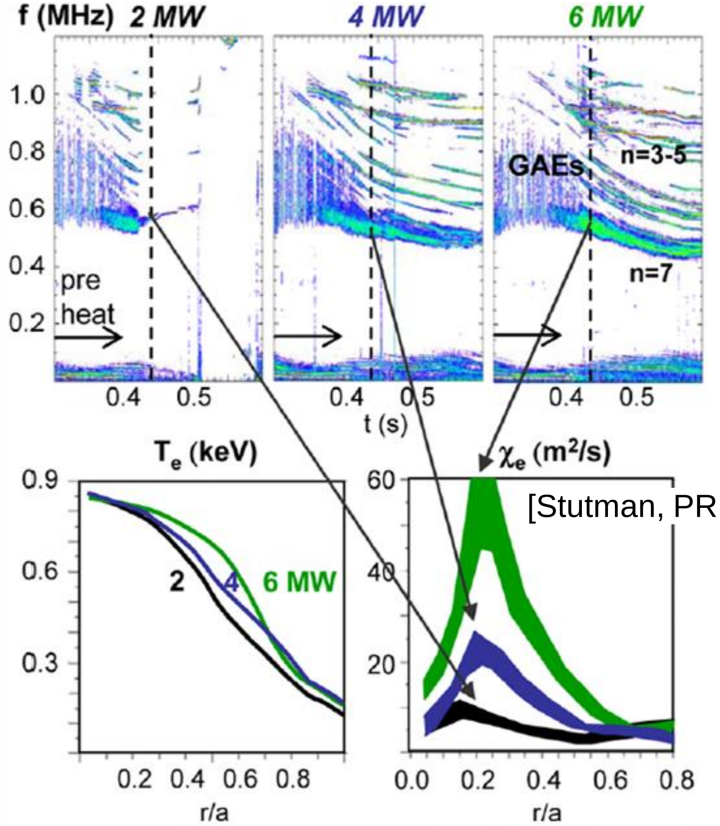


# Overview

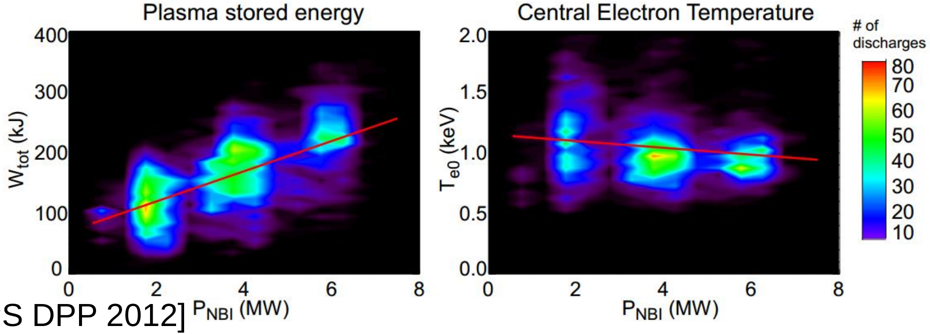
- CAEs and GAEs have previously been linked to anomalous $T_e$ flattening in NSTX
- 3D hybrid simulations of NSTX-like plasmas find a rich spectrum of high frequency ($\omega < \omega_{ci}$) Alfvén modes for a wide range of fast ion parameters ($v_0/v_A$, $\lambda_0$)
- CAEs are strictly more stable than GAEs for $v_0/v_A < 4$ in simulations, consistent with the relative abundance of GAEs in experiment
- Co-GAEs seen in simulations are not often observed and analyzed in experiments
- GAE frequencies vary significantly with beam parameters without clear corresponding changes in mode structure
- Initial comparisons of mode spectrum between experiment and simulations are indirect, yielding fair agreement

# CAE/GAE May Limit ST Performance

- High beam power NSTX discharges exhibit anomalously flat $T_e$ profiles
  - Correlates with increased beam power, strong CAE/GAE activity
- Vital to understand how properties of fast ion distribution affect excitation of these modes
  - Anomalously low $T_e$ imperils future ST development



[Stutman, PRL 2009]



[Tritz, APS DPP 2012]

# CAE/GAEs for the Uninitiated

## Mutual properties

- Typical frequencies: $\omega_{TAE} < 0.1\omega_{ci} < \omega_{CAE,GAE} < \omega_{ci}$
- MHD modes radially localized between magnetic axis and LFS
  - Both may have large compressional component near edge, making experimental classification challenging
- May be driven unstable by resonant energetic particles
  - Regular resonance $\omega - k_\parallel v_\parallel = 0$
  - Doppler-shifted cyclotron resonance $\omega - k_\parallel v_\parallel = \pm\omega_{ci}$
  - Most generally expressed $\omega - n\langle\omega_\phi\rangle + p\langle\omega_\theta\rangle = l\langle\omega_{ci}\rangle$

## Compressional AE (CAE)

a.k.a fast magnetosonic mode

- Compressional polarization
  - $\delta b \approx \delta b_\parallel \gg \delta B_\perp \approx 0$
- Dispersion: $\omega \approx k v_A$
- Mode converts to KAW at Alfvén resonance location $\omega = \omega_A(r_0)$

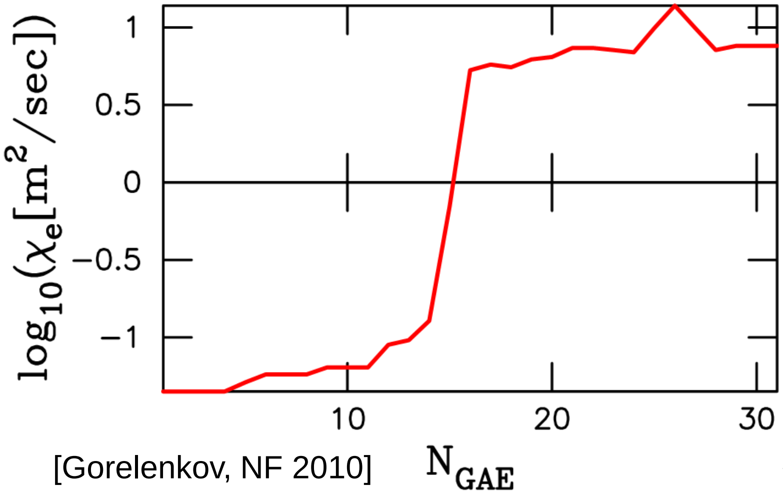
## Global AE (GAE)

- Shear polarization
  - $\delta b \approx \delta b_\perp \gg \delta B_\parallel \approx 0$
- Dispersion: $\omega \leq (k_\parallel v_A)_{min}$
- Exists below an extremum of the Alfvén continuum (e.g. near low magnetic shear)

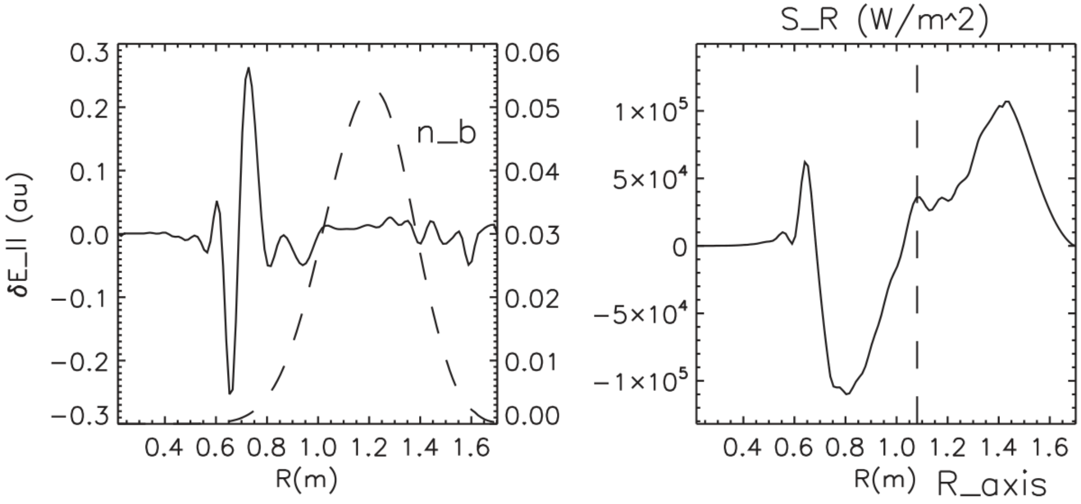
# Theoretical Explanations for $T_e$ Flattening

- Enhanced electron transport due to orbit stochasticity induced by many overlapping GAE (and CAE?)
  - Must generate $\chi_e \sim 10\text{-}50\ \mathrm{m^2/s}$ to match inferred experimental rate

- Energy channeling via mode conversion from core CAE (and GAE?) to edge KAW
  - Predicts up to ~0.5MW power deposition per eigenmode



[Gorelenkov, NF 2010]



[Belova, PRL 2015]

# HYbrid MHD/Particle Code (HYM)

- Hybrid initial value code in 3D toroidal geometry
- Single fluid MHD thermal plasma + particle fast ions
- Full-orbit kinetic ions in $\delta f$ numerical scheme
- Linear and nonlinear capabilities
  - Linear simulations linearize fluid equations + evolve energetic particle weights along equilibrium trajectories
- Self consistently solves for equilibrium including energetic particle effects
- Typical run at NERSC: 100CPU x 10hrs = 1k CPUhrs

# Fast Ion Distribution Model

Equilibrium distribution function $F_0 = F_1(v)\, F_2(\lambda)\, F_3(p_\varphi, v)$

$$F_1(v) = \frac{1}{v^3 + v_*^3}, \quad \text{for} \quad v < v_0$$

$$F_2(\lambda) = \exp(-(\lambda - \lambda_0)^2 / \Delta\lambda^2)$$

$$F_3(p_\phi, v) = \frac{(p_\phi - p_0)^\beta}{(R_0 v - \psi_0 - p_0)^\beta}, \quad \text{for} \quad p_\phi > p_0$$

where $v_0$ = 2 - 6 $v_A$, $v_* = v_0/2$, $\lambda = \mu B_0/\varepsilon$ – pitch angle parameter, $\lambda_0$ = 0.1 - 0.9

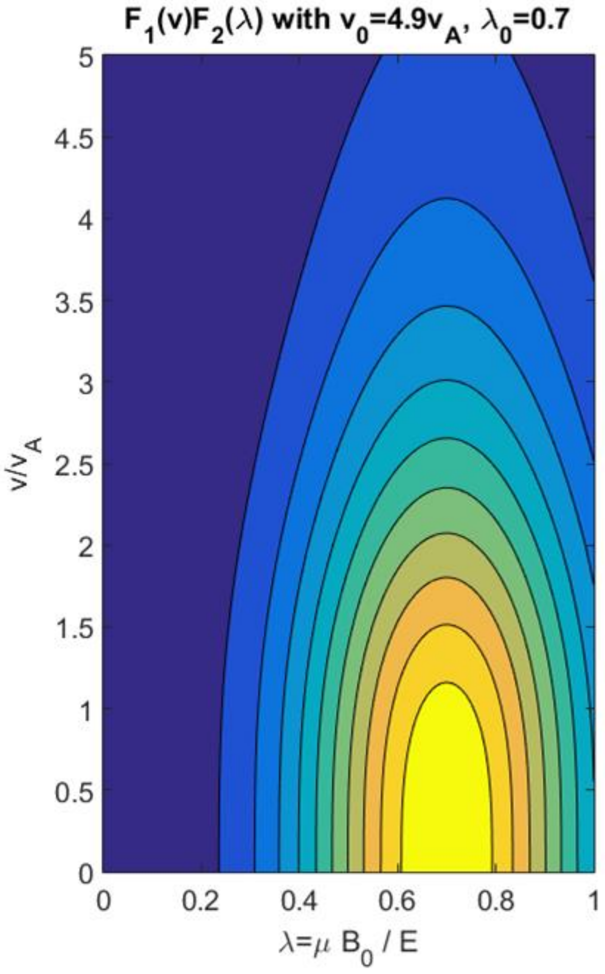
and $\mu = \mu_0 + \mu_1$ includes first-order corrections [Littlejohn'81]:

$$\mu = \frac{(\mathbf{v}_\perp - \mathbf{v}_\mathbf{d})^2}{2B} - \frac{\mu_0 v_\parallel}{2\mathrm{B}}[\hat{b} \cdot \nabla \times \hat{b} - 2(\hat{a} \cdot \nabla \hat{b}) \cdot \hat{c}]$$

$\mathbf{v}_d$ is magnetic gradient and curvature drift velocity, $\hat{c} = \mathbf{v}_\perp / v_\perp$, $\hat{a} = \hat{b} \times \hat{c}$

