Resonance Broadened Quasi-linear (QL) model (RBQ) for fast ion relaxation in the presence of Alfvénic instabilities

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in collaboration with
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with acknowledgments to
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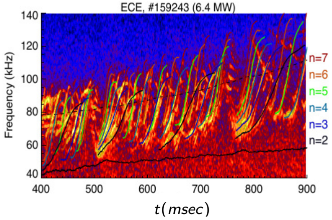
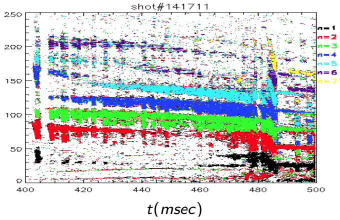
Two regimes of Alfvén mode fast ion relaxation need to be addressed

steady state DIII-D & *chirping NSTX*





- How can we model energetic particle confinement in both regimes: current drive, losses.
  - Steady state regime is understood but not chirping frequency regime.
- We target numerically efficient resonance broadened quasi-linear (RBQ) model for whole device modeling simulations in steady state regimes: *Berk et al.'95, Ghantous, PhD'14, Duarte, PhD'17 .*

- **Steady-state** (compatible with RBQ) together with (marginally expected) **chirping** frequency **regimes** are predicted in burning plasmas, ITER! *(Duarte et al., APS'18 oral talk at ITER BPO session)*

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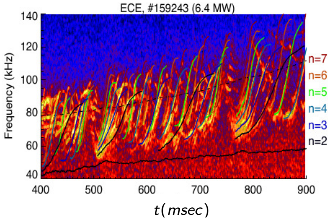

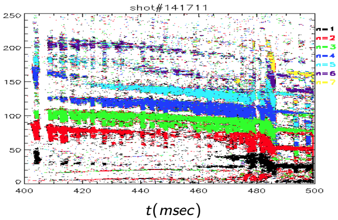

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    - rigorous verifications are undertaken
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Flux coordinate NOVA-K formalism, consistent with Hamiltonian approach

3 constants of motion (COM): Canonical toroidal momentum

$$P_\varphi = \frac{e\psi}{mc} - \sigma_\parallel v \sqrt{1-\lambda B/B_0}\, R\frac{B_\varphi}{B},$$

magnetic moment

$$\mu = \frac{\mathscr{E}_\perp}{B} = \frac{\lambda \mathscr{E}}{B_0},$$

kinetic energy

$$\mathscr{E} = \frac{v^2}{2}.$$

Wave particle interaction results from energy exchange due to an eigenmode electric field ($\boldsymbol{E} = -\nabla\phi$):

$$\boldsymbol{v}\cdot\boldsymbol{E} \simeq \boldsymbol{v}_{dr}\cdot\boldsymbol{E}_\perp \sim \boldsymbol{v}_{dr}\cdot\sum_m \frac{m\phi_m}{r} \sim \sum_m \left\langle \left(2\mathscr{E} - 3\mu B\right) J_0\left(\frac{m}{r}\rho_\perp\right)\frac{m\phi_m}{r}\right\rangle_{orbit},$$

supplemented by the resonance condition:

$$\Omega\left(P_\varphi, v, \mu\right) = \omega - n\dot{\varphi} - m\dot{\theta} - l\omega_b = 0.$$

NOVA-K framework allows:

Realistic representation of EP drift orbits.

Orbit averaging for wave particle interaction matrices: $\langle \boldsymbol{E}\cdot\boldsymbol{v}_d\rangle_{drift}$.

