Continuum Gyrokinetic Simulations of NSTX SOL Turbulence with Sheath-Limited Model Geometries

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Goal: Tokamak edge physics from first-principles

- Developing a quantitative understanding of the edge plasma is crucial for optimization of fusion performance; design of reactor-grade burning plasma machines
- Although relatively narrow, edge plasma is very challenging computationally: need to handle a wide range of scales; large amplitude fluctuations; interactions with a sheath model; complex atomic and surface physics.
- We present the *first* continuum simulations of turbulence in sheath-limited SOL geometry.
- Divertor heat-fluxes, blob statistics and plasma profiles are studied for different poloidal (vertical) magnetic fields. Results show heat-flux widths consistent with Eich/Goldston scalings and significant intermittency in blob dynamics.

Gyrokinetic model

Gkeyll solves gyrokinetic equations (GKE) as a conservation law

$$\frac{\partial}{\partial t}(\mathcal{J} f) + \frac{\partial}{\partial z^\alpha}(\mathcal{J}\dot{z}^\alpha f) = \mathcal{J} C[f].$$

Long-wavelength limit of GK Hamiltonian is

$$H = \frac{1}{2}mv_\parallel^2 + \mu B + q\langle\phi\rangle$$

and the non-canonical Poisson Bracket operator is

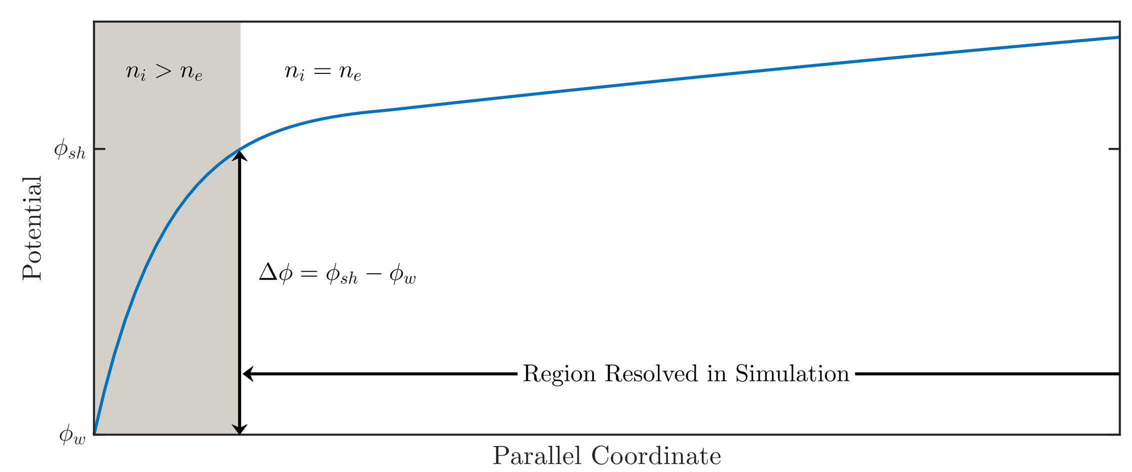
$$\{F, G\} = \frac{\mathbf{B}^*}{mB_\parallel^*}\cdot\left(\nabla F\frac{\partial G}{\partial v_\parallel} - \frac{\partial F}{\partial v_\parallel}\nabla G\right) - \frac{c\,\hat{\mathrm{b}}}{qB_\parallel^*}\times\nabla F\cdot\nabla G.$$

where $\mathbf{B}^* = \mathbf{B} + (Bv_\parallel/\Omega)\nabla\times\hat{\mathrm{b}}$ and $B_\parallel^* = \hat{\mathrm{b}}\cdot\mathbf{B}^*$. Field is evolved with GK Poisson equation

$$-\nabla\cdot(\epsilon_\perp\nabla_\perp\phi) = \rho_c = \sum_s q\int d^3v f$$

where $\epsilon_\perp$ is the perpendicular plasma polarization coefficient. Collisions handled with Lenard-Bernstein operator (drag+diffusion but no velocity dependent collision frequency).