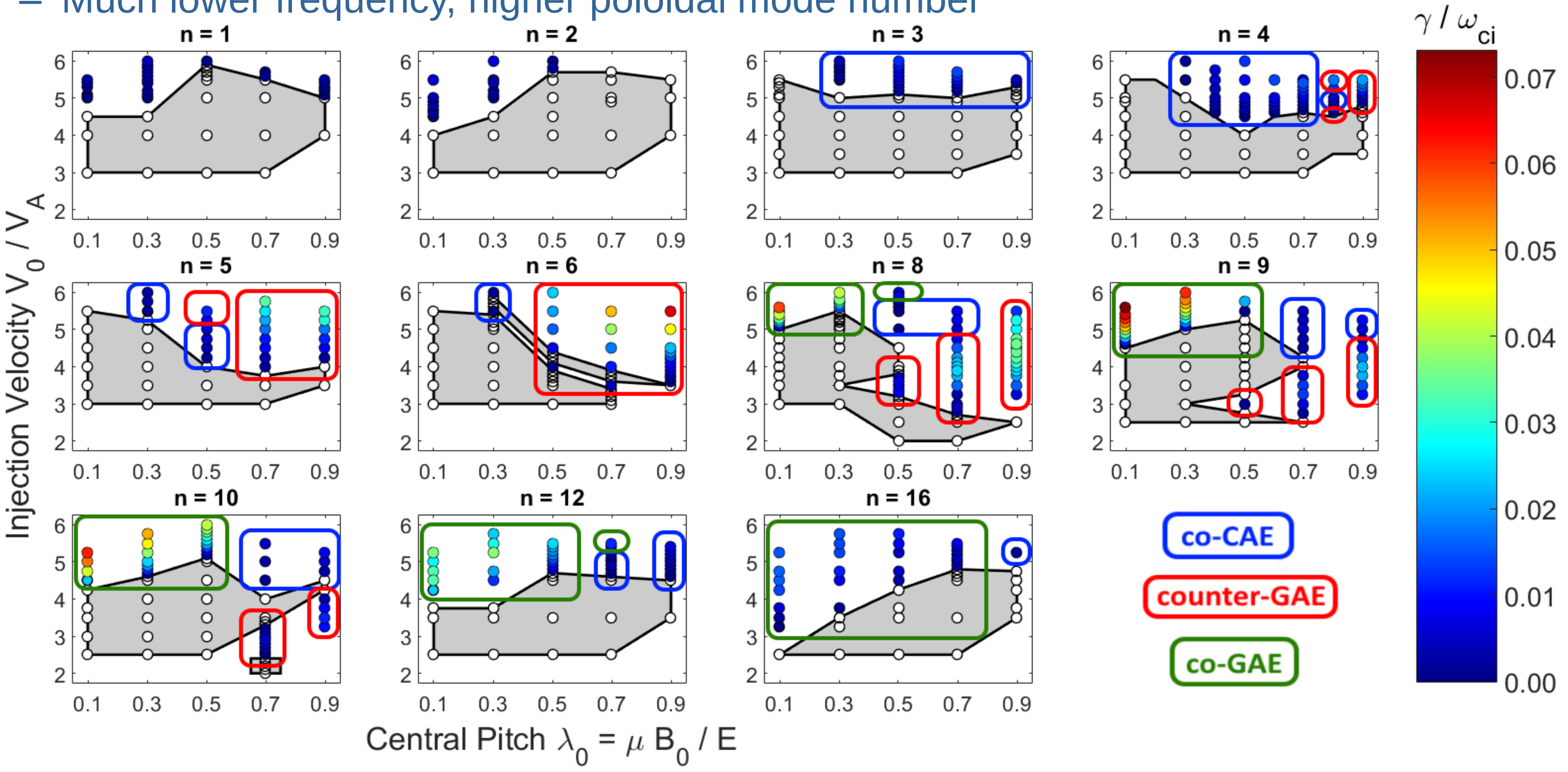
Parameters are chosen to match TRANSP beam profiles.

$$\boxed{\begin{array}{rcll} 0 & < \lambda & < 1 - \epsilon & \text{passing} \\ 1 - \epsilon & < \lambda & < 1 + \epsilon & \text{trapped} \end{array}}$$



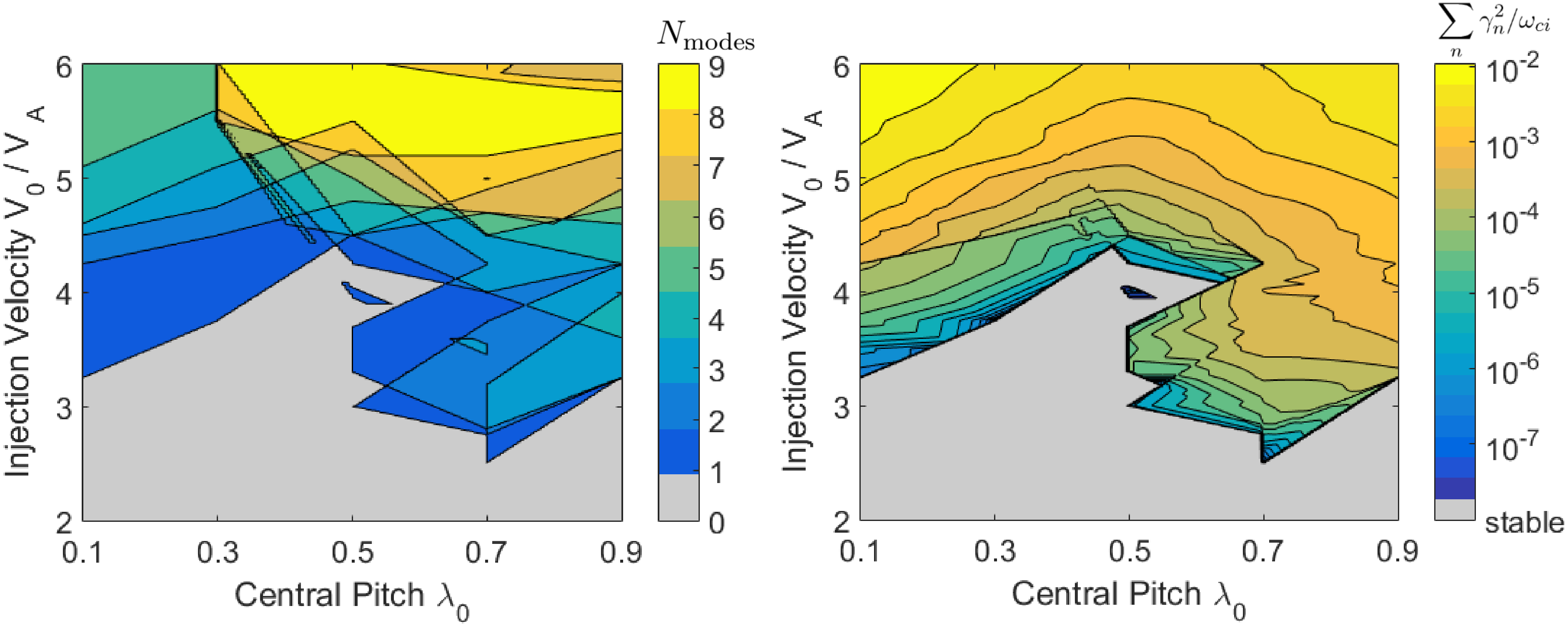
# Variety of Unstable Modes Found

- Most unstable: n=8-10 co-GAE, then n=5-8 cntr-GAE, then n=3-4 co-CAE
  - Colored circles: linear growth rate of most unstable mode
  - White circles: no unstable mode of any type
- n = 1,2 modes are much different from CAE/GAE
  - Much lower frequency, higher poloidal mode number



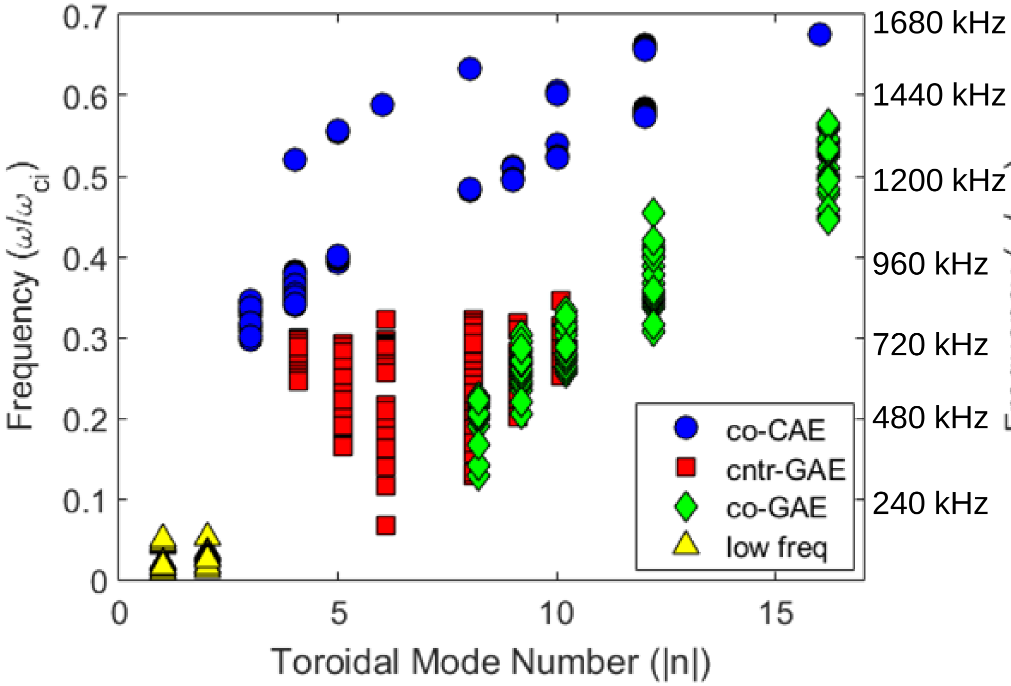
# Number of Excited Modes in Phase Space

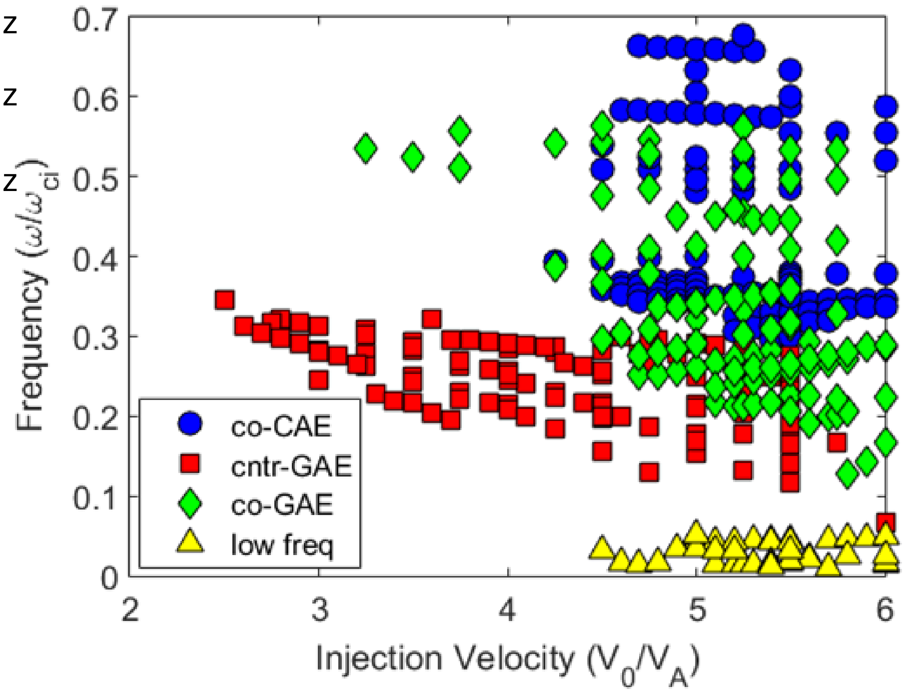
- Both the number of unstable modes and the total amplitude of all unstable modes increase sharply with $v_0/v_A$
- Modes prefer large $v_\perp$ (co-CAEs, cntr-GAEs) or $v_{||}$ (co-GAEs)
  - Prediction: $\lambda_0$~0.5 should not lead to substantial $T_e$ flattening except at very large $v_0/v_A$
- Left: number of unstable toroidal harmonics at each point in phase space
- Right: $\gamma^2$ sum of unstable modes (approximation for total amplitude)



# Mode Spectrum Depends Strongly on n & $v_0$

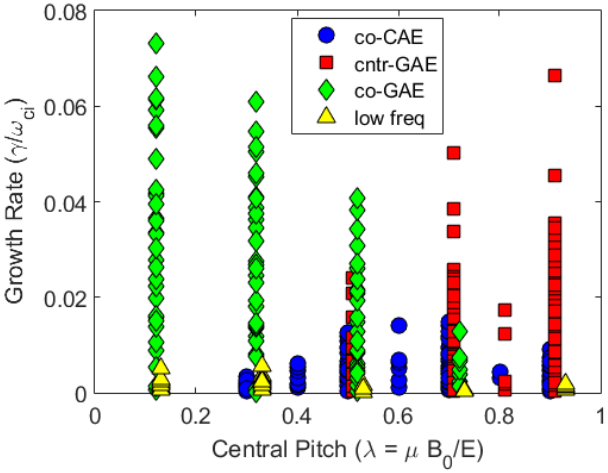
- At each n, CAE and GAE modes appear at distinct frequencies
- Co-GAE seen in simulations at large |n| (>7) are yet to be thoroughly investigated experimentally
  - Weakly unstable near the boundary of realistic NSTX beam geometry
- Spectrum becomes much more rich as $v_0/v_A$ increases past 4.5

