Gyrokinetic Analysis of Electrostatic Turbulence and Transport Properties in Tokamaks and Spherical Tori

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in collaboration with

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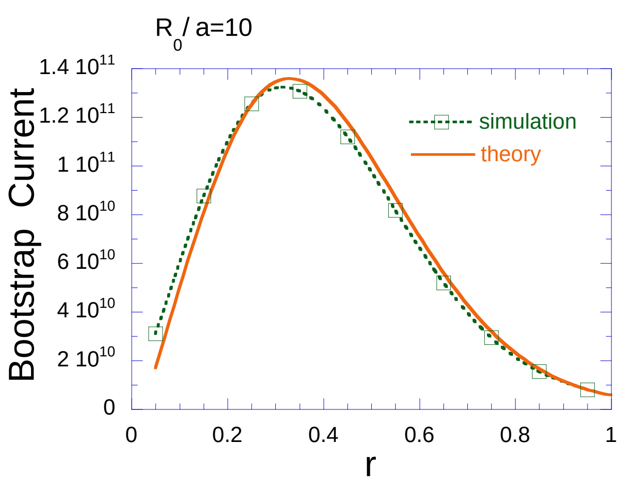
Outline

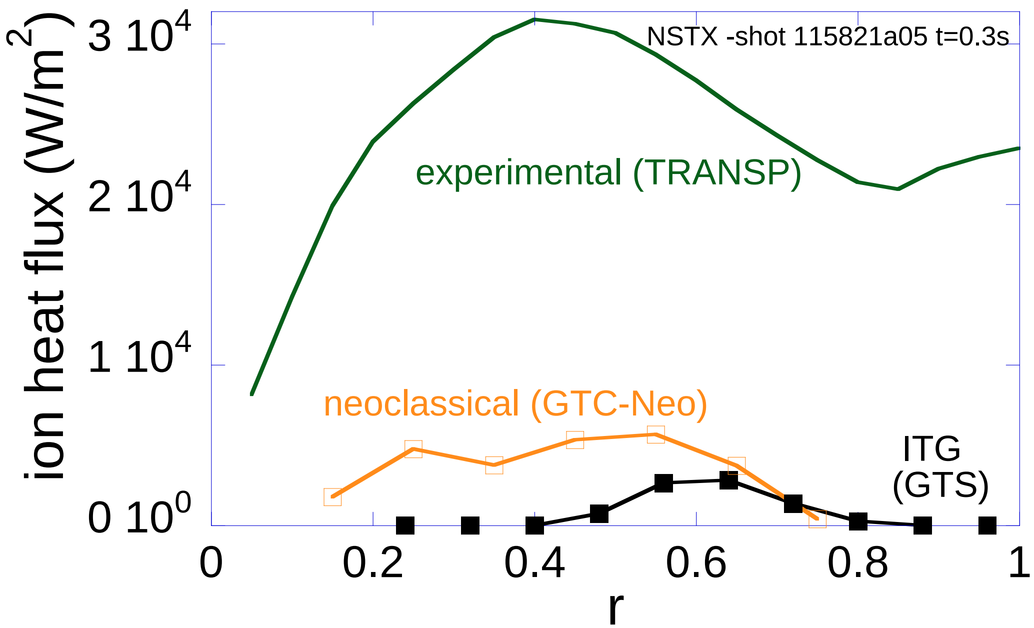
- Properties of ITG turbulence driven ion transport in NSTX plasmas, comparison with DIII-D
- Simulation of toroidal momentum transport
- Anisotropic properties of neoclassical equilibrium
- Recent NSTX-physics-oriented development of GTS with respect to electron physics

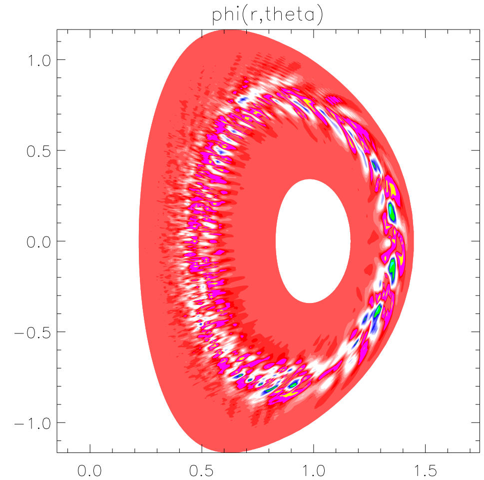
Gyrokinetic PIC Simulations of Neoclassical Transport

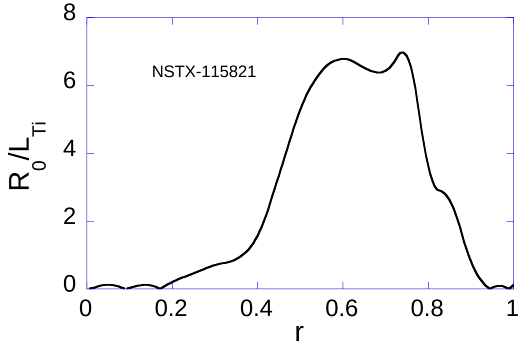
- GTC-Neo: δf global Gyrokinetic PIC Code
  [Wang et al., Comput. Phys. Commun. (2004); Phys. of Plasmas (2006)]
- Calculates neoclassical fluxes, E_r, j_b, etc
- Nonlocal physics due to large ion orbits
- Two species now: ions + electrons
- Momentum, energy and particle number conserving collisions
- Interfaced with MHD equilibrium codes and TRANSP data base
- It is routinely applied to the analysis of NSTX discharges



GTS Turbulence Simulations Show That ITG Modes Have Low Contributions to Energy Transport

