Interaction between high harmonic fast waves and fast ions in NSTX/NSTX-U plasmas

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NSTX-U

Abstract

This paper examines the interaction between fast waves and fast ions generated by NBI in NSTX plasmas by using the recent extension of the RF full-wave code TORIC to include non-Maxwellian ions distribution functions. Tests on the RF kick-operator implemented in the Monte-Carlo particle code NUBEAM is also discussed in order to move towards a self consistent evaluation of the RF wave-field and the ion distribution functions in the TRANSP code.

TORIC v.5 code

- The TORIC v.5 code solves the wave equation for the electric field $\mathbf{E}$:
- TORIC v.5 uses a Maxwellian plasma dielectric tensor

$$\varepsilon \equiv \mathbf{I} + \frac{4\pi i}{\omega}\sigma = \mathbf{I} + \chi$$

- Two TORIC v.5's versions:
  - TORIC: IC minority regime
    - FLR corrections only up to the $\omega = 2\omega_{ci}$
  - TORIC-HHFW: High Harmonic Fast Wave regime
    - Full hot-plasma dielectric tensor employed
    - The k^2 value in the argument of the Bessel functions is obtained by solving the local dispersion relation for FWs
- Principal author M. Brambilla (IPP Garching, Germany)
- TORIC v.5 is implemented in TRANSP

HHFW regime: Maxwellian

Local coordinate frame $(\hat{\mathrm{x}}, \hat{\mathrm{y}}, \hat{\mathrm{z}})$ with $\hat{\mathrm{z}} = \hat{\mathrm{b}}$ and $\mathbf{k}\cdot\hat{\mathrm{y}} = 0$ (Stix).

The susceptibility for a hot plasma with a Maxwellian distribution function can be evaluated analitically (as originally implemented in TORIC)

$$\chi_s = \left[\hat{\mathrm{z}}\hat{\mathrm{z}}\frac{2\omega_p^2}{\omega k_\parallel v_{th}^2}\langle v_\parallel\rangle + \frac{\omega_p^2}{\omega}\sum_{n=-\infty}^{+\infty} e^{-\lambda}\mathrm{Y}_n(\lambda)\right]_s$$

where

$$\mathrm{Y}_n = \begin{pmatrix} \frac{n^2 I_n}{\lambda}A_n & -in(I_n - I_n')A_n & \frac{k_\perp}{\omega_c}\frac{nI_n}{\lambda}B_n \\ in(I_n - I_n')A_n & \left(\frac{n^2}{\lambda}I_n + 2\lambda I_n - 2\lambda I_n'\right)A_n & \frac{ik_\perp}{\omega_c}(I_n - I_n')B_n \\ \frac{k_\perp}{\omega_c}\frac{nI_n}{\lambda}B_n & -\frac{ik_\perp}{\omega_c}(I_n - I_n')B_n & \frac{2(\omega - n\omega_c)}{k_\parallel v_{th}^2}I_n B_n \end{pmatrix}$$

$$A_n = \frac{1}{k_\parallel v_{th}}Z_0(\zeta_n), \quad B_n = \frac{1}{k_\parallel}(1 + \zeta_n Z_0(\zeta_n)), \quad Z_0(\zeta_n) \equiv \text{plasma dispersion func.}$$

$$\zeta_n \equiv \frac{\omega - n\omega_c}{k_\parallel v_{th}}, \quad \lambda \equiv \frac{k_\perp^2 v_{th}^2}{2\Omega_c^2}$$

HHFW regime: Beyond Maxwellian

The susceptibility for a hot plasma with an arbitrary distribution function:

$$\chi_s = \frac{\omega_{ps}^2}{\omega}\int_0^{+\infty} 2\pi v_\perp dv_\perp \int_{-\infty}^{+\infty} dv_\parallel \hat{\mathrm{z}}\hat{\mathrm{z}}\frac{v_\parallel^2}{\omega}\left(\frac{1}{v_\parallel}\frac{\partial f}{\partial v_\parallel} - \frac{1}{v_\perp}\frac{\partial f}{\partial v_\perp}\right)_s + \frac{\omega_{ps}^2}{\omega}\int_0^{+\infty} 2\pi v_\perp dv_\perp \int_{-\infty}^{+\infty} dv_\parallel \sum_{n=-\infty}^{+\infty}\left[\frac{v_\perp U}{\omega - k_\parallel v_\parallel - n\Omega_{cs}}\mathrm{T}_n\right]$$

