NSTX-U

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Multi-beam effects on compressional Alfvén eigenmode stability

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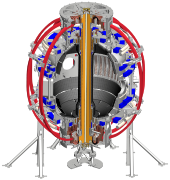
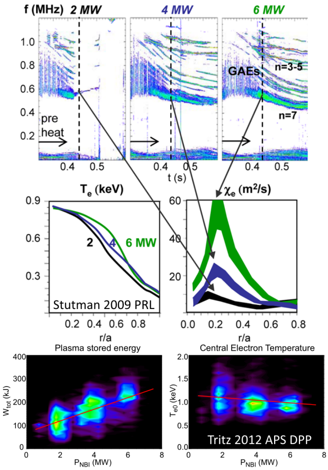


High Frequency Alfvén Eigenmodes and Anomalous Electron Temperature Flattening

- High frequency Alfvén eigenmodes are often observed in spherical tokamak experiments
  - Low field → large v_{beam}/v_A
- In NSTX, these modes have been linked to anomalous T_e flattening at high beam power[1]
- Understanding stability properties is required to test theories of Alfvénic electron energy transport
- New beam sources on NSTX-U provide new degrees of freedom for phase-space engineering

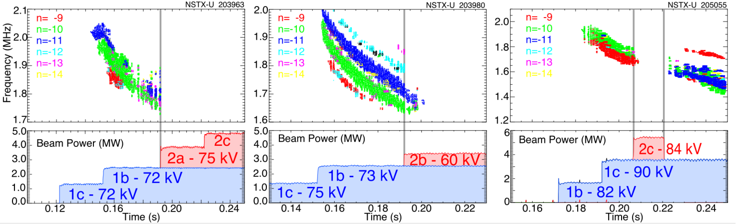


[1] D. Stutman *et al*. Phys. Rev. Lett. **102**, 115002 (2009)

GAE Suppression on NSTX-U

- NSTX-U found robust suppression of GAEs with addition of new, outboard/tangential beams[2]
- Experimental observations reproduced by numerical modeling and supported by analytic theory[3]

What predictions can be made about **CAE** suppression?

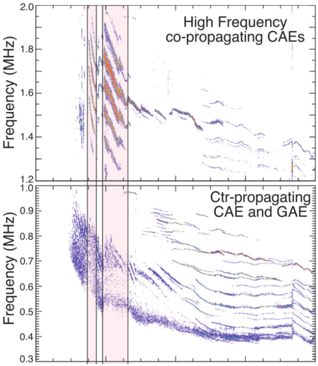


[2]E. Fredrickson *et al*. Nucl. Fusion **58**, 082022 (2018)

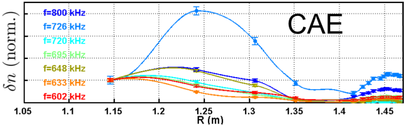
[3]E. Fredrickson *et al*. Phys. Rev. Lett. **118**, 265001 (2017)

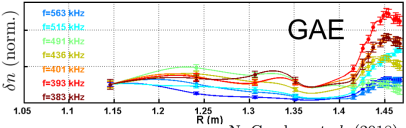
Sub-cyclotron Alfvén Eigenmodes in NSTX

- High frequency Alfvén eigenmodes routinely excited in NSTX(-U) plasmas by neutral beam injection
  - Driven by Doppler-shifted cyclotron resonance with fast ions $\omega - \langle k_{\|} v_{\|} + k_{\perp} v_{\mathrm{Dr}} \rangle = \ell \langle \omega_{ci} \rangle$
- Identified as combination of compressional (CAE) and global (GAE) Alfvén eigenmodes
  - Co-/cntr-propagating $|n| \approx 3 - 14$
  - $\omega/\omega_{ci} \approx 0.1 - 0.7$ ($\gg \omega_{TAE}$)