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Resonance Broadened QL diffusion through pitch angle scattering

Action-angle formalism through flux variables results in a set of equations for fast ion DF: *(Kaufman,PhFl'72, Berk, Breizman, NF'95)* and adapted for RBQ *(Duarte, PhD'17, Gorelenkov, NF'18)*

$$\frac{\partial}{\partial t} f = \pi \sum_{\mathbf{l},k} \frac{\partial}{\partial P_\varphi} C_k^2 \mathscr{E}^2 \frac{G^*_{m'p} G_{mp}}{\left|\partial \Omega_{\mathbf{l}} / \partial P_\varphi\right|_{res}} \mathscr{F}_{\mathbf{l}} \frac{\partial}{\partial P_\varphi} f + \nu_{eff}^3 \left| \frac{\partial \Omega_{\mathbf{l}}}{\partial \bar{P}_\varphi} \right|^{-2} \frac{\partial^2}{\partial P_\varphi^2} (f - f_0),$$

where EP distribution is evolved due to scattering terms on RHS amended by the scattering "source" operator.

AE amplitude satisfies

$$C_k(t) \sim e^{(\gamma_L + \gamma_d)t} \Rightarrow \frac{dC_k^2}{dt} = 2(\gamma_L + \gamma_d) C_k^2.$$

RBQ does not evolve *AE mode structure, only amplitudes.

Critical for RBQ platform *for multiple mode cases (Dupree'66, Berk'95, White'18) is* resonant frequency and its broadening by nonlinear bounce ω_{bNL} and effective scattering ν_{eff}:

$$\delta\left(\Omega = \omega + n\dot{\varphi} - m\dot{\theta} - l\omega_b\right) \rightarrow window\ \ function\ \ \mathscr{F}\left(\Delta P_\varphi\right).$$

RBQ1D benefits are:

- Time efficient.
- Realistic computations of current drive, loss distribution over the first wall, intermittency.

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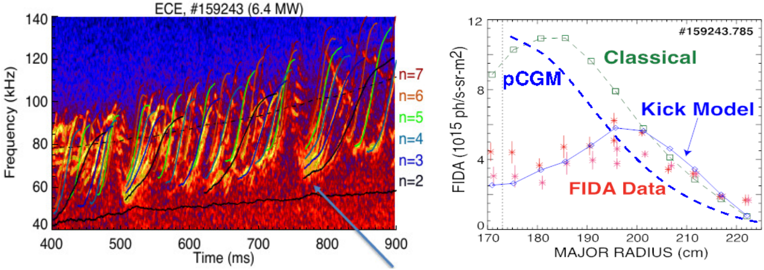
Apply to DIII-D case studied in critical gradient experiments

Detailed analysis of DIII-D experiment #159243 & 6.4MW

(Collins et al, PRL'16, Heidbrink et al., PoP'17).

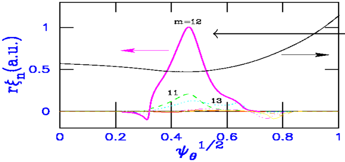


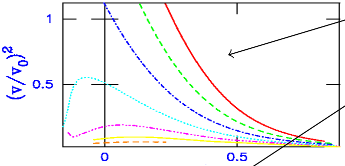
805msec is chosen near rational q_{min} for detailed study

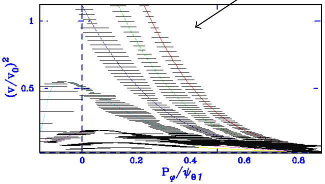
Earlier developed critical gradient model (pCGM) does not reproduce hollow EP profiles and underestimates neutron deficit by two times.

Need velocity space resolution such as in the Kick Model *(Podesta et al., PPCF'14)*

RBQ workflow illustration for $n = 4$ Reversed Shear Alfvén Eigenmode (at q_{min})







- Ideal MHD NOVA finds Reversed Shear Alfven Eigenmode structure $f = 84kHz$ *(Collins, PRL'16)*.
    - This mode provides a channel for ion diffusion and an inverted fast ion pressure profiles: resonant particles are close to the injected pitch angle near the axis.
- NOVA-K code computes resonancies for particle interactions with the mode and $\langle \boldsymbol{v} \cdot \boldsymbol{E} \rangle$ matrices.
- RBQ broadens those resonancies along P_φ direction using QL presciptions for each mode. Shown is the broadening at measured amplitude $\delta B_\theta / B = 7 \times 10^{-3}$.
- Monte-Carlo TRANSP package post-processes RBQ diffusion to compute the fast ion distribution function evolution.
    - The Probability Density Function for ion diffusion in the velocity space for further processing within TRANSP is evaluated.
    - Three versions of RBQ1D are developed:
        - Interpretive, predictive with single mode saturation amplitude, and predictive with multiple mode amplitudes computed selfconsistently.
- RBQ1D is the solver to find the diffusion in the constant of motion space.
    - Employ kick model probability density function to describe QL diffusion.

