Predictions and observations of global beta-induced Alfvén-acoustic modes in JET and NSTX

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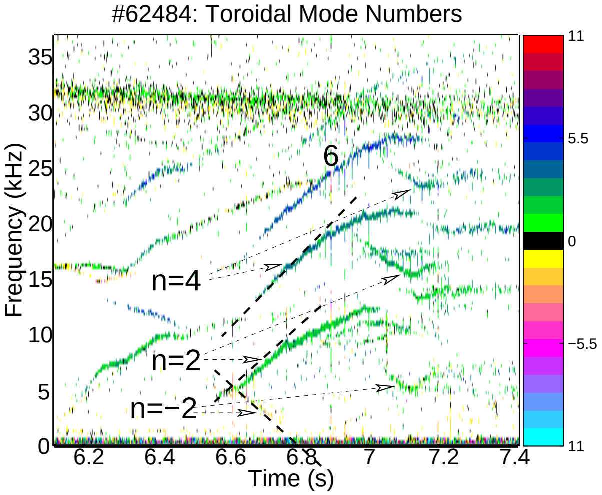
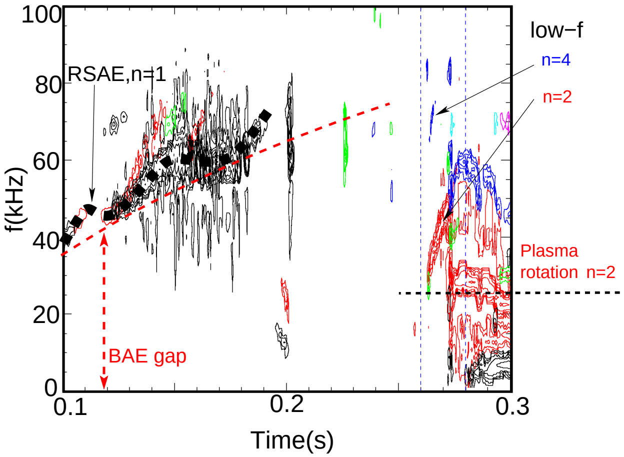
EFDA EUROPEAN FUSION DEVELOPMENT AGREEMENT
JET

PPPL
PRINCETON PLASMA
PHYSICS LABORATORY

New experimental observations on JET and NSTX motivate low frequency mode study

	JET	$NSTX$
$B(T)$	2.7	0.45
$R/a(m)$	$2.95/0.95$	$0.85/0.66$
$\beta_{pl}/\beta_{fast}\ (\%)$	$1/1$	$34/15$
$fast\ ions$	$2MW\ ICRH$	$2MW\ NBI$





Frequencies sweep up from $f_{pl} = 0 - 20kHz$

Only even n's were observed.

q-profile: monotonic in JET, reversed in NSTX.

Frequency is much lower than RSAE/TAE frequency, v_A/qR and $\omega_* < 1kHz$

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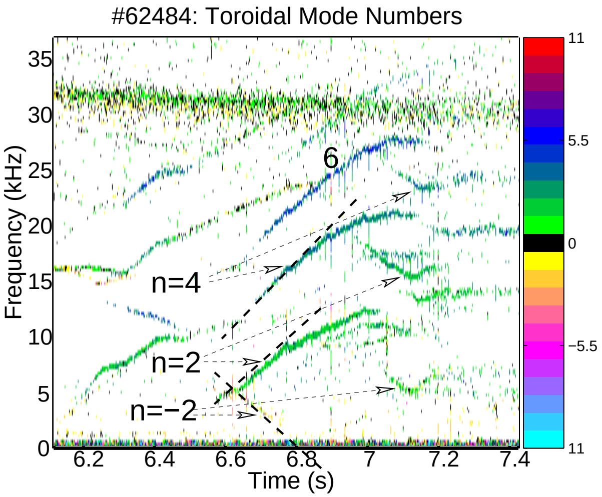

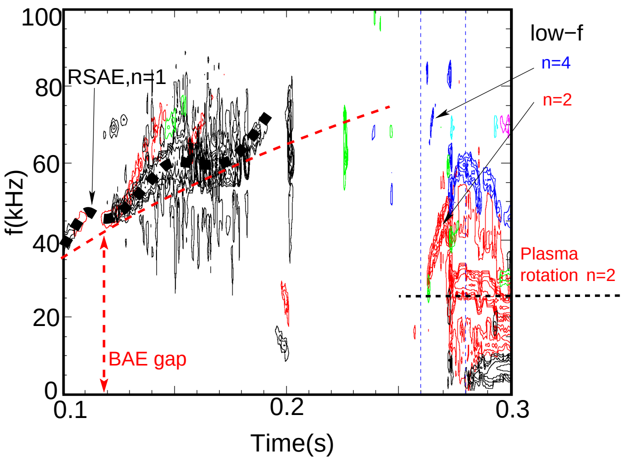

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What are these modes: EPMs, fishbones, KBM, TAEs?

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- Due to coupling to acoustic branch strong interaction with thermal ions is expected:
  - $\Rightarrow$ strong drive due to fast ions and strong damping due to thermal ions,
  - $\Rightarrow$ potential for energy channeling from beam ions directly to thermal ions (α-channeling, Fisch'93, hot-ion mode, LiWall).

TALK OUTLINE

1. Theory of Alfvén - acoustic continuum in ideal MHD
2. JET analysis and data comparison
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Theory of Alfvén/acoustic continuum

Simplified shear Alfvén and acoustic coupled equations capture main effects in low-β, large aspect ratio plasma, low ω_*, (Cheng, Chance '86):

$$\Omega^2 y + \partial_\parallel^2 y \quad + \gamma\beta \sin\theta\, z \quad = 0 \ (Alfvenic) \tag{1}$$

$$\Omega^2 \left(1 + \frac{\gamma\beta}{2}\right) z + \frac{\gamma\beta}{2} \partial_\parallel^2 z \quad + 2\Omega^2 \sin\theta\, y \quad = 0 \ (acoustic)\,, \tag{2}$$

where $\Omega \equiv \omega R_0 / v_A$, $y \equiv \xi_s \varepsilon / q$, $\xi_s \equiv \vec{\xi} \cdot \frac{[\mathbf{B} \times \nabla\psi]}{|\nabla\psi|^2}$ and $z \equiv \nabla \cdot \vec{\xi}$, $\hat{k}_\parallel \equiv i\partial_\parallel$.

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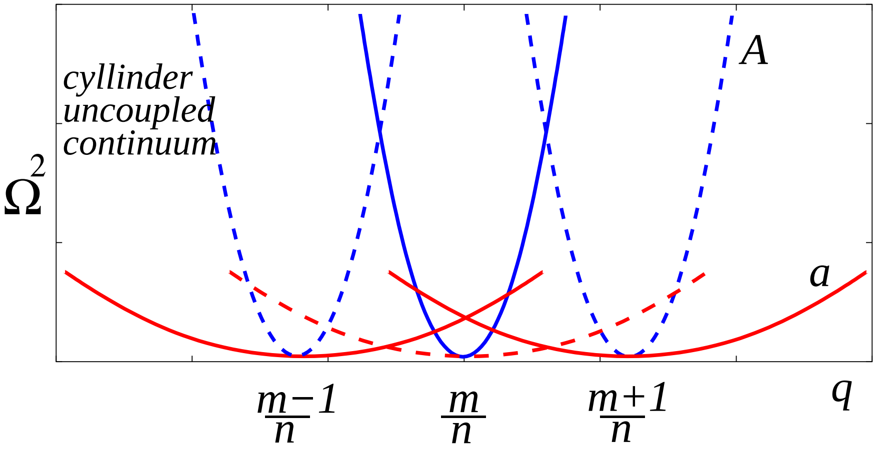
Various solutions follows (Winsor'68, Goedbloed'75, Mikhailovski'75,'98, Chu'92, Turnbull '92, Zonca'96, van der Holst'00, Breizman'05, Berk'06):

- Pure acoustic modes (AMs) $\Omega^2=\frac{1}{2}\gamma\beta k_\parallel^2$.
- Pure Alfvénic branch $\Omega^2=k_\parallel^2+\gamma\beta\left(1+1/2q^2\right)$.
- GAMs: $\Omega^2=\gamma\beta\left(1+1/2q^2\right)$ in the assumption of $\Omega^2\geq\gamma\beta$.
- Modified shear Alfvén branch $\Omega^2=k_0^2/\left(1+2q^2\right)$ exists for $\Omega^2\ll\gamma\beta$.

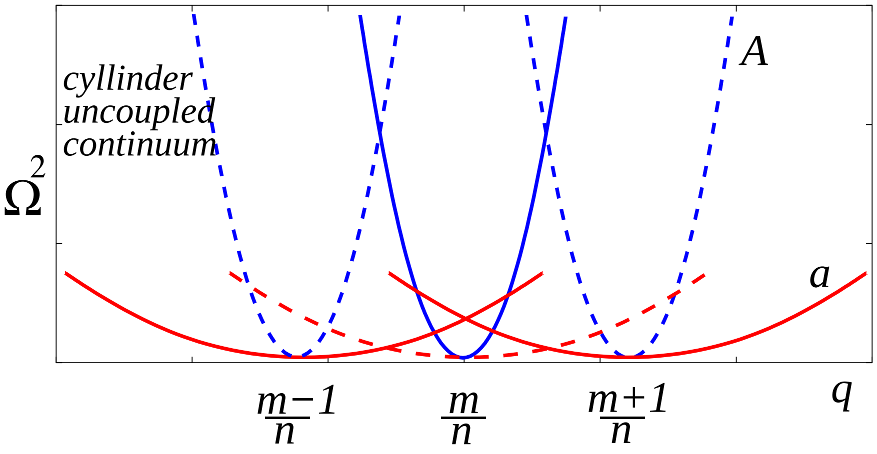
Alfvén/acoustic coupling in toroidal equilibrium (schematic)

- Alfvén (A) continuum at low frequency: $\Omega^2 = k^2_{0,\pm1}$
- Acoustic (a) branch $\Omega^2 = \gamma\beta k^2_{0,\pm1}/2\,(1+\delta)$

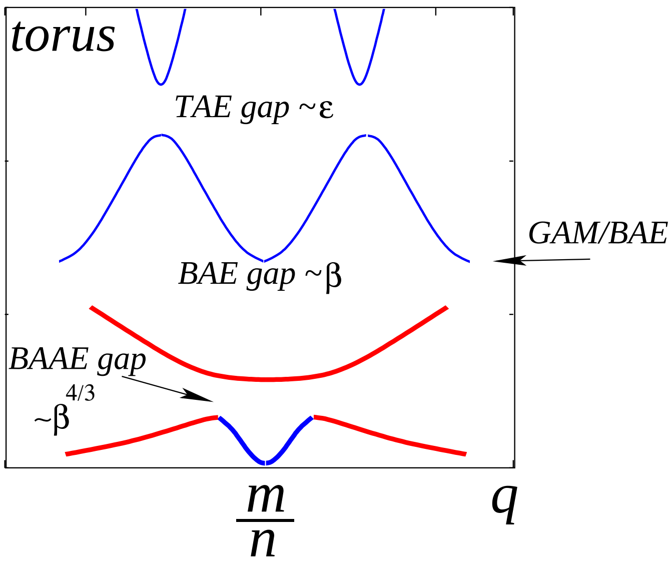


Alfvén/acoustic coupling in toroidal equilibrium (schematic)

- Alfvén (A) continuum at low frequency: $\Omega^2 = k_{0,\pm1}^2 \,/\, \left(1+2q^2\right)$ (modified)
- Acoustic (a) branch $\Omega^2 = \gamma\beta k_{0,\pm1}^2/2\,(1+\delta)$ is coupled via $m\pm1$ sidebands with modified Alfvén continuum (m harmonic) due to geodesic curvature and pressure.

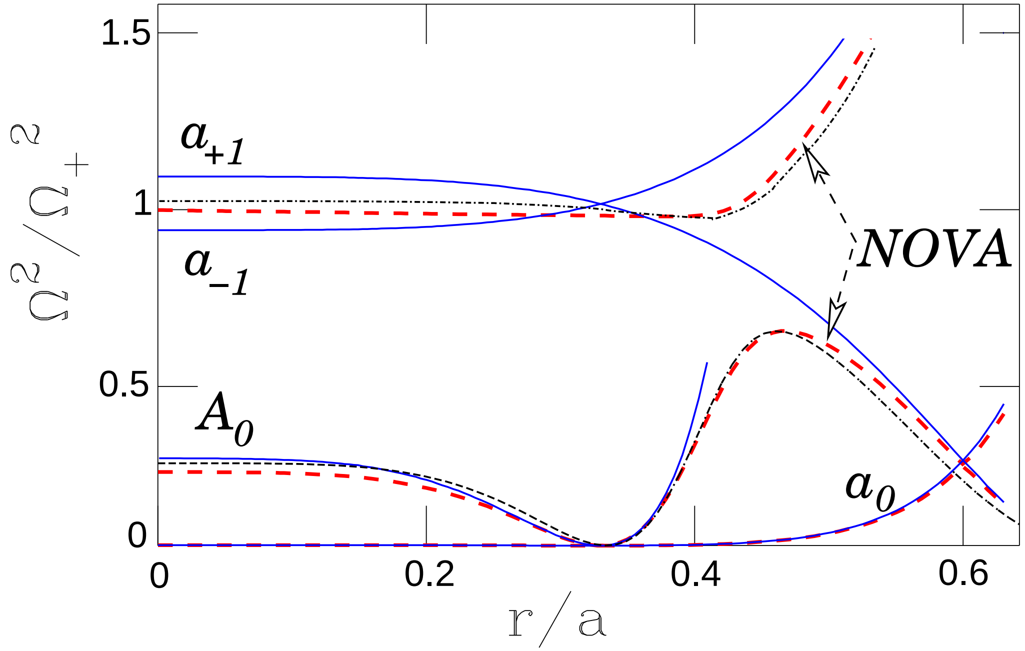


$\Rightarrow$



Analytic dispersion for Alfvén/acoustic continuum gap is derived

- Consider JET, monotonic q-profile, ten times higher aspect ratio.
- $\Omega_+ = \sqrt{\gamma\beta/2}/q_r$ (compare with GAM $\Omega = \sqrt{\gamma\beta\left(1+1/2q^2\right)}$).



NOVA - - - -

Theory:

- torus —-
- cyllinder ——

Exact MHD (NOVA) continuum is in good agreement with theory.

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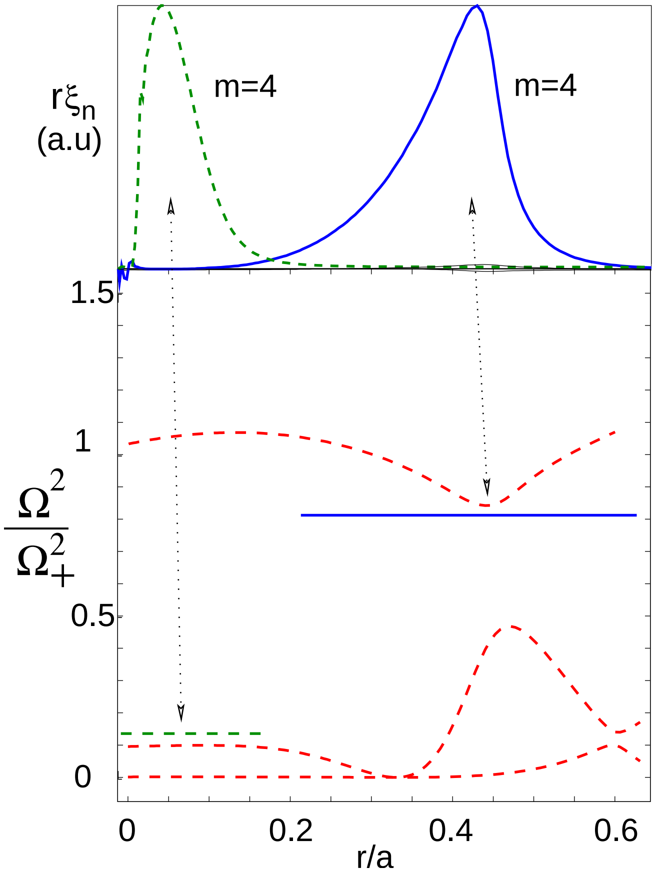
JET plasma analysis: two global BAAE modes are found numerically



- Core localized and gap BAAEs are found with one dominant poloidal harmonic.
  - Monotonic $q-$profile (EFIT): $q_0 \simeq 1$, $q_a = 4$.
- Core localized BAAE (A): $\omega = v_A k_{\|} / \sqrt{1+2q^2_{min}}|_{r=0}$.
- Gap BAAE (A-a): $\Omega_+ \simeq v_A \sqrt{\gamma\beta/2}/q_{min}R$.
- $n = 4$, $r\xi_n$ is shown.
- $\nabla\xi$, $m \pm 1$ sidebands are present ($\sim \xi_\theta/a$).

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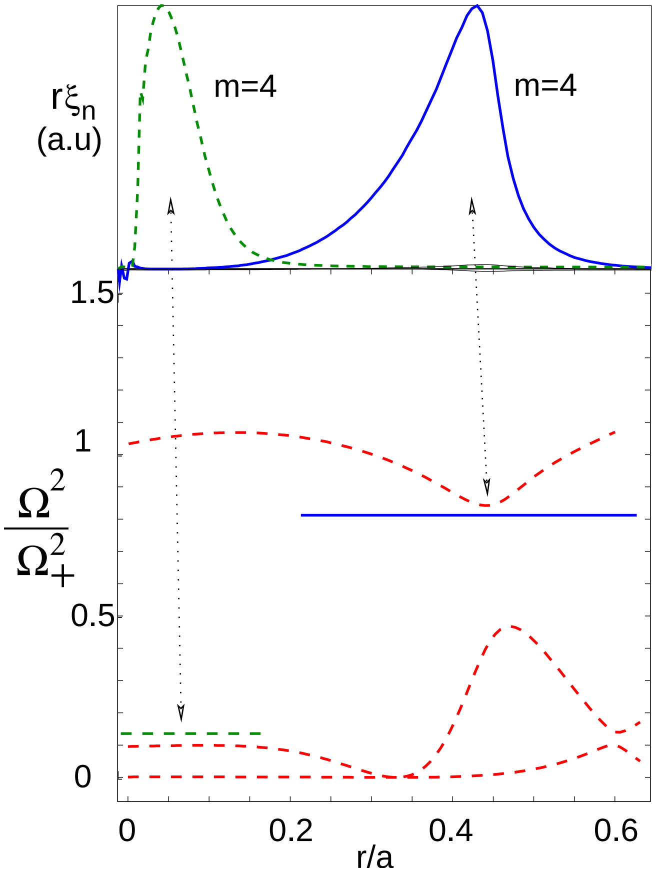

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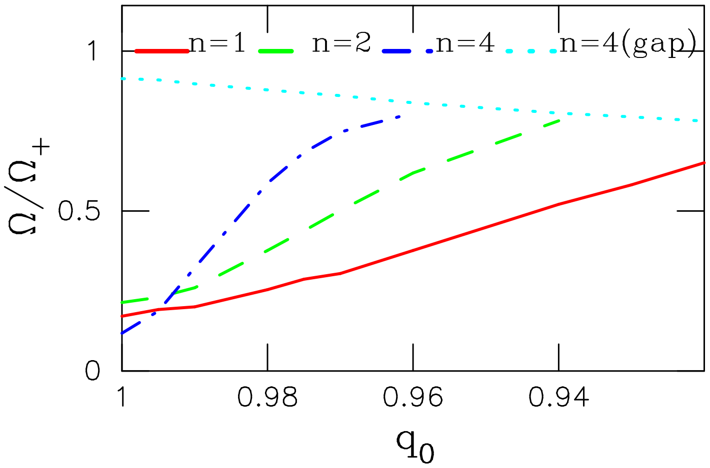
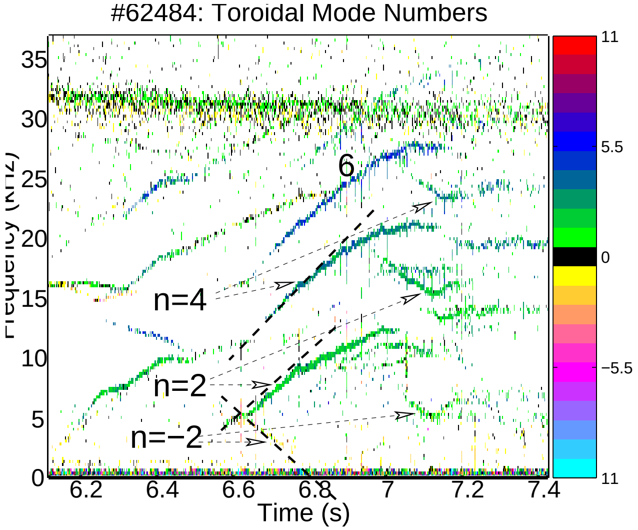
BAAE frequency is related to q_{min} value ⇒ useful for diagnostic

Relaxing q-profile results in BAAE frequency up-sweep

- Core BAAE activity is predicted to have sweeping frequency ($T_e > T_i$)
- Up-chirp is limited by the gap, $\Omega_+ \simeq v_A\sqrt{\gamma\beta/2}/qR$.
  $\Omega_+ = 31kHz$ for pure electron plasma.
- Core BAAE evolution frequency is close to modified Alfvén branch.
- Rotation is inferred $f_{rot} = 2.5kHz$.





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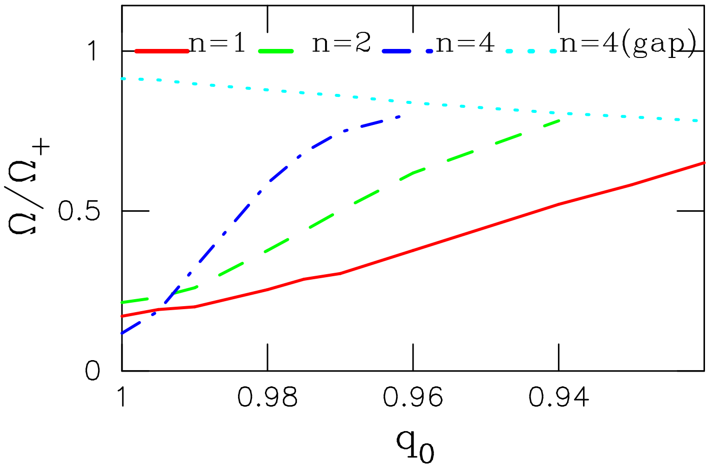

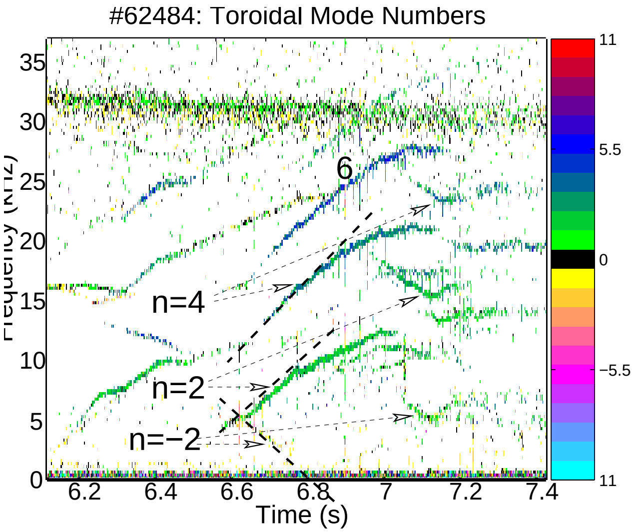

NOVA predicts gap $f_{BAAE} = 24.8kHz$ against observed $14kHz$, all n's exist ($q_0 = 1$).

Possible way to resolve this is to assume local negative shear with $q_{min} = 3/2$:

1) frequency $\sim q^{-1}$, goes down to $\sim 16kHz$,

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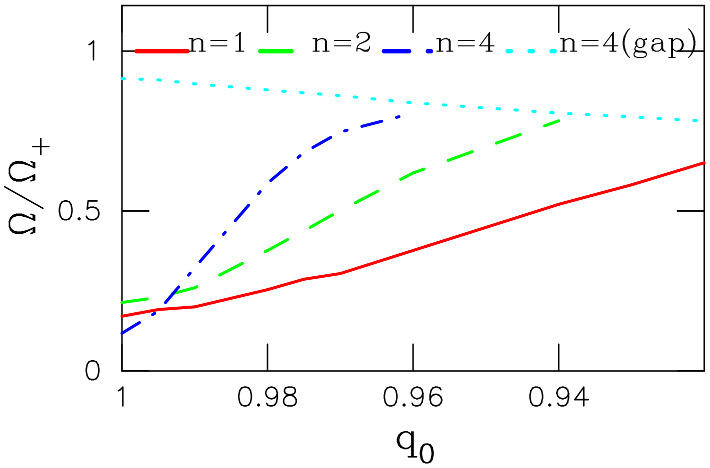

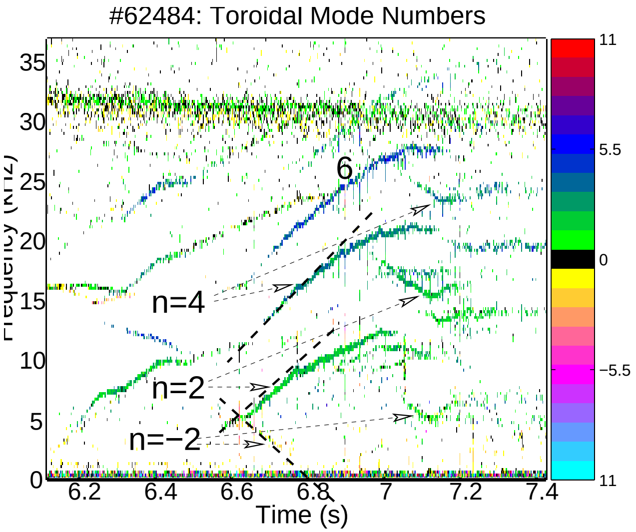

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MSE was available only later in the shot.

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NSTX experiments with MSE address frequency mismatch

- Low density $n_e \simeq 3 \times 10^{19} m^{-3}$, $P_{NBI} = 2MW$, $E_{NBI} = 90keV$.
- 12 channel MSE measures q profile (reversed shear).
- Need to test the theory.

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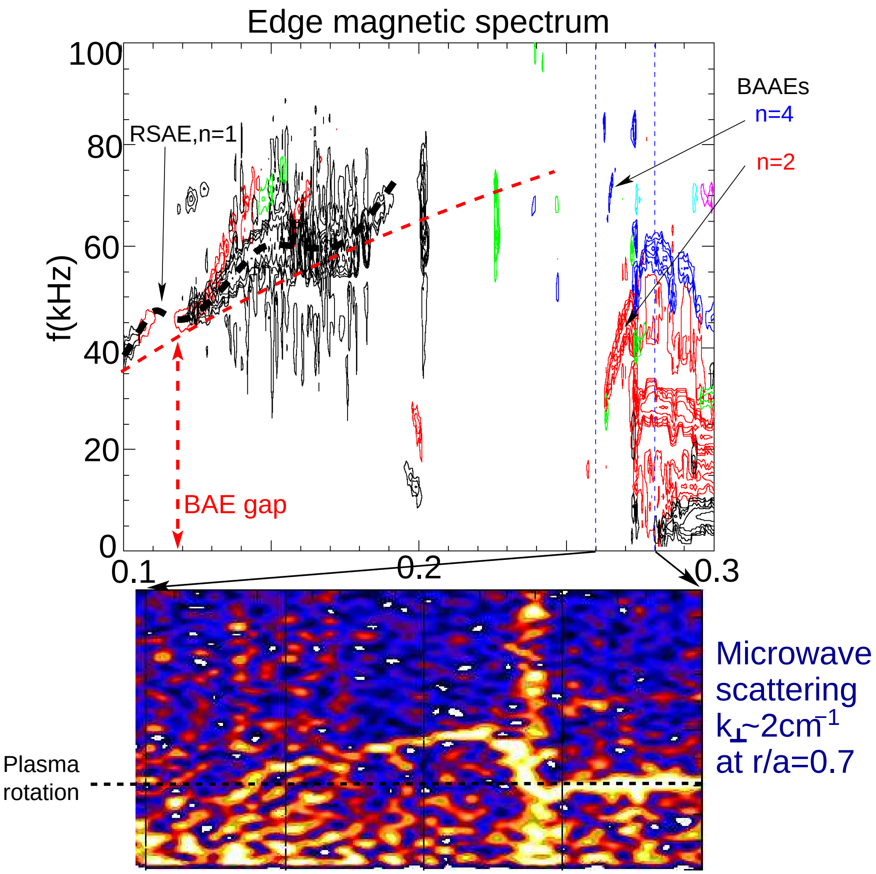
- Low frequency oscillations (BAAEs) are seen unstable:
  - Characteristic upshift frequency evolution from zero (plasma frame).
  - BAAEs reside in wider BAE gap $f \sim \sqrt{\beta_{pl}}$.
- High-k component of BAAE at $r/a = 0.7 \Rightarrow$ conversion to KAW (H.Park, P2.045).



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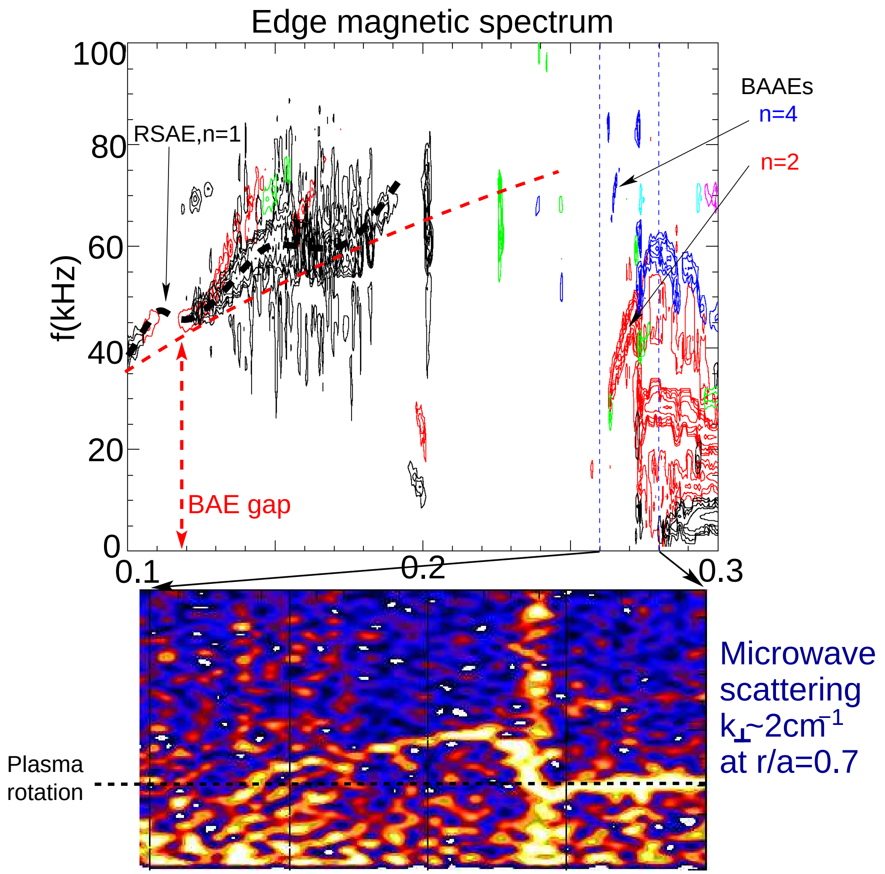
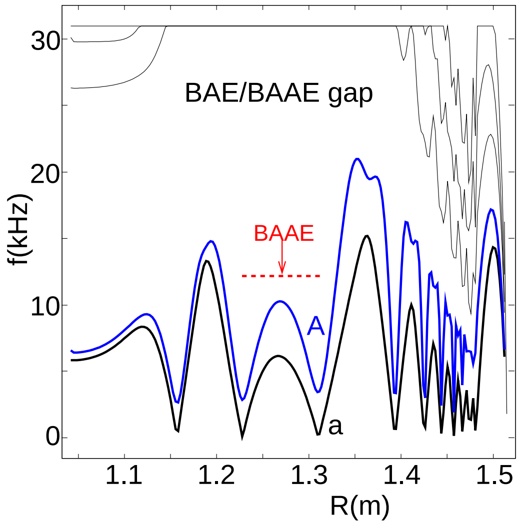
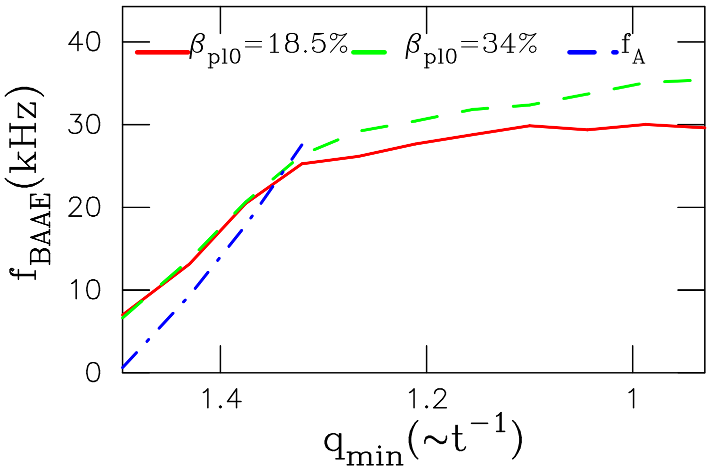


TAE/RSAEs are suppressed (see E. Fredrickson poster) and BAAEs are excited by beams in high-β NSTX plasmas (typically $\beta_{pl} >\sim 15\%$).

Numerically global BAAE modes are found at q_{min} surface in NSTX





MSE measured inversed q-profile is used in NOVA modeling.

- At high-$\beta_0 = 34\%$, BAE is wide, up to TAE frequency.
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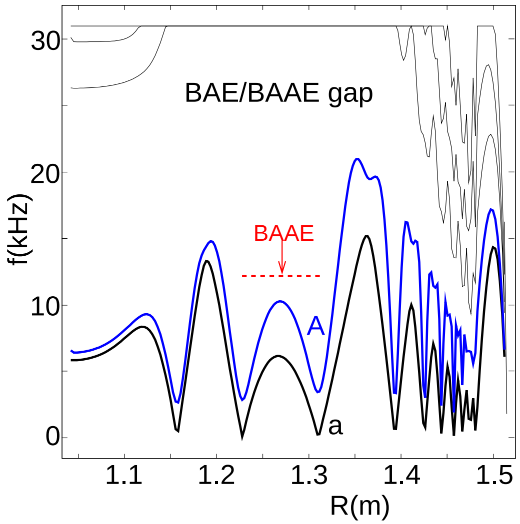

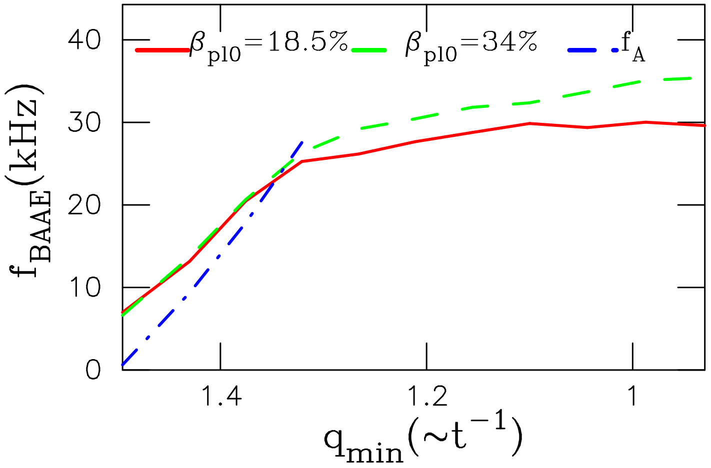

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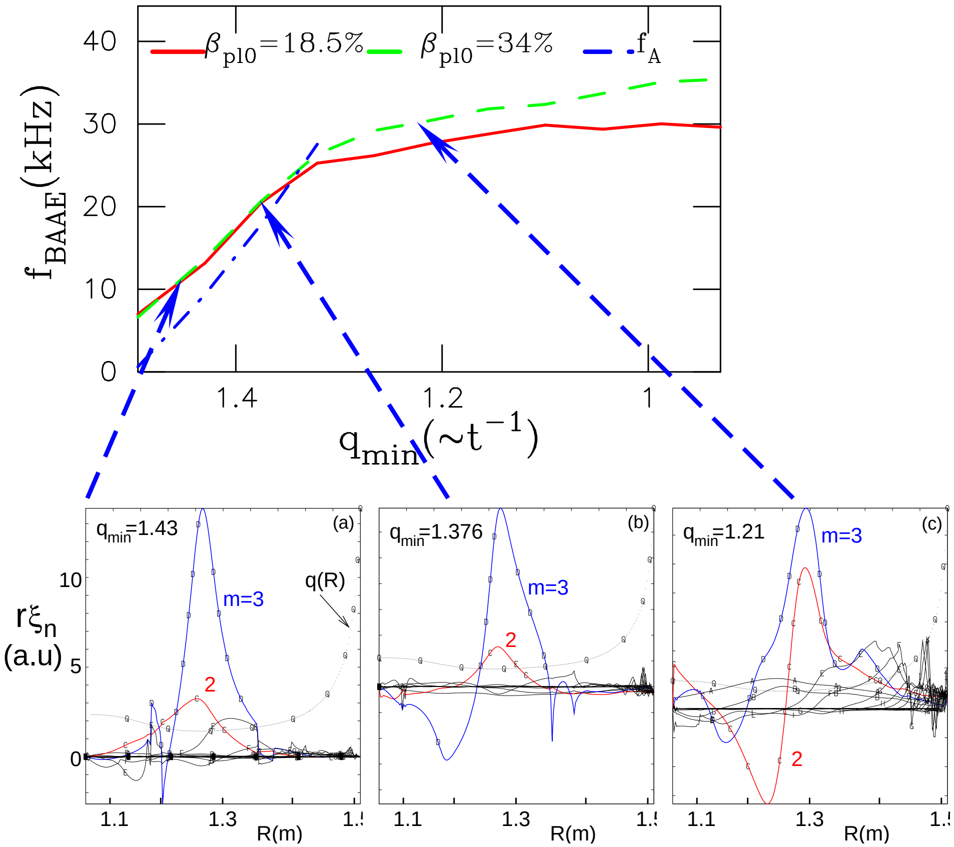
- Two Alfvén/acoustic (A/a) continuum branches are found with $\Omega^2 < \gamma\beta$, $n = 2$
- Low shear BAAE frequency
  - does not depend on β for q close to rational
  - continuously transporms to gap mode (due to higher β, strong coupling)
  - f_{BAAE} is close to modified Alfvén branch $f_A = v_A k_{\|} / \sqrt{1 + 2q^2_{min}}|_{r=0}$.

NOVA: BAAE broadens radially as q_{min} decreases

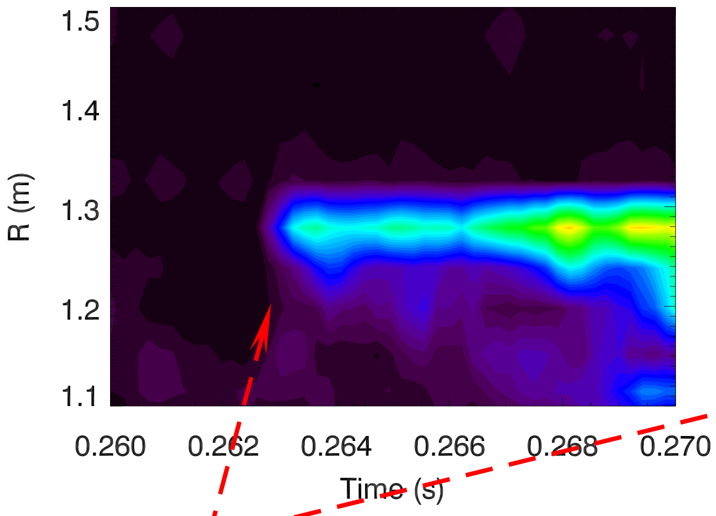


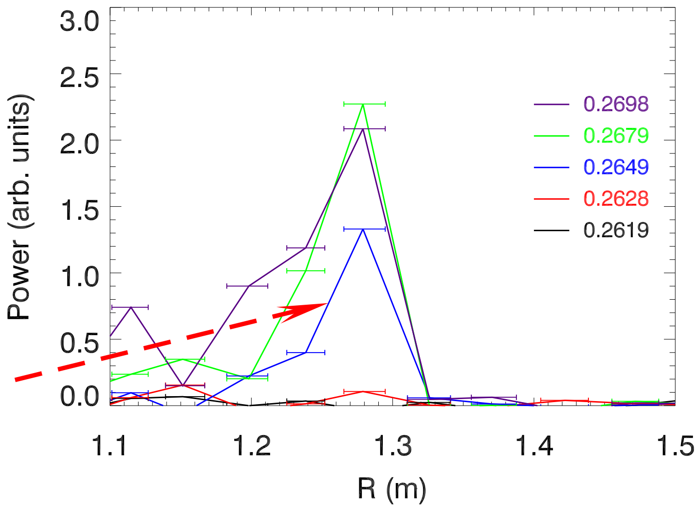
- BAAE frequency sweeps as q-profile relaxes.
- One dominant harmonic $m = nq_{min} = 3$.
- BAAEs interact with the continuum.

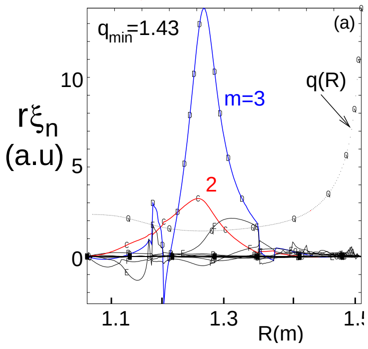
Ultra SXR measures the same radial structure broadening

Raw USXR signal (∼BAAE structure)

Radial profile evolution







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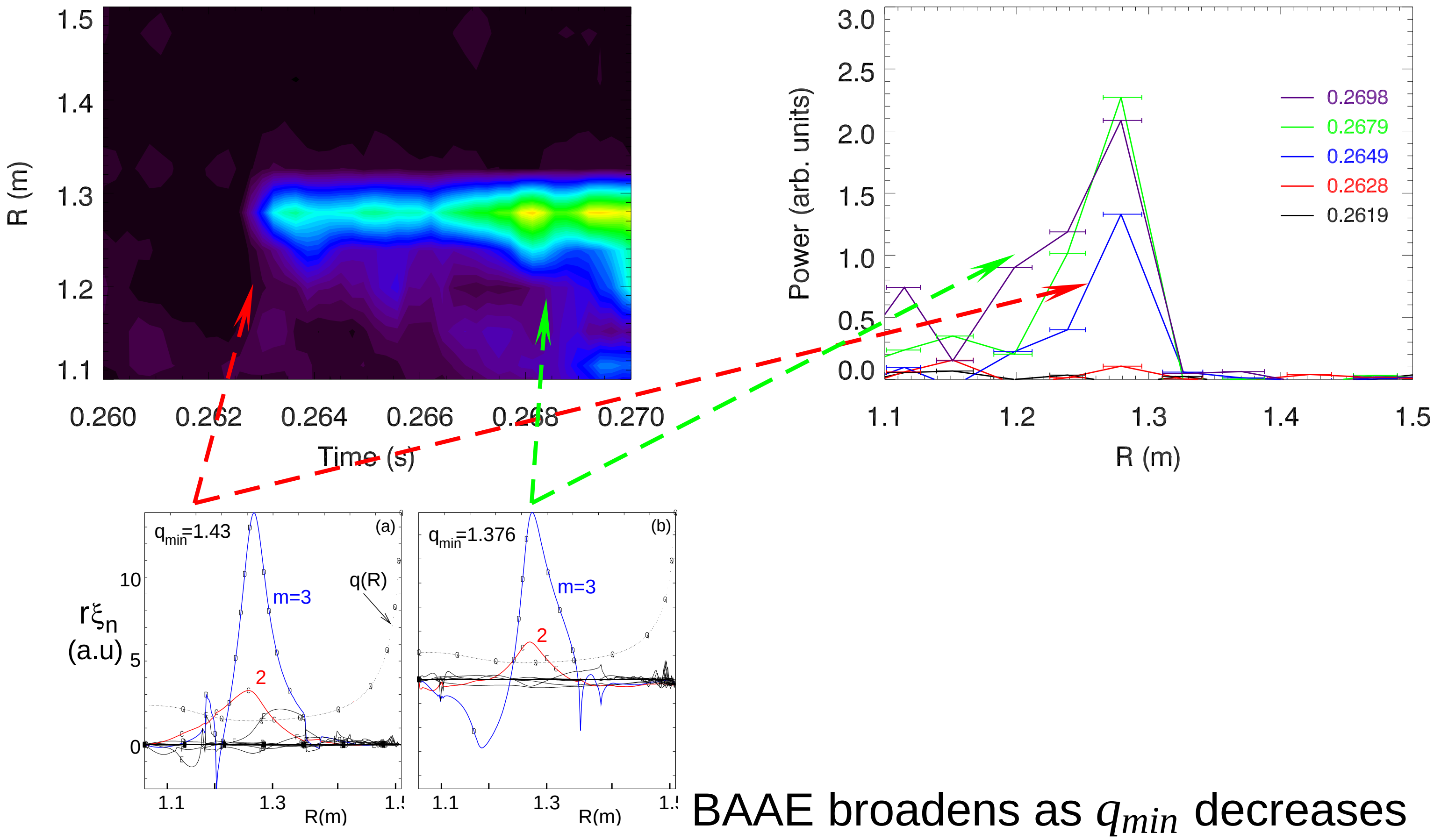

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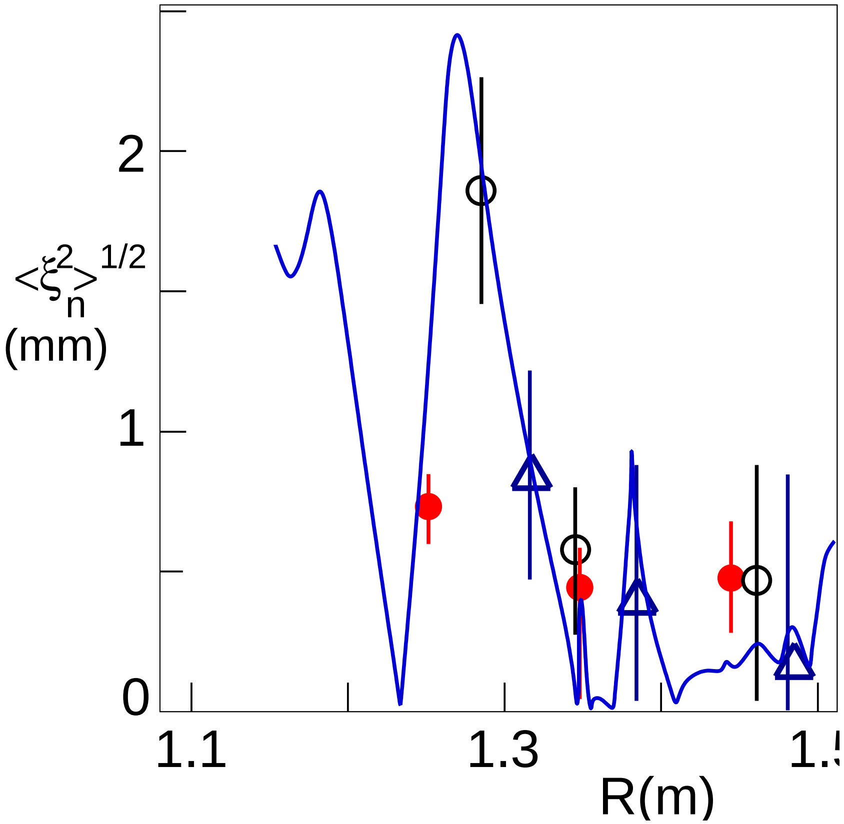
Reflectometer confirms localized BAAE structure



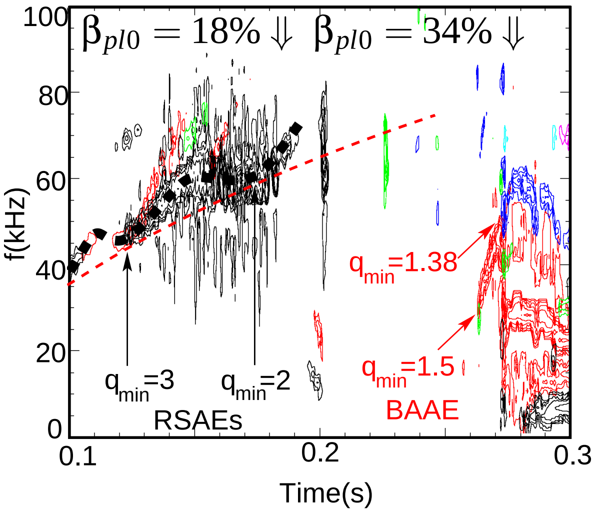
- Three plasmas, 3 points each:

  ● $n_e = 3.3 \times 10^{19} m^{-3}$

  O $n_e = 3.6 \times 10^{19} m^{-3}$

  Δ $n_e = 3.8 \times 10^{19} m^{-3}$
- Vertical axis refers to points ● - #123816.
- Measurements are taken at signal maximum.
- Internal fluctuations level $\delta n/n \sim 2 \times 10^{-3}$.

In high-β plasma BAAEs may be the only MHD spectroscopy tool for determining q-profile



- RSAE/TAEs can be used to infer q_{min} in low-, medium-β plasma
- Zero BAAE frequency point (plasma frame) indicates rational q_{min}.
- BAAE activity is terminated at $t = 0.275s$.

  Potential interplay of beam driven instabilities with internal $m = 3/n = 2$ kink-like instability - similar to TAE/sawtooth nonlinear interplay (Bernabei'01, Sharapov'06).
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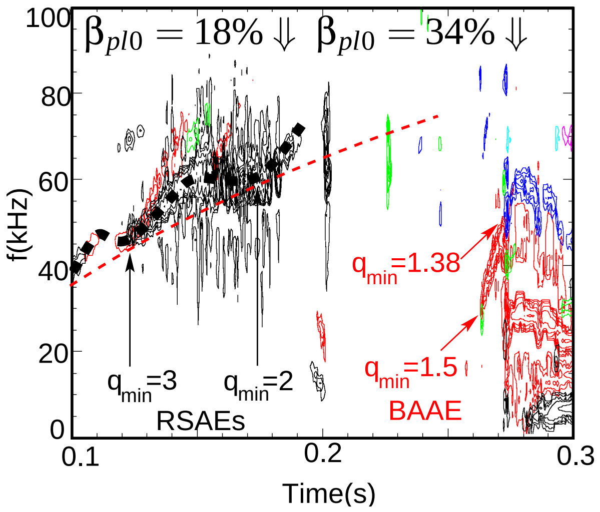

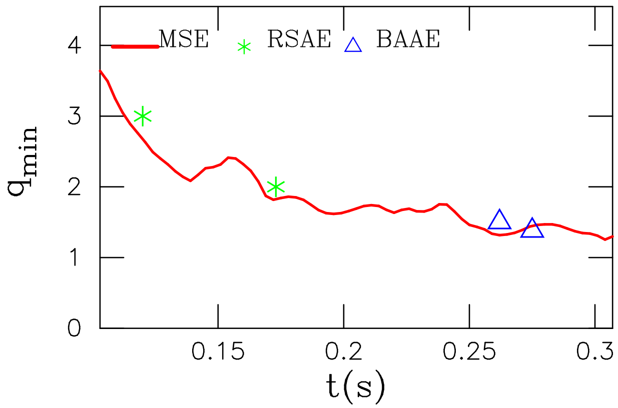

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TALK OUTLINE

1. Theory of Alfvén - acoustic continuum in ideal MHD
2. JET analysis and data comparison
3. NSTX analysis and data comparison
4. **Discussion and Summary**

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- Theory and numerical analysis show:
  - the existence of geodesic curvature induced gaps in the Alfvén/acoustic continuum below GAM frequency (van der Holst'00),
  - low-n global beta-induced Alfvén/acoustic eigenmodes - BAAE are found,
  - BAAEs exist in finite beta plasma within wider BAE gap.

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  - frequency is lower $0 < \Omega < \sqrt{\gamma\beta/2}/q_{min}$ vs. $\Omega = \sqrt{\gamma\beta\left(1+1/2q_{min}^2\right)}$ for BAE/GAM.
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- Kinetic modification of MHD theory is required for new global modes (Zonca'96, Mikhailovski'98):
  - damping is expected to be strong due to phase velocity of acoustic component close to thermal ion velocity.
  - dominant electron plasma is expected to be favorable for BAAE existence.

Summary (continued)

- NOVA shows existence of BAAEs in ICRH JET and NBI NSTX plasmas.
- Qualitatively NOVA predicts BAAE frequency evolution in agreement with observations on both tokamaks.
- In NSTX $n = 2$ low shear BAAE internal structure, frequency and their evolution are in agreemnt with NOVA.
  - MSE measurements on NSTX seem to validate theory and MHD (q_{min}) spectroscopy via BAAEs.
  - Maybe useful for burning plasmas, ITER.
- For pure electron plasma (lowest f) gap (sound wave effect) BAAE frequency is above the measured value in JET by factor ~ 1.77 (if $T_i \ll T_e$).
- Need to reconcile theory and experiment via kinetic theory and/or:
  - may imply local reversed shear with $q_{min} = 1.5$ but strong indications exist for $q_0 = 1$,
  - possible redistribution of the current drive due to:
    - MHD activity H-minority transport,
    - ICRH current drive,
    - runaway electrons in low density JET plasma.
- BAAEs are expected in plasmas with $T_e > T_i$ and strong drive from fast ions and/or η_i (ITG-like drive)

Thank you !