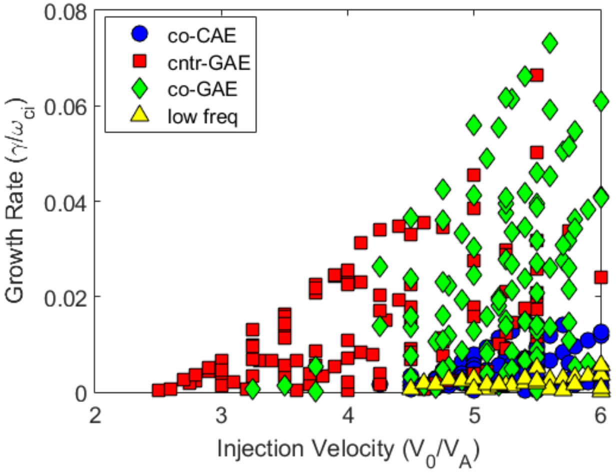


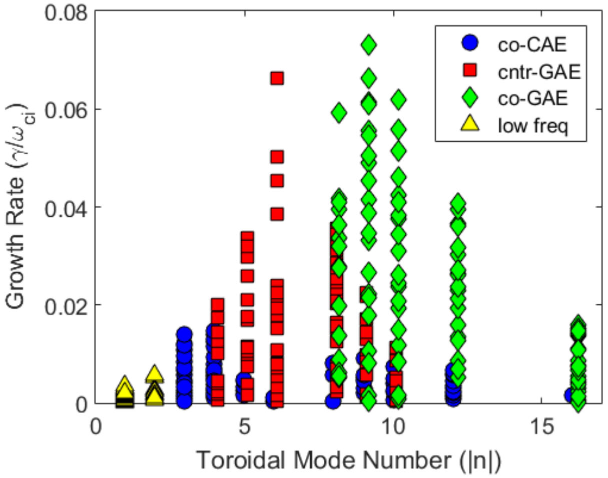


# GAE Typically More Unstable than CAE

- Largest growth rates occur for n ~ 6 – 10
- GAE mostly co-rotating when injecting low $\lambda_0$ beam (tangential), counter-rotating for large $\lambda_0$ (perpendicular)
- Almost exclusively cntr-GAE for $2.5 < v_0/v_A < 4$
- CAE are strictly more stable than GAE for $v_0/v_A < 4$
  - Implications for $T_e$ flattening mechanism
    - A difference in the amount of $T_e$ flattening near this value of $v_0/v_A$ could indicate which type of mode (CAE vs GAE) is most responsible for the unexplained thermal energy transport

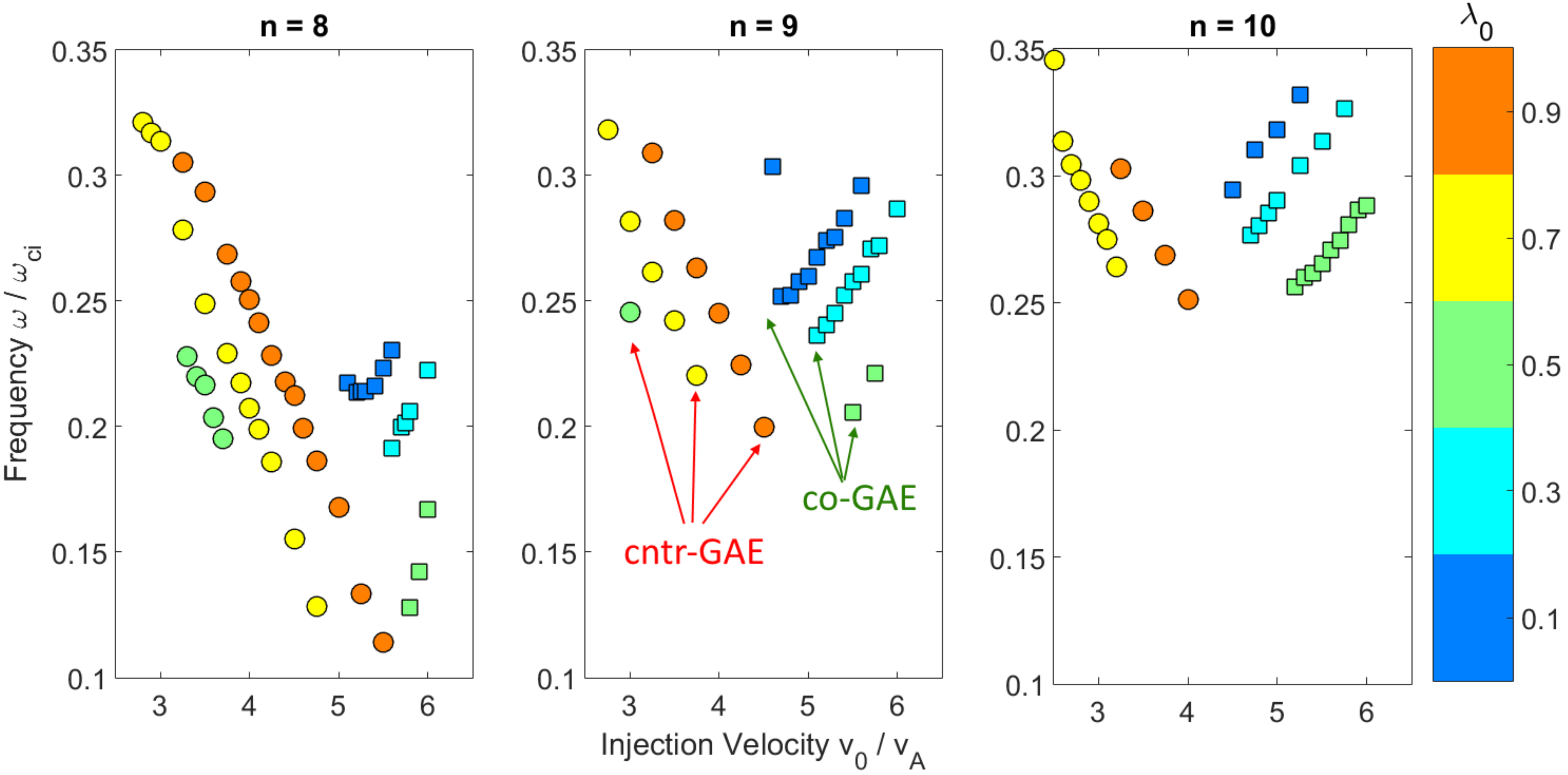






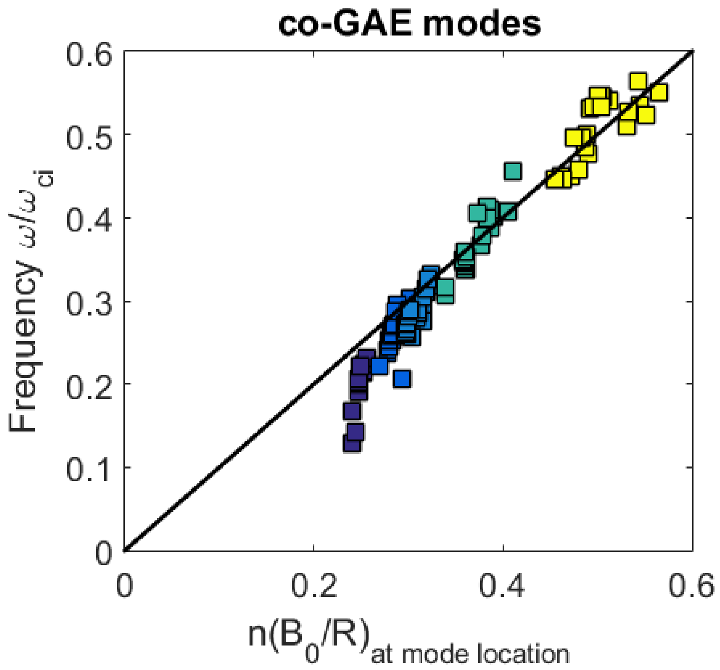
# GAE Frequencies Shift with $\lambda_0$ & $v_0$

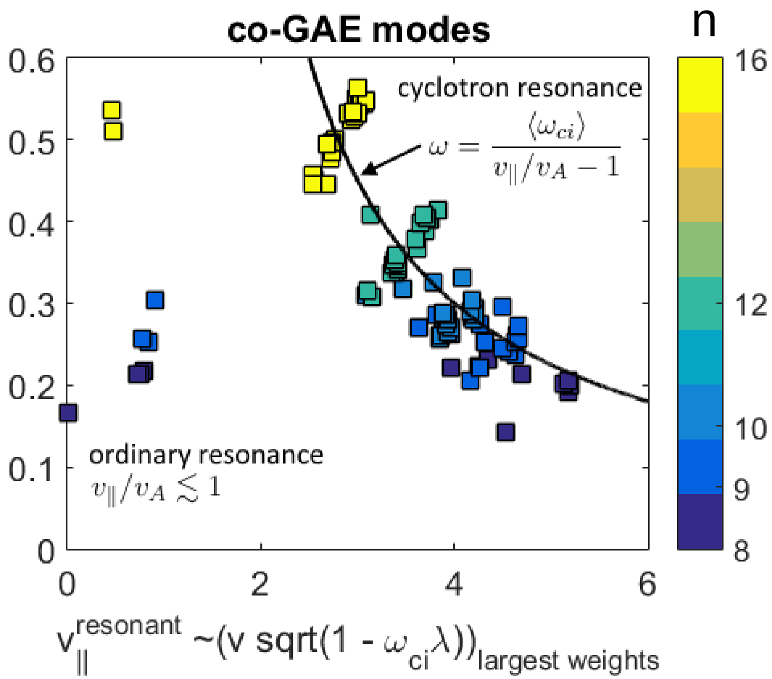
- Opposite trends for cntr-GAEs (circles) and co-GAEs (squares)
- Are these true MHD modes or high frequency EPM?
- In contrast, CAE frequencies change only slightly



# Co-GAE Dispersion and Resonance

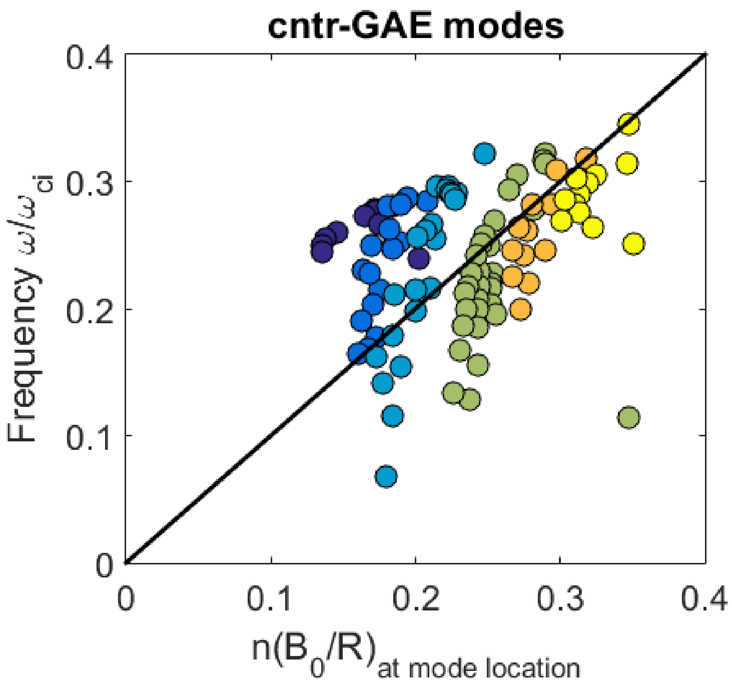
- Approximate dispersion $\omega \approx k_{\parallel} v_A$ is excellent fit to n dependence of frequencies
  - – Fit is much worse when using B,R on-axis vs B,R at mode location
- Cyclotron resonance $\omega - k_{\parallel}\langle v_{\parallel}\rangle = -\langle \omega_{ci}\rangle$ decent fit to $v_{\parallel}$ dependence of frequencies
  - – Resonant $v_{\parallel}$ not necessarily near the injected $v_{\parallel}$

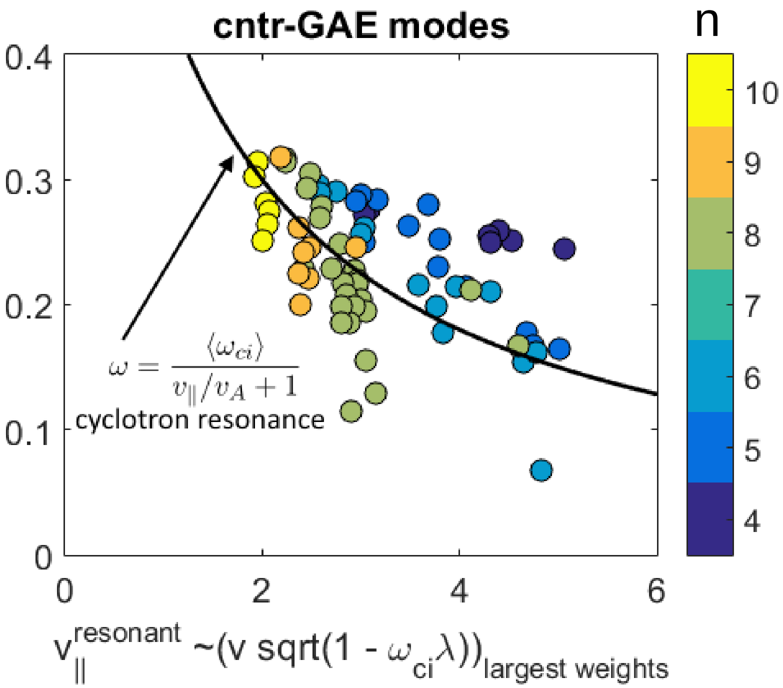




# Counter-GAE Dispersion and Resonance

- Approximate dispersion $\omega \approx k_{\parallel} v_A$ is very rough fit to n dependence in frequencies
  - $k_{\parallel} = (n - m/q)/R \approx n/R$ is worse approximation for cntr-GAEs than co-GAEs due to lower n and higher m harmonics
- Cyclotron resonance $\omega - k_{\parallel}\langle v_{\parallel}\rangle = \langle \omega_{ci} \rangle$ captures qualitative $v_{\parallel}$ dependence of frequencies
  - Resonant $v_{\parallel}$ not necessarily close to the injected $v_{\parallel}$