Sheath boundary conditions



- Need to model sheath using BCs due to GK quasineutrality condition
- Get ϕ_{sh} from solving GK Poisson equation, then use $\Delta\phi = \phi_{sh} - \phi_w$ to reflect low $v_\parallel$ electrons entering sheath
- Allows currents in and out of walls ($\oint \mathbf{J}\cdot d\mathbf{x} = 0$).

Discontinuous Galerkin scheme to discretize GKE

Discontinuous Galerkin schemes use function spaces that allow *discontinuities* across cell boundaries.

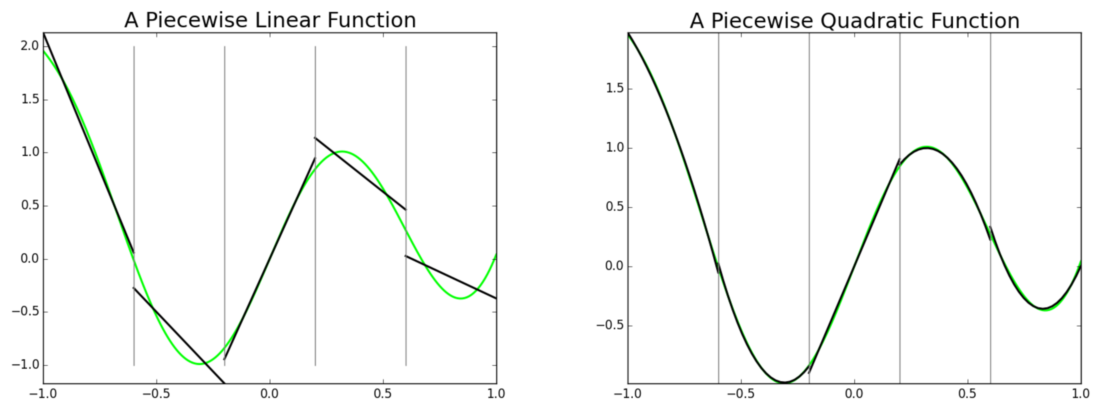


Figure 1: The best L_2 fit of $x^4 + \sin(5x)$ with piecewise linear (left) and quadratic (right) basis functions.

- First introduced by Reed and Hill in 1973 for neutron transport in 2D. General formulation in paper by Cockburn and Shu, JCP 1998. More than 1000 citations.
- DG combines key advantages of finite-elements (low phase error, high accuracy, flexible geometries) with finite-volume schemes (limiters to produce positivity/monotonicity, locality)

DG combined with FV schemes may lead to excellent algorithms for GKE, specially in the edge region.