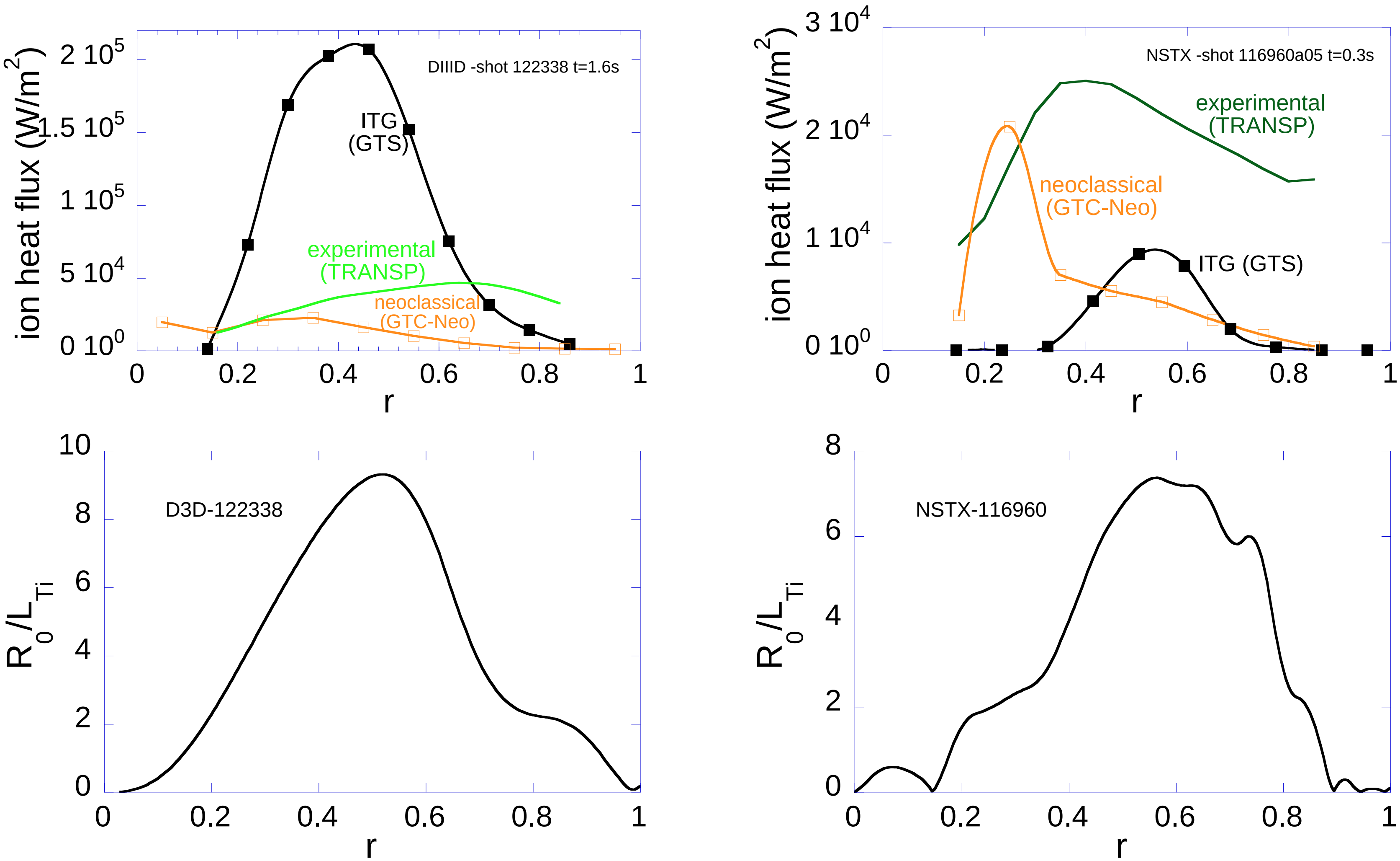






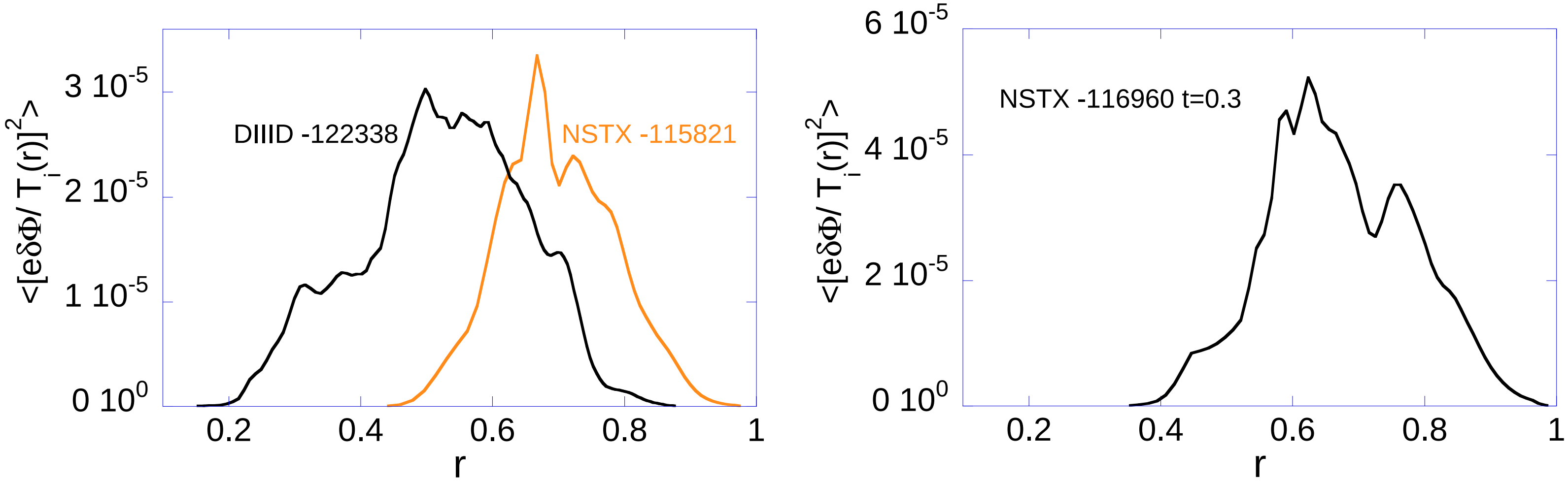
- – simulation radial domain: $0.2 \leq r \leq 1.0$; – adiabatic electrons; – equilibrium $\mathbf{E} \times \mathbf{B}$ shear flow not included; – ion-ion collisions included
- ITG turbulence has significant fluctuation amplitudes, but drives small ion energy transport in NSTX plasmas (sometimes below neoclassical level)!