E. Fredrickson *et al.* (2013) Phys. Plasmas **20**, 042112





N. Crocker *et al.* (2018) Nucl. Fusion **58**, 016051

Hybrid Simulation Method

- Hybrid MHD and Particle code (HYM)[4]
  - Single fluid resistive MHD thermal plasma
  - Full orbit kinetic fast ions with δF scheme
- Initial value code in 3D toroidal geometry
- Linear fluid equations and unperturbed particle trajectories
  - Optional nonlinear physics (not used for this study)
- Self-consistent equilibrium includes energetic particle effects via current-coupling

$$\frac{\partial^2\psi}{\partial z^2} + R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right) = -R^2P' - HH' \underbrace{-GH' + RJ_{b\phi}}_{\text{self-consistent EP terms}}$$

$$\boldsymbol{B} = \nabla\phi\times\nabla\psi + h\nabla\phi \qquad h(R,z)\equiv H(\psi)+G(R,z) \qquad \boldsymbol{J}_{\boldsymbol{b},\mathrm{pol}} = \nabla G\times\nabla\phi$$

$\longrightarrow$ pressure anisotropy, increased Shafranov shift, more peaked current

- Non-self-consistent equilibrium allows investigation of fast ion drive independent of changes to equilibrium

[4] E. Belova *et al.* Phys. Plasmas **10**, 3240 (2003)

HYM Physics Model

Fluid thermal plasma

$$\rho \frac{d\boldsymbol{V}}{dt} = -\nabla P + (\boldsymbol{J} - \boldsymbol{J_b}) \times \boldsymbol{B} - en_b(\boldsymbol{E} - \eta\delta\boldsymbol{J}) + \mu\Delta\boldsymbol{V}$$

$$\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B} + \eta\delta\boldsymbol{J}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}$$

$$\mu_0 \boldsymbol{J} = \nabla \times \boldsymbol{B}$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{V})$$

$$\frac{d}{dt}\left(\frac{P}{\rho^\gamma}\right) = 0$$

Kinetic fast ions

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{v}$$

$$\frac{d\boldsymbol{v}}{dt} = \frac{q_i}{m_i}(\boldsymbol{E} - \eta\delta\boldsymbol{J} + \boldsymbol{v} \times \boldsymbol{B})$$

δF Scheme

$$F = F_0(\mathcal{E}, \mu, p_\phi) + \delta F(t)$$

$$w \equiv \delta F / F$$

$$\frac{dw}{dt} = -(1 - w)\frac{d \ln F_0}{dt}$$

- ρ, $\boldsymbol{V}$, P are plasma mass density, velocity, and pressure
- n_b, $\boldsymbol{J_b}$ are beam ion density and current
  - Assuming $n_b \ll n_e$ but allowing $\boldsymbol{J_b} \approx \boldsymbol{J_{th}}$

Fast Ion Distribution Model

- Equilibrium dist: $F_0 = \sum_i A_i F_1(v; v_{0,i}) F_2(\lambda; \lambda_{0,i}) F_3(p_\phi, v)$
  - Energy $\mathcal{E} = \frac{1}{2} m_i v^2$
  - **Trapping parameter $\lambda = \mu B_0/\mathcal{E} \approx \mathcal{E}_\perp B_0/\mathcal{E} B$**
    - Passing: $0 < \lambda < 1 - r/R$
    - Trapped: $1 - r/R < \lambda < 1 + r/R$
  - Canonical angular momentum $p_\phi = -q_i \psi + m_i R v_\phi$

$$F_1(v; v_{0,i}) = \frac{1}{v^3 + v_c^3} \quad \text{for } v < v_{0,i}$$

$$F_2(\lambda; \lambda_{0,i}) = \exp\left(-\left(\lambda - \lambda_{0,i}\right)^2 / \Delta\lambda^2\right)$$

$$F_3(p_\phi, v) = \left(\frac{p_\phi - p_{\min}}{m_i R_0 v - q_i \psi_0 - p_{\min}}\right)^\alpha \quad \text{for } p_\phi > p_{\min}$$