Problem setup for model NSTX-SOL

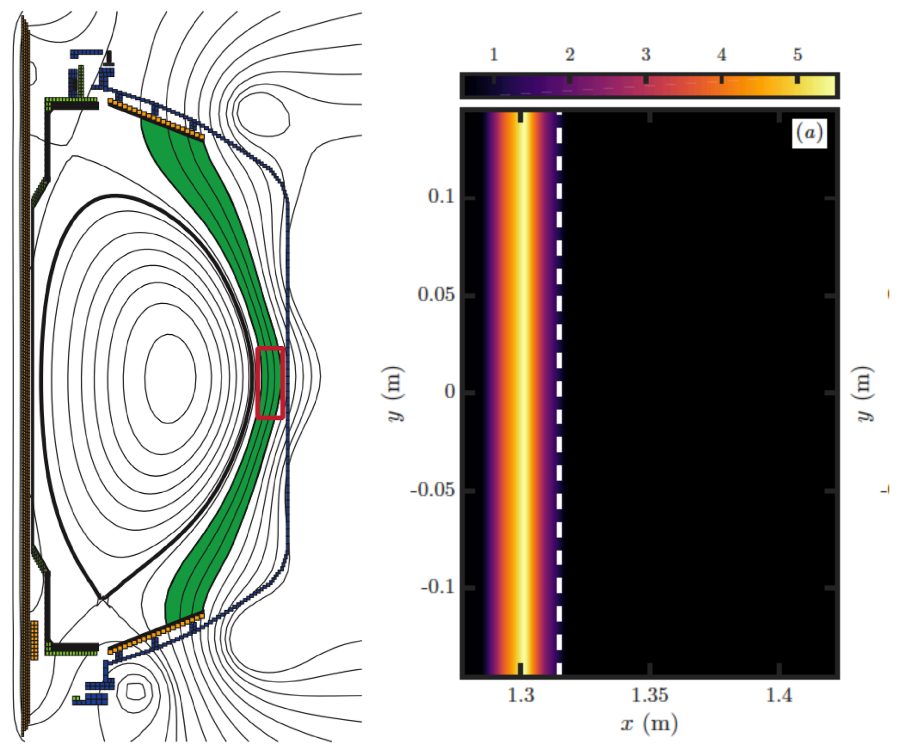


Figure 2: Left: Cross-section of an MHD equilibrium in NSTX. The SOL region shaded in green is the simulation domain. A helical model of the SOL magnetic field is used. Right: Density source rate (in 10^{23} m^{-3} s^{-1}) at the midplane of the SOL. Temperature of source is 50 eV on left of dashed white line and 20 eV on the right.

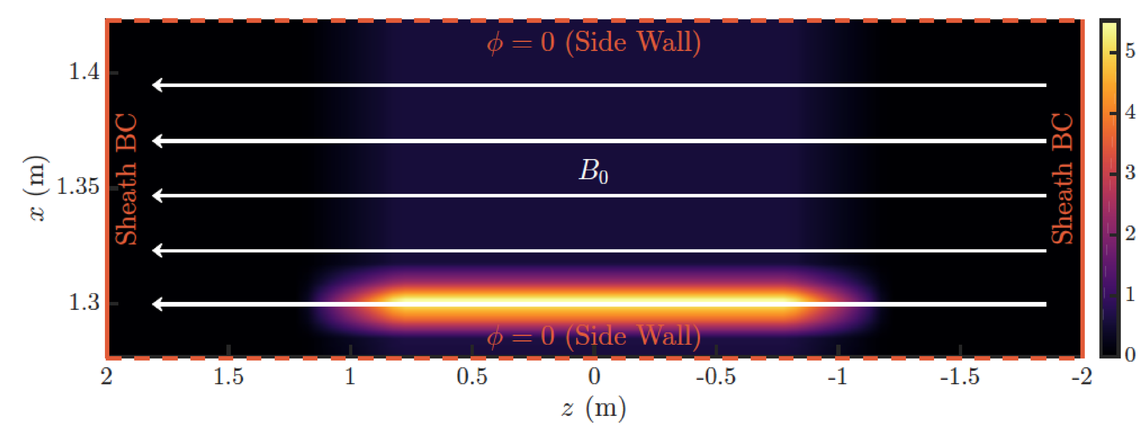


Figure 3: Plasma density source rate (in 10^{23} m^{-3} s^{-1}) in $x - z$ plane. Source is uniform in the periodic y direction.

Turbulence in NSTX-like SOL

Typical NSTX SOL parameters were used. The ratio B_v/B_z was varied. Magnetic shear is absent but is indirectly accounted for by varying connection length.

