Improved measurement of Te and Zeff on NSTX-U using Integrated Data Analysis

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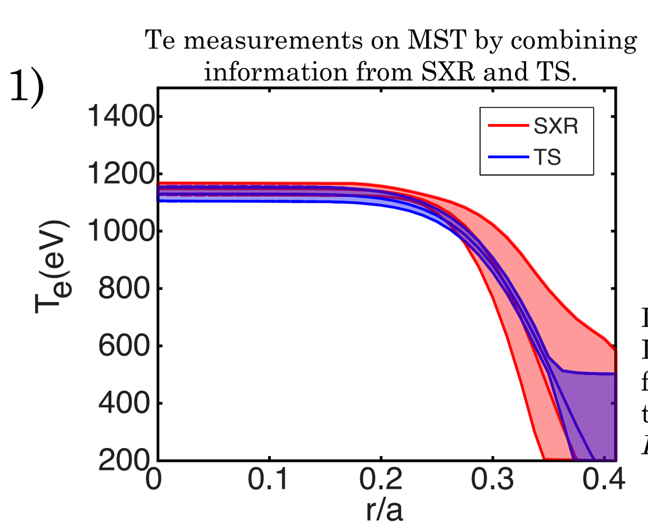
Overview

- We have begun to develop IDA to improve the precision and temporal resolution of electron temperature (Te) profiles and increase the reliability of effective ionic charge (Zeff) measurements on NSTX-Upgrade, using Integrated Data Analysis (IDA).
- Bayesian probability theory provides a natural framework for IDA, and will be applied to this project.
- We will combine data from the multi-energy soft x-ray (ME-SXR) diagnostic and charge exchange recombination spectroscopy measurements to improve Zeff estimation.
  - Zeff governs many plasma dynamics, so a more precise determination will aid validation
  - This will help enable measurements of Zeff once metal tiles are install in NSTX-U
- We will combine ME-SXR and Thomson scattering data (and others as appropriate and available) to improve Te measurement.
  - Improved precision will allow better transport and confinement studies.
  - Improved spatial and temporal resolution will aid in pedestal evolution studies.

IDA

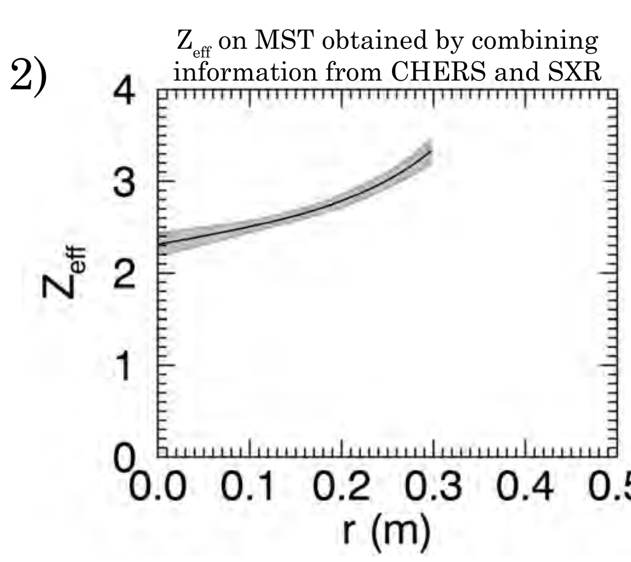
- As we transition to fusion experiments that are full nuclear environments severe limitations will be imposed on diagnostics
- Integrated Data Analysis (IDA) provides a framework to maximum the scientific information extracted from available data
- The goal of IDA is to
  - combine data from heterogeneous and complementary diagnostics
  - consider all dependencies within and between diagnostics
  - obtain the most reliable results in a transparent and standardized way.
- IDA
  - Typically results in improved precision/resolution [1].
  - Enables "meta-diagnostics," information from various instruments combined to produce unique measurements [2].

1)



L. M. Reusch, et. al. "An Integrated Data Analysis Tool for Improving Measurements on the MST RFP." *Rev. Sci. Instrum.* **85**, 11D844 (2014)

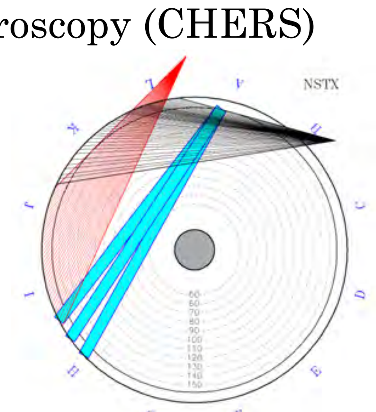
2)



M.E. Galante, *et. al.* "Determination of Z_{eff} by integrating measurements from x-ray tomography and charge exchange recombination spectroscopy." *Nucl. Fusion* **55** (2015).