- NSTX: $v_0/v_A \lesssim 5$, $v_c \approx v_0/2$, $\lambda_0 = 0.7$, $\Delta\lambda = 0.3$, $\alpha = 6$
- NSTX-U: $v_0/v_A \lesssim 2$, $\lambda_0 = 0$ for new beam source

Theoretical Framework

- Perturbative linear growth rate derived by Gorelenkov in 2003[5]
  - δf from integrating gyrokinetic equation along equilibrium orbits
  - includes finite Larmor radius (FLR) effects analytically
    - can be significant in spherical tokamaks
  - requires *slow* resonance: $\gamma \ll \omega_b$
- Restrict to 2D velocity space: ignore p_ϕ, $\boldsymbol{r}$ dependence
  - may be incorporated with ω_* effect and integration over space
- Goal: simple stability criteria due to fast ion drive without assumptions about bulk profiles, mode structure, orbits, *etc*
  - upper bound on growth rate, since neglecting bulk damping
    - primarily: electron Landau and radiative/continuum damping
  - including finite $\omega/\omega_{ci} < 1$ and all $k_\parallel/k_\perp$ terms

[5] N. Gorelenkov *et al.* Nucl. Fusion **43**, 228 (2003)

Growth Rate Calculation

$$\gamma/\omega \propto -\, \boldsymbol{E}^* \cdot \Im\epsilon_b^A \cdot \boldsymbol{E} \big/ \, \boldsymbol{E}^* \cdot \epsilon \cdot \boldsymbol{E}$$

$$\propto -\int d\mathcal{E}\, d\mathcal{E}_\perp l^2 \delta(\theta - \theta_{\text{res}}) \left(\boldsymbol{G}'_\ell \cdot \boldsymbol{E}\right)^* \left(\boldsymbol{G}_\ell \cdot \boldsymbol{E}\right) \hat{\Pi} f_b$$

Transform to new variables: $x = \mathcal{E}_\perp / \mathcal{E}\, (= \langle\bar{\omega}_{ci}\rangle\, \lambda),\ \mathcal{E}_\| = \mathcal{E} - \mathcal{E}_\perp$

1. Jacobian. $d\mathcal{E}\, d\mathcal{E}_\perp = \mathcal{E}_\| d\mathcal{E}_\| dx/(1-x)^2$
2. Resonance. $l_{res}^2 \delta(\theta - \theta_{\text{res}}) \propto \delta(\mathcal{E}_\| - \mathcal{E}_\|^{res}) / \left|k_\|\right|$
3. FLR terms. $\boldsymbol{G}_\ell = \boldsymbol{G}'_\ell = v_\perp \left(\ell J_\ell(z)/z, iJ'_\ell(z)\right), z = k_\perp \rho_{\perp b}$
   $\left(\boldsymbol{G}'_\ell \cdot \boldsymbol{E}\right)^* \left(\boldsymbol{G}_\ell \cdot \boldsymbol{E}\right) \equiv |\boldsymbol{E}|^2 \mathcal{E}_\perp \mathscr{J}_\ell^m(z)$
4. Gradients. $\hat{\Pi} f_b = \left[\frac{\partial}{\partial \mathcal{E}} + \frac{\ell}{\bar{\omega}} \frac{\partial}{\partial \mathcal{E}_\perp}\right] f_b = \frac{1}{\mathcal{E}} \left[\mathcal{E} \frac{\partial}{\partial \mathcal{E}} + \left(\frac{\ell}{\bar{\omega}} - \lambda\right) \frac{\partial}{\partial \lambda}\right] f_b$
5. Resonance condition $\omega - k_\| \left\langle v_{\|,res} \right\rangle = \ell \left\langle \omega_{ci} \right\rangle$
   allows trivial integration over $\mathcal{E}_\|$, leaving

$$\frac{\gamma}{\omega_{ci}} \propto \int dx \frac{x}{(1-x)^2} \mathscr{J}_\ell^m(z) \left[\mathcal{E} \frac{\partial}{\partial \mathcal{E}} + \left(\frac{\ell}{\bar{\omega}} - \lambda\right) \frac{\partial}{\partial \lambda}\right] f_b(\mathcal{E}, \lambda) \Big|_{\mathcal{E} = \mathcal{E}_\|^{res}/(1-x)}$$