GTS Simulations Show that ITG Modes Are Relevant for DIII-D, But Not for NSTX



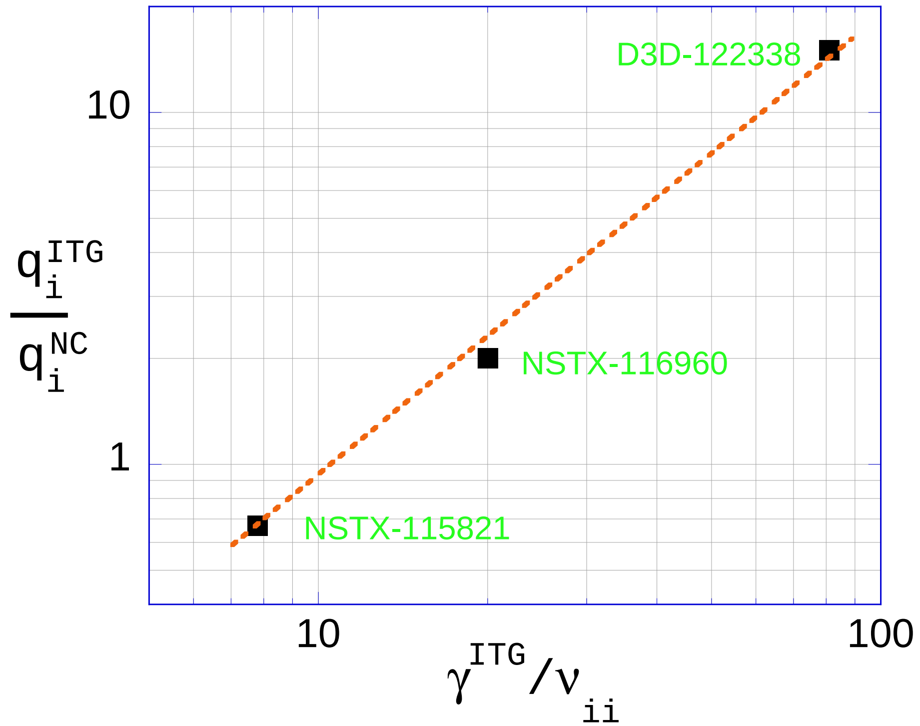
- In contrast, in DIII-D plasmas, ITG turbulence can drive large transport ($\times$10 neoclassical level)

ITG Modes Drive Significant Potential Fluctuations



- Turbulence fluctuation levels for two machines are actually comparable $e\delta\phi/T_i < 1\%$

Mixed Scaling between ITG and Neoclassical Transport

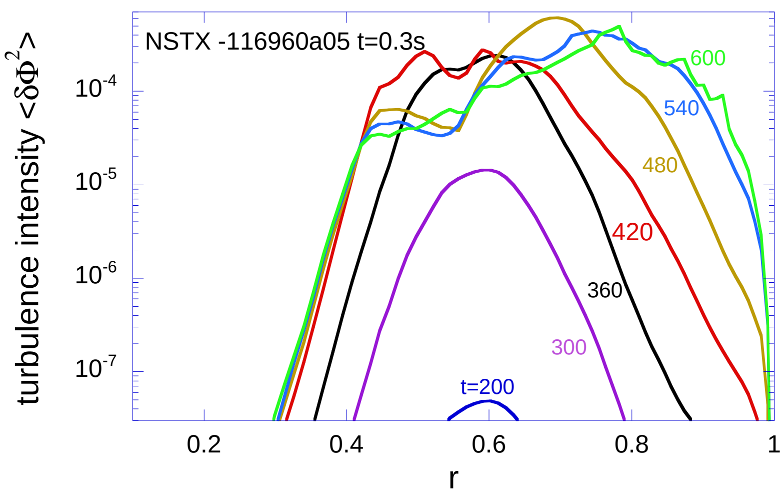


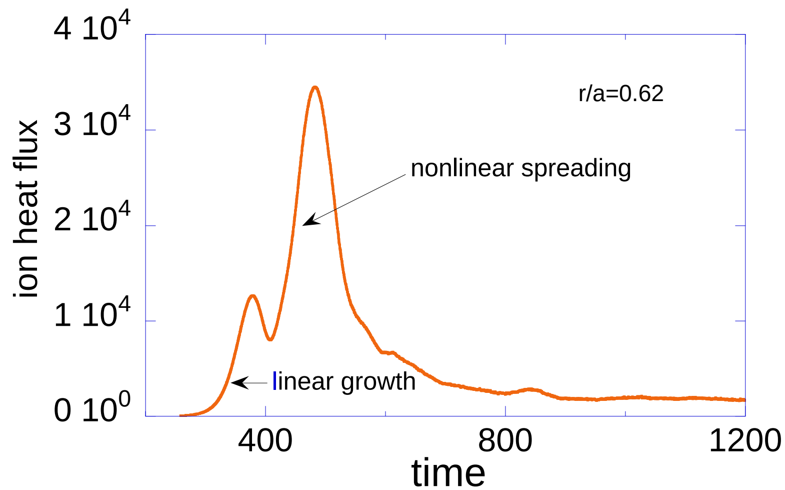
- Measured at locations around maximum R_0/L_{T_i}:

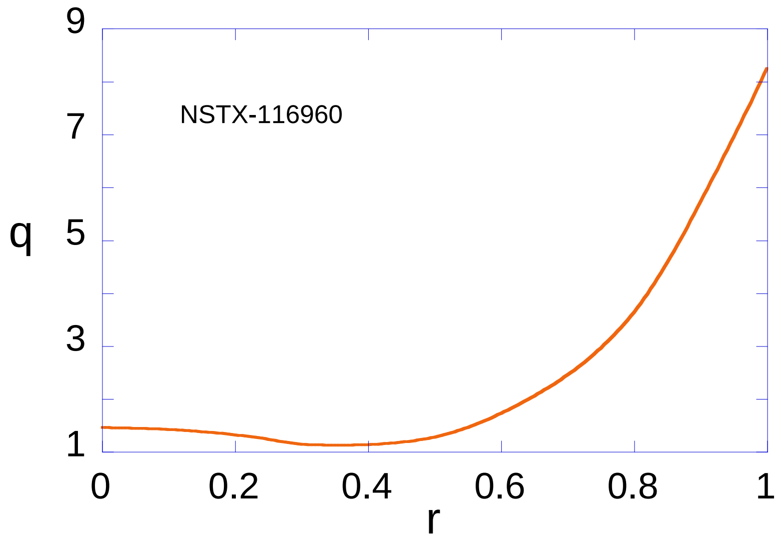
$$\frac{q_i^{\mathrm{ITG}}}{q_i^{\mathrm{NC}}} = \frac{\chi_i^{\mathrm{ITG}}}{\chi_i^{\mathrm{NC}}} \propto \frac{\gamma^{\mathrm{ITG}}}{\nu_{ii}} \qquad ?$$

- To compare neoclassical and turbulent transport, we need to discuss them using a unified language