where

$$U \equiv \frac{\partial f}{\partial v_\perp} + \frac{k_\parallel}{\omega}\left(v_\perp\frac{\partial f}{\partial v_\parallel} - v_\parallel\frac{\partial f}{\partial v_\perp}\right) \quad \text{and}$$

$$\mathrm{T}_n = \begin{pmatrix} \frac{n^2 J_n^2(z)}{z^2} & \frac{inJ_n(z)J_n'(z)}{z} & \frac{nJ_n^2(z)v_\parallel}{zv_\perp} \\ -\frac{inJ_n(z)J_n'(z)}{z} & (J_n'(z))^2 & -\frac{iJ_n(z)J_n'(z)v_\parallel}{v_\perp} \\ \frac{nJ_n^2(z)v_\parallel}{zv_\perp} & \frac{iJ_n(z)J_n'(z)v_\parallel}{v_\perp} & \frac{J_n^2(z)v_\parallel^2}{v_\perp^2} \end{pmatrix}, \quad z \equiv \frac{k_\perp v_\perp}{\Omega_{cs}}$$

- Integrals in the $v_\parallel$-space with the singularity function $(\omega - k_\parallel v_\parallel - n\Omega_{cs})^{-1}$
- Sum over the harmonic number n and the $k_\perp$ dependence in the argument of the Bessel functions
- Evaluate six components of T_n

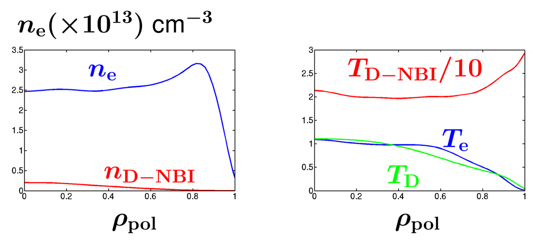
χ is pre-computed to reduce TORIC runtime

- A set of N_ψ files is constructed, each containing the principal values and residues of χ for a single species on a uniform mesh ($(v_\parallel, \theta)$ and $(v_\parallel, \theta, N_\perp)$ for ICRH and HHFW regimes, respectively), for a specified flux surface
- $f(v_\parallel, v_\perp)$, is specified in functional form at the minimum field strength point $B(\theta) = B_{\min}$ on ψ_j
- An interpolator returns the components of χ .

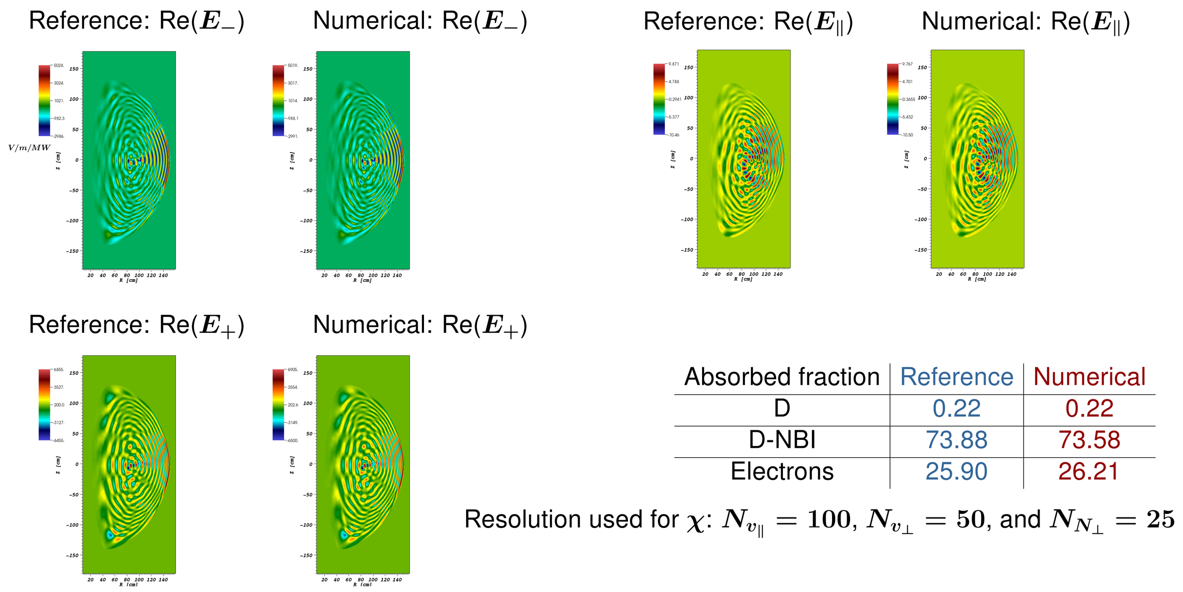
NSTX case

Main parameters:

