Reduced Energetic Particle Transport Models Enable Comprehensive Time-Dependent Tokamak Simulations

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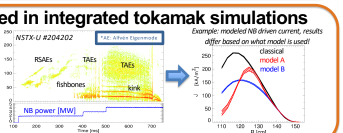
Verified, validated EP models are required in integrated tokamak simulations

- EPs (alphas, NB ions, RF tails) provide main source of heating, momentum, current drive in burning plasmas
- *But: EPs drive instabilities, instabilities affect EPs*

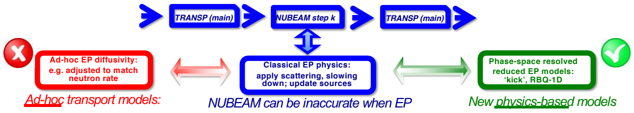
This work: reduced EP transport models being developed, validated for time-dependent predictive simulations



Example: modeled NB driven current, results differ based on what model is used!

TRANSP is the main platform for testing EP models in Integrated Simulations

- NUBEAM module in TRANSP accounts for (neo)classical EP physics
- Includes scattering, slowing down, atomic physics



TRANSP (main) → *NUBEAM step k* → *TRANSP (main)*

Ad-hoc EP diffusivity: e.g. adjusted to match neutron rate

Classical EP physics: apply scattering, slowing down; update sources

Phase-space resolved reduced EP models: 'kick', RBQ-1D

Ad-hoc transport models:
-> often unphysical
-> no predictive capabilities!

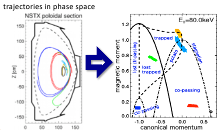
NUBEAM can be inaccurate when EP transport is enhanced by instabilities

New physics-based models
-> enable predictive capabilities

Constants of Motion variables are used to describe resonant wave-particle interaction

Each **particle orbit** characterized by:
- E, energy
- $P_\zeta \sim mRv_{par} - q\Psi$, canonical momentum
- $\mu \sim v_{perp}^2/B$, magnetic moment

Complex orbits in real space translate in simple trajectories in phase space



Wave stability (drive):

$$\gamma \propto \omega \frac{\partial F_{nb}}{\partial E} + n\frac{\partial F_{nb}}{\partial P_\zeta}$$

Resonant interactions obey simple rule:

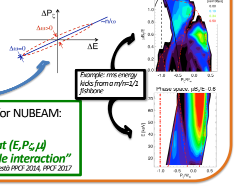
$$\omega P_\zeta - nE = const.$$

$\omega = 2\pi f$: mode frequency
n : toroidal mode number

$$\Delta P_\zeta / \Delta E \propto n/\omega$$

Define ***transport probability matrix(es)*** for NUBEAM:
$p(\Delta E, \Delta P_\zeta \mid E, P_\zeta, \mu)$
"Conditional probability that a particle at (E,P_ζ,μ) receives kicks ΔE, ΔP_ζ from wave-particle interaction"
Podestà PPCF 2014, PPCF 2017

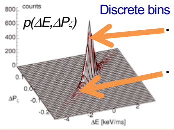
R. B. White, Theory of toroidally confined plasmas, Imperial College Press (2001)



Example: rms energy kicks from a m/n=1/1 fishbone

Probability matrices describe enhanced transport in Monte Carlo module NUBEAM

Discrete bins in (P_ζ,E,μ) can contain both *resonant* and *non-resonant* particles



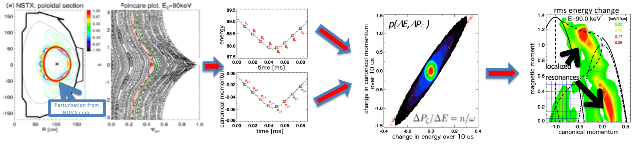
- 'Non-resonant' particles have small fluctuations around initial (E, P_ζ)
- 'Resonant' particles can experience large ΔE, ΔP_ζ variations

- Probability matrix approach not limited to "diffusive" transport
- Can account for convective transport
  - Skewed PDF
- Can be used to introduce different sources of EP transport
  - MHD instabilities
  - Microturbulence
  - 3D fields (still not explored)

RBQ-1D and *kick* models distill physics of wave-particle interaction for inclusion in *p(ΔE,ΔPζ)* transport matrix for NUBEAM

- Both models use mode structure, damping rate from MHD codes, e.g. NOVA/NOVA-K
  - Input: thermal profiles, equilibrium
- **RBQ-1D** based on 'resonance-broadened quasi-linear' *theory* for wave-particle interaction: *Gorelenkov NF 2018*
  - Use "diffusive transport" approximation -> gaussian $p(\Delta P_\zeta \mid E,P_\zeta,\mu)$
  - 1D: assume that transport along canonical momentum P_ζ dominates
  - Computationally efficient
- **'Kick' model**: particle-following code ORBIT used to infer transport matrix *numerically*