# $\omega_{GAE}$ also Changes with Fixed Equilibrium

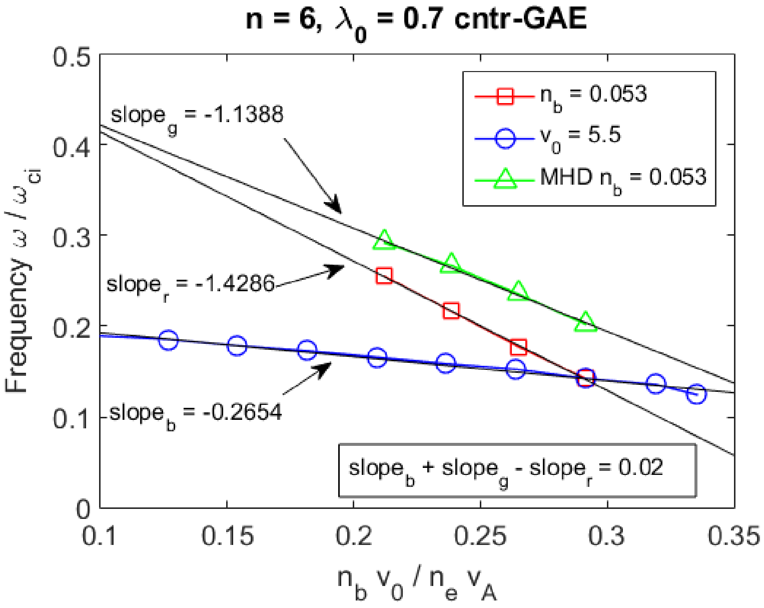
- Equilibrium solver is self-consistent → increasing $v_0/v_A$ increases $J_{beam}/J_{plasma}$, modifying the equilibrium. Must disentangle “equilibrium effects” from “EP phase space effects” on frequencies
  - Method 1: Fix beam energy and vary beam density to make the equilibrium more MHD-like without changing the energetic particle phase space
  - Method 2: Remove energetic particles from equilibrium equations (no longer self consistent), fix beam density and vary beam energy to manipulate the EP phase space
- **Large frequency changes are still observed when varying $v_0/v_A$ with identical equilibria**

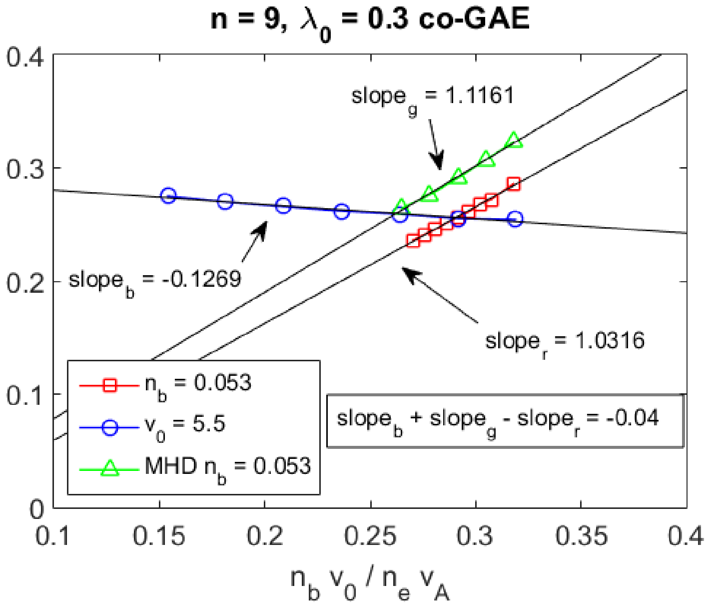
**LEGEND**

Self-consistent equilibrium, varying $v_0$ with constant $n_b$

Self-consistent equilibrium, varying $n_b$ with constant $v_0$

**NON** self-consistent, “MHD-like” equilibrium, varying $v_0$ with constant $n_b$



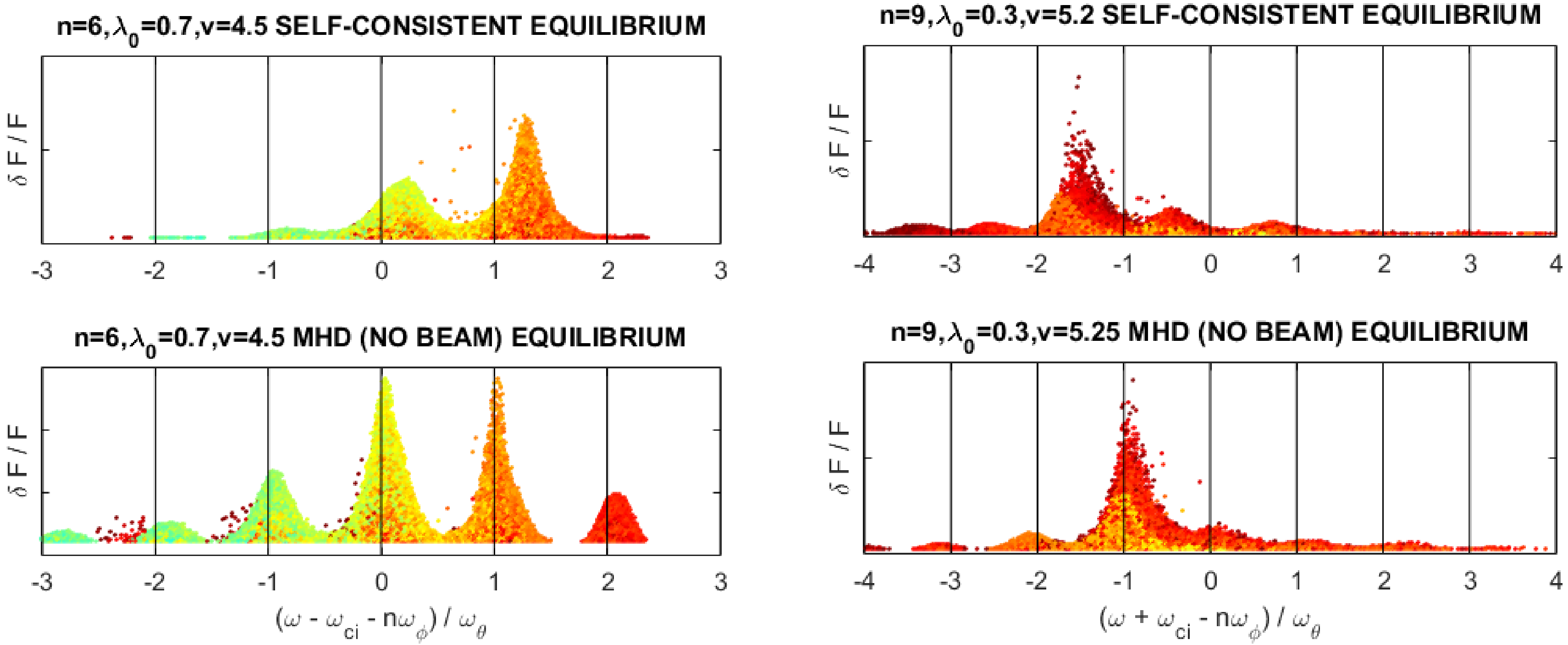


# EPMs in Disguise?

- Large and unexpected changes in frequency with changes in EP distribution is uncharacteristic of MHD modes
- Even when excluding EP effects on equilibrium, different EP distributions with the same equilibrium lead to large changes in frequency
  - Quantitatively described by resonance condition
- Frequencies are usually near expected GAE freq.
- Yet there are not always (or even typically?) clear and substantial modifications to the mode structure corresponding to these large changes in frequency
- Are these modes more accurately regarded as high frequency energetic particle modes (EPM)?

# GAE resonances nicer in MHD equilibrium

- Resonances are often shifted from integers for GAEs simulated with self-consistent equilibrium
  - In contrast, CAEs line up extremely well with integers
    - Key physics difference; CAEs obey ordinary resonance $\omega = k_\parallel v_\parallel$ vs the more complicated Doppler-shifted cyclotron resonances for the GAEs.
- Incidental finding in non self-consistent simulations of GAEs: resonances are much better aligned with integers when beam effects are excluded from equilibrium



n=6,$\lambda_0$=0.7,v=4.5 SELF-CONSISTENT EQUILIBRIUM

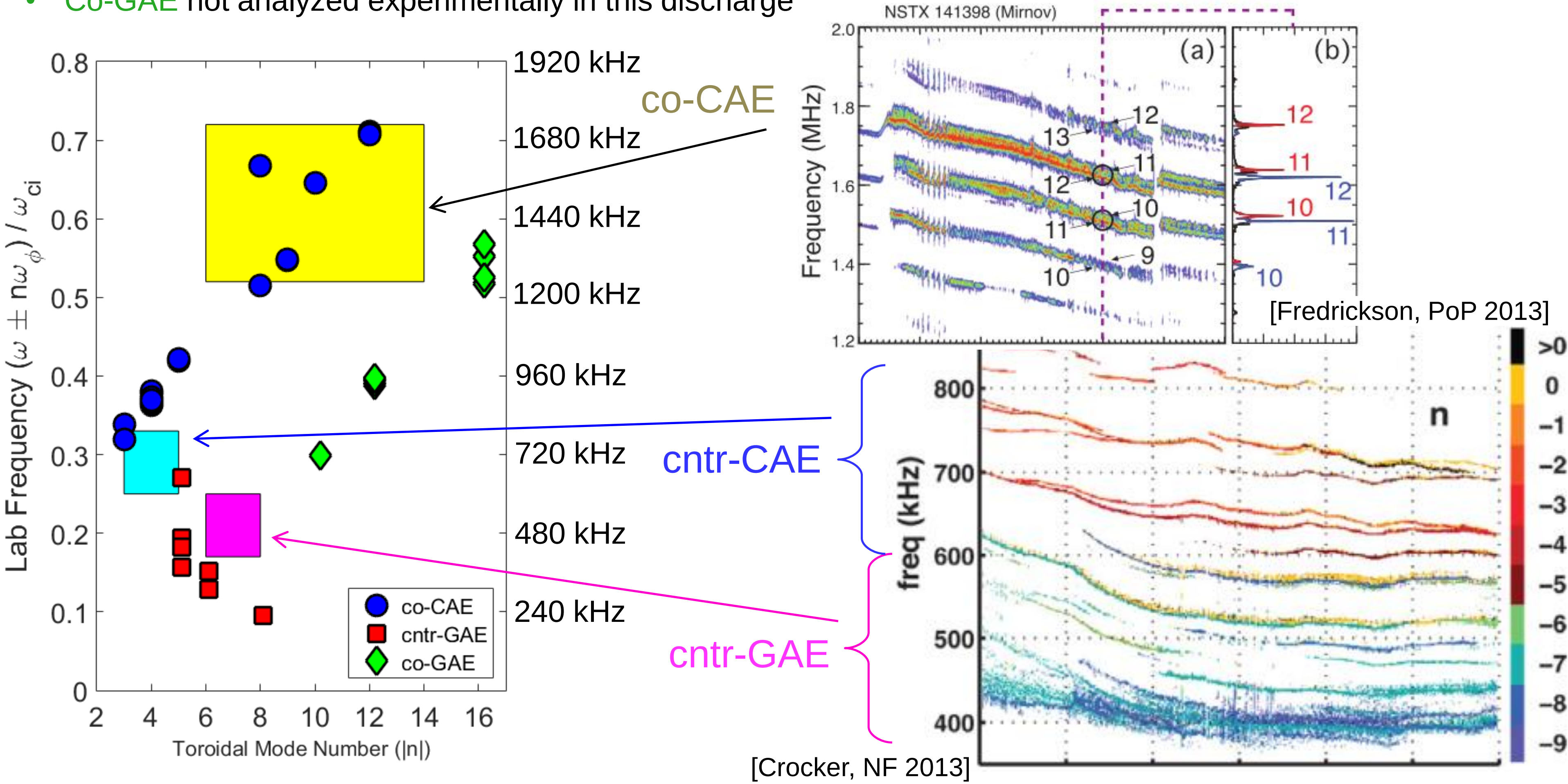
n=9,$\lambda_0$=0.3,v=5.2 SELF-CONSISTENT EQUILIBRIUM

n=6,$\lambda_0$=0.7,v=4.5 MHD (NO BEAM) EQUILIBRIUM

n=9,$\lambda_0$=0.3,v=5.25 MHD (NO BEAM) EQUILIBRIUM

# Experimental vs Simulated Mode Spectrum

- From TRANSP, realistic parameters for NSTX shot 141398 are $\lambda_0$ = 0.5 – 0.7, $v_0/v_A$ = 4.75 – 5.25
- Co-CAE agree with high frequency observations, <u>disagree on direction</u> at lower frequency range
- Cntr-GAE simulations near but below experimental measurements
- Co-GAE not analyzed experimentally in this discharge