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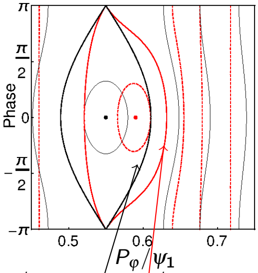
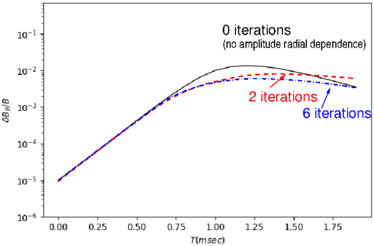
I. Resonant ion island dynamics is accounted for using Hamiltonian technique

EP islands for “Gaussian” mode



RBQ needs ˜2 iterations to converge well. Lowers saturation ampl.



0^{th} iteration 6^{th} iteration ("new" island) accounting for RSAE radial structure

(Berk-Breizman approach)

- Low amplitude $\Delta P_\varphi \sim \Delta\Omega = 4\omega_b$ at $\delta B_\theta/B \lesssim (1 \div 5)\times 10^{-4}$ *(via ORBIT modeling, G.Meng, NF'18).* Supports resonant frequency approach for nonlinear wave particle interaction.
- Radial amplitude structure limits NL resonance frequency *(R.White et al., PoP'18).*

II. Analytic solution for amplitude evolution near threshold

Amplitude A vs time t for full cubic equation (green) and its analytical solution (black)

Near marginal stability, the amplitude governed by

$$\frac{dA(t)}{dt} = A(t) - \frac{1}{2}\int d\Gamma \mathscr{H} \left\{ \int_0^{t/2} dz z^2 A(t-z) \times \right.$$
$$\left. \times \int_0^{t-2z} dy e^{-\hat{\nu}_{eff}^3 z^2 (2z/3+y)} A(t-z-y) A^*(t-2z-y) \right\}.$$

(Berk et al., PRL'96)

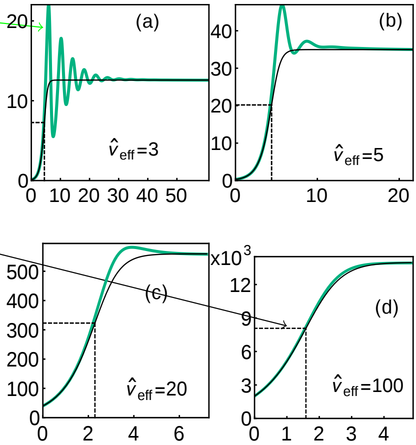
At large ν_{eff} ($>$ net growth rate) only recent time history dictates the WPI dynamics, i.e. when $y, z \to 0$:

$$A(t) = \frac{A(0)e^t}{\sqrt{1 - bA^2(0)(1-e^{2t})}}$$

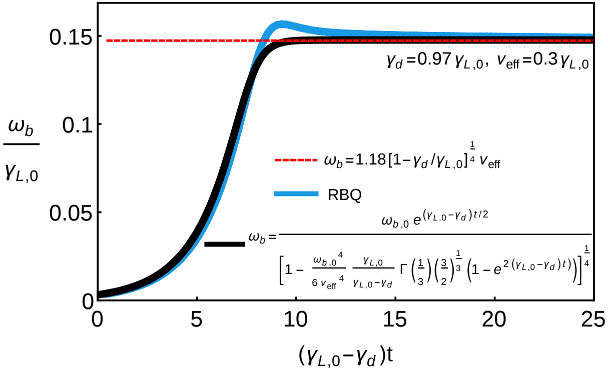
where $A(0)$ is the initial amplitude and

$$b \equiv \left[\int d\Gamma \mathscr{H} \frac{\Gamma(1/3)}{6\hat{\nu}_{eff}^4} \left(\frac{3}{2}\right)^{1/3} \right]$$

(V.Duarte et al., submitted to NF).