- TRANSP Run ID: 134909B01
- Plasma species: electron, D, D-NBI
- $B_T = 0.53$ T
- $I_p = 868$ kA
- $T_e(0) = 1.1$ keV
- $n_e(0) = 2.5\times10^{13}$ cm^{-3}
- $T_D(0) = 1.1$ keV, $T_{D-NBI}(0) = 21.4$ keV from TRANSP $\left(T_{bi} = \frac{2}{3}\frac{U}{n_{bi}}\right)$
- $n_{D-NBI}(0) = 2.01\times10^{12}$ cm^{-3}
- TORIC resolution: $n_{mod} = 31$, $n_{elm} = 200$



Test HHFW: Maxwellian reference case



Absorbed fraction	Reference	Numerical
D	0.22	0.22
D-NBI	73.88	73.58
Electrons	25.90	26.21

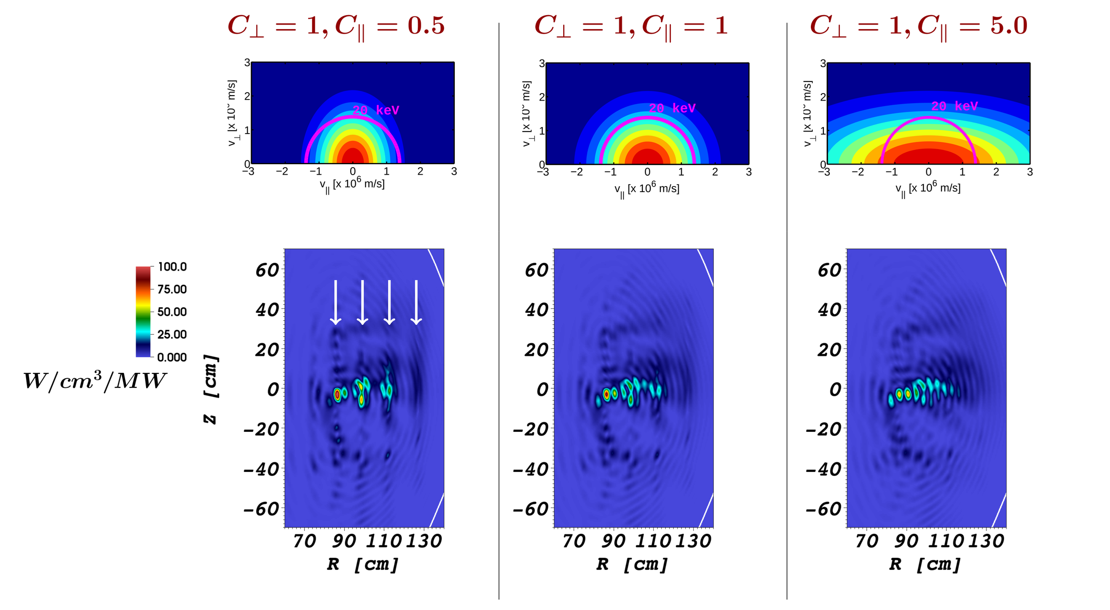
Resolution used for χ: $N_{v_\parallel} = 100$, $N_{v_\perp} = 50$, and $N_{N_\perp} = 25$