Nonlocal Features due to Turbulence Spreading

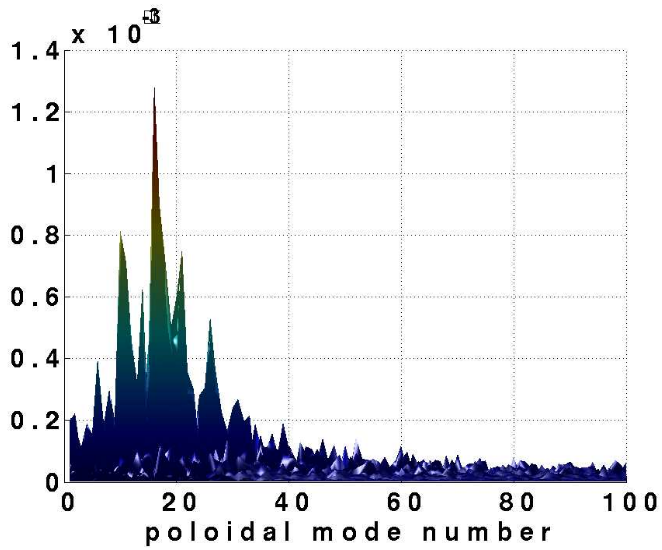


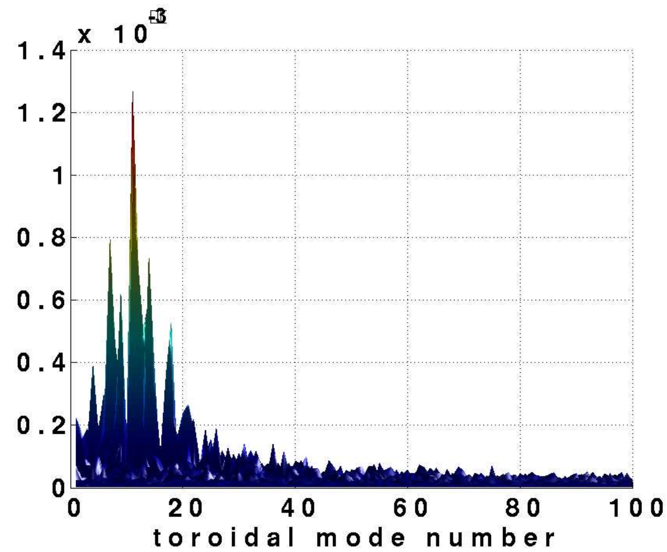


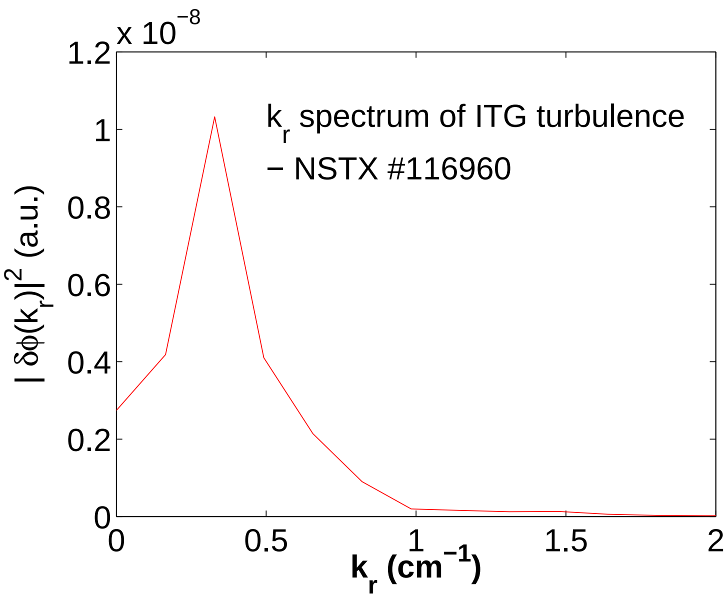


- Spreading in outward direction is more significant
- The reversed magnetic shear in the inner side may provide stronger damping

ITG Turbulence Spectrum in NSTX Plasma

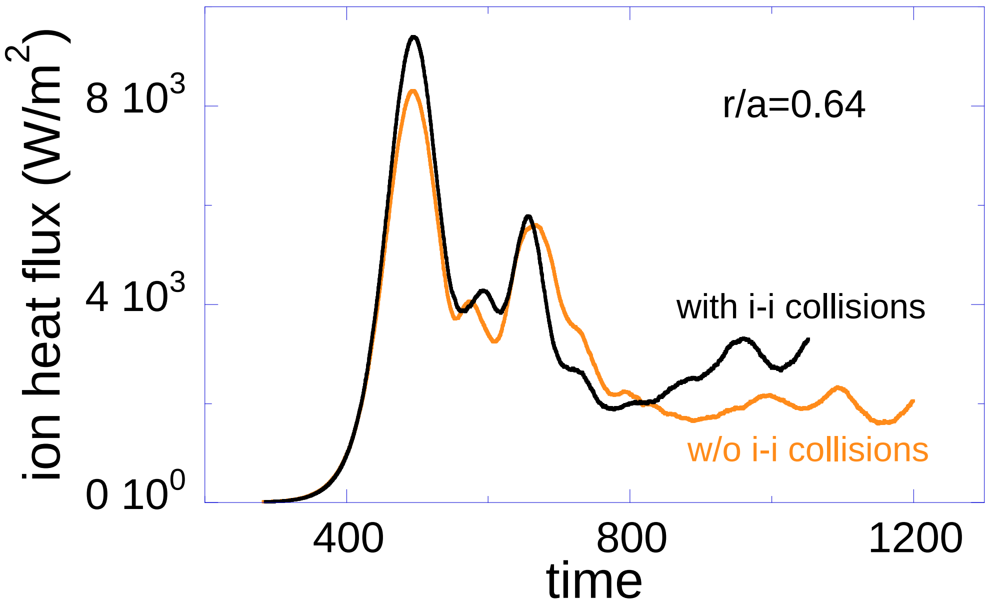






- Nonlinear toroidal couplings are responsible for the formation of a down-shifted toroidal spectrum in the fully developed ITG turbulence regime.
- Low-k measurements can tell if ITG exists and drives any transport in NSTX $\Longrightarrow$ to validate simulation