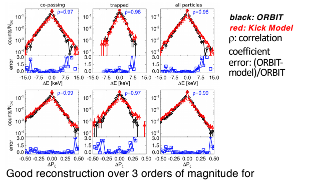
Initialize test particles uniformly in phase space — Track energy, momentum variations (*kicks*) at fixed time intervals — Combine ΔE, ΔP_ζ from same (E,P_ζ,μ) phase space *bin* into $p(\Delta E,\Delta P_\zeta)$ — Repeat for all (E,P_ζ,μ) *bins* to infer 5D matrix -> input for NUBEAM: $p(\Delta E,\Delta P_\zeta \mid E,P_\zeta,\mu)$



Podestà PPCF 2014, 2017

Models are being verified against theory & first-principles codes

- Kick model: good agreement with ORBIT preserved when evolving F_{nb} over 5 ms, typical macro-step of NUBEAM

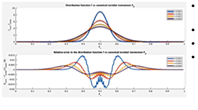


black: ORBIT
red: Kick Model
ρ: correlation coefficient
error: (ORBIT-model)/ORBIT

Good reconstruction over 3 orders of magnitude for co-, trapped (, counter- not present in *this* input F_{nb}) particles

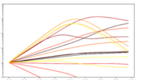
Podestà PPCF 2014, PPCF 2017, NF 2018

- RBQ-1D diffusion solver benchmarked against known analytical solutions



- Example: evolution of 'test distribution'
- RBQ-1D: solid
- Analytical: dashed
- Bottom plot: evolution of relative error

- RBQ-1D computes the expected saturation amplitudes and corresponding diffusivities
- Include capability of treating multiple AE modes simultaneously
- Compute relaxed distribution

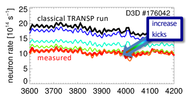


Gorelenkov NF 2018, PoP 2018 (in preparation)

Models can be used for both *interpretive* and *predictive* simulations

- Focus on energetic particle evolution, stability of EP-driven instabilities (e.g. Alfvén Eigenmodes), EP transport by instabilities

Interpretive runs: *reduced input from experiment*
- ➢ *To validate EP models, analyze actual discharges*
- Use experimental info to set ΔE, ΔP_ζ
  - E.g. based on neutron rate, internal measurements of mode amplitude



Predictive runs:
- ➢ *To optimize/explore new scenarios*
- Use saturation condition to set ΔE, ΔP_ζ
  - Impose drive = damping vs time

$$\frac{\delta E_{wave}}{\delta t} = P_{EP} - 2\gamma_{damp} E_{wave} = 0$$

drive from NUBEAM or RBQ-1D
damping from NOVA-K

Main limitation:
- *Can be only as good as damping rate estimates!*

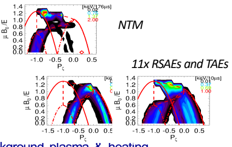
- **Many practical cases lie in between 'fully interpretive' & 'fully predictive'**
- For example, only *relative* mode amplitudes may be known from experiment
- Or: parameters for predictions are adjusted based on experimental information
  - e.g. limit frequency and mode number range
- Also: thermal profiles are assumed to be known in this work!
- *For truly predictive simulations, thermal profiles would need to be recomputed as sources change*

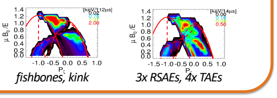
Podestà PPCF 2017

NSTX-U and DIII-D scenarios challenge models over broad set of conditions

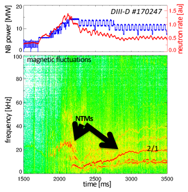
complexity

- DIII-D: NTM-only scenario
  - Single (dominant) instability
  - Limited number of resonances
- DIII-D: AEs-only scenario
  - Large number of weaker AEs
  - "Sea" of resonances
- NSTX-U: multi-mode scenario
  - Transient scenario, variations in background plasma & heating sources
  - Multiple types of instabilities
  - *Need to account for possible synergy between different modes (e.g. fishbones + TAEs + kink)*





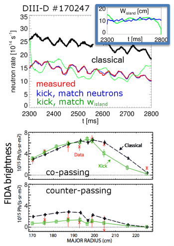
DIII-D discharges with large NTM provides a good test bed for EP transport models



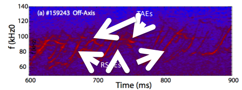
No instabilities in the AE frequency range observed during the time of interest

Heidbrink NF 2018 Bardóczi PoP 2017
*Poli NF 2017 **Bardóczi PPCF 2018 (submitted)***