HHFW regime: Bi-Maxwellian distribution

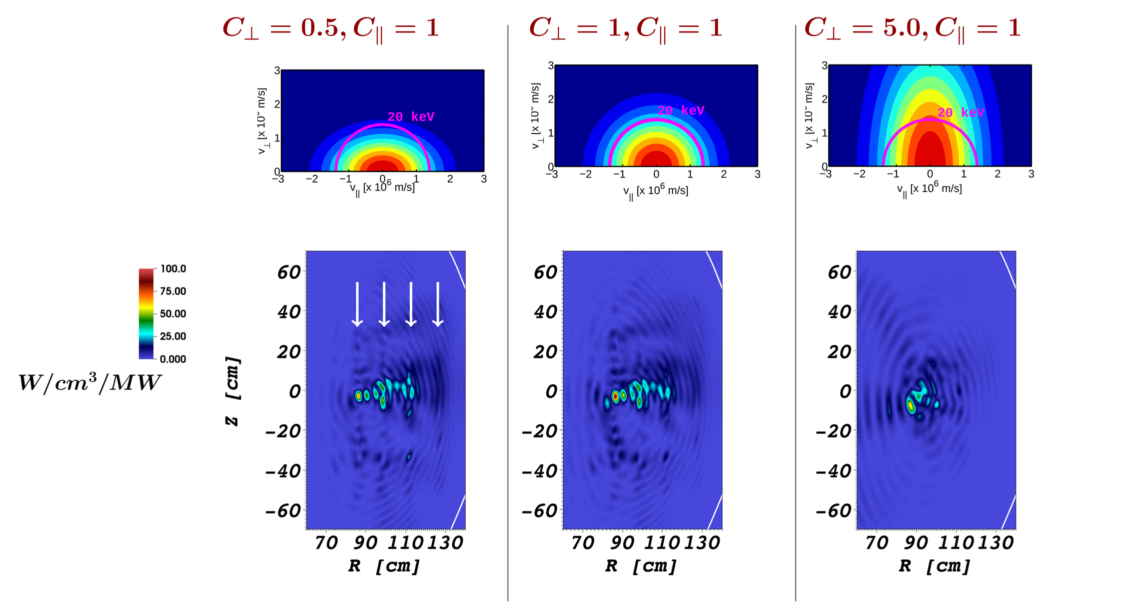
$$f_D(v_\parallel, v_\perp) = (2\pi)^{-3/2}(v_{th,\parallel}v_{th,\perp}^2)^{-1}\exp[-(v_\parallel/v_{th,\parallel})^2 - (v_\perp/v_{th,\perp})^2]$$

with $v_{th,\parallel} = \sqrt{2C_\parallel T(\psi)/m_D}$, $v_{th,\perp} = \sqrt{2C_\perp T(\psi)/m_D}$, with constants $C_\parallel$ and $C_\perp$

- For $C_\perp = 1$ and $C_\parallel = \{.5, 1., 3., 5.\}$, P_{D-NBI}, varied by less than 1%
  - for small $C_\parallel$, the absorption profile tends to be localized to the resonant layers



- For $C_\parallel = 1$ and $C_\perp = \{.5, 1., 3., 5.\}$, the corresponding $P_{D-NBI} = \{70.06\%, 73.56\%, 62.84\%, 48.48\%\}$



Slowing-down distribution

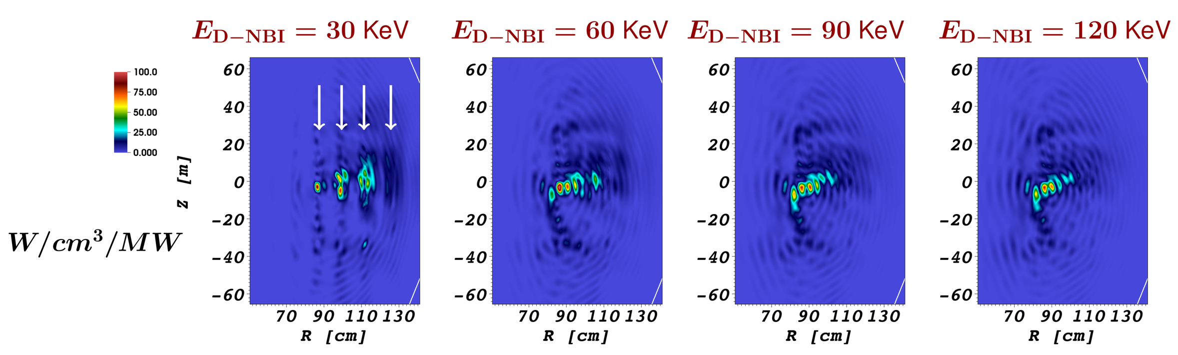
$$f_D(v_\parallel, v_\perp) = \begin{cases} \frac{A}{v_c^3}\frac{1}{1+(v/v_c)^3} & \text{for } v < v_m, \\ 0 & \text{for } v > v_m \end{cases} \qquad v_m \equiv \sqrt{2E_{D-NBI}/m_D}$$

$$A = 3/[4\pi\ln(1+\delta^{-3})], \quad \delta \equiv \frac{v_c}{v_m}, \quad v_c^3 = 3\sqrt{\pi}(m_e/m_D)Z_{eff}v_{th}^3, \quad Z_{eff} \equiv \sum_{ions}\frac{Z_i^2 n_i}{n_e}$$