Growth Rate for Beam Distribution

For multi-beam distribution, and letting $u \equiv \mathcal{E}/\mathcal{E}_0 = \mathcal{E}_{\parallel}^{res}/\mathcal{E}_0(1-x)$

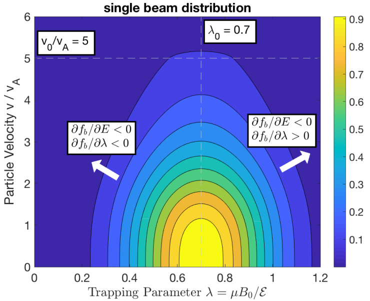
$$\frac{\gamma}{\omega_{ci}} \propto -\sum_{\text{beams}} \int_0^{1-\mathcal{E}_{\parallel}^{res}/\mathcal{E}_0} dx \, \underbrace{\overbrace{\frac{x}{(1-x)^2}}^{\text{Jacobian}} \overbrace{\mathscr{J}_{\ell}^{m}(z)}^{\text{FLR terms}} \overbrace{\frac{e^{-(x-\lambda_0\langle\bar{\omega}_{ci}\rangle)^2/\Delta\lambda^2\langle\bar{\omega}_{ci}\rangle^2}}{1+(4u)^{3/2}}}^{f_b}}_{\text{non-negative}}$$

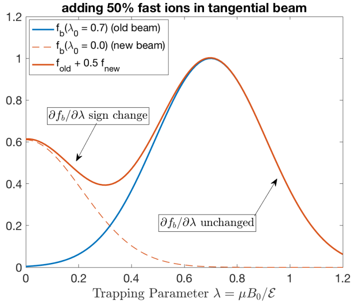
$$\Bigg[\underbrace{\frac{3}{2}\left(1-\frac{1}{1+(4u)^{3/2}}\right)}_{\substack{\partial f_b/\partial\mathcal{E}\text{ damping} \\ (\text{negligible for } \ell\neq 0)}} + \underbrace{\frac{2}{\Delta\lambda^2\langle\bar{\omega}_{ci}\rangle^2}\left(x-\lambda_0\langle\bar{\omega}_{ci}\rangle\right)\left(\frac{\ell}{\bar{\omega}}-x\right)}_{\substack{\partial f_b/\partial\lambda\text{ drive/damping} \\ (\text{has sign of } x-\lambda_0\langle\bar{\omega}_{ci}\rangle)}}\Bigg]$$

If $1-\mathcal{E}_{\parallel}^{res}/\mathcal{E}_0 < \lambda_0\langle\bar{\omega}_{ci}\rangle$, then the integrand does not change sign. Then co-propagating modes ($\ell = 0, -1$) will be **damped**, and cntr-modes ($\ell = +1$) will be **driven** by the fast ions.

One and Two Beam Distributions

- Single beam distribution has opposite sign $\partial f_b/\partial\lambda$ for $\lambda < \lambda_0$ vs. $\lambda > \lambda_0$
  - Left: single beam with $\lambda_0 = 0.7$ and $v_0/v_A = 5.0$
- Adding second beam at new λ_0 may change sign $[\partial f_b/\partial\lambda]$
  - Right: λ dependence of distribution resulting from adding a second beam with $\lambda_0 = 0$ and 50% of first beam's density