RBQ verification against analytical solution for amplitude evolution near threshold

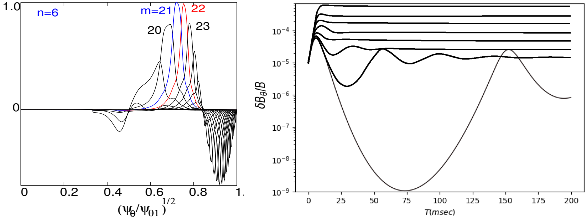


Both RBQ diffusion solver and the analytical solution recover:

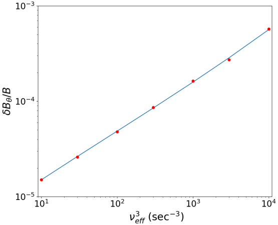
- time evolution
- expected saturation level near marginal stability *[Berk, Breizman '90]*!

III. RBQ verification via Coulomb collisions

Global $n = 6$ TAE saturates over $\sim$ *msec*



RBQ shows $\delta B_\theta/B \propto \nu_{eff}^{\mathbf{1.65}}$



- TAE amplitude scales with fast ion Coulomb scattering frequency, $\delta B_\theta/B \sim \nu_{eff}^{\mathbf{2}} \sim \nu_{\perp}^{\mathbf{2/3}}$, where $\nu_{eff}^{\mathbf{3}} = \nu_\perp \left|\frac{\partial\Omega}{\partial\chi}\right|^{\mathbf{2}}$ *(Berk et al.,Phys. Fluids B'90).*
- Dirichlet boundary conditions, $f_h(\bar{\psi}_\theta \to 0) = const$ and $f_h(\bar{\psi}_\theta \to 1) = 0$, are required to account for Coulomb scattering.

- Intermittency (fluctuations in losses) is expected in predictive RBQ simulations!!

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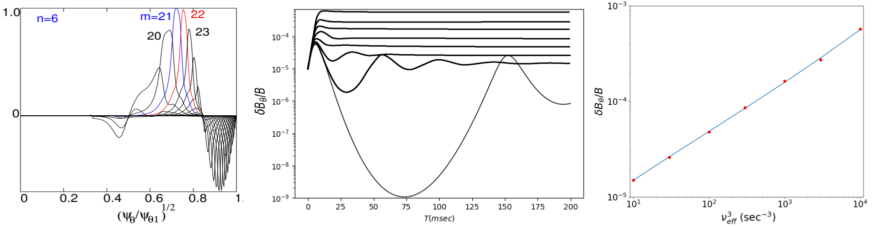

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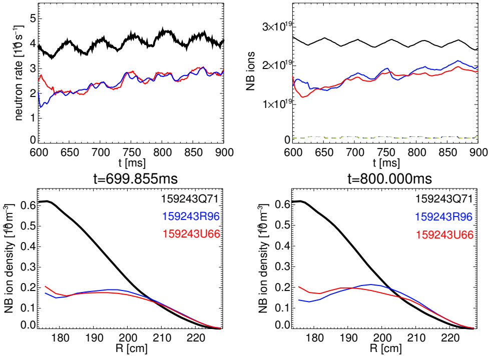
*11 observed modes are employed similar to kick model **interpretive** version*

Color-coded evolution/profiles

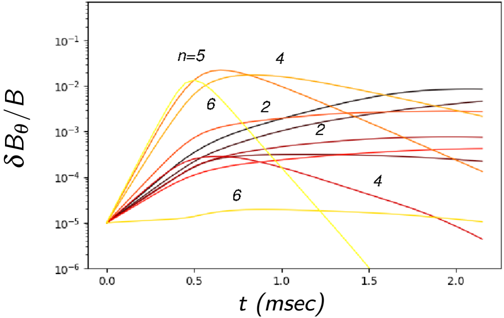


- Black curve is TRANSP calculations without AE effects
- Blue: kick model with neutron flux constraint
- Red: RBQ with the same constraint
- AE amplitudes are used in RBQ1D as inferred at $t = 805 msec$ time.