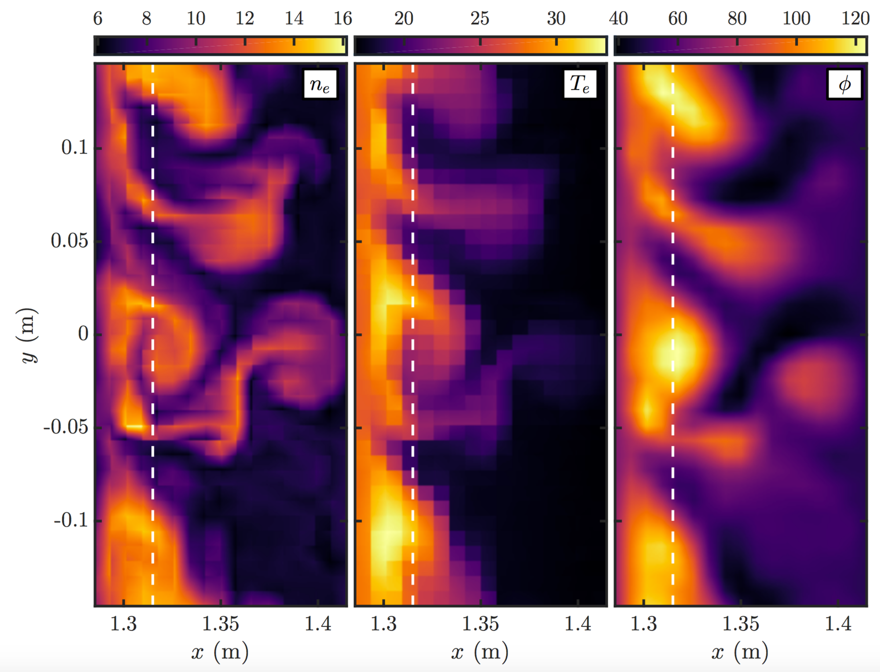


Figure 4: Snapshots of the electron density (in 10^{18} m^{-3}), electron temperature (in eV), and electrostatic potential (in V) in the plane perpendicular to the magnetic field at $z = 0$ m. This simulation has $B_v/B_z = 0.3$. Mushroom structure in blob density is observed at large x.

Mean density, density fluctuations and particle flux

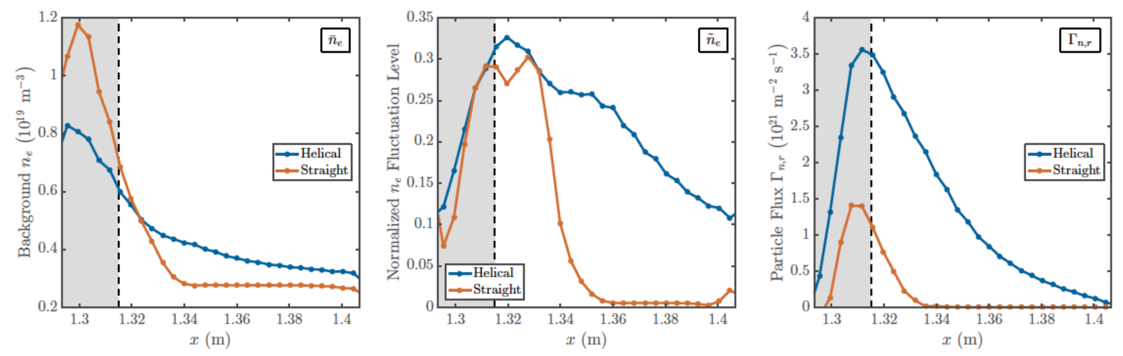


Figure 5: Radial profiles of background electron density, normalized electron-density fluctuations and radial $E\times B$ particles fluxes $\Gamma_{n,r}$ in a helical SOL compared to a straight field-line geometry. The helical simulation shows higher peak fluctuations and particle fluxes due to drive from bad curvature.