Narrow Beam Approximation

For a very narrow beam, growth rate can be integrated by expanding near $x = x_0$ $(= \lambda_0 \langle \bar{\omega}_{ci} \rangle)$

$$\frac{\gamma}{\omega_{ci}} \underset{\sim}{\propto} - \int_{x_0-\epsilon}^{x_0+\epsilon} \underbrace{\frac{x}{(1-x)^2} \left(\frac{\ell}{\bar{\omega}} - x \right) \mathscr{J}_\ell^m(z)}_{h(x) \approx h(x_0) + (x-x_0) h'(x_0)} (x - x_0)\, e^{-(x-x_0)^2/\Delta x^2} dx$$

$$= -h'(x_0) \underbrace{\Delta x^2 \left(-\epsilon \exp(-\epsilon^2/\Delta x^2) + \Delta x \sqrt{\pi} \mathrm{Erf}(\epsilon/\Delta x)/2 \right)}_{\text{positive}}$$

- leads to stability condition on $k_\perp \rho_{\perp b}$ only
  - similar to conclusions drawn in Gorelenkov 2003 NF
- requires $\Delta\lambda \lesssim 0.05$ for validity
  - much narrower than realistic beam distributions $(\Delta\lambda \approx 0.3)$

Wide Beam Approximation

- Two assumptions make integration tractable
  1. For large $\Delta\lambda$, approx $\partial_x e^{-(x-x_0)^2/\Delta x^2} \approx -2(x-x_0)/\Delta x^2$
     - Reasonable for $x_0 - \Delta x/\sqrt{2} < x < x_0 + \Delta x/\sqrt{2}$
  2. For $\zeta \lesssim 1$, use small argument expansion for $\mathscr{J}_\ell^m(z)$
     - $z = k_\perp \rho_{\perp b} = \frac{k_\perp v_{\|,res}}{\omega_{ci}} \sqrt{\frac{x}{1-x}} \equiv \zeta \sqrt{\frac{x}{1-x}}$
     - For $\zeta \gg 1$, use asymptotic expansion, but atypical
- Yields sufficient conditions for net drive from anisotropy:

  $\ell = 0$ co-CAE requires $v_0 > v_{\|,res}/(1-x_0)^{5/8}$

  $\ell = -1$ co-CAE requires $v_0 > v_{\|,res}/(1-x_0)^{3/4}$

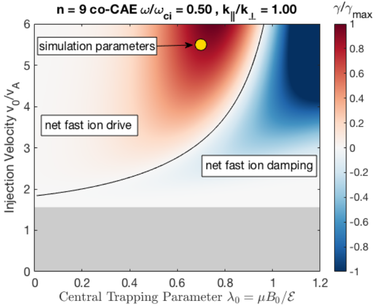
  $\ell = +1$ cntr-CAE requires $v_0 < v_{\|,res}/(1-x_0)^{3/4}$
  - $\ell = \pm 1$ GAE has same conditions as CAEs
- $\partial f_b/\partial\mathcal{E}$ damping is non-negligible for $\ell = 0$, requiring instead:

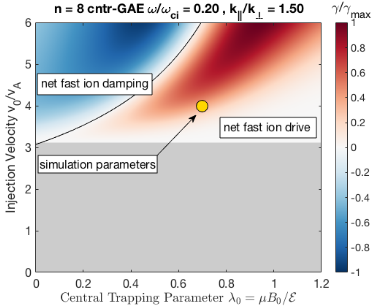
$$v_0 > v_{\|,res}/\left(1 - \frac{x_0}{2}\left[1 + \sqrt{1 + 12\Delta x^2/5x_0^2}\right]\right)^{5/8}$$

- Typical NSTX(-U) beams have $\Delta x \approx 0.3$ – sufficiently large

Growth Rate via Numerical Integration

- Numerically integrate full analytic expression for growth rate
  - Depends on beam parameters v_0/v_A and λ_0
  - Depends on mode parameters ω/ω_{ci}, $k_{||}/k_{\perp}$, and ℓ_{res}
- red: net fast ion drive, blue: net fast ion damping, gray: insufficient beam velocity for resonant interaction
- black curve: approximate boundary derived analytically