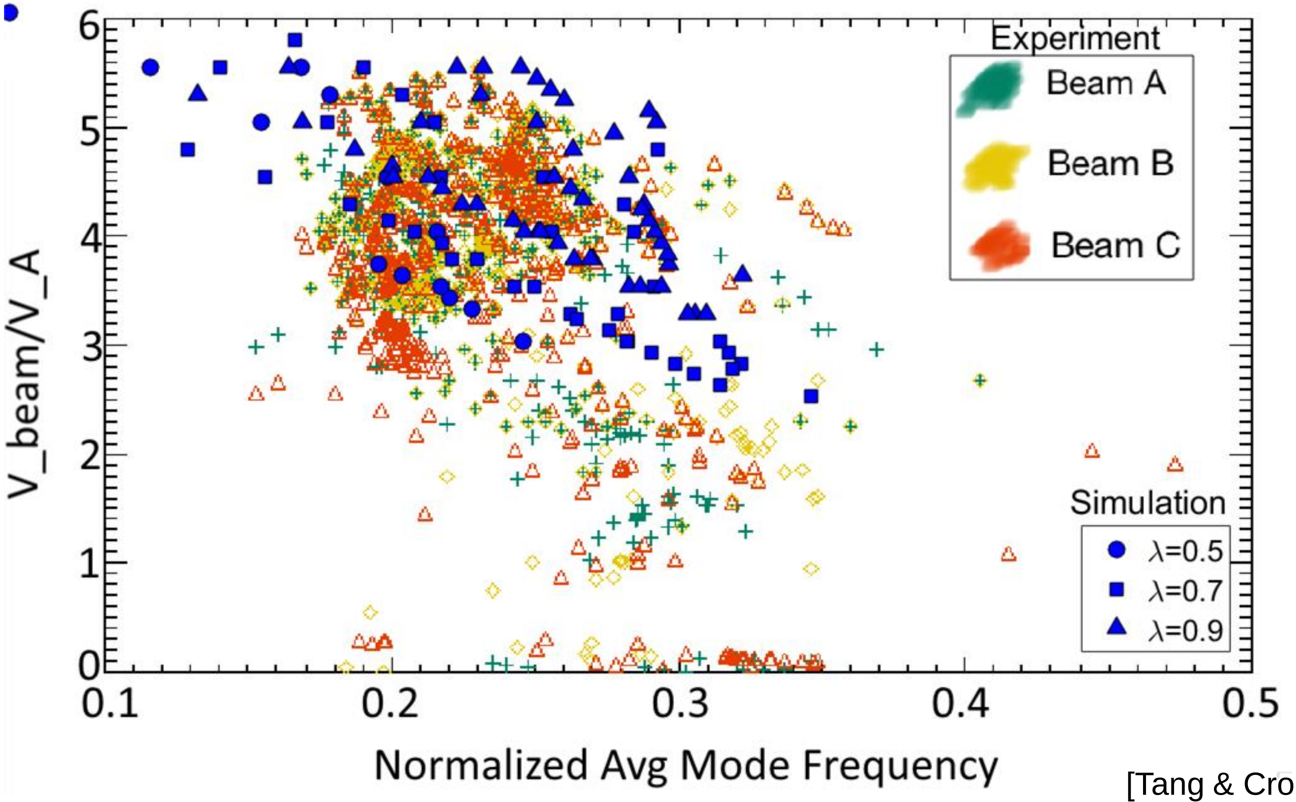


[Fredrickson, PoP 2013]

[Crocker, NF 2013]

# Mode Spectrum Dependence on $v_0/v_A$

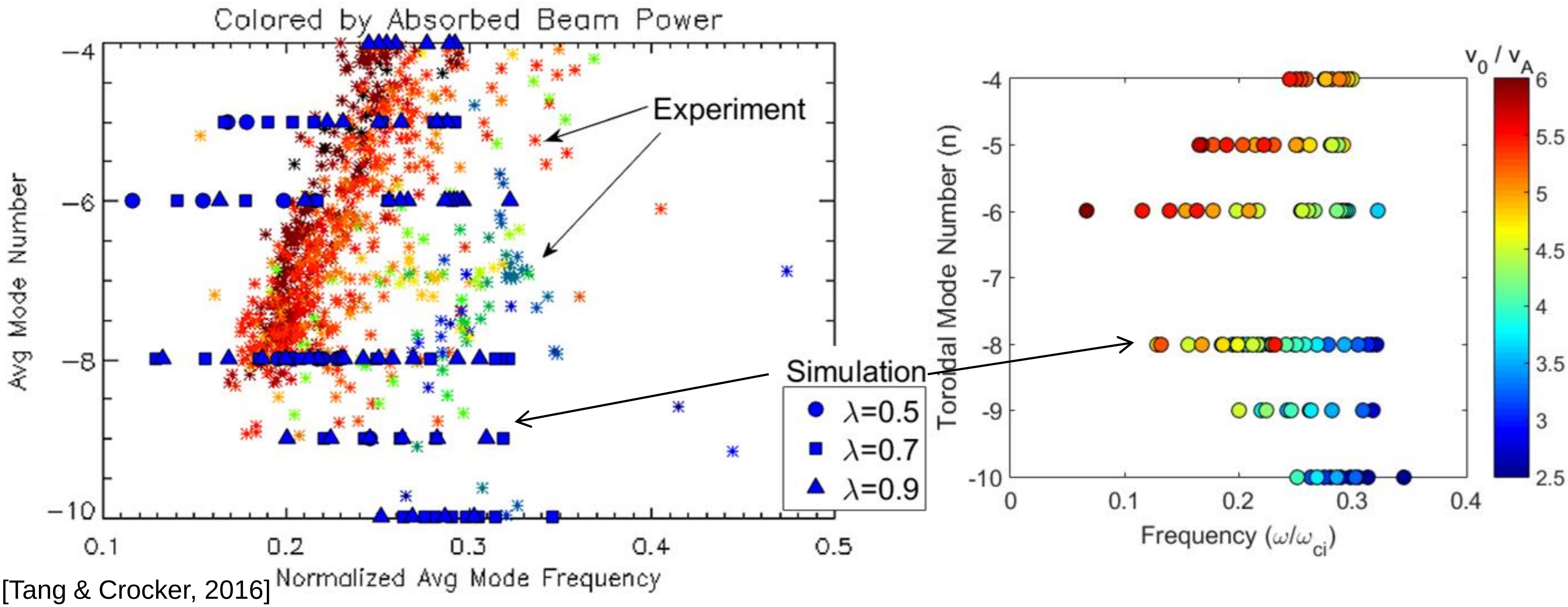
- Red/yellow/green points from experimental survey of 50ms intervals in NSTX discharges satisfying:
  - (1) $T_e > 500$ eV, (2) <f> > 200 kHz, (3) -10 < <n> < -4
- Blue marks: *individual* cntr-GAE modes from simulations
- Some qualitative agreement, but very preliminary – requires more direct comparison
  - Individual mode comparison can test simulation predicted stability boundaries in $v_0/v_A$



[Tang & Crocker, 2016]

# Mode Spectrum Dependence on n

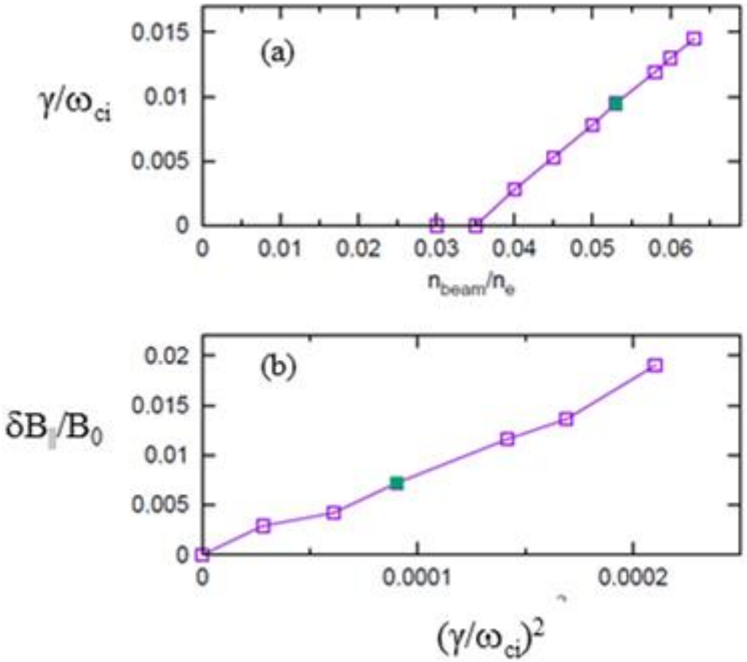
- Red/yellow/green points from experimental survey of 50ms intervals in NSTX discharges satisfying:
  - (1) $T_e > 500$ eV, (2) $<f> > 200$ kHz, (3) $-10 < <n> < -4$
- Blue marks: *individual* cntr-GAE modes from simulations
- Reasonable agreement for $|n| > 7$, and correct trends along $v_0/v_A$ contours
- Also requires more direct comparison



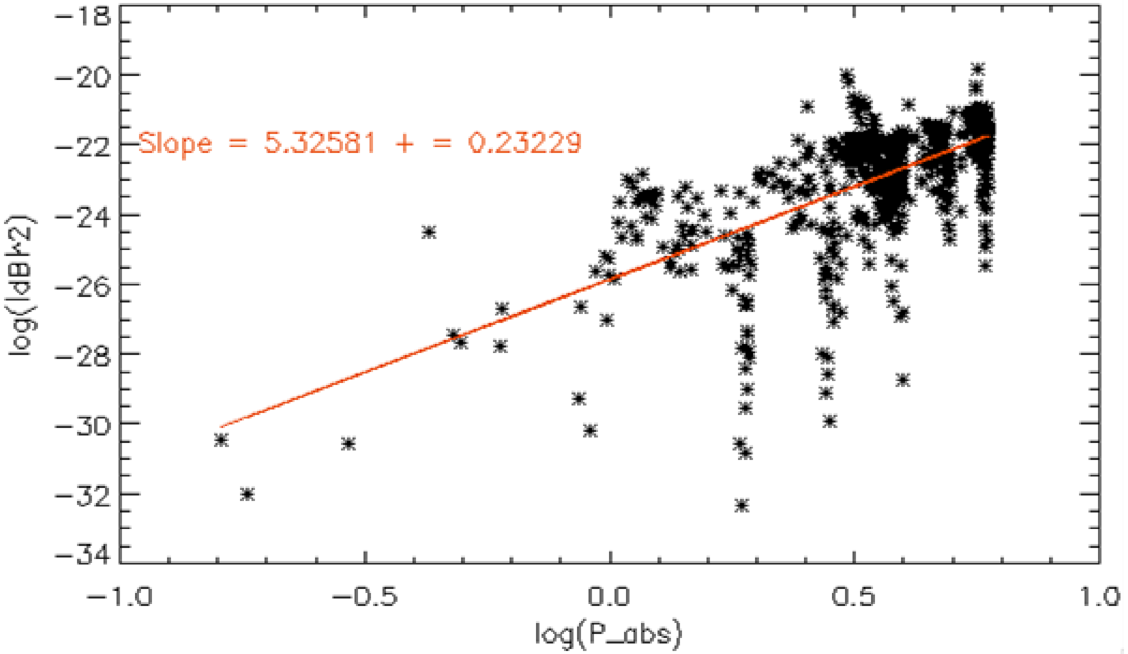
[Tang & Crocker, 2016]

# Modes Saturate with $\gamma^2$

- Nonlinear simulations of n = 4 CAE reveal $\delta b \propto \gamma^2 \propto P_b^2$
  - Consistent with saturation via particle trapping
- Experimental database shows $\delta b \propto P_b^{2.6}$ when fitting all CAE/GAE modes
- Hence power transferred to KAW scales $\propto \gamma \delta b^2 \propto \gamma^5 \propto P_b^5$
  - Very strong beam power dependence implies energy channeling should be strong effect in high power NSTX-U discharges
    - Though CAE could be more stable in general due to larger nominal $B_0$



[Tang & Crocker, 2016]

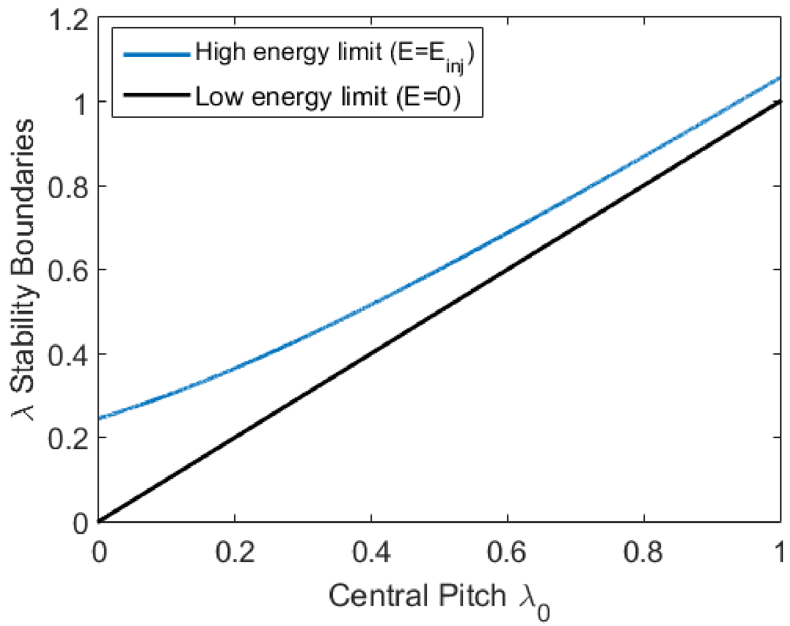


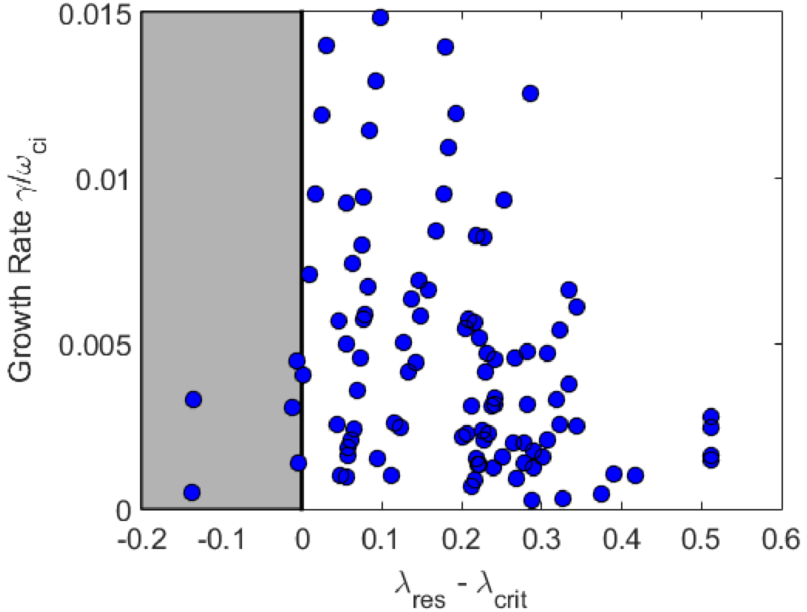
# Qualitative Co-CAE Stability Theory

- Drive for $l = 0$ resonance is proportional to [Belikov, PoP 2003] $\nabla_{\mathbf{v}} \mathrm{F_b}(\mathrm{E}, \mu) \propto \frac{\partial F_b(E,\mu)}{\partial E} = \frac{\partial F_b}{\partial E} - \frac{\lambda}{E} \frac{\partial F_b}{\partial \lambda} > 0$ for instability
- Simulated $\mathrm{F_b}$ is slowing down in E, Gaussian in pitch
  - First term is stabilizing for the entire phase space
  - Second term is stabilizing for $\lambda < \lambda_0$ and **destabilizing** for $\lambda > \frac{\lambda_0}{2}\left(1 + \sqrt{1 + \frac{8}{3}\left(\frac{\Delta\lambda}{\lambda_0}\right)^2}\right) \equiv \lambda_{crit}$
- Resonance condition $\omega = k_{\|} v_{\|}$ with approximate relation $v_{\|} \approx v\sqrt{1 - \langle\omega_{ci}\rangle\lambda}$ yield the resonant $\lambda$:
  - $\lambda_{res} \approx \frac{1}{\langle\omega_{ci}\rangle}\left(1 - \frac{\omega^2}{k_{\|}^2 v^2}\right)$