Heat-flux as function of poloidal field

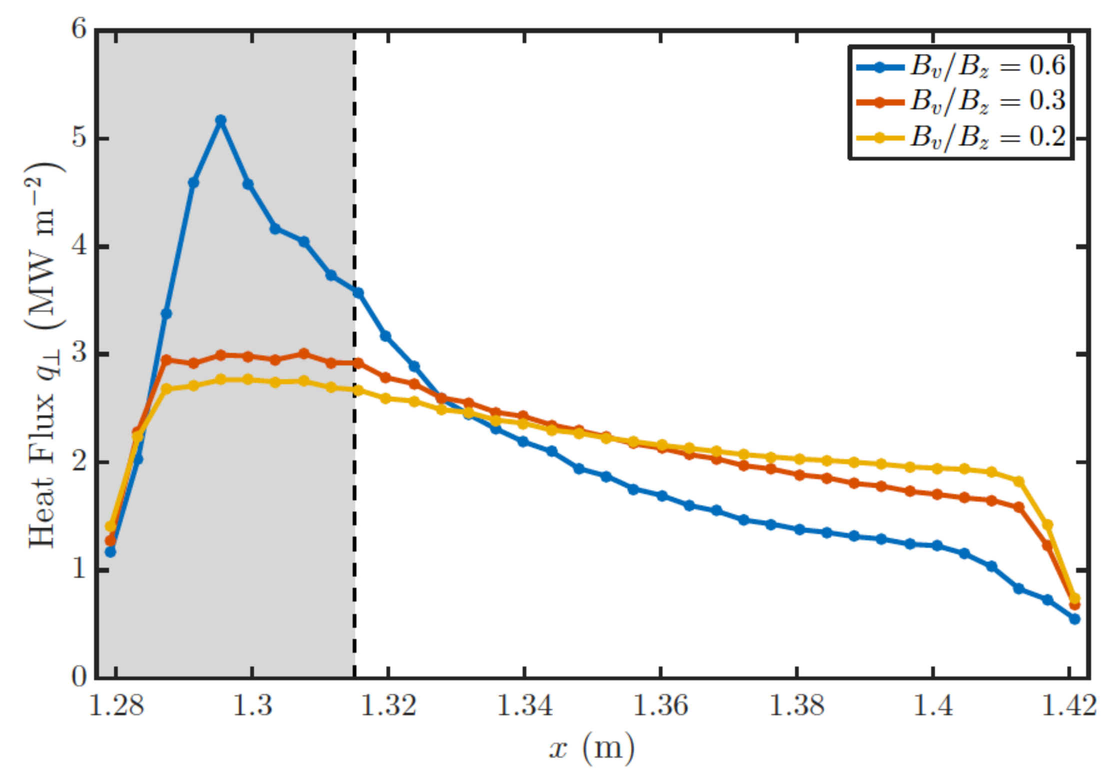


Figure 6: Comparison of the steady-state parallel heat-flux normal to divertor plate for three cases with different pitch angles (poloidal field). Heat flux profile broadens as B_v/B_z is decreased, qualitatively similar to experimental "Eich" and "Goldston" scalings, though the physical mechanism sometimes invoked may differ, since there are no magnetic drifts across flux surfaces in our present geometry.

Particle-flux as function of poloidal field

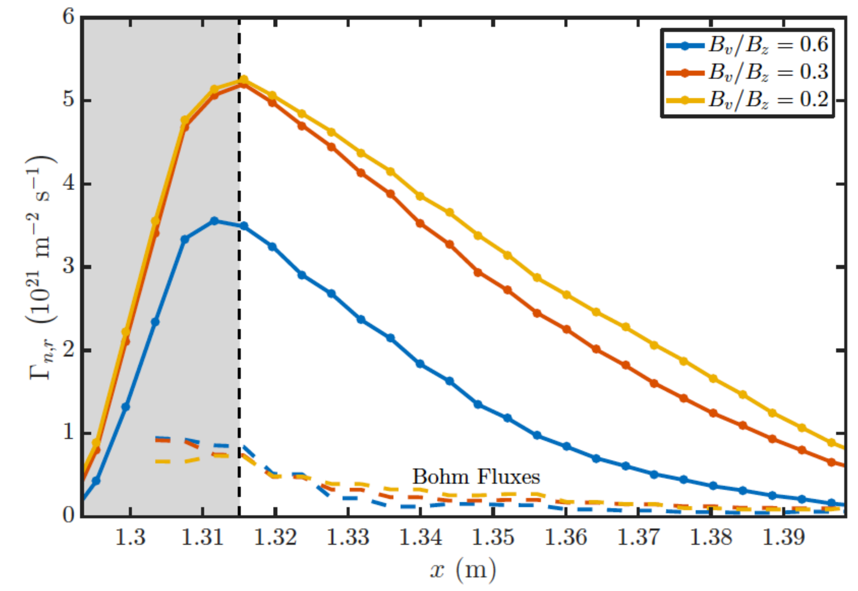


Figure 7: Comparison of radial $E\times B$ particle flux evaluated at the midplane for three different poloidal fields. Increasing the poloidal field decreases the radial flux, consistent with the heat-flux profiles on the divertor plate. For comparison, Bohm fluxes estimates are shown as dashed lines.