Effects of Coulomb Collisions

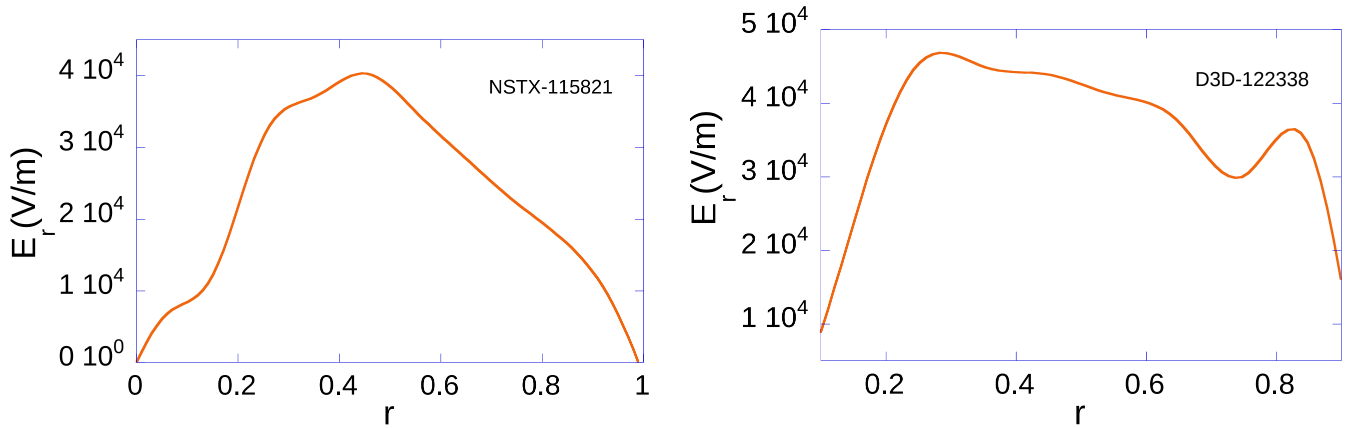


- Linear ion-ion collisions:

$$C_{ii}^{l}(\delta f) = \underbrace{C(\delta f, f_0)}_{\text{(drag \& diffusion)}} + \underbrace{C(f_0, \delta f)}_{\text{(effect of perturbed field particles)}}$$

- Collisions enhance ITG driven ion heat flux, but not significantly
- We may expect more sensitive dependence on collisions in marginal instability regime

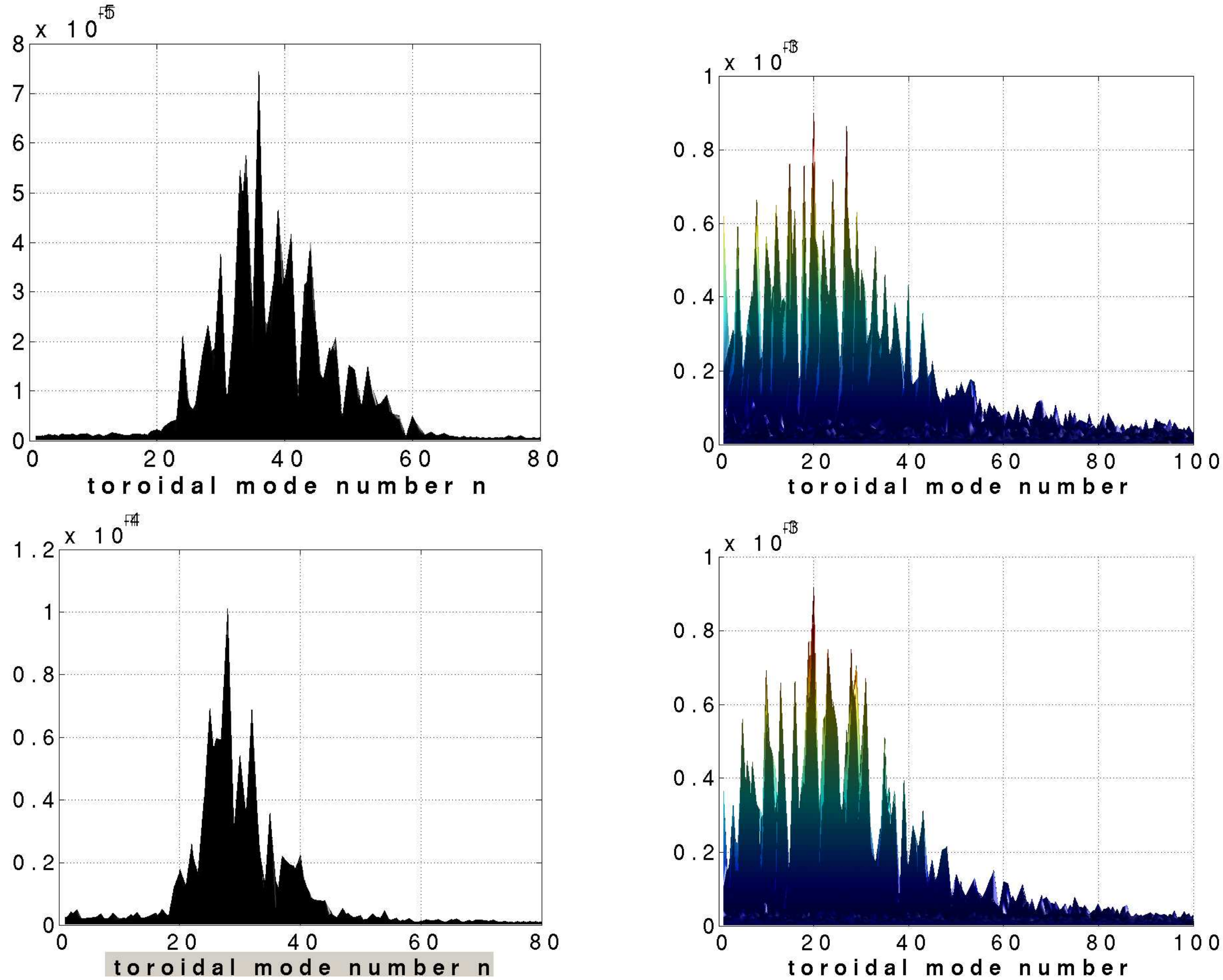
Effects of Equilibrium E × B Shear Flows



- Equilibrium radial electric field is determined by neoclassical dynamics, and is calculated by GTC-Neo self-consistently
- ITG is stabilized when equilibrium $\mathbf{E} \times \mathbf{B}$ shear flow is included