Diagnostics

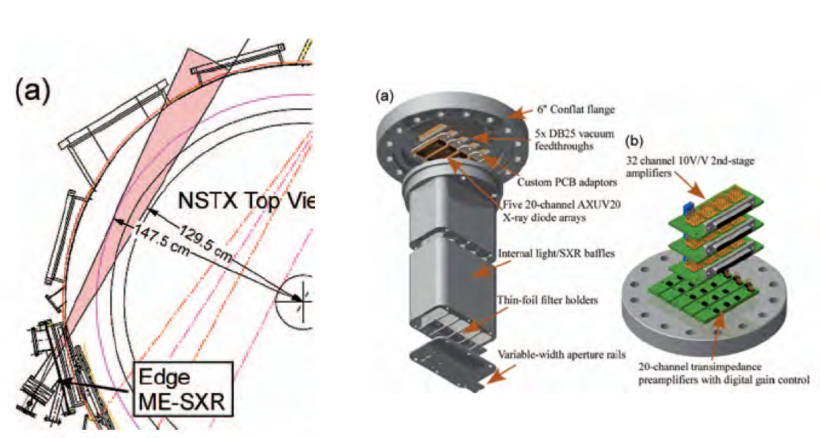
Charge Exchange Recombination Spectroscopy (CHERS)



R. E. Bell, "Carbon ion plume emission produced by charge exchange with neutral beams on National Spherical Torus Experiment." *Rev. Sci. Instrum.* **77**, 10E902 (2006).

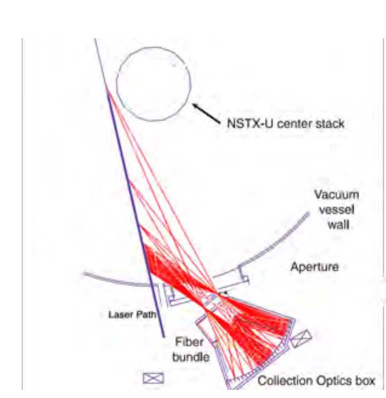
R. E. Bell and R. Feder, "Measurement of poloidal velocity on the National Spherical Torus Experiment (invited)." *Rev.Sci. Instrum.* **81**, 10D724 (2010).

Multi-energy soft x-ray diagnostic (ME-SXR)



K. Tritz, D. J. Clayton, D. Stutman, and M. Finkenthal, "Compact diode-based multi- energy soft x-ray diagnostic for NSTX," *Rev. Sci. Instrum.* **83**, 10E109 (2012).

Thomson Scattering (TS)



A. Diallo, B. P. LeBlanc, G. Labik, and D. Stevens, "Prospects for the Thomson scattering system on NSTX-Upgrade." *Rev. Sci. Instrum.* **83**, 10D532 (2012).

B. P. LeBlanc A. Diallo, G. Labik, and D. R. Stevens, "Radial resolution enhancement of the NSTX Thomson scattering diagnostic." *Rev. Sci. Instrum.* **83**, 10D527 (2012).

Bayesian Probability Theory

- Bayesian analysis relies on Bayes' Rule:

$$P(x|y,I) \propto P(x|y,I)P(x|I)$$

  - where x and y are different (conditional) events, and I is the background information
  - read as "the probability of x given y and I is proportional to the probably of y given x and I"
- and marginalization:

$$P(x|y,I) = \int P(x,\alpha|y,I)d\alpha$$

  - here x, y, and α are parameters and I is the background information
  - allows for the removal of nuisance parameters through integration over all possible values for α.