For $Z_{eff} = 2$ and $E_{D-NBI} = 30, 60, 90, 120$ keV $\Longrightarrow$

$P_{D-NBI} = \{77.84\%, 75.85\%, 70.97\%, 64.71\%\}$

- Similar behavior when varied $C_\perp$ in the bi-Maxwellian case
- Fast ions absorption should decrease with something like $T_{\text{fast ions}}^{-3/2}$ (?)
  - This is due to the behavior of the function $f(\lambda) = \lambda I_n e^{-\lambda}$, which reached a maximum value at $\lambda = n^2/3$.
    For large λ, $f(\lambda) \propto \lambda^{-3/2}$ [See Stix's book & Ono, PoP 1995]



P2F code: from particles list to distr. function

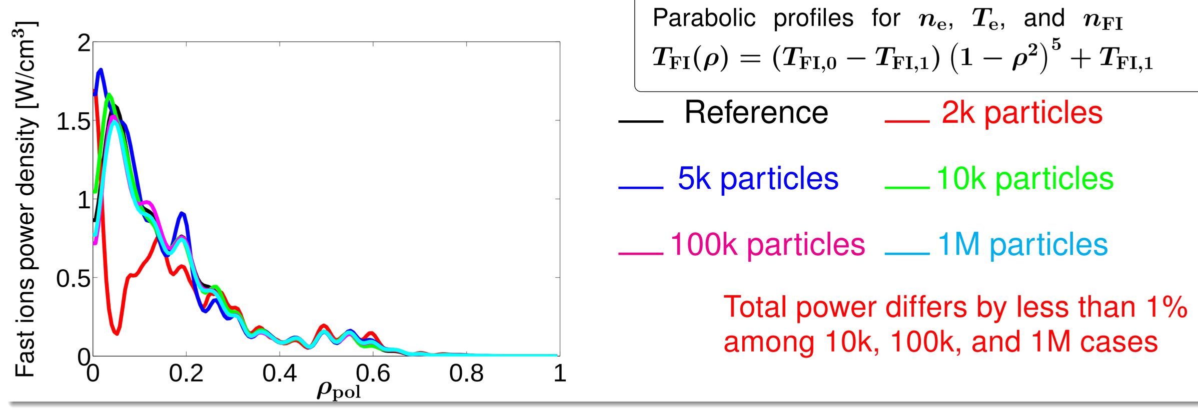
- P2F code was developed by D. L. Green (ORNL)
- P2F takes a particle list and creates a 4D $(R, z; v_\parallel, v_\perp)$ distribution function for use in a continuum code like TORIC
- At present it has essentially three modes:
  1. build a 4D histogram giving a noisy distribution
  2. uses Gaussian shape particles in velocity space to give smooth velocity space distributions at each point in space
  3. distribute each particle along its orbit according to the percentage of bounce time
- Successfully tested starting with a particles list representing a Maxwellian

TORIC + P2F code: Maxwellian case

Procedure:

1. generate particle list representing a Maxw.
2. run P2F to obtain a distribution function
3. pre-compute χ with f above
4. run TORIC with pre-computed χ
5. compare TORIC with reference

$n_e(\rho = 0) = 2.5\times10^{13}$ cm^{-3}
$n_e(\rho = 1) = 2.5\times10^{12}$ cm^{-3}
$T_e(\rho = 0) = 1$ keV; $T_e(\rho = 1) = 0.1$ keV
$n_{FI}(\rho = 0) = 2.0\times10^{12}$ cm^{-3}
$n_{FI}(\rho = 1) = 2.0\times10^{11}$ cm^{-3}
$T_{FI}(\rho = 1) = 20$ keV; $T_e(\rho = 1) = 5$ keV
Parabolic profiles for n_e, T_e, and n_{FI}
$T_{FI}(\rho) = (T_{FI,0} - T_{FI,1})\left(1 - \rho^2\right)^5 + T_{FI,1}$



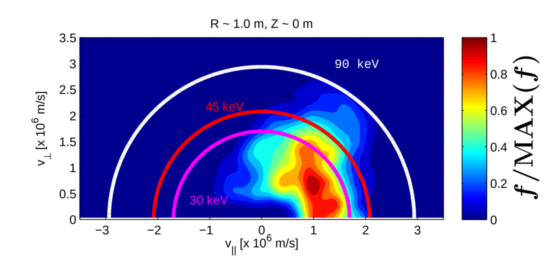
Total power differs by less than 1% among 10k, 100k, and 1M cases