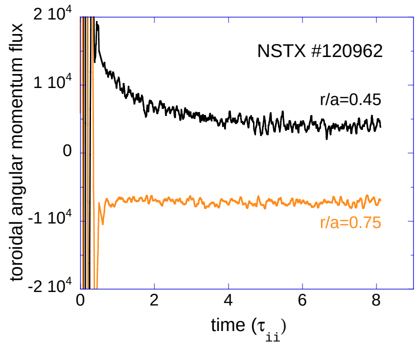
Fluctuation Spectrum Shift due to Applied Equilibrium $\mathbf{E} \times \mathbf{B}$ Flow

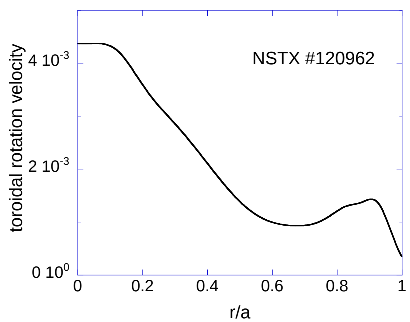
linear phase nonlinear phase

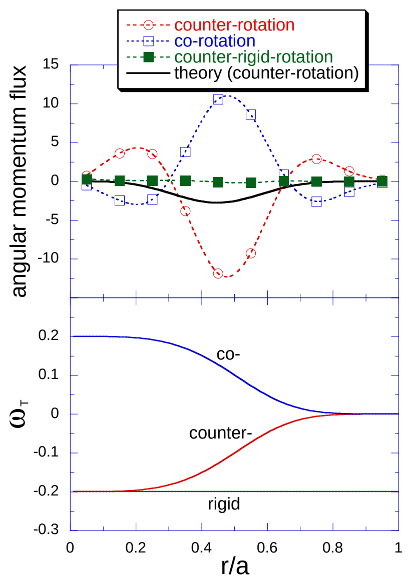


- Using an equilibrium $\mathbf{E} \times \mathbf{B}$ flow with $E = E_r/3$

Neoclassical Momentum Transport Is Significantly Enhanced for Steep Toroidal Rotation Gradient







- GTC-Neo is used to calculate an accurate baseline of momentum transport, which is required to understand any anomalous momentum transport and torque
  - nonlocal properties of neoclassical angular momentum transport associated with steep gradients, large orbits
  - off-diagonal momentum transport associated with temperature and pressure gradients in collisionless regime
  - poloidal electric field in rotating plasma and effect on momentum transport
- Simulations with steep rotation gradient obtained angular momentum transport 5-6 times larger than theory(Hinton-Wong).

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Toroidal Rotation Model in GTS: Turbulence Driven Γ_ϕ

Turbulence fluctuation part δf:

$$\frac{D\delta f}{Dt} = -\vec{v_E} \cdot \nabla f_0 + \hat{b^*} \cdot \nabla(\frac{e}{m_i}\bar{\phi})\frac{\partial f_0}{\partial v_\parallel} + C_i^l(\delta f_i).$$

Neoclassical equilibrium f_0:

$$\frac{\partial f_0}{\partial t} + (v_\parallel \hat{b} + \vec{v_{E_0}} + \vec{v_d}) \cdot \nabla f_0 - \hat{b^*} \cdot \nabla(\mu B + \frac{e}{m_i}\Phi_0)\frac{\partial f_0}{\partial v_\parallel} = C_i(f_0, f_0).$$

Lowest order equilibrium solution consistent with plasma rotation:
$f_0 = f_{SM} = n(r,\theta)(\frac{m_i}{2\pi T_i})^{3/2} e^{-\frac{m_i}{T_i}[\frac{1}{2}(v_\parallel - U_i)^2 + \mu B]}$

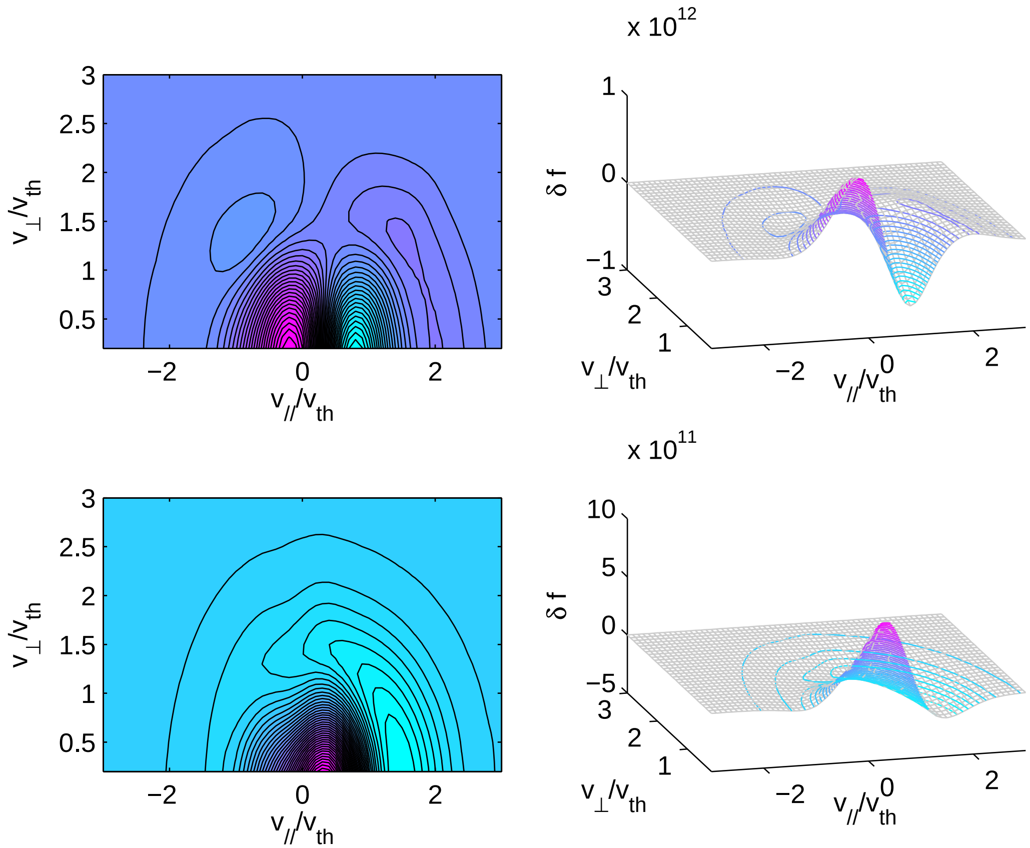
parallel flow: $U_i = I\omega_t / B$, density: $n(r,\theta) = N(r) e^{\frac{m_i U_i^2}{2T_i} - \frac{e\tilde{\Phi}_0}{T_i}}$