RBQ applications in the interpretive mode are consistent with neutron rate data.

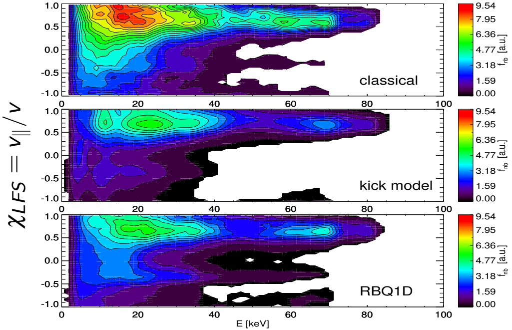
Used probability density function will be substituted with actual diffusion in the COM phase space.

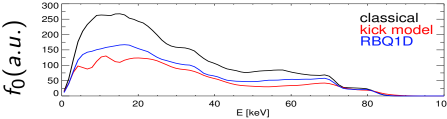
Predictive capability results from selfconsistent evolution of all 11 modes



- RBQ1D computes selfconsistent Alfvén Eigenmode amplitudes consistent with measured values $O\left(10^{-4}-10^{-2}\right)$ *(Collins, PRL'16)*.
- Shown are AE evolutions with modified prescriptions for broadening *(White et al., PoP'18)*.
- Amplitudes (diffusion coefficients) at saturation are sensitive to growth rate values: growth rates need to be robustly computed!

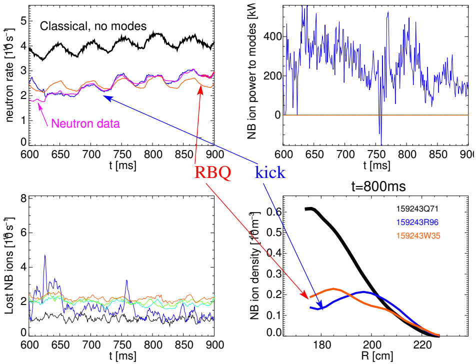
Distribution function has similar properties with the kick model distribution





- Co-going passing ions are strongly redistributed.
- Amplitudes are kept constant throughout observed times.
- Neutron rate includes radial and energy dependence within TRANSP simulations.
- (Near) hollow EP density is due COM location sensitive diffusion.
- Rotation is ignored!!
  It can be significant and could lead to EP energy shift $\sim E_0/2$ in DIII-D.

Compare RBQ1D, kick simulations with neutron deficit using TRANSP



- Distributions are evolved by TRANSP Monte-Carlo package.
- Kick model agrees with FIDA data over the velocity space region.
- RBQ1D and kick model simulations are consistent.

Status and plans for Quasi-Linear model development

1. RBQ model is formulated for *realistic numerically efficient simulations* based on earlier Quasi-Linear theories *(Berk, NF'95,PoP'96, Dupree, PhFl'66, Kaufman, PhFl'72).*
2. Broadening technique is included for non-slanted 1D mesh in P_φ.
3. RBQ1D employs full diffusion solver to resolve island dynamics near the resonance.
4. RBQ1D is applied to DIII-D cases for V&V *(Gorelenkov, et al., IAEA TCM'17, NF'18)*. Low rotation is critical for present RBQ applicability by ignoring energy change.
   Can we apply RBQ to chirping frequency scenarios? Zonal flow, convective transport?
5. 2D extension will be developed within ISEP SciDAC.
   - 1D problem requires direct information about the diffusion coefficients in whole device modeling simulations.
   - Conservative estimates for RBQ approach imply $(0.1 \div 1)\,PFlop$ computer power needed for burning plasma devices to allow a reasonable time to run.