NUBEAM particles list (NSTX shot 141711 WITHOUT HHFW)

Procedure:

1. get NUBEAM particle list
2. run P2F to obtain a distribution function
3. pre-compute χ with f above
4. run TORIC with pre-computed χ
5. compare TORIC run with standard TORIC

NSTX shot 141711
$P_{NBI} = 2$ MW
$I_P = 900$ kA
particles number = 53115



Abs. fraction	f Maxw.	f non-Maxw.
Electrons	41.80 %	37.99 %
D-NBI	53.94 %	58.12 %

Larger P_{D-NBI} is expected due to a larger f tail formed by the RF application

RF kick operator in NUBEAM

- RF resonance condition in terms of E, μ, and B

$$\Lambda^{rf}(B, n, k_\parallel, E, \mu) \equiv n\frac{qB}{m_i} + k_\parallel\sqrt{\frac{2}{m_i}(E - \mu B)} - \omega_{rf}$$

- Monte-Carlo implementation of RF diffusion operator [X. Q. Xu and M. N. Rosenbluth, PoF B **3**, 627 (1991), C. F. Kennel and F. Engelmann, PoF **9**, 2377 (1966)]

$$Q_{rf} = D_{rf}\left[\frac{\partial}{\partial V_\parallel}(\nu_{\parallel,s}^{rf}f) + \frac{\partial}{\partial V_\perp^2}(\nu_{\perp,s}^{rf}f) + \frac{\partial^2}{\partial V_\parallel\partial V_\perp^2}(\nu_{\parallel\perp}^{rf}f) + \frac{1}{2}\frac{\partial^2}{\partial V_\parallel^2}(\nu_\parallel^{rf}f) + \frac{1}{2}\frac{\partial^2}{\partial(V_\perp^2)^2}(\nu_{\perp,s}^{rf}f)\right]$$

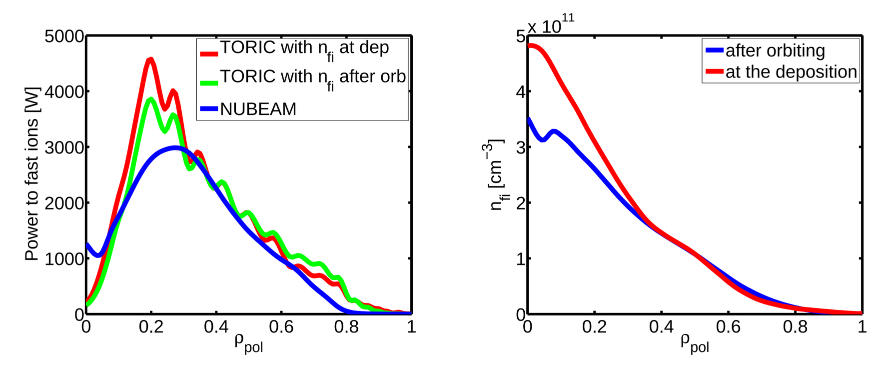
$$V_\perp'^2 = V_{\perp,0}^2 - \nu_{\perp,s}^{rf}\Delta t + 2\sqrt{3}(R_2 - 0.5)\sqrt{\left[\nu_\perp^{rf} - \frac{(\nu_{\parallel\perp}^{rf})^2}{\nu_\parallel^{rf}}\right]\Delta t} + 2\sqrt{3}(R_1 - 0.5)\frac{\nu_{\parallel\perp}^{rf}}{\nu_\parallel^{rf}}\sqrt{\nu_\parallel^{rf}\Delta t}$$

$$V_\parallel' = V_{\perp,0}(1 - \nu_{\parallel s}^{rf}\Delta t) + 2\sqrt{3}(R_1 - 0.5)\frac{\nu_{\parallel\perp}^{rf}}{\nu_\parallel^{rf}}\sqrt{\nu_\parallel^{rf}\Delta t}$$

- RF wave field information from TORIC: E_+, E_-, and $k_\perp$ for each n_ϕ

First and preliminary test of RF kick operator

Comparison between TORIC and NUBEAM + RF kick operator assuming a Maxw. distribution at the deposition



Work in progress