# Unstable CAE Modes vs Theory

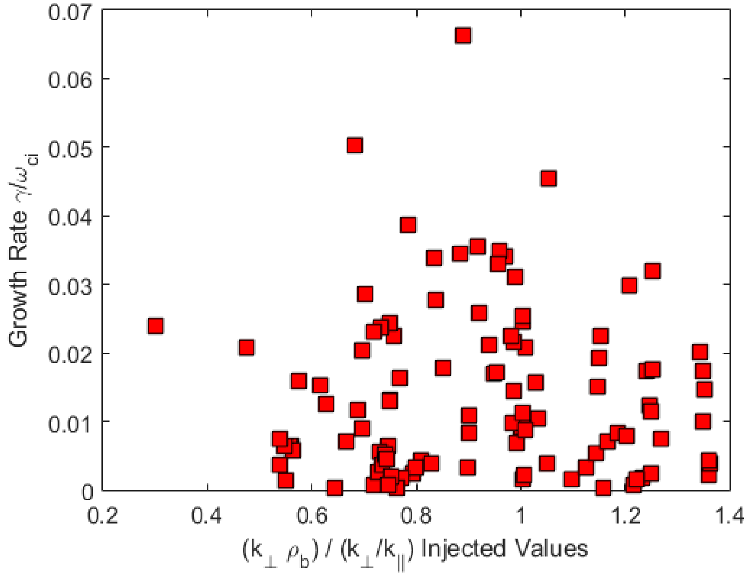
- $\lambda_{res} > \lambda_{crit}$ is a necessary condition for instability
- Check against simulations with assumptions:
  - $\langle\omega_{ci}\rangle \approx 0.9\omega_{ci0}$ (good), $k_{\parallel} \approx \frac{n}{R}$ (okay), $v_{res} \approx v_0$ (unreliable)
- CAEs in simulation usually satisfy this instability condition (predicts unshaded region to be unstable)
- Beilkov *et. al.* also claim $v_0/v_A > 4$ is necessary for co-CAEs to be preferentially driven by *trapped* particles – remarkably similar to the stability boundary demonstrated in these simulations (earlier slide) [Belikov, PoP 2004]

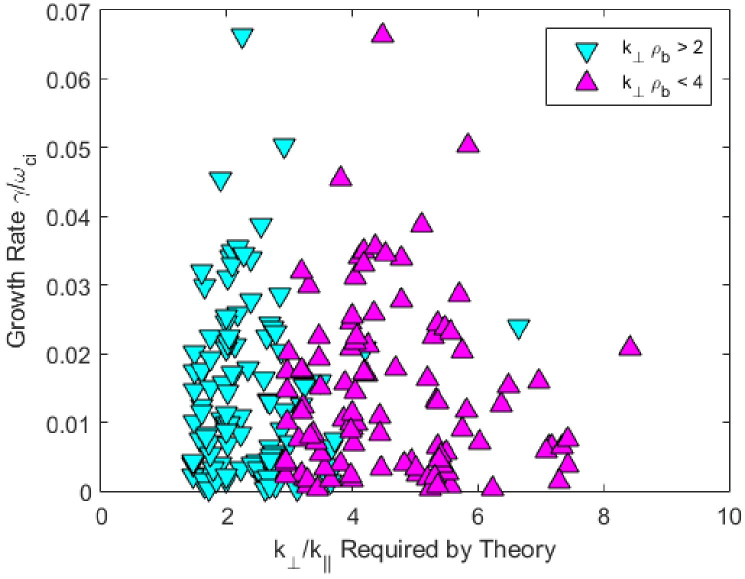




# Cntr-GAE Stability Theory

- According to theory, $2 < k_\perp \rho_b < 4$ required for cntr-GAE instability with NSTX-like EP distributions [Gorelenkov, NF 2003]
- From dispersion, $k_\perp \approx \frac{\omega}{v_A}\left(\frac{k_\perp}{k_\parallel}\right)$, hence $k_\perp \rho_b = \frac{\omega}{\omega_{ci}} \frac{v_\perp}{v_A}\left(\frac{k_\perp}{k_\parallel}\right) \approx \frac{\omega}{\omega_{ci}} \frac{v_0\sqrt{\langle \omega_{ci} \rangle \lambda_0}}{v_A}\left(\frac{k_\perp}{k_\parallel}\right)$
- Unfortunately, $k_\perp / k_\parallel$ is not known for each mode, though it is expected to be much larger than 1 (usual tokamak limit)
- Encouragingly, the other factors in $k_\perp \rho_b$ show a clustering in growth rates (left plot)
- The inferred values of $k_\perp / k_\parallel$ necessary for the unstable modes to obey the theoretical instability conditions are generally $k_\perp / k_\parallel$ = 2 – 6 (right plot)
  - Challenging to verify because mode structure is not usually well-aligned with $\psi$ contours → obfuscates poloidal mode number and thus expressions for $k_\parallel$ and $k_\perp$





# Summary and Conclusions

- 3D hybrid simulations were performed for a wide range of beam parameters to investigate CAEs/GAEs in NSTX plasmas
- Cntr-GAEs have best overall agreement between simulations and experimental observations
- Co-GAEs are observed in simulations to be quite unstable for very tangential beam distributions, but little experimental or analytic work exists on these modes
- CAEs in simulation disagree with direction of propagation observed in experiment in moderate frequency band
- CAEs are more stable than GAEs for $v_0/v_A < 4$
  - Implications for dominant mechanism of anomalous $T_e$ flattening
- GAE frequencies depend strongly on beam parameters without clearly corresponding changes in mode strucure
  - Quantitatively explained by GAE dispersion + resonance, but may be better described as a new, high frequency EPM?

# Ongoing and Future Work

- Direct single mode simulation/experiment comparison
- Continued investigation of GAE vs EPM question
- What causes the GAE resonances to be shifted, and why are the shifts so diminished when neglecting the (often large) EP contributions to the equilibrium?
- Analytic descriptions of stability boundaries
- Compare relative importance of enhanced electron diffusion and energy channeling in various regimes
- Predictions for ITER and other devices which may routinely access the EP parameter space necessary to excite these modes

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# Backup Slides

# HYM Physical Equations

## Background plasma - fluid:

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla p + (\mathbf{j} - \mathbf{j}_b) \times \mathbf{B} - n_b (\mathbf{E} - \eta \mathbf{j})$$

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{j}$$

$$\mathbf{B} = \mathbf{B}_0 + \nabla \times \mathbf{A}$$

$$\partial \mathbf{A} / \partial t = -\mathbf{E}$$

$$\mathbf{j} = \nabla \times \mathbf{B}$$

$$\partial p^{1/\gamma} / \partial t = -\nabla \cdot (\mathbf{V} p^{1/\gamma})$$

$$\partial \rho / \partial t = -\nabla \cdot (\mathbf{V} \rho)$$

## Fast ions – delta-F scheme:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \mathbf{E} - \eta \mathbf{j} + \mathbf{v} \times \mathbf{B}$$

$w = \delta F / F$ - particle weight

$$\frac{dw}{dt} = -(1 - w) \frac{d(\ln F_0)}{dt}$$

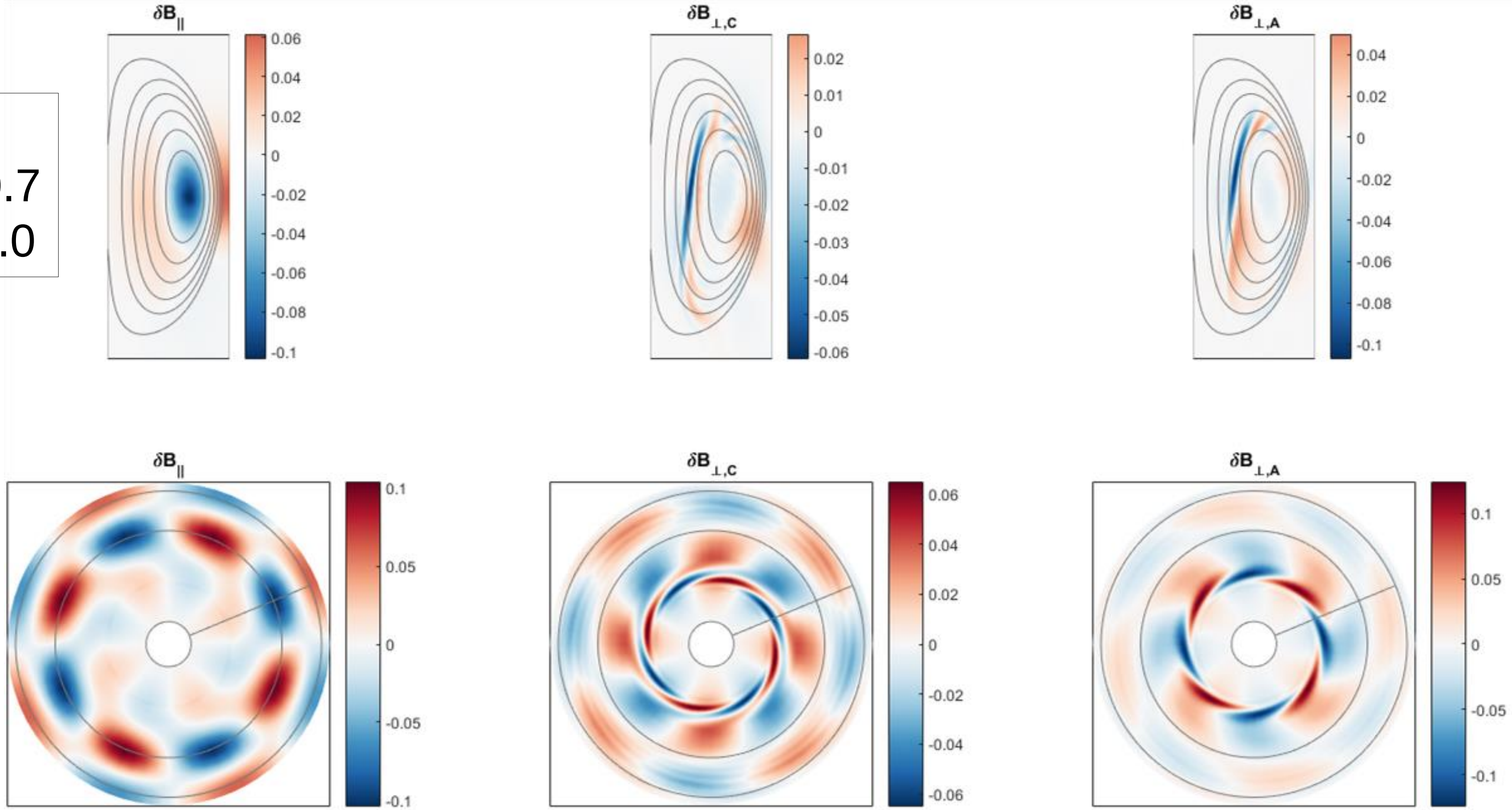
$$F_0 = F_0(\varepsilon, \mu, p_\phi)$$

$\rho$, **V** and $p$ are thermal plasma density, velocity and pressure, $n_b$ and $\mathbf{j}_b$ are beam ion density and current, and $n_b << n_e$ – is assumed.