Larger amplitude, more intermittent blobs in SOL

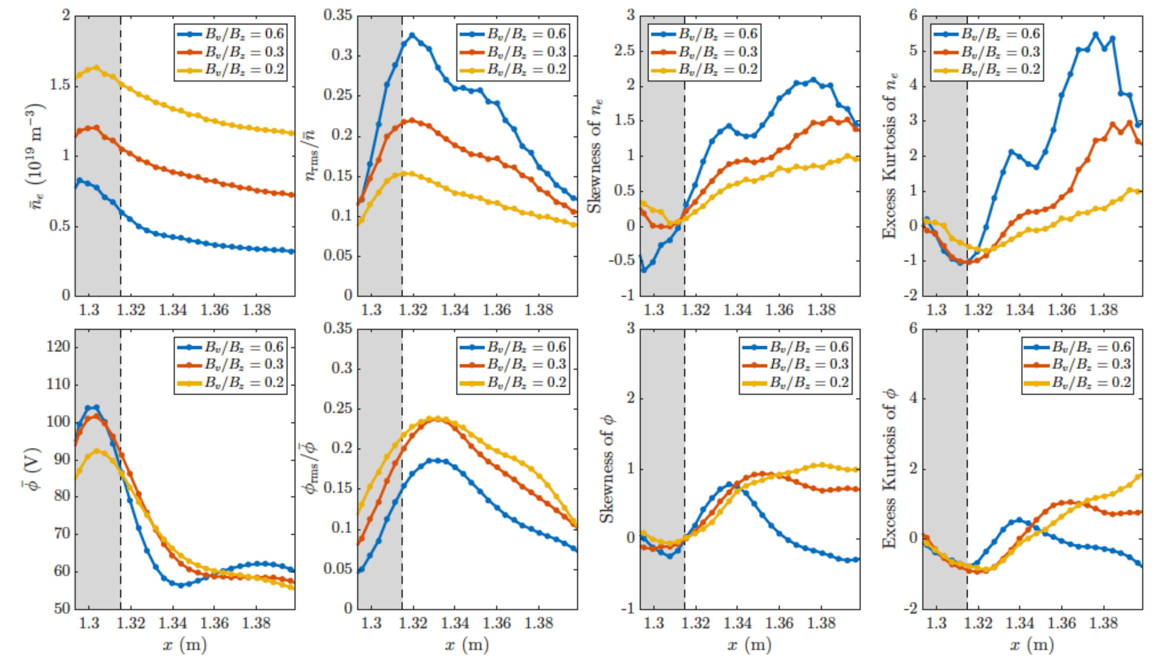


Figure 8: Comparison of electron-density fluctuations (top row) and electrostatic fluctuations (bottom row) at mid-plane. The density fluctuations (blobs) are larger amplitude and more intermittent than the potential fluctuations which show much smaller skewness and kurtosis.

Ion and electron temperatures are not in equilibrium

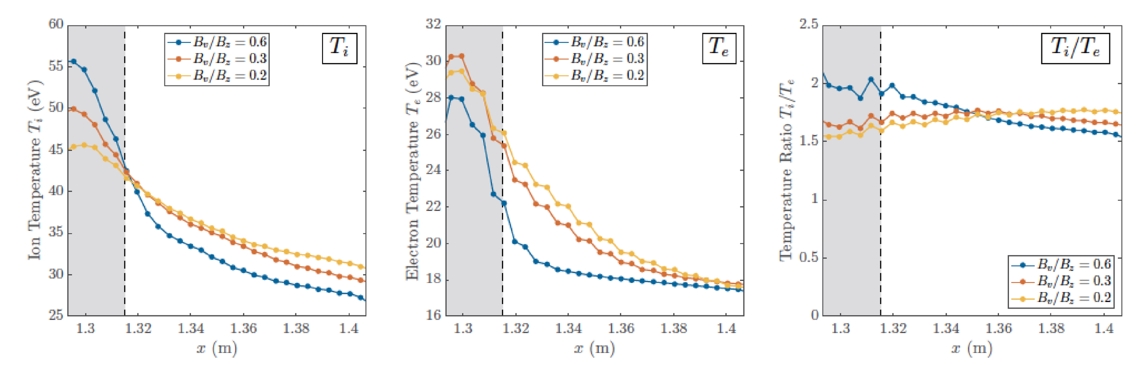


Figure 9: Radial profiles of steady-state ion (left) and electron (middle) profiles near midplane. Right plot shows ion-to-electron temperature ratio. Although both electrons and ions are sourced at the same temperature, the sheath allows rapid loss of high energy electrons to wall, resulting in lower electron temperatures in the SOL.

Conclusion and current work

We have demonstrated the *first* continuum GK simulations of SOL turbulence in sheath-limited geometries. Our current work focuses on improved representation of geometry, further parameter scans and inclusion of atomic and surface physics.