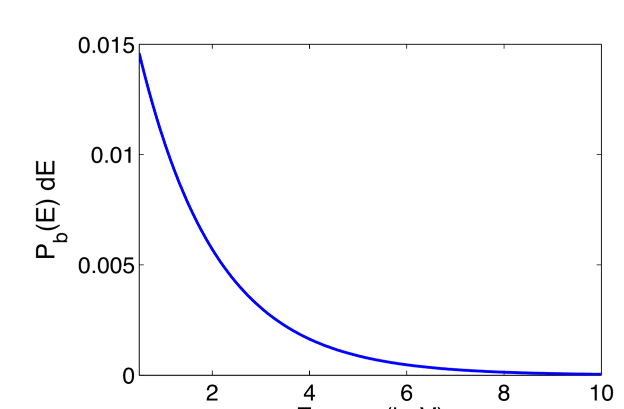
Bayesian Analysis of Data

- Bayesian analysis of a single diagnostic data takes the form:
  - Z_{eff}: $P(Z_{eff}|D,I) \propto P(D|Z_{eff},I)P(Z_{eff}|I)$
  - T_e: $P(T_e|D,I) \propto P(D|T_e,I)P(T_e|I)$
  - D is the diagnostic data from an individual diagnostic.
  - $P(Z_{eff}|D,I)$, the posterior probability, is the answer: an estimate of Z_{eff} given the diagnostic data.
  - $P(D|Z_{eff},I)$, the likelihood function, relates the diagnostic data to Z_{eff} through a diagnostic forward model
  - The forward model of the system includes all physical processes generating signal and instrumentation effects.
  - $P(Z_{eff}|I)$ represents our prior knowledge about possible values for Z_{eff}, such as expected ranges or physical constraints.
- Bayesian analysis of multiple diagnostics takes the form:
  - Z_{eff}: $P(Z_{eff}|D_1,D_2,\ldots,I) \propto P(Z_{eff}|I) \times \prod_i P(D_i|Z_{eff},I)$
  - T_e: $P(T_e|D_1,D_2,\ldots,I) \propto P(T_e|I) \times \prod_i P(D_i|T_e,I)$
  - where D_i is the diagnostic data from an individual diagnostic and $P(D_i|T_e,I)$ is the likelihood function for each diagnostic.
  - $P(Z_{eff}|D_1,D_2,\ldots,I)$ gives the single best estimate for Z_{eff} given all the diagnostic data
  - This is how IDA is accomplished using Bayesian analysis.

SXR Forward Model

Total x-ray power come from multiple sources:

- Bremsstrahlung radiation:
  $P_b\, dE = K\sum_i \frac{n_e n_i Z_i^2}{\sqrt{T_e}} e^{-\frac{E}{T_e}} dE$
- K is a physical constant and n_i is the density for a single ionic charge state

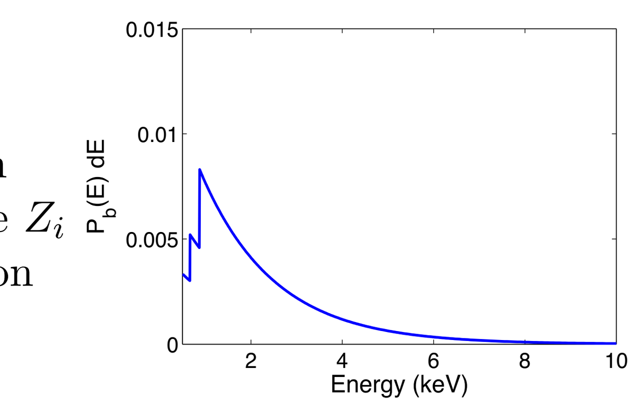


SXR Forward Model, Cont.

- Recombination radiation:

$$P_r\, dE = A\sum_i \frac{n_e n_i Z_i^2}{\sqrt{T_e}} e^{-\frac{E}{T_e}} \times \left[\frac{\xi}{n^3}\frac{\chi_i}{T_e}\exp\left(\frac{\chi_i}{T_e}\right) + \sum_{\nu=n+1} \frac{Z_i^2 R_y}{T_e}\frac{2}{\nu^3}\exp\left(\frac{Z_i^2 R_y}{n^2 T_e}\right)\right] dE$$

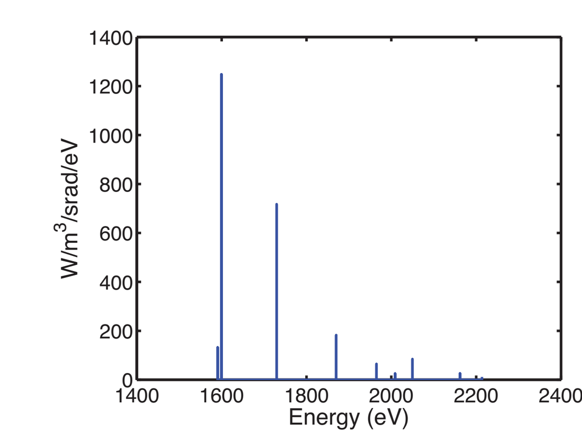
  - A is a physical constant.
  - n_i is the density for single ionic charge state with associated charge Z_i and recombination energy χ_i.



- Atomic Line radiation:

$$P_l\, dE = \sum_i\sum_j E_{i,j} n_e n_i \frac{PEC_{i,j}}{4\pi}\delta(E - E_{i,j})dE$$

  - n_i is the density for single ion charge state
  - j denotes the j^{th} transition of ion charge state i



  - No line radiation from Low-Z impurities (They are all fully stripped).
  - When metal tiles are installed, taking into account line radiation will be important.
- The sum is taken over all charge states of all ions.
- n_i for the various charge states depends on T_e and can be found assuming ionization balance.
- implemented in ADAS, which assumes coronal equilibrium.

Next Steps

- Benchmark ADAS SXR model against SXR data
  - preliminary comparison against USXR:

Filter	USXR Signal	SXR Model
5 μm Be	3575 W/m^2	4400 W/m^2
100 μm Be	375 W/m^2	710 W/m^2

  - Uses experimental profiles for T_e, n_e, and n_i.
  - Models a chord looking through the core.
- Implement instrumentation model for USXR and ME-SXR system
- Assess systematic and statistical uncertainties.
- Define/build likelihood functions for all diagnostics.

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