# Co-CAE Mode Converts to KAW

- CAE typically m = 0.5 – 2, peaking on axis
- KAW structure visible in $\delta B_\perp$ fluctuation on HFS

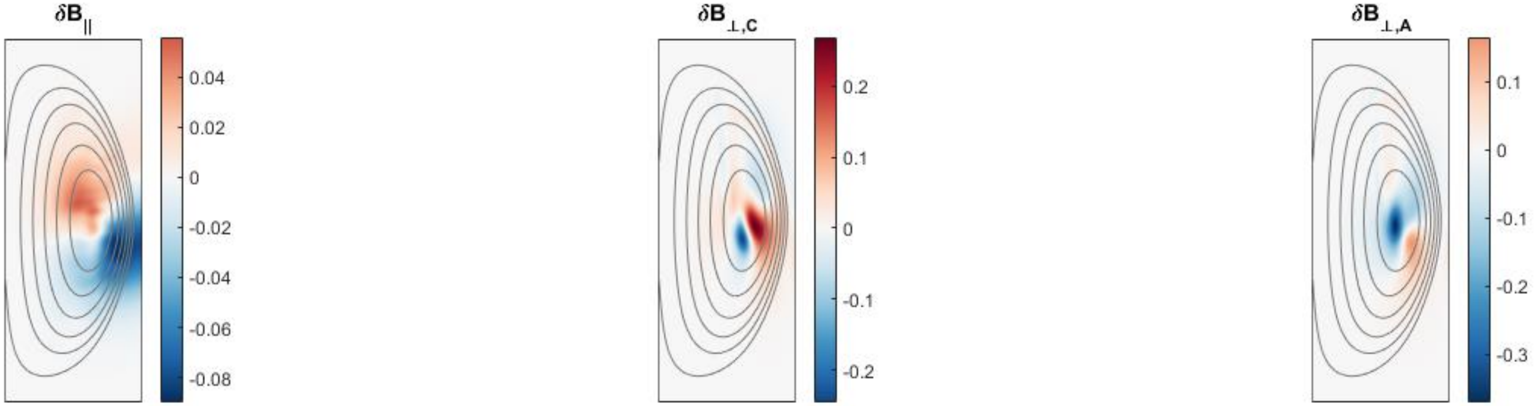
n=4
$\lambda_0$=0.7
$v_0$=5.0

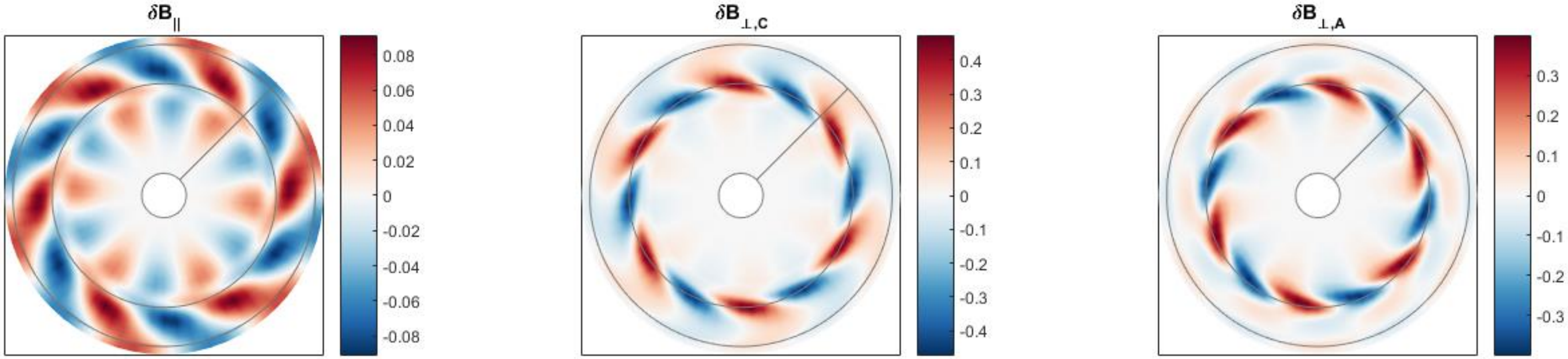


# Counter-GAE are Core-Localized

- Typically m = 1.5 – 2.5
- Often has large compressional component near edge

n=6
$\lambda_0$=0.7
$v_0$=4.5

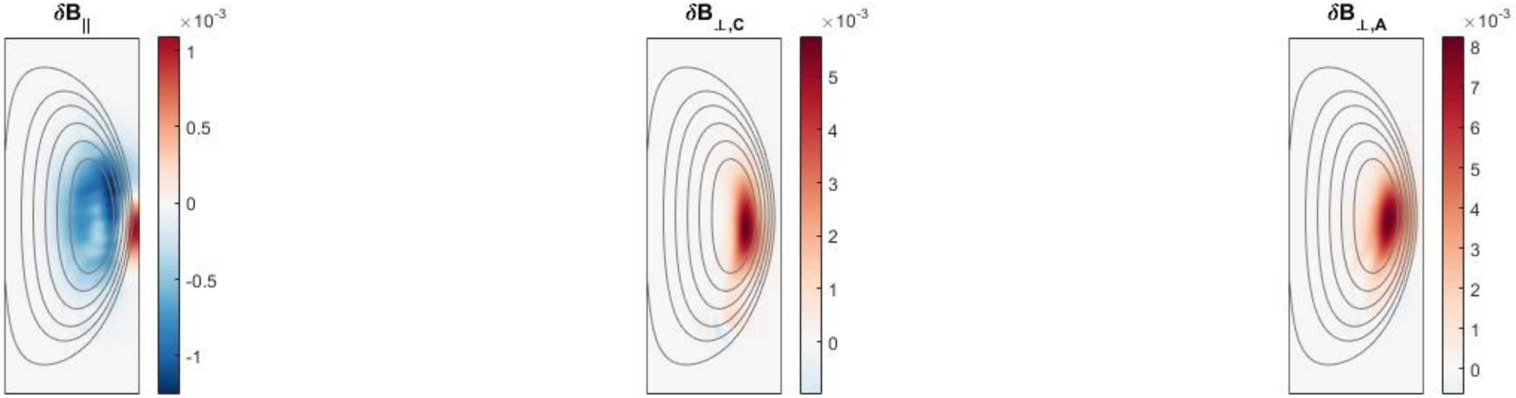


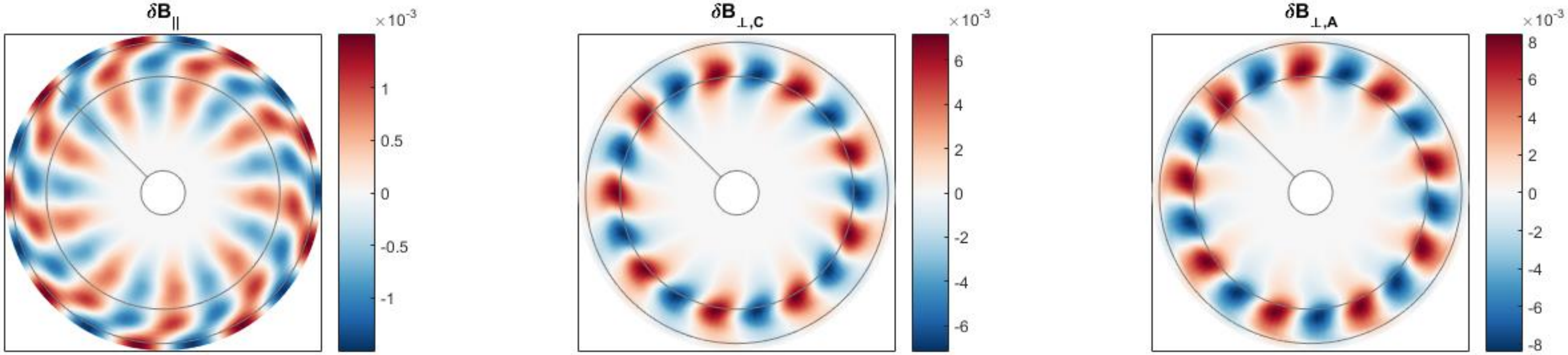


# Co-GAE are also Core-Localized

- Typically very low m = 0.5 – 1
- Not commonly observed in experiment?

n=9
$\lambda_0$=0.3
$v_0$=5.2

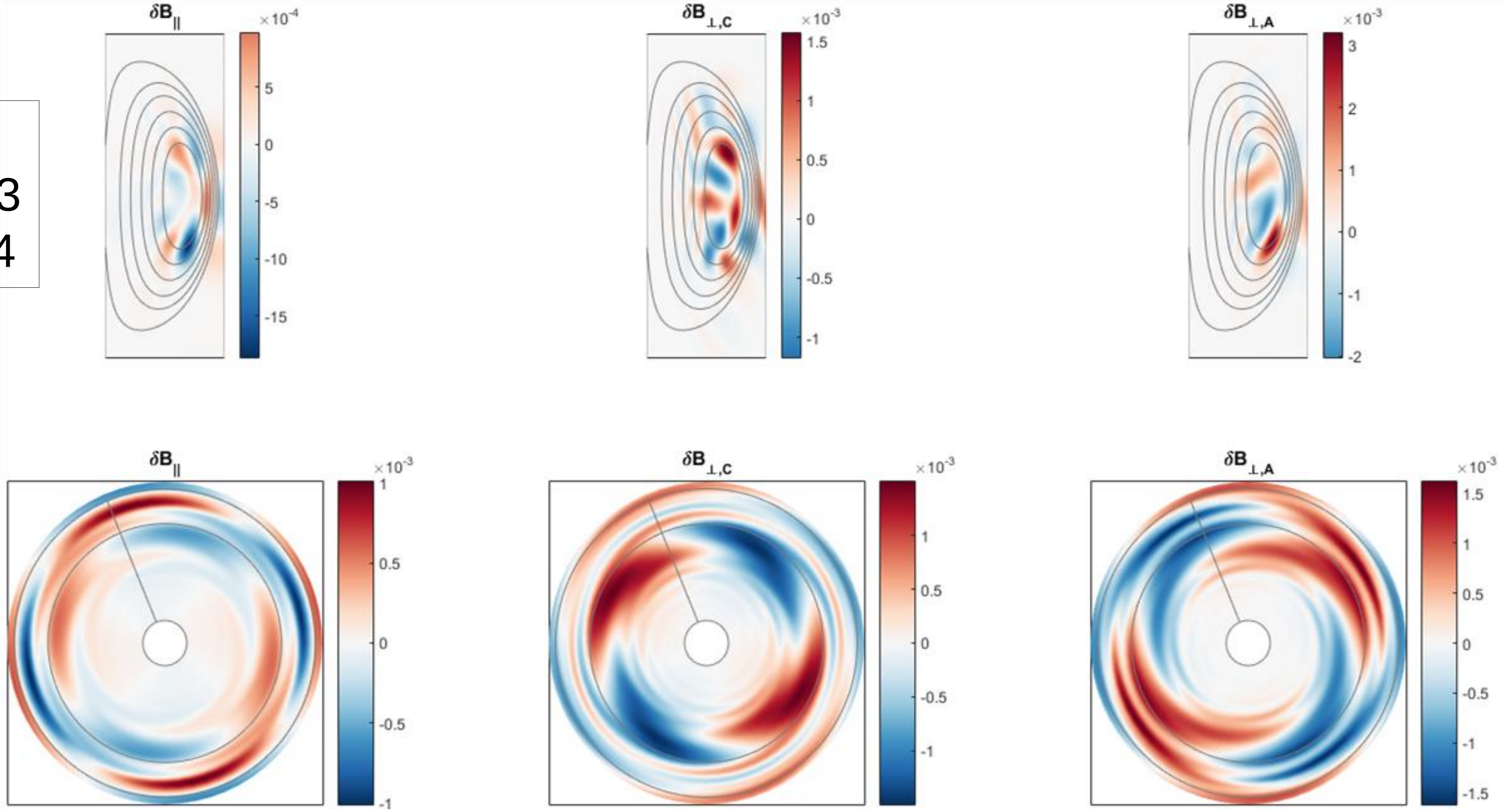




# Low Frequency Modes are Qualitatively Different

- Typically m ≥ 3 (as large as m~6)
- What are they? TAE/fishbone/EPM/???

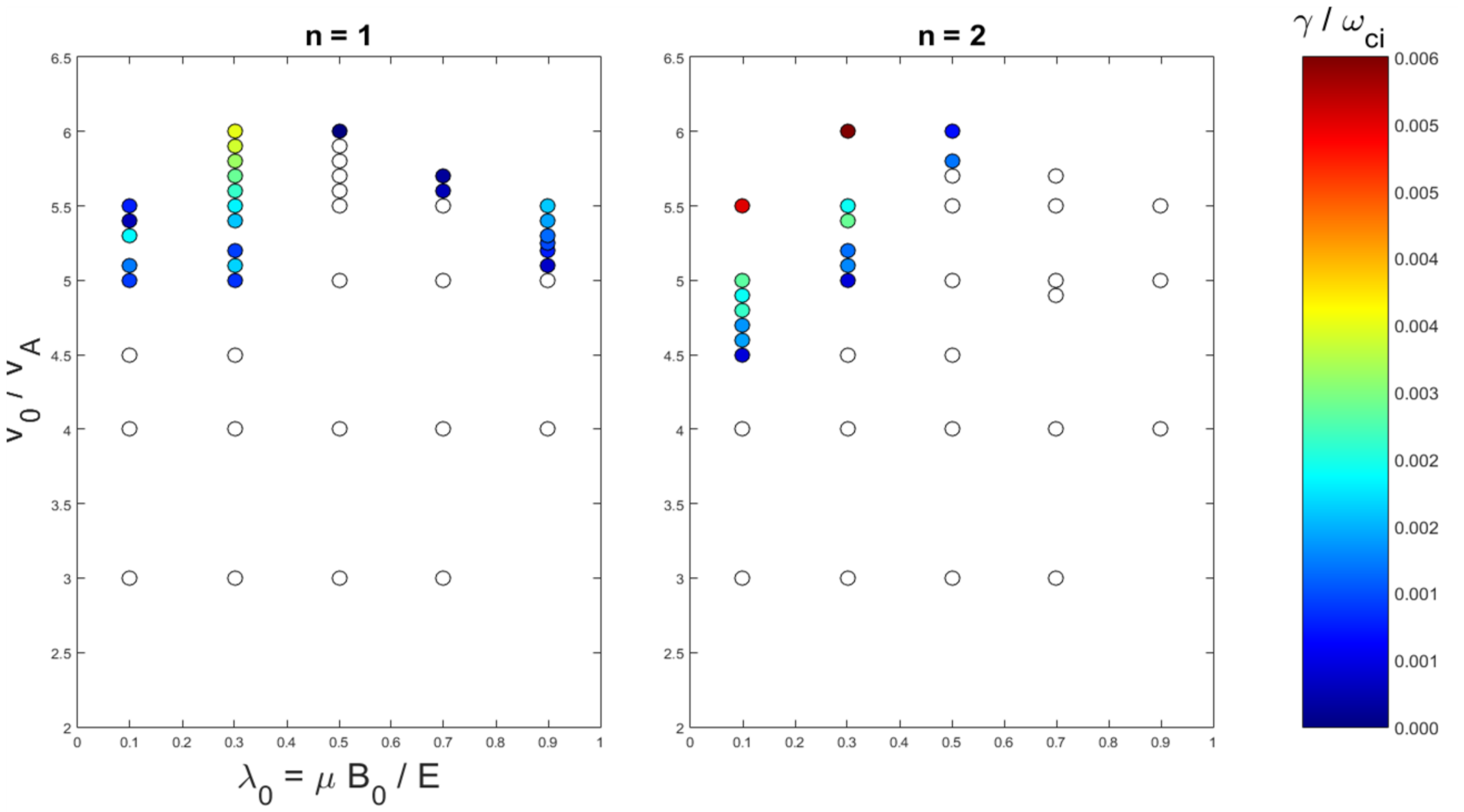
n=2
$\lambda_0$=0.3
$v_0$=5.4



# Unexpected Low-Frequency Modes

- **Frequencies : $\omega \sim 0.01 - 0.05\omega_{ci} = 2\pi(25 - 125)$ kHz**
- **Growth rates : $\gamma/\omega = 0.013 - 0.37$, $\gamma_{max} = 0.073\omega_{ci} = 90$ kHz**
- **Require large $v_0$ and either very high or low $\lambda_0$**