- NTMs destabilized by step-up in NB power
  - Dominant 2/1 in this case
- Large NTM amplitude causes EP confinement degradation
  - Clear drop in neutron rate
- Kick "interpretive" run:
  - Scale kicks to match measured neutrons
  - Mode amplitude related to *Wisland*
- Inferred NTM island width agrees with measured *Wisland* from ECE
  - *a posteriori* check, validation
  - **Path towards 'predictive' simulations** with *Wisland* from Modified Rutherford Equation
- Favorable comparison with phase-space resolved data (FIDA)
  - Acceptable for co-passing, good for counter-passing
  - ***Key exercise for model validation***
    - *Ad-hoc diffusion would give same drop for co/cntr*



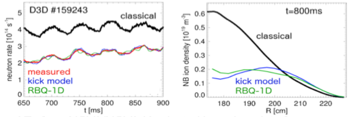
Kick & RBQ-1D application to DIII-D scenario with multiple unstable AEs



Heidbrink PoP 2016
Collins PRL 2016
Collins NF 2017

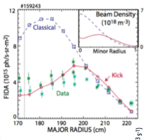
- Kick & RBQ-1D successfully benchmarked for scenario with multiple unstable AEs

Gorelenkov NF 2018

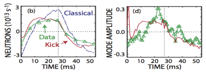


- AEs from NOVA/NOVA-K selected based on data from ECE, Mirnov coils for mode frequency & mode number
- Amplitudes inferred around t=800ms
  - Adjusted vs time to match measured neutron rate

- Successful comparison with phase-space resolved data (FIDA, NPA) validates models



✓ Simulation also reproduces dynamic response to NB modulation
- *Neutron rate: time constants for rise/fall consistent with kick model results*
- *Mode amplitude modulation roughly consistent with kick model*

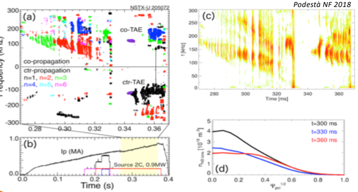


☒ *However: less favorable comparison with FIDA using updated calibration*
- *Need to work closely with experimentalists*

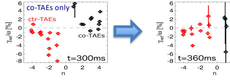
Retaining phase space resolution is critical for validation

Kick model application to NSTX-U scenarios with *counter*-propagating AEs

Podestà NF 2018



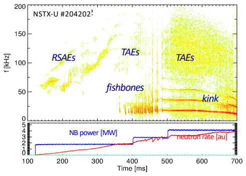
- Transition from co- to counter-TAEs as NB ion density profile becomes flat/hollow
- Most quantities evolve in time, not suitable for "single-time-slice" analysis



- Main features of the experiment can be reproduced
  - Reproduces transition co- to counter-TAEs
  - Capture time evolution of unstable modes, spectrum, …

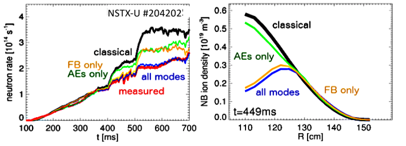
Kick model application to NSTX-U multi-mode scenario

Towards predictive simulations: need estimate of unstable spectrum, saturated amplitudes



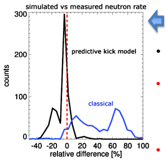
- Need estimate for relative AE amplitudes:
  - Use saturation condition (drive=damping) to infer AE amplitudes vs time
- Then, rescale fishbone & kink amplitudes to match measured neutron rate
  - No damping available (yet)

- Analysis provides assessment of role of different instabilities on EP transport, NB driven current



- AEs and fishbones/kinks cause comparable drop in neutrons
  - Fishbones, kinks are mostly responsible for NB ion density depletion
  - AEs have larger effect on NB ion energy redistribution
- *Synergy between modes is observed*, e.g. in total EP losses

Initial assessment of *predictive* capabilities for AE-induced fast ion transport



Relative difference from interpretive simulations: NSTX, NSTX-U and DIII-D database

- *Predictive* analysis (AEs only) results generally agree within +/-15% with *interpretive* simulations
- However: in some cases, predictive runs fail to reproduce experiments!
  - Predicted AE spectrum differs from experiment
  - *Key role of damping rate from MHD codes*
    - Affects inferred AE saturation amplitude
- ***More validation is required to assess model limitations, missing physics***

Conclusions and future work

- Verification & Validation being extended for kick & RBQ-1D
  - Part of the US Joint Research Target milestone in 2018: "Assess predictive capability of reduced EP transport models"
- Plan to extend RBQ to 2D (canonical momentum & energy)
- Extending kick model to low-f instabilities, e.g. sawteeth, kink/fishbones, NTM

- Reduced models enable efficient simulations retaining (most of) the relevant EP physics
  - Including *predictive* capabilities (ITER & beyond)
- Phase-space resolution is required to move beyond *ad-hoc* models
  - Critical for heating, current drive, thermal transport
- ***Goal: develop framework to streamline TRANSP analysis including effects of instabilities on EPs***

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