Potential Suppression Techniques

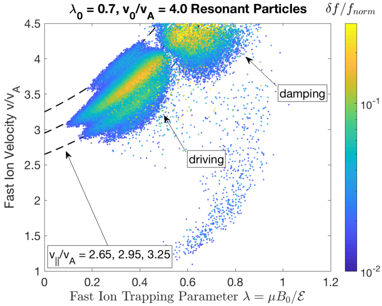
Adding a second beam in the stable region should **damp** the mode

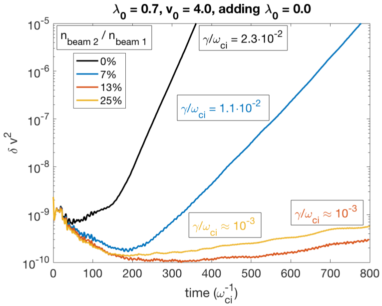
1. Add a beam in a different geometry
   - ▶ To suppress cntr-GAEs, add a more tangential beam (low λ_0)
     - explains NSTX-U GAE suppression observations
   - ▶ To suppress co-CAEs, add a more *radial* beam (high λ_0)
     - testable in future NSTX-U experiments
   - – To suppress either, counter-inject a new beam at the same λ_0
     - potential experiments on DIII-D
2. Add a beam with a different injection energy
   - – To suppress cntr-GAEs, add a beam with a higher voltage
     - unrealistic due to hardware constraints
   - – To suppress co-CAEs, add a beam with a *lower* voltage
     - possible for limited voltage range

Items marked with ▶ are included in this poster.

GAE suppression via Tangential Injection

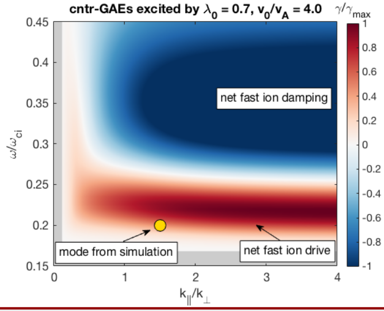
- To stabilize cntr-GAE, add second beam with small $\lambda_0 = 0$
  - Adding second beam with 7% density of original beam reduces γ/ω_{ci} by 50% – adding 13% in new beam reduces by 10x
    - Simulated mode: $n = 8, \omega/\omega_{ci} = 0.2, k_{||}/k_{\perp} = 1.5$
- Previously shown in experiments and simulations of NSTX-U
  - this confirms the phenomena in the NSTX-like model equilibrium before pursuing the speculative CAE suppression

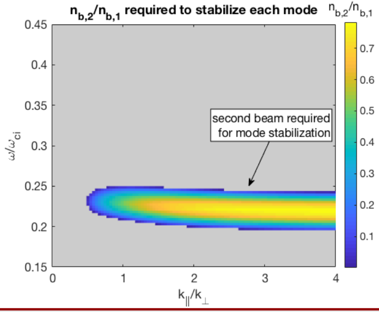




GAE suppression via Tangential Injection

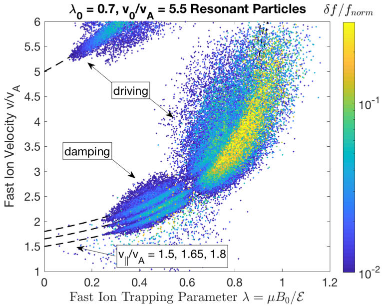
- Previous case is for a single mode with fixed ω/ω_{ci} and $k_{||}/k_{\perp}$
- Theory can estimate second beam power required to suppress *all* GAEs excited by a beam with fixed λ_0 and v_0/v_A
  - Right figure: gray region indicates mode damped by first beam or *ad-hoc* bulk damping rate inferred from simulations
- Suppression may be achieved more efficiently with λ_0 small but nonzero, due to where fast ion damping peaks