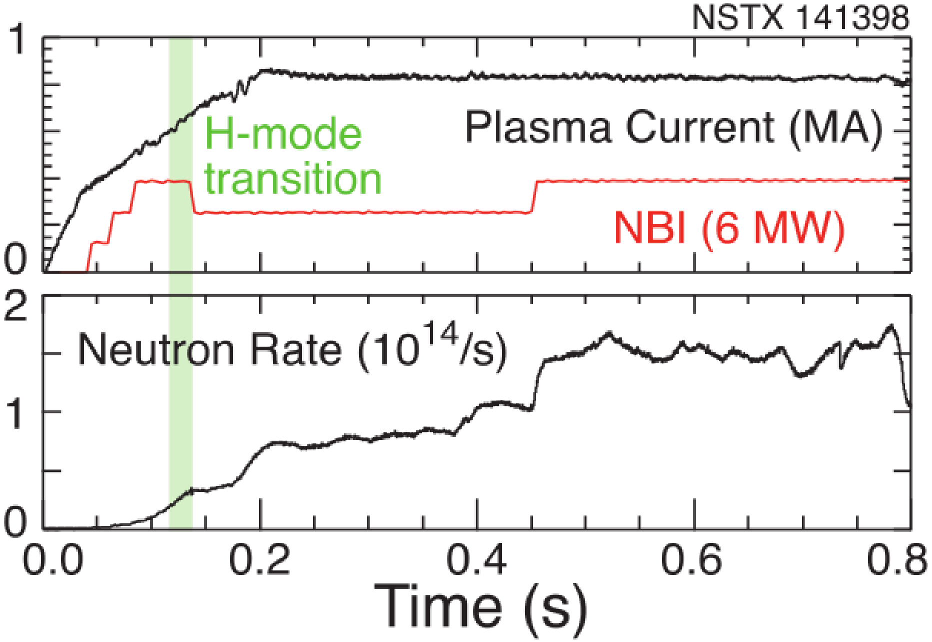
- Colored circles: low n mode is most unstable mode
- Gray circles: Different mode has larger growth rate
- White circles: No unstable modes of any type

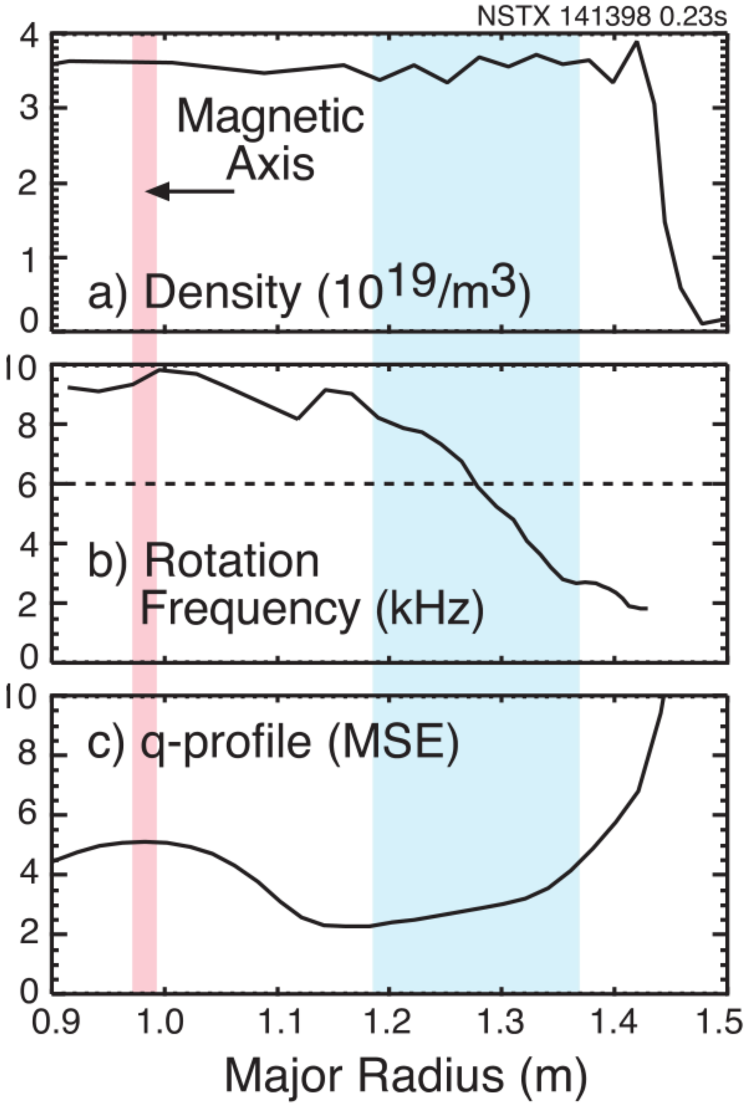


# NSTX H-mode Shot 141398

- $n_e = 6 \times 10^{19}\ \mathrm{m}^{-3}$
- $B_{tor,0}$=0.325 T
- $I_p$ = 0.8 MA
- $f_{ci,0}$=2.4 MHz

- 90 keV NBI at 6 MW
- $v_0 = 4.9\ v_A$
- $\lambda_0 = 0.7$
- $n_b/n_e \sim 5\%$



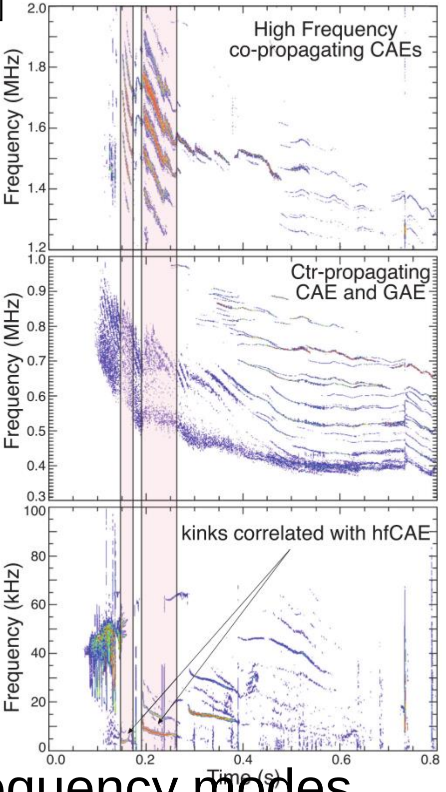


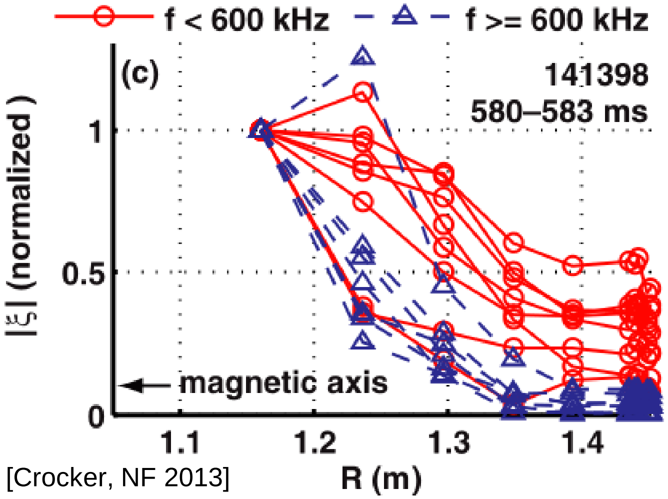
[Figures from Fredrickson, PoP 2013]

# CAE & GAE Observed in Experiment

- Fredrickson observes 3 groups of modes
  - Co-propagating CAE
    - n=6-14, $0.5\omega_{ci} < \omega < 0.75\omega_{ci}$
  - Counter-propagating CAE & GAE
    - $0.15\omega_{ci} < \omega < 0.35\omega_{ci}$
  - Kinks correlated with high frequency co-CAE
    - $\omega \sim .005\omega_{ci}$

[Fredrickson, PoP. 2013]



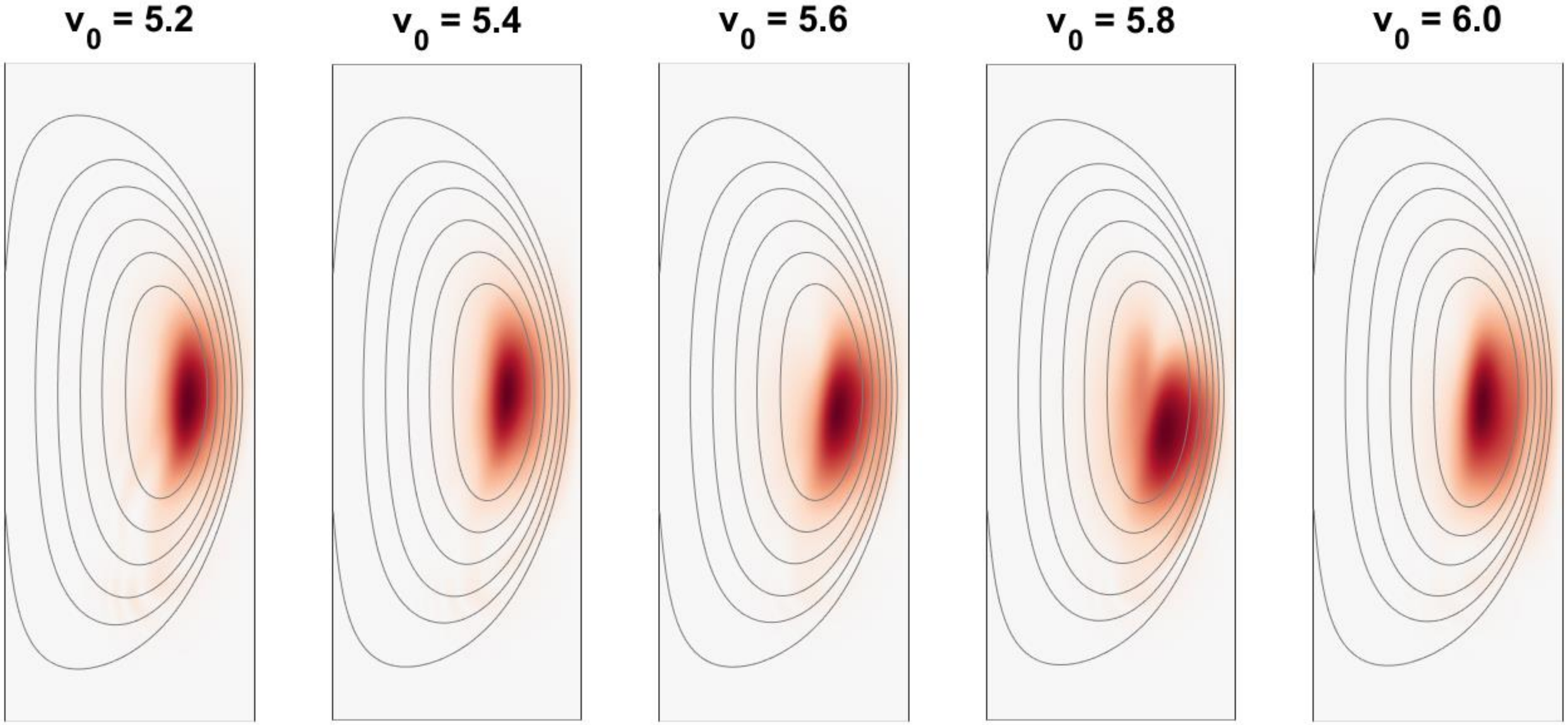


[Crocker, NF 2013]

- Crocker studies moderate frequency modes
  - $\omega > 0.25\omega_{ci}$ CAE
    - Mostly -3 < n < -5, more core-localized
  - $\omega < 0.25\omega_{ci}$ GAE
    - Mostly -6 < n < -8, broad mode structure

# Co-GAE structure mostly unchanged

- Below: $\delta b_\perp$ for n=9, $\lambda_0$=0.3 co-GAE with $v_0/v_A$=5.2-6.0
- $\omega/\omega_{ci}$ increases linearly from 0.24 to 0.29
- Mode structure does not change qualitatively
  - Same eigenmode or different in subtle way?



# Resonant Quantities

- To do: if time permits, would be nice to comment on the sometimes large variance between injection quantities and beam quantities
  - E.g. for CAEs, resonant lambda is typically opposite injected lambda (inject passing particles → modes driven by trapped particles, and the converse)