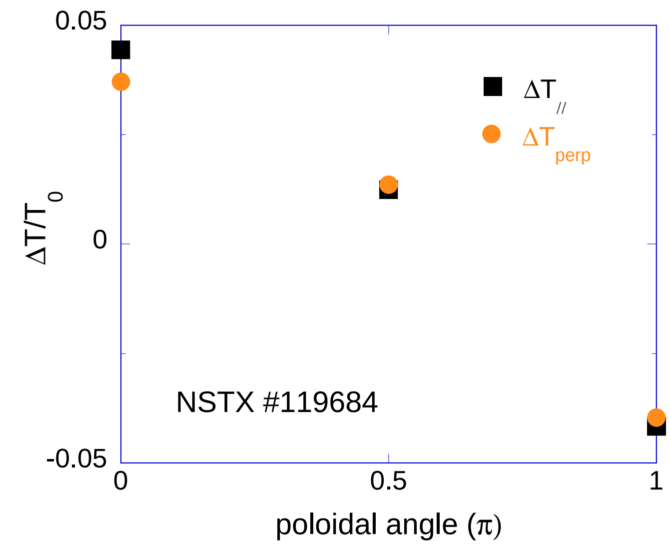
$$\frac{D\delta f}{Dt} = \Big[(\cdots)\vec{v_E} \cdot \nabla \ln T - \vec{v_E} \cdot \nabla \ln n(r,\theta) - \frac{m(v_\parallel - U_i)}{T_i}\vec{v_E} \cdot \nabla U_i(r,\theta)$$

$$+\frac{mU_i}{T_i v_\parallel}\vec{v_E} \cdot \mu \nabla B - \frac{1}{T_i}(v_\parallel \hat{b} + \vec{v_d}) \cdot \nabla(e\bar{\phi})(1 - \frac{U_i}{v_\parallel})\Big] f_0$$

$\{\langle n(r,\theta)\rangle$, $T(r)$, $\Phi_0(r)$, and $\omega_t(r)\} \Longrightarrow$ turbulence-driven fluxes

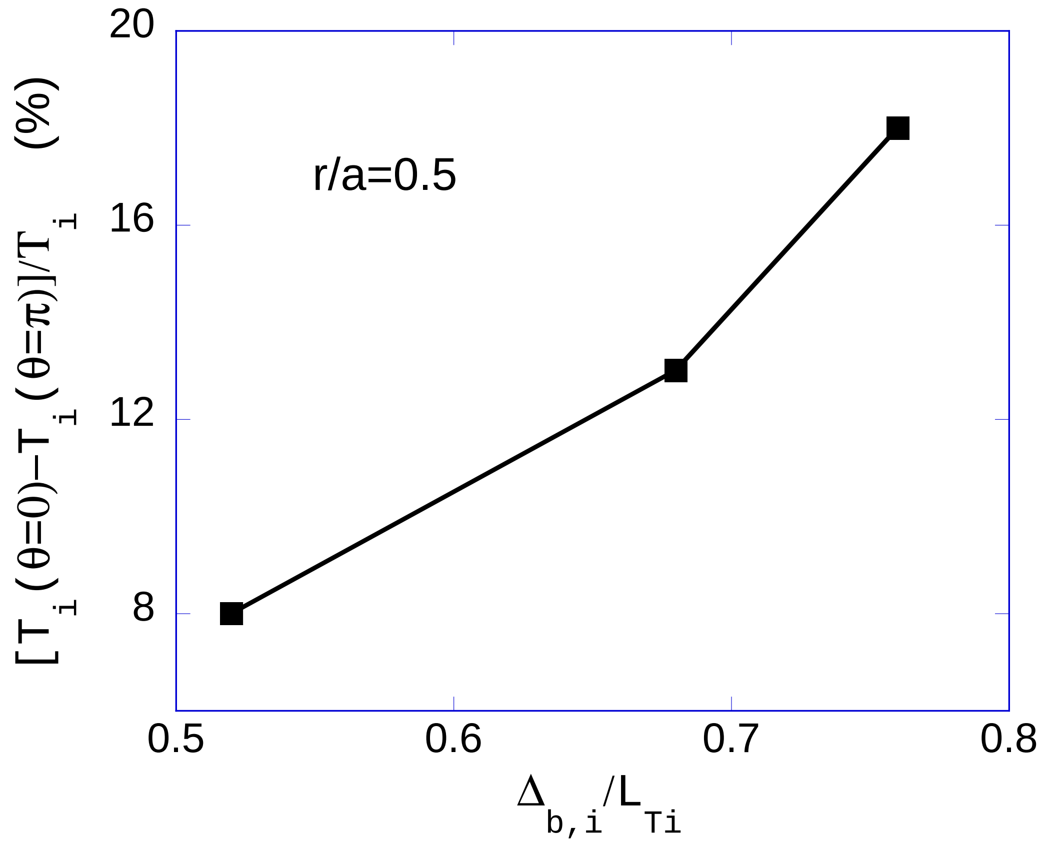
Anisotropic Properties and Structure of Neoclassical Equilibrium of NSTX Plasmas





- Plasma anisotropy with respect to $T_{\|}$ and $T_{\perp}$ is insignificant
- However, there exists considerable variation of T_i on magnetic surface (up to $\sim 20\%$ difference between outer and inner sides on mid-plane)
- $\delta f_{nc}/f_{SM} \sim 10\%$

Anisotropic Properties and Structure of Neoclassical Equilibrium of NSTX Plasmas



- Difference increases with $\Delta_{b,i}/L_{T_i}$, but insensitive to density gradient
- As a consequence, pressure iso-surfaces are different than magnetic surfaces