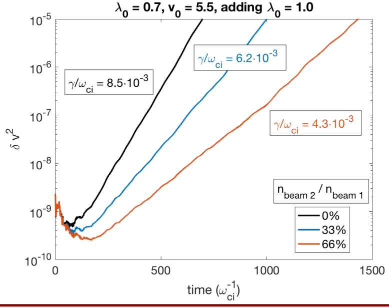




CAE suppression via Radial Injection

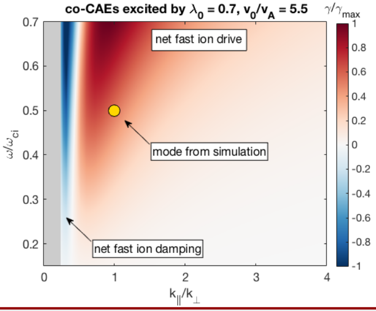
- To stabilize co-CAE, add second beam with large $\lambda_0 = 1$
  - Adding second beam with 66% density of original beam reduces γ/ω_{ci} by 50% but does not totally suppress.
    - Simulated mode: $n = 9, \omega/\omega_{ci} = 0.5, k_{\|}/k_{\perp} = 1.0$
- Drive/damping regions of phase space are *relative* to central beam pitch – can change sign with new beam
  - $\gamma \sim \sum_i \int \partial f_i/\partial\lambda d\lambda \sim \sum_i \int (\lambda - \lambda_{0i}) f_i d\lambda$

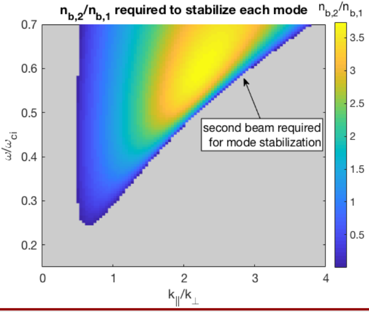




CAE suppression via Radial Injection

- Previous case is for a single mode with fixed ω/ω_{ci} and $k_\parallel/k_\perp$
- Theory can estimate second beam power required to suppress *all* CAEs excited by a beam with fixed λ_0 and v_0/v_A
  - Right figure: gray region indicates mode damped by first beam or *ad-hoc* bulk damping rate inferred from simulations
- Agrees with simulations indicating CAE damping is less robust
  - More effective damping with even larger $\lambda_0 \approx 1.2$





Summary and Conclusions

- A 2D stability theory is developed for CAEs/GAEs driven by beam-like distributions
- Marginal stability boundaries can be derived in two regimes
  - For wide beams ($\Delta\lambda \gtrsim 0.2$), realistic NSTX(-U) case:
    - $\ell = 0$ co-CAEs require $v_0 > v_{\|,res}/(1 - \langle\bar{\omega}_{ci}\rangle\,\lambda_0)^{5/8}$
    - $\ell = 1$ cntr-GAEs require $v_0 > v_{\|,res}/(1 - \langle\bar{\omega}_{ci}\rangle\,\lambda_0)^{3/4}$
  - For narrow beams ($\Delta\lambda \lesssim 0.05$), idealized case:
    - net fast ion drive depends on value of $k_\perp \rho_{\perp b}$
- Hybrid simulations confirm CAE/GAE stabilization via additional beam injection with specific geometry
  - mitigate cntr-GAEs via additional tangential injection
  - mitigate co-CAEs via additional radial injection
    - but less effective than for GAEs

Future Work

- Simulate CAE suppression in a self-consistent equilibrium
- Model an NSTX-U discharge which observed co-CAEs
  - make predictions for CAE suppression in a future experiment
- Further investigate why second beam suppression is less effective for CAEs than GAEs
- Determine optimal λ_0 to add new beam with for a given marginal stability boundary and mode properties
- Explore CAE suppression via additional beam at lower voltage
- Study potential CAE/GAE suppression with counter-beams