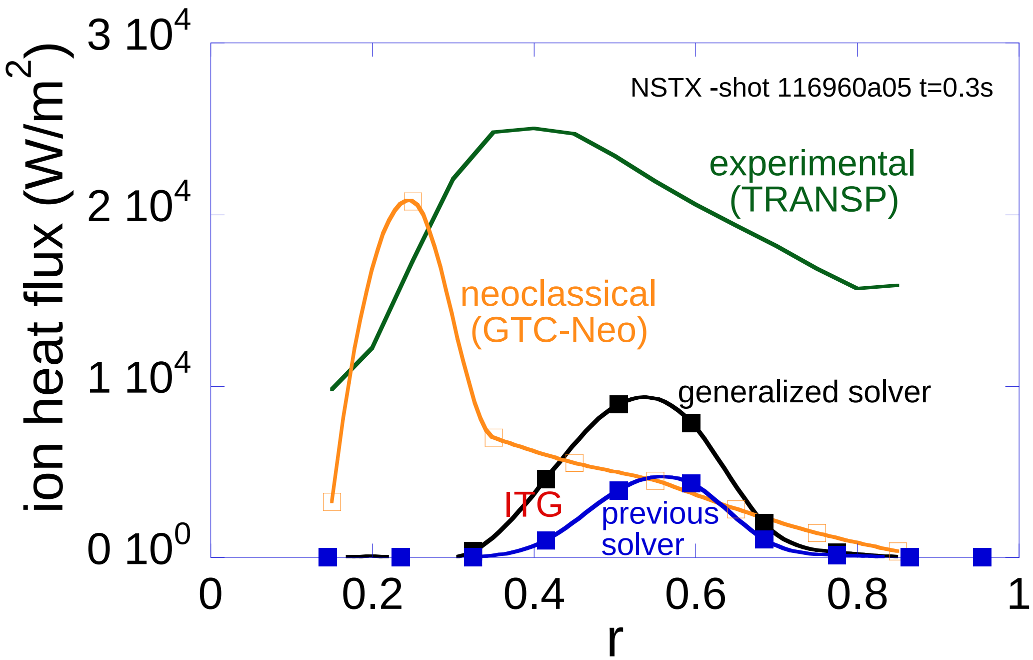
Recent Development of GTS and NSTX-physics-oriented Algorithm

- Generalized Poisson Solver to solve integral equation for total potential $\Phi = \delta\Phi + \langle\Phi\rangle$ using superLU/PETSc

  – previous solver solves for $\delta\Phi$ and $\langle\Phi\rangle$ separately using approximations:

  i) Pade approximation $\Gamma_0(b) \equiv I_0(b)e^{-b} \approx 1/(1+b)$ and

  ii) $\left\langle \tilde{\Phi} \right\rangle \approx \widetilde{\langle\Phi\rangle}$ – not justified for NSTX geometry!



Recent Development of GTS and NSTX-physics-oriented Algorithm

- Electron physics via split-weight scheme

$$f_e = f_{e0} - \frac{e\delta\Phi}{T_e} f_{e0} + \delta h_e$$

$$\frac{\partial \delta h_e}{\partial t} + \vec{v_D} \cdot \nabla \delta h_e - \hat{b^*} \cdot \nabla(\mu B + \frac{e}{m_e}\Phi_0 + \frac{e}{m_e}\Phi)\frac{\partial \delta h_e}{\partial v_\parallel}$$

$$= -\vec{v_E} \cdot \nabla f_{e0} + \hat{b^*} \cdot \nabla(\frac{e}{m_e}\Phi)\frac{\partial f_{e0}}{\partial v_\parallel} + \frac{e}{T_e}\frac{\partial \delta\Phi}{\partial t} f_{e0} + \vec{v_D} \cdot \nabla \left(\frac{e\delta\Phi}{T_e}\right) f_{e0} + C_e^l(\delta h_e).$$

$\vec{v_D} \equiv v_\parallel \hat{b} + \vec{v_{E_0}} + \vec{v_E} + \vec{v_d}$

$\frac{\partial \delta\Phi}{\partial t}$ on RHS can cause numerical instability if using direct numerical derivative

Iterative method taking NA electrons as higher order correction – not justified for NSTX because of high fraction of trapped electrons

Recent Development of GTS and NSTX-physics-oriented Algorithm

Need to introduce additional equation for $\frac{\partial \delta \Phi}{\partial t}$:

$$\frac{e}{T_i}(\frac{\partial \Phi}{\partial t} - \widetilde{\frac{\partial \Phi}{\partial t}}) = \frac{1}{n_0}(-\nabla \cdot \vec{\Gamma_i} + \nabla \cdot \vec{\Gamma_e}) + \vec{v}_{\delta E} \cdot (2\frac{\nabla B}{B} - \frac{e}{T_e}\nabla \Phi_0)$$

$$\vec{\Gamma_e} = \int (v_{\parallel}\hat{b} + v_{\vec{E}_0} + v_{\vec{E}} + \vec{v_d})\delta h_e d^3v$$

$$\vec{\Gamma_i} = \int (v_{\parallel}\hat{b} + v_{\vec{E}_0} + v_{\vec{E}} + \vec{v_d})\delta f_i d^3v$$

The corresponding linear problem is solved using superLU/PESTc

Summary

Our gyrokinetic simulation studies contribute several interesting remarks to the observation that the ion transport is at neoclassical levels in NSTX.

- In NSTX plasmas, ITG driven ion energy transport is at neoclassical level.
- In contrast, for DIII-D discharges, ITG turbulence is shown to drive large transport ($\times 10$ neoclassical level).
- Turbulence fluctuation levels for two machines are actually comparable ($\sim e\delta\Phi/T_i < 1\%$; $\rho_{*,NSTX} \sim 2\rho_{*,DIII-D}$).
- A mixed transport scaling (?) $\dfrac{q_i^{\mathrm{ITG}}}{q_i^{\mathrm{NC}}} = \dfrac{\chi_i^{\mathrm{ITG}}}{\chi_i^{\mathrm{NC}}} \propto \dfrac{\gamma^{\mathrm{ITG}}}{\nu_{ii}}$
- Self-consistent equilibrium $\mathbf{E} \times \mathbf{B}$ flows can strongly stabilize ITG.
- Effect of collisions is weak.

Summary – continued

Neoclassical momentum transport and equilibrium properties:

- Neoclassical angular momentum transport is strongly enhanced for steep toroidal rotation gradient
- Significant T_i variation on magnetic surface is found due to the neoclassical effects associated with large ion orbits and steep T_i profile.

Gyrokinetic PIC Simulations of Turbulent Transport

- GTS: generalized gyrokinetic simulation model implemented using GTC [Lin et al, Science (1998)] architecture to simulate tokamak experiment [Wang et al., Phys. of Plasmas (2006)]
- Shaped cross-section; experimental profiles; consistent rotation and equilibrium $\mathbf{E} \times \mathbf{B}$ flow; linear Coulomb collisions; $\cdots$
- Interfaced with MHD equilibrium codes (based on ESI interface by Zakharov and White) and TRANSP data base
- Benchmarked